CHALMERS TEKNISKA HÖGSKOLA 2003-12-10
GÖTEBORGS UNIVERSITET Kl: 09-13
Mikroteknologi och nanovetenskap Sal: VV13

Tentamen i LÅGTEMPERATURFYSIK för F4, Master, Ph.D. Students

Lärare: Per Delsing, Tord Claeson, tel 772 3317

Hjälpmedel: Tefyma, Physics Handbook, Stand Math Tables och liknande handböcker, valfri räknedosa.

Answer 5 of the following problems. Motivate your answer in a logical way. You are welcome to illustrate with readable diagrams. Answer in Swedish or English.

The 2003 Nobel Prize in Physics was awarded to VL Ginzburg, A Abrikosov, and A Leggett for their theoretical work on superconductivity and superfluidity. This exam will take up a few of these aspects.

1. Characteristic lengths in superconductors as derived from the Ginzburg-Landau theory. The penetration depth λ and the coherence length ξ are two important lengths in superconductors that follow from the Ginzburg-Landau equations, which can be written:
$\alpha\Psi + \beta|\Psi|^2\Psi + (1/2m^*)(-i\Sigma\nabla + e^*A)^2\Psi = 0$
$J_S(r) = i(e^*\Sigma/2m^*)(\Psi\nabla\Psi^* - \Psi\nabla\Psi^*) - (e^*2/m^*)\Psi^*\Psi A$
(where e* and m* are charge and mass of an electron pair)

(a) Define and describe the two lengths. How do you derive them from the equations? (2p)
(b) What are typical values of these
(i) in a pure type I superconductor (like Al)?
(ii) in a typical high T_c (cuprate) superconductor (type II superconductor)? (1p)
(c) What happens to the two lengths λ and ξ as you decrease the mean free path of the normal electrons by alloying a pure superconductor? What are the temperature dependencies of λ and ξ close to T_c? (1p)

2. Type II superconductors. Abrikosov employed the Ginzburg-Landau equations to show that superconductivity can be nucleated at a critical magnetic field, H_{c2}, which is much higher than the thermodynamic critical field, H_c. The Ginzburg-Landau equations can be written (with $e^*=2e$, $m^*=2m_e$):
$\alpha\Psi + \beta|\Psi|^2\Psi + (1/2m^*)(-i(h/2\pi)\nabla + e^*A)^2\Psi = 0$
$J_S(r) = i(e^*(h/2\pi)/2m^*)(\Psi\nabla\Psi^* - \Psi\nabla\Psi^*) - (e^{*2}/m^*)(\Psi^*\Psi)A$
(a) Indicate the derivation of the upper critical field, H_{c2}, from the G-L equations. (1p)
(b) Draw a phase diagram of critical field versus temperature for a type II superconductor and indicate the different phases. Would the corresponding phase diagram be very different for a so called high temperature superconductor? (1p)
(c) Above a lower critical field, H_{c1}, magnetic flux penetrates a type II superconductor in the form of quantized fluxons (flux lines). These form a lattice. What is the value of the flux quantum? Discuss at least two experimental methods to map the flux lattice.What are the properties that you use in these methods? (2p)

3. Superfluids. Both ^{3}He and ^{4}He display superfluid phases. The charged electron fluid can also be considered as a superfluid, in this case a superconductor. It is your task, in this problem, to discuss the similarities and the differences, with emphasis on differences, between superfluid ^{3}He, superfluid ^{4}He, and a superconductor. Treat in particular ^{3}He, the topic of this year´s Nobel prize. The discussion should include both theoretical concepts like statistics, condensation, symmetry of the macroscopic wave function, interactions leading to the condensed states, different phases that may occur, phase

diagrams, etc, and experimental properties like heat capacity, viscosity, anisotropy, vortices, coherence lengths, etc. (4p)

4. Second sound. You have measured the velocity of second sound in superfluid helium during a laboratory session.

a. What is second sound? What are the conditions that must be fulfilled for its occurrence? (1p)

b. How do you measure it? Discuss at least one experimental set-up to detect second sound waves and the speed of their propagation. What is the speed at, say, 1.5K? (1p)

c. Imagine that you rotate superfluid helium. Will there be any difference between the propagation (and damping) of second sound waves propagating parallel and perpendicular to the axis of rotation? If so, why? If not, why? (1p)

d. There are also other "orders of sound". What is zero sound? third sound? fourth sound? What are the conditions for these to occur? (1p)

5. Applications of superconducting tunneling (alt. Josephson effects).

There are many practical applications of superconducting tunneling and Josephson effects. Your task is to pick one possible application and discuss:

- principle (physics behind the application)
- conditions to be fulfilled, design criteria, ultimate prestanda
- practical set-up
- possibly circuit designs
- examples of results and prospects for the future

You can pick any of the possible applications, e.g.:

a. dc-SQUID
b. voltage standard
c. high frequency detector (e.g. direct detector, mixer or parametric amplifier)
d. superconducting digital electronics
e. so called qu-bits for quantum computing (4p)

6. Dilution Refrigerator. ^{3}He/^{4}He dilution refrigerators have become "work horses" to reach mK temperature. What is the physical principle behind the cooling? What are the crucial points in a practical realization? How are they solved in principle? How is the dilution refrigerator built up in practice – give a sketch of the different parts. Which temperature region can be reached? How does the cooling capacity compare with those of other types of very low temperature cryostats? What are the advantages and drawbacks using this type of refrigerator? (4p)

7. Low temperature techniques. Treat either alt.A or alt.B:

7A A piece of 10cm^3 copper is attached to the cooling stage of a dilution refrigerator (T=50mK) via a 30mm long, 2mm diameter stainless steel rod.

a) Describe the different contributions to the heat capacity of the copper piece. What are their temperature dependences and which one dominates at this temperature (1p)

b) Describe the different contributions to the heat conduction in the stainless steel. What are their temperature dependencies, and which one dominates at this temperature (1p)

c) Calculate the heat capacity of the copper piece, assuming that copper has a specific heat of 0.04 mJ/(mole K) and calculate the thermal conductance in the stainless steel rod if the heat conductivity of the 0.01 W/(K·m) (1p)

d) The copper piece is heated to 60 mK and then allowed to cool down again via the stainless steel rod. Calculate after what time the copper piece has reached 52mK. You can assume that the heat capacity and the thermal conductance do not change very much between 50 mK and 60 mK. (See data sheet for data on elements) (1p)

7B. Design of Cryostat. You want to cool an experiment with the aid of liquid helium (4.2 K). You want to insert a sample, with its holder which has a diameter of Ø=58 mm and height 60 mm, into the tail of a cryostat (where the main storage of helium is above the tail). It is desirable to have about 3 l liquid helium around (or rather above) the sample after the transfer of liquid is completed. It is your task to design a suitable cryostat that will allow you measurements during one day (say 8 hours) after you have filled the cryostat with helium.
Design a suitable cryostat and estimate heat leaks and how long time it will take until the helium has boiled off. Heat leaks are due to your cryostat, to the tube that supports your sample holder (it is suggested that you choose a stainless steel tube with diameter Ø=12 mm and thickness t=0.2 mm, the length according to your design), and to the experiment itself. The latter heat leaks are mainly due to 10 electrical leads of pure copper, each with a diameter of 0.1 mm (you choose appropriate lengths and possible thermal anchors from room temperature to the cold space). Joule heating in a thermometer and measuring coils amounts to about 2 mW. Assume that the insulation vacuum is perfect so that you can neglect conduction through gas in the insulation space.
Account for how you design your cryostat, chosen dimensions and layout, chosen materials, etc, and discuss how to optimize the performance. (4p)

8. Short questions to test the understanding of concepts. Give short descriptions or definitions (use diagrams if appropriate) of the following:

(a) Sketch the phase diagram (p-T) of liquid ^{3}He, in particular in the range of about 1 mK <T<3 mK, denote different phases and list their most characteristic properties.

(b) What is Pomeranchuk cooling to reach mK temperature?

(c) What is photon or phonon stimulated tunneling in a superconducting tunnel junction?

(d) What is the relation between frequency and voltage in the ac Josephson effect?

(e) What is the current distribution over the length of a Josephson junction in a magnetic field that is applied within the plane of the junction and perpendicular to the direction of the current? The length of the junction is assumed to be no longer than the Josephson penetration depth.

(f) What is the current distribution over the length of a Josephson junction in a magnetic field that is applied within the plane of the junction and perpendicular to the direction of the current? The length of the junction is assumed to be considerably larger than the Josephson penetration depth.

(g) What is the Josephson penetration depth? What determines its value?

(h) What is the superconducting energy gap? Any difference between a "low temperature (classical) superconductor and a high temperature (cuprate) superconductor?

(i) Sketch the low temperature phase diagram (T vs concentration) of a ^{4}He-^{3}He liquid mixture. Give most important temperatures and concentrations of the phase diagram for the functioning of a dilution refrigerator. (0.5p per sub-problem, max 4p)