Theoretical Physics
Chalmers & Göteborg University

Midterm Exam in Quantum Mechanics, FKA081/FYN190

Friday October 25 2002, 13.15 - 18.15, VV

Examiner: Henrik Johannesson, phone: 3185.

No reference/calculator is allowed.

Please put your name on *each solution sheet*, and don't forget to include also your e-mail address on the cover sheet.

Please structure your solutions carefully. State precisely which assumptions, theoretical results, approximations, etc. you use. The logic of your arguments must be transparent, and you should strive for optimal readability! All problems are equally weighted (6p/problem).

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1. Observables and measurements

Consider a system whose state and two observables are given by

$$|\psi(t_0)> = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \qquad A = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

a) What is the probability that a measurement of A at time t_0 yields the value -1?
b) Let us carry out a set of two measurements where at time t_0 B is measured first and then, immediately afterwards, A is measured. Find the probability of obtaining a value of 0 for B and a value of 1 for A.
c) Assume that we instead measure A first (at time t_0) and then, immediately afterwards, B. Find the probability of obtaining a value of 1 for A and a value of 0 for B.
d) Compare the results of b) and c). Explain!

2. Correlation functions

Consider a function, known as the *correlation function*, defined by

$$C(t) = < X(t) X(0) >,$$

where $X(t)$ is the position operator in the Heisenberg picture. Evaluate the correlation function explicitly for the groundstate of a one-dimensional simple harmonic oscillator. Try to interpret your result!

Hint: $X = (\hbar/2m\omega)^{1/2}(a + a^\dagger)$ *in the Schrödinger picture, where* a $(a^\dagger)$ *is a destruction (creation) operator.*

3. Angular momentum

Let $J_\pm = J_x \pm iJ_y$ be the usual ladder operators, with J_x and J_y the x- and y-components respectively of the total angular momentum operator.

a) Show that $<J_x> = <J_y> = 0$ in a state $|j, m>$.
b) Derive the relation

$$J_\pm \,|j, m> = \sqrt{(j \mp m)(j \pm m + 1)}\hbar \,|j, m \pm 1> .$$

c) Check that $\Delta J_x \Delta J_y$ satisfies the uncertainty relation (where, for a given state $|\,j, m>$, $\Delta J_i \equiv\, < j, m \,|\, (J_i - < J_i >)^2 \,|\, j, m >^{1/2}, i = x, y$, is the usual mean square deviation of the corresponding expectation value).
d) Show that the minimum uncertainty is obtained in the state $|j, \pm j>$.

4. Time evolution

A particle of mass m, which moves freely inside a one-dimensional quantum box* of length a, has the following initial wave function at $t = 0$:

$$\psi(x, 0) = \frac{A}{\sqrt{a}} \sin(\frac{\pi x}{a}) + \frac{1}{\sqrt{5a}} \sin(\frac{5\pi x}{a})$$

where A is a real constant.

a) Find A so that $\psi(x, 0)$ is normalized.
b) If a measurement of the energy is carried out at $t = 0$, what are the values that can be found and what are the corresponding probabilities? Calculate the average energy of the particle.
c) Find the wave function $\psi(x, t)$ at any later time t.
d) What is the probability that the particle will return to its initial state at a later time t? Does your answer agree with your "classical intuition"? Discuss!

.*) A *quantum box* is a region in which particles are confined by an infinite potential at the boundaries.

5. The classical limit

Imagine that you are going to a party tonight, relaxing after a long week of exams. A friend of yours show up, a guy who dropped out from Chalmers last year. You tell him that you took a quantum mechanics exam earlier today, but your friend is not easily impressed:

" Ohh... that nonsense! What really turned me off from quantum mechanics is that you can't connect to classical physics. Look! I *know* that classical physics is OK for things around us, the world we see and touch. But how on earth do you get Newton's second law or Maxwell's equations out of the Schrödinger equation? It just doesn't work! There's something rotten with quantum mechanics... ".

What do you tell your friend?