Helium 4

Phase diagram of Helium 4, two different liquid phases, He-I and He-II.

Temperatures	Boiling	4.21 K
	λ-point	2.17K
	Critical	5.2 K

Clasius Clapeyron

$$\frac{\partial P_m}{\partial T} = \frac{\Delta S_m}{\Delta V_m} \approx 0,\ T < 1\mathrm{K}$$

$$\Rightarrow \ \Delta S_m \to 0,\ T \to 0$$

in accord with the

3rd law of thermodynamics

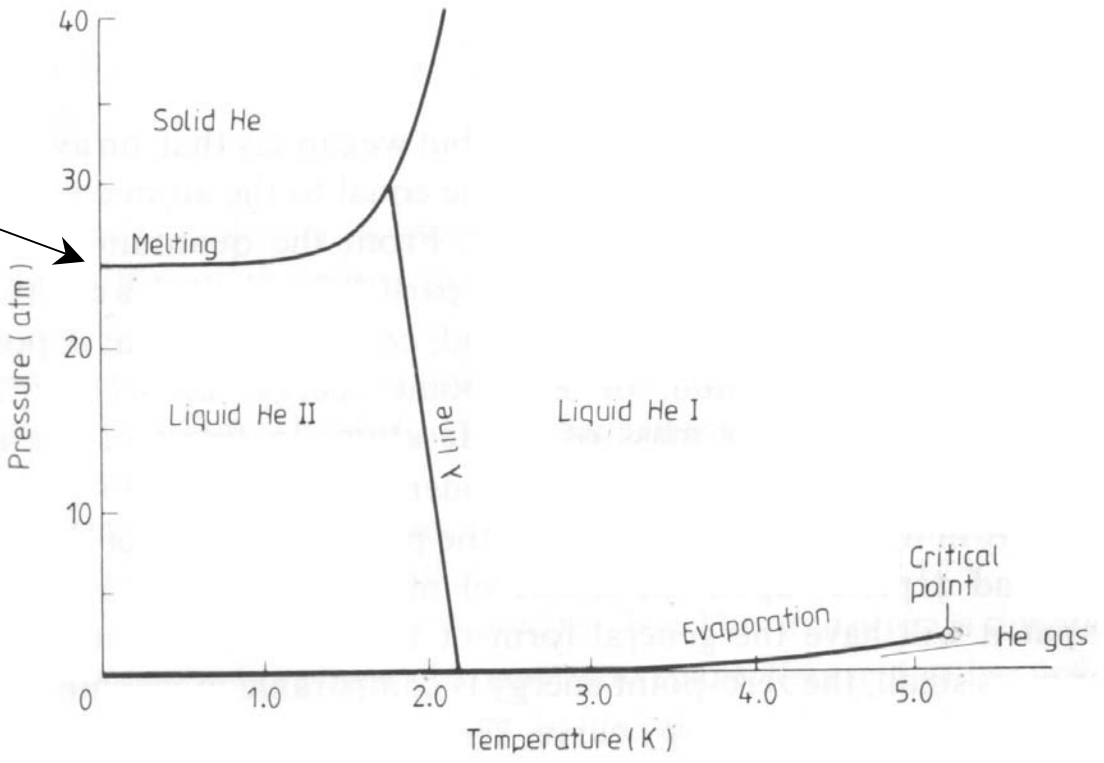


Figure 1.1 Phase diagram of ^{4}He (after London 1954).

Why is helium liquid even at zero temperature

Helium is the only substance that does not liquefy at zero temperature

Zero point fluctuations. The helium atom is confined by its neighboring atoms in a volume *V*, with the radius $R \sim V^{1/3}$. From Heisenbergs uncertainty in momentum

$$\Delta p \sim \frac{\hbar}{R}, \; E_0 \sim \frac{\Delta p^2}{2m_4} \approx \frac{\hbar^2}{2m_4 V^{2/3}}$$

Potential energy: attractive due to van der Waal forces, repulsive due to ”hard core”

Leonard Jones potential $E_{LJ} \sim \left(\frac{\sigma}{R}\right)^6 - \left(\frac{\sigma}{R}\right)^{12} \sim \frac{1}{V^2} - \frac{1}{V^4}$

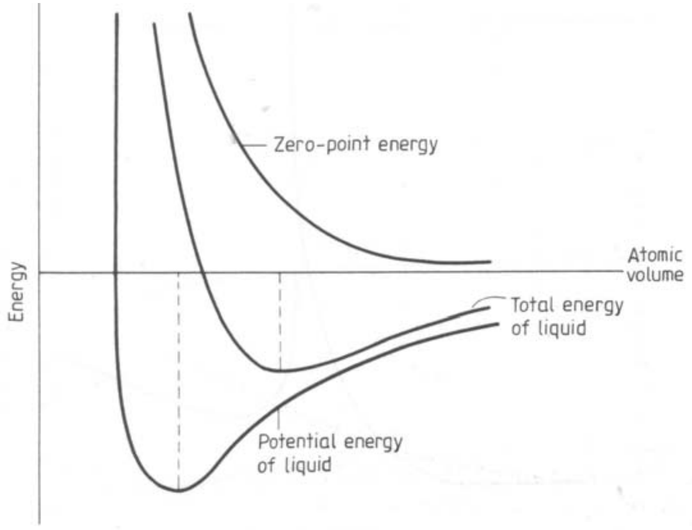


Figure 1.3 Energy of liquid helium. Total energy is sum of potential energy and zero-point energy.

The thermal fluctuations have to be reduced substantially to make helium-4 liquefy, 4.2K. Even at zero temperature the zero point fluctuations keep helium-4 from solidifying.
By applying a large pressure to the liquid, the atoms can be brought close enough to form a liquid. P≈25 bar

Helium 3 has even lower mass increasing the zero point fluctuations even more.
Thus Helium 3 liquefies at somewhat lower temperature 3.4 K, and solidifies at somewhat higher pressure ~32 bar

Compare with Hydrogen and other gases
Why does hydrogen solidify ?
(stronger van der Waals forces)

Properties below the Lamda transistion

Specific heat, Keeson and Clusius 1932

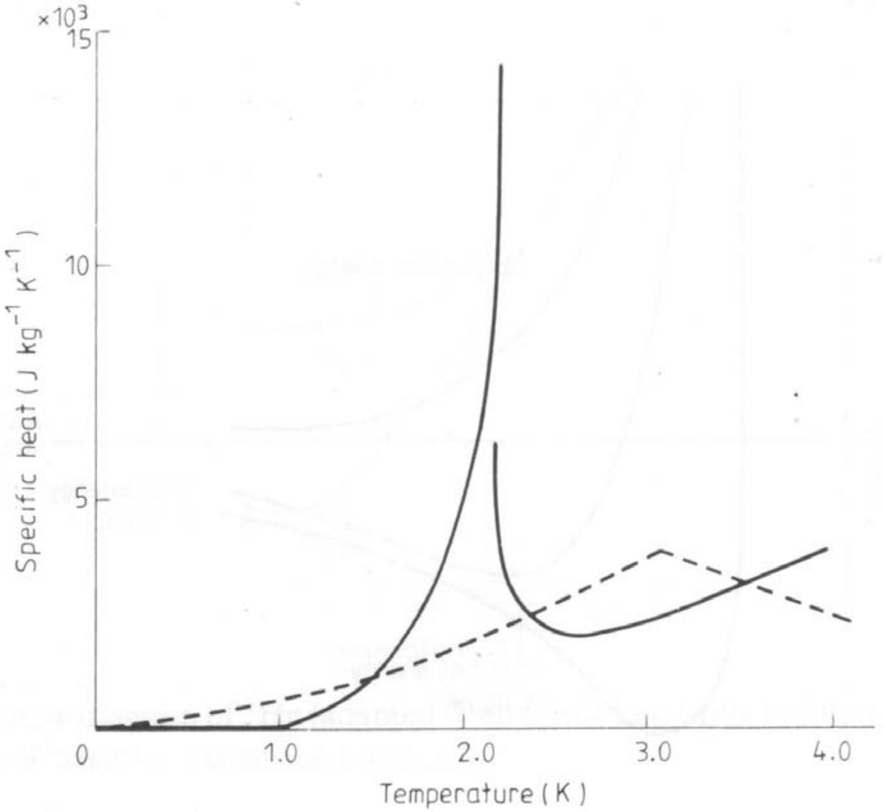


Figure 1.4 Specific heat of liquid ^{4}He (after Atkins 1959). Broken line shows specific heat of ideal Bose–Einstein gas having same density as liquid ^{4}He.

It takes longer time to lower the temperature just below the λ-point

Mixtures of Helium-4 and Helium-3

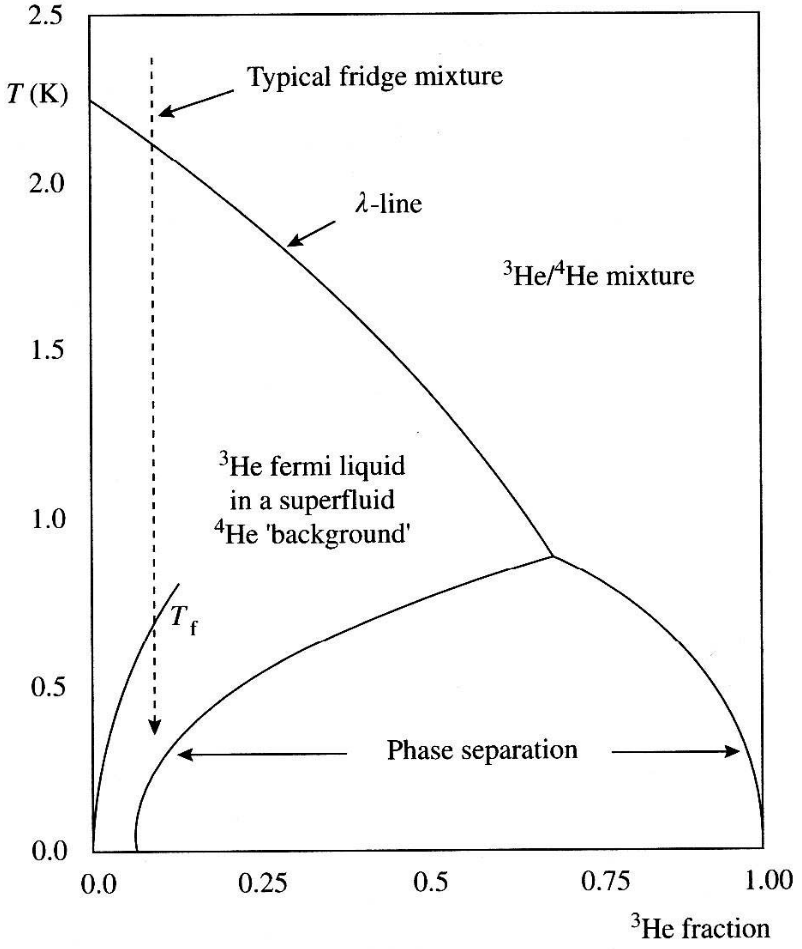


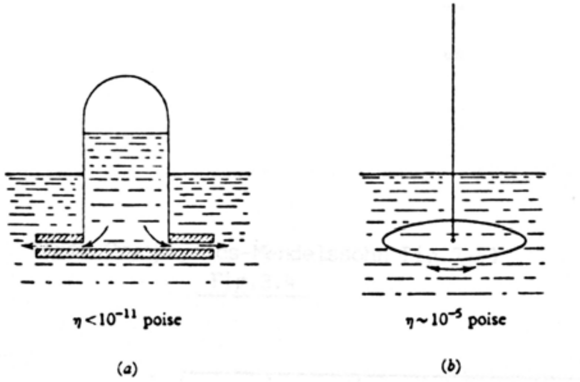
FIG. 8.3 The schematic phase diagram of liquid helium mixtures.

Increasing the concentration of He-3 decreases the transition temperature

Below 0.8K the liquid can separate into two phases one helium 4 rich and one helium-3 rich. The He-3 rich phase will float on top of the He-4 rich phase.

At zero temperature the He-3 rich phase becomes “pure” i.e. 100% He-3, whereas the He-4 rich phase always contains some He-4 (6.4% or more).

Measuring viscosity in two different ways gave different results !!!



. The two different methods of measuring the viscosity of liquid helium II. (*a*) Flow through narrow channels; (*b*) damping of an oscillating disk.

Measuring viscous resistance to flow

Measuring viscous drag on a body moving in a liquid

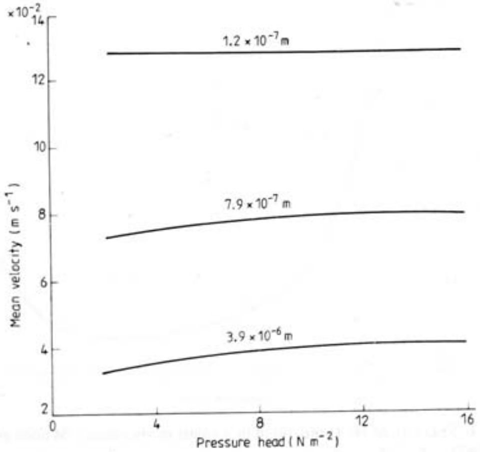


Figure 1.5 Flow velocity of He II through channels of various widths at 1.2 K. (After Allen and Misener 1939 and Atkins 1952.)

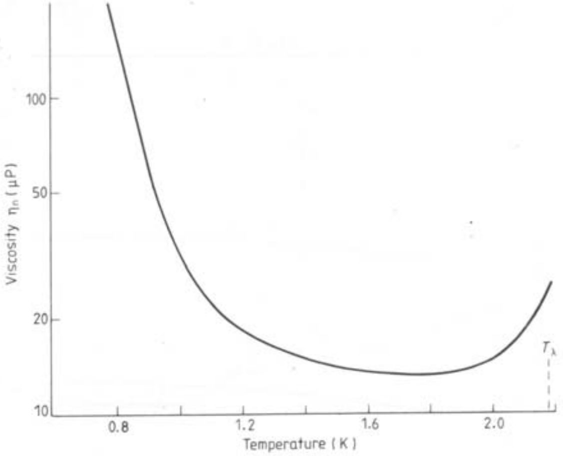


Figure 1.6 Viscosity of He II measured in a rotation viscometer (Woods and Hollis Hallett 1963).

Velocity almost independent of pressure
Indicates zero viscosity

Clearly shows that viscous drag exists in Helium II

What is the explanation of this contradiction ?

The two-fluid model

The two-fluid model which we have discussed earlier for superconductors can also be used to describe the properties of superfluid helium.

The total density is the sum of the superfluid density and the normal density

$$\rho = \rho_S + \rho_N$$

Super fluid	Normal fluid
ρ_S No viscosity No entropy	ρ_N Nonzero viscosity Nonzero entropy

The mass flow is given by

$$\bar{j} = \rho_S \bar{v}_S + \rho_N \bar{v}_N$$

Temperature dependence of the densities

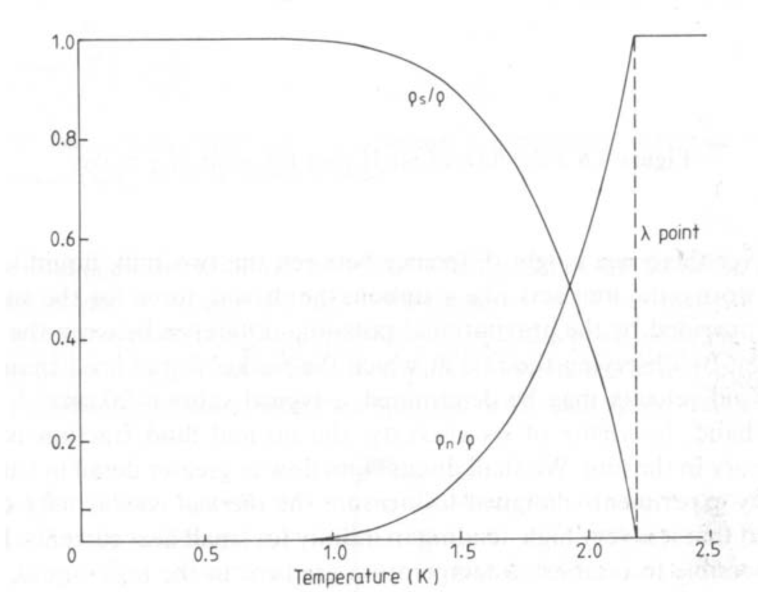


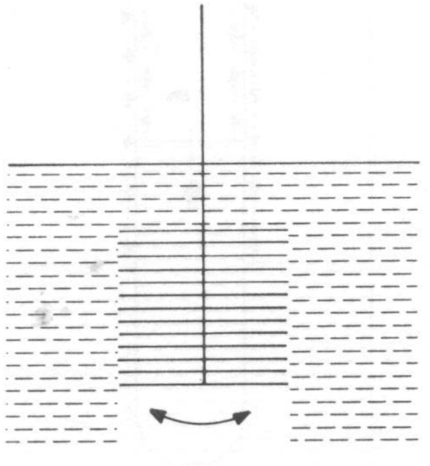
Figure 1.7 Andronikashvili's experiment (after Atkins 1959).

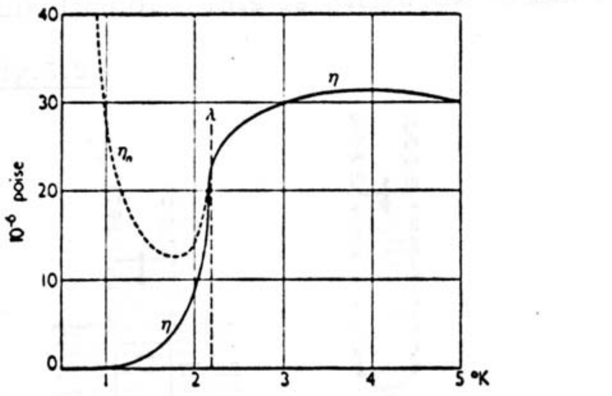
The super fluid moves towards warm surfaces, where as the normal fluid moves to cold surfaces to reach the equilibrium density.

Since the superfluid can flow with no viscosity the thermal conductivity becomes very large below the λ-point, it increases by a factor of ~10^6. However a thermal conductivity is hard to define since it depends on the geometry.

This can be seen when cooling Helium-4 through the λ-transition, boiling stops completely below the λ-point. At temperatures below 1 K the thermal conductivity decreases again since there is almost only one liquid.

The Experiment by Andronikashviili





Total viscosity η of liquid helium and viscosity η_n of the normal component of He II.

A set of metal discs suspended by a torsion wire
Close enough to drag all the normal liquid along.

This allowed to measure the fractions of super fluid and normal fluid

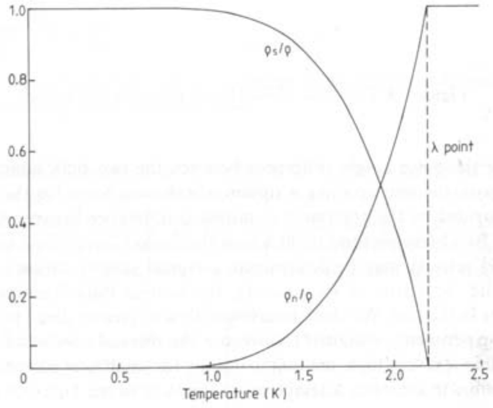


Figure 1.7 Andronikashvili's experiment (after Atkins 1959).

$$\eta_{tot} = \underbrace{\frac{\rho_s}{\rho_{tot}}\eta_s}_{=0} + \frac{\rho_n}{\rho_{tot}}\eta_n = \frac{\rho_n}{\rho_{tot}}\eta_n$$

$$\frac{\rho_s}{\rho_{tot}} = 1 - \frac{\rho_n}{\rho_{tot}}$$

Rolling film

If an empty beaker is immersed half ways into superfluid He, a thin film of helium condenses on the walls of the beaker. This film can fill the beaker even if it is only immersed half way into the superfluid. The thin helium film is about **30 nm** thick and it actually flows upwards and fills the beaker until the levels are equal inside and outside the beaker. The film acts as a siphon. This can be explained from the fact that it would cost energy to break the film and it costs less energy for the film to flow upwards.

If the beaker is then lifted out of the superfluid the reverse process occur, and small droplets can be seen at the bottom of the beaker as it empties. The flow velocity of the film is about 0.2 m/s.

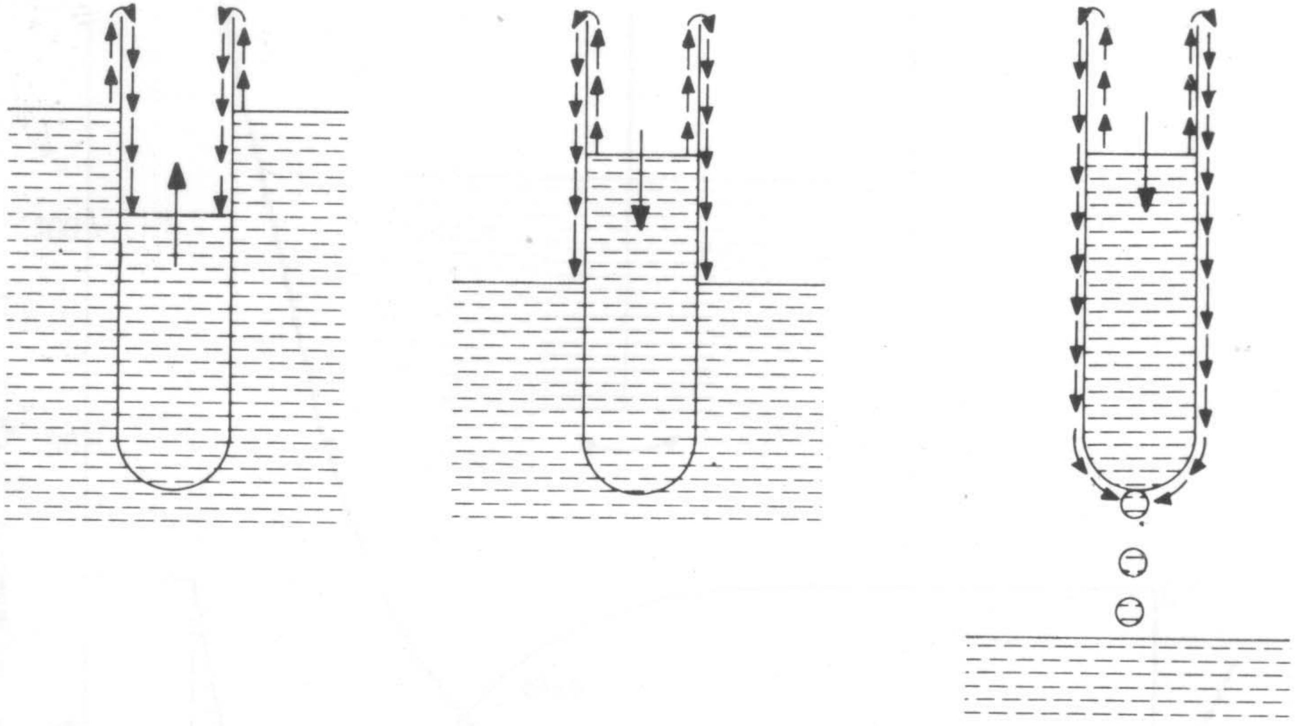
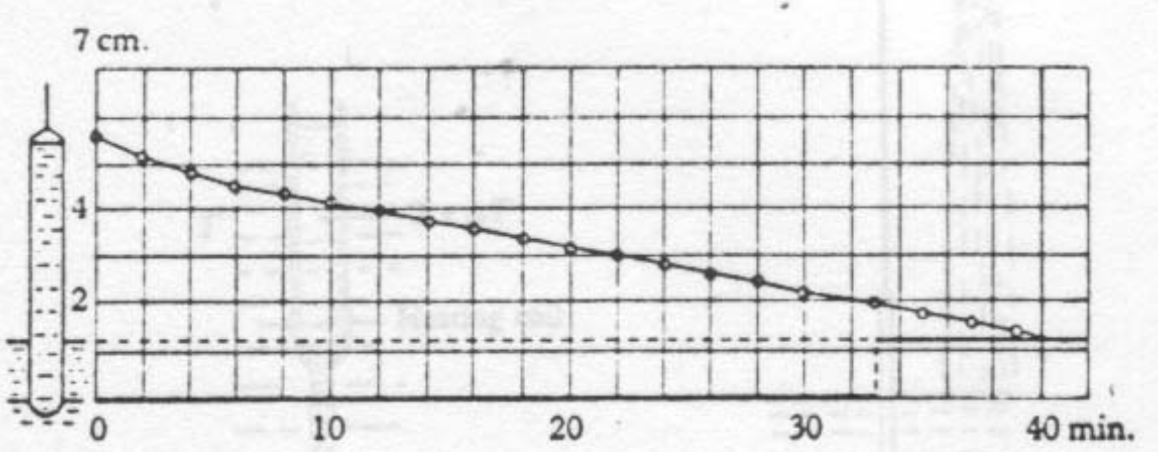


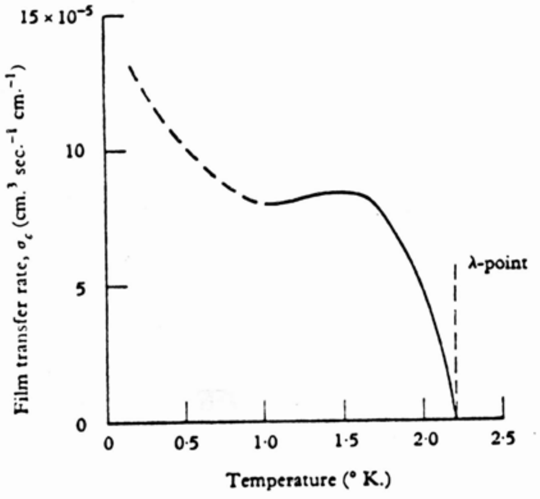
Figure 1.8 Film flow of He II over the walls of a beaker.

How fast does the film flow ? 0.2m/s



Almost constant rate of film transfer from a long beaker (after Daunt and Mendelssohn, 1939*b*).

Film flow versus temperature



Rate of transfer through the helium film as a function of temperature.

Heat transport in a super fluid

If a tube is densely packed with a fine powder, the powder will prevent the normal fluid from passing, but the super fluid, which has no viscosity can pass. This is called a super leak. Thus if we heat one side of the tube the level will increase there.

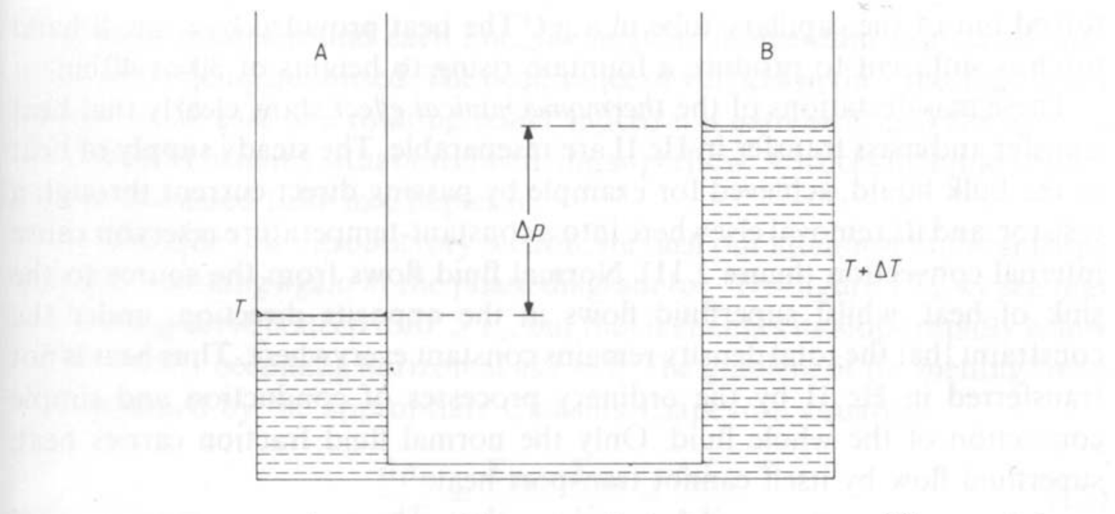


Figure 1.9 Two vessels connected by a superleak. A temperature difference between the two is accompanied by a pressure head.

$$\Delta W = \Delta Q$$

$$\Delta W = \Delta p \cdot \Delta V = \Delta p \frac{\Delta m}{\rho} \qquad \sigma \equiv \frac{S}{m}$$

$$\Delta Q = S \cdot \Delta T \equiv \Delta m \cdot \sigma \cdot \Delta T$$

$$\Delta p \frac{\Delta m}{\rho} = \Delta m \cdot \sigma \cdot \Delta T$$

$$\frac{\Delta p}{\Delta T} = \rho \cdot \sigma$$

$$\Delta p = \rho \cdot g \cdot \Delta h$$

$$\frac{\Delta h}{\Delta T} = \frac{\sigma}{g}$$

Transport of heat and mass are inter dependent

Importance in Dilution refrigerators

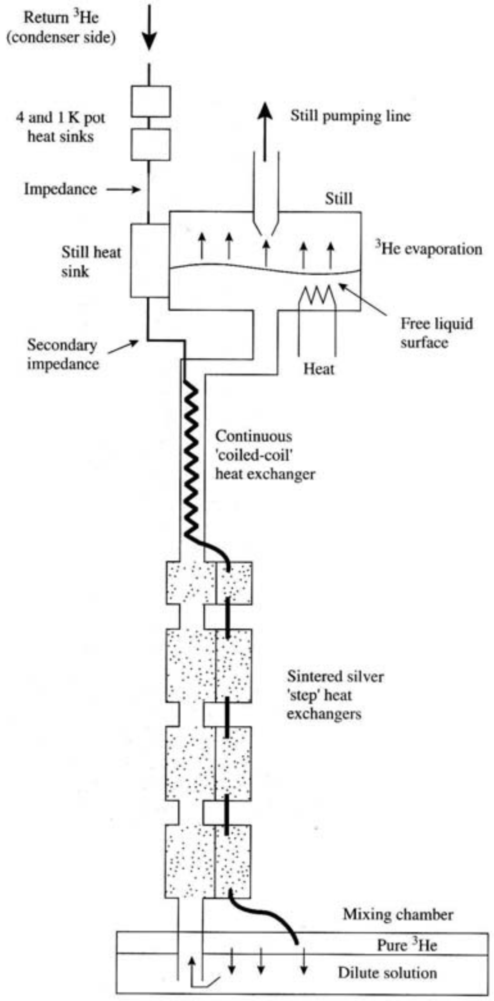


FIG. 8.1 A dilution refrigerator in schematic form.

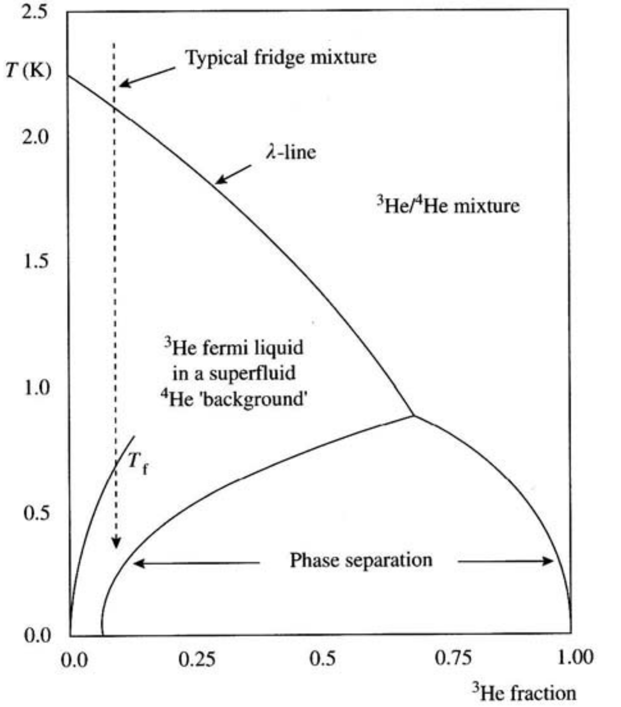


FIG. 8.3 The schematic phase diagram of liquid helium mixtures.

This effect is important in the dilution refrigerator since there is a thermal gradient between the Still and the mixing chamber, and the helium 4 rich phase which is in the heat exchangers is super fluid

The fountain effect

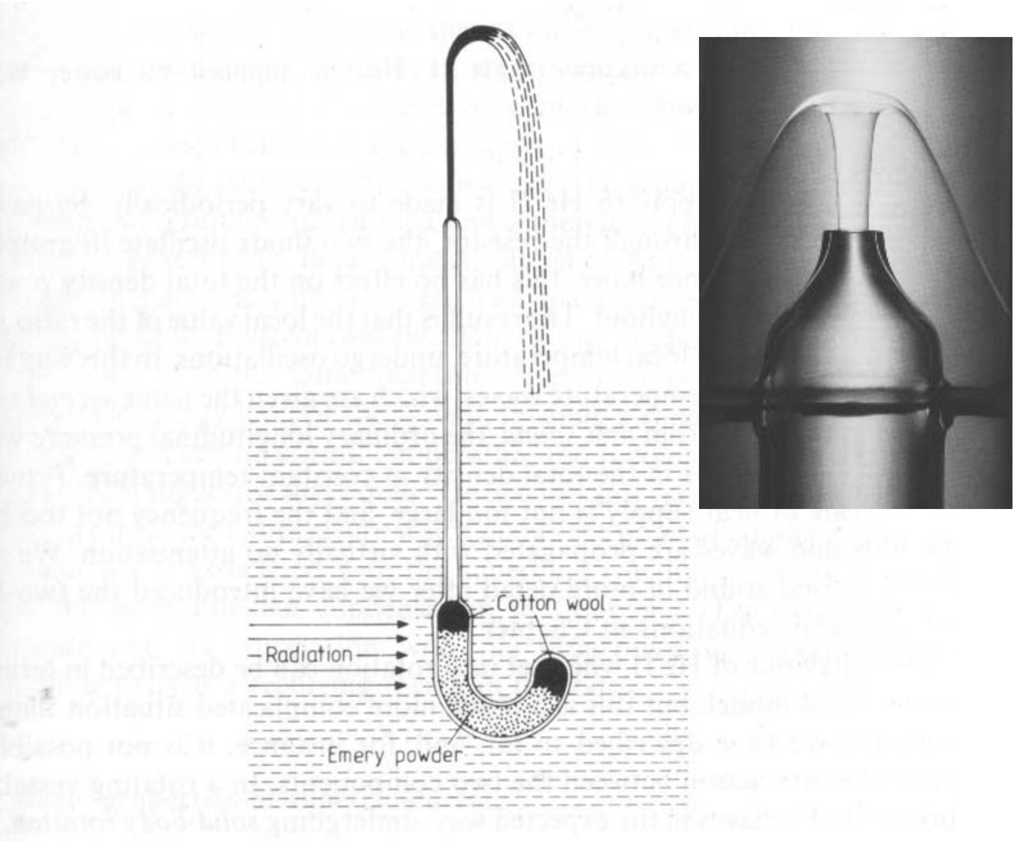
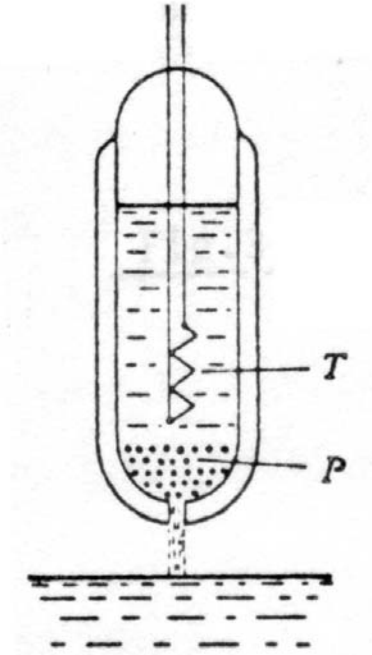


Figure 1.10 The helium fountain (Wilks 1967, after Allen and Jones 1938).

By increasing the temperature in the tube superfluid moves into the tube and increases the pressure, the Helium then sprays out at the top. As long as you supply heat inside the tube and keep cooling the bath this can go on until you run out of helium.

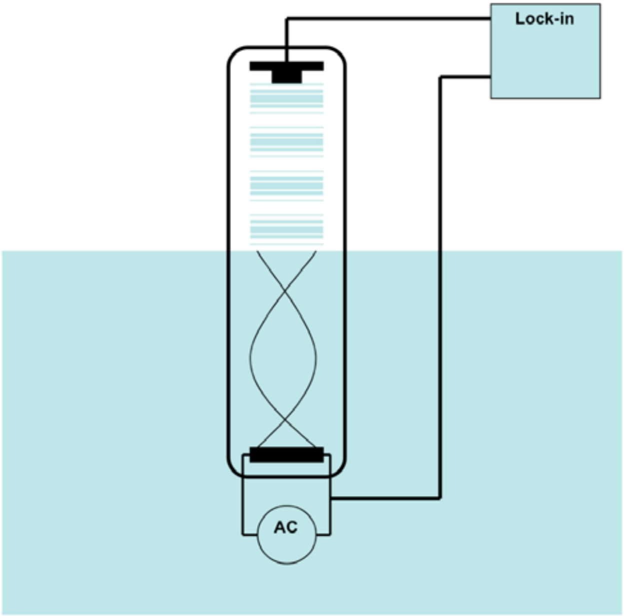
The mechano-chaloric effect



Apparatus to demonstrate the mechanocaloric effect (after Daunt and Mendelssohn, 1939*a*).

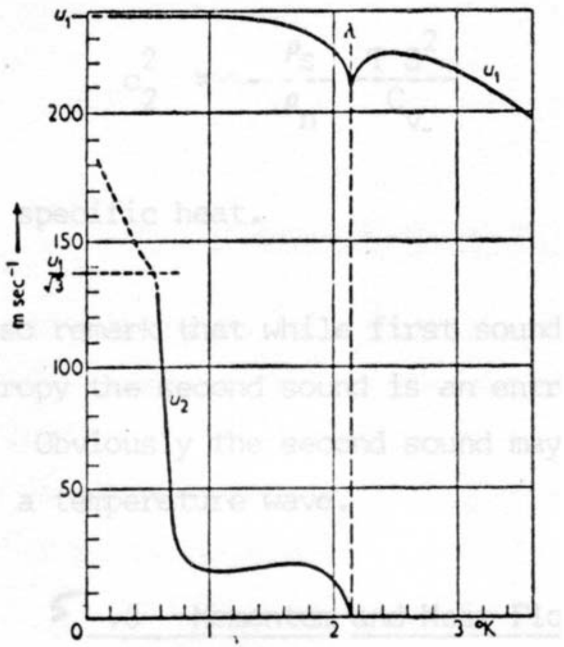
If a beaker with a super leak at the bottom contains superfluid helium, is lifted out of a bath, the superfluid part of the liquid will leak out but not the normal liquid. Thus the temperature increases as the beaker empties.

Second Sound
Entropy density waves



A heater is driven with an ac signal which generates an oscillating temperature entropy wave directed towards the surface. At the surface the increased temperature generated increased evaporation which leads to an increased preassure and thus a sound signal.

Note that the frequency of this signal is twice that of the ac voltage on the heater since heating is generated for both positive and negative voltages.



Velocities of first and second sound in dependence on temperature.

Velocities are relatively independent of frequency

1st sound	Ordinary pressure density wave
2nd sound	Temperature entropy wave
3rd sound	Surface wave on a He-II film
4th sound	Pressure wave inside a superleak

Excitations

There are two types of excitations in the superfluid, phonons and rotons

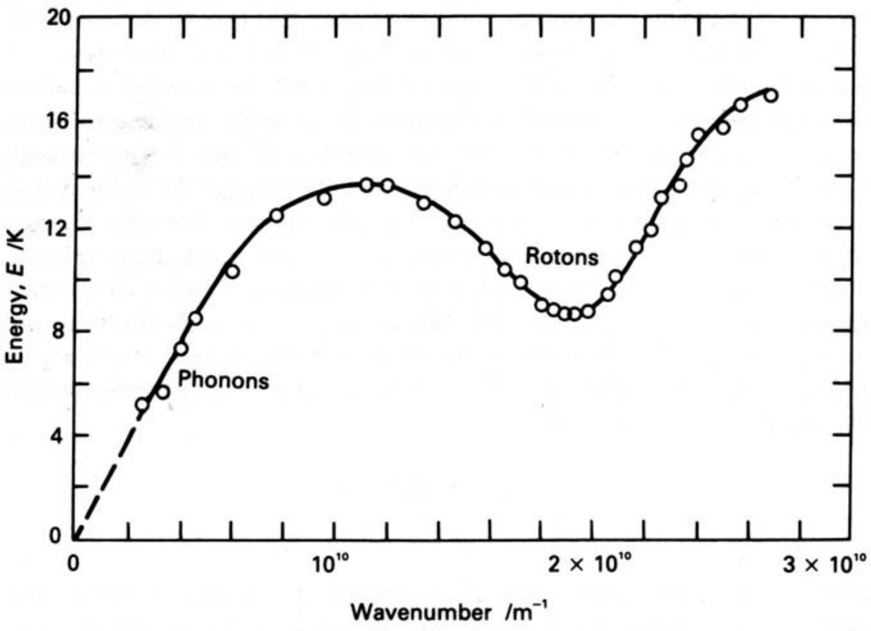


Figure 3.8 *Dispersion curve of excitations in helium-II, as deduced from neutron scattering measurements [after Henshaw, D. G. and Woods, A. D. B. (1961)* Phys. Rev., ***121***, *1266]*

Dispersion relations

$$E_{ph} = C_1 p$$

$$E_{rot} = \Delta + \frac{(p - p_0)^2}{2\mu}$$

C_1=239 m/s	Sound velocity
Δ/k_B=8.65 K	energy gap
μ= 0.16 m_4	effective mass
$p_0/\hbar$=19.1/nm	wave number