10 GHz oscillator with ultra low phase noise

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30th March 2004

Abstract

In this paper we present our first efforts to make an ultra low phase noise oscillator at 10.4GHz. The oscillator is based on a sapphire loaded cavity resonator. We describe a numerical procedure to accurately calculate the high Q quasi TM Whispering Gallery Modes. Based on these calculations a resonator has been made with a very sharp resonance (the WGH_{511} mode) at 10.4GHz. Based on Leeson's model for the oscillator phase noise, the integration of the sapphire resonator in an oscillator loop, is expected to yield a phase noise below -140dBc/Hz at 1kHz offset.

1 Introduction

In this paper a description of a *Whispering Gallery Mode* (WGM) sapphire oscillator will be given. The simplest topology of this oscillator is presented in figure 1. Here, we have shown the three essential components which are present in the loop: A resonator characterized by coupling factors β_1 and β_2 at ports 1 and 2 and loaded quality factor Q_L. An amplifier with gain factor G at steady oscillation, which counteracts the loss in the passive part of the loop. Finally, we need a component to be able to tune the frequency of oscillation. Here, this component has been chosen to be a voltage controlled phaseshifter [7]. The simple loop arrangement allows for a very accurate analysis of the resulting phase noise of the

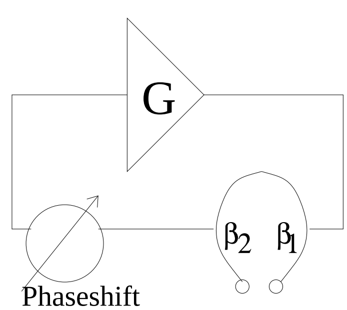


Figure 1: Simple oscillator circuit.

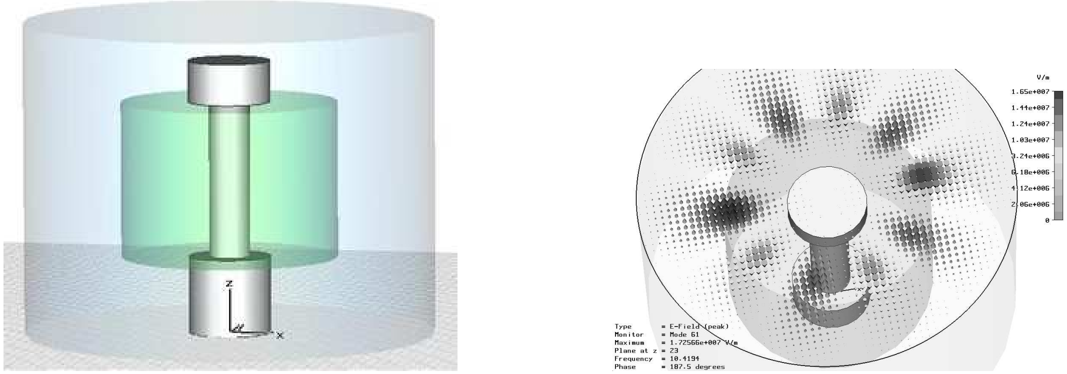


Figure 2: Left: An illustration from CST Microwavestudio of the inside of the SLC. The surrounding cavity has not been shown for clarity. Right: The electric field of the WGM_{511} mode shown at a z cross section at the top of the sapphire cylinder.

oscillator, however, there exist more advanced loop topologies for an improved phase noise performance [9]. The simplest model to illustrate the importance of the various parameters in the oscillator loop is Leeson's model [6] giving an expression for the single sideband phase noise (SSPN):

$$SSPN(\Delta f) = \frac{1}{2}\Big((\frac{f_0}{2Q_L\Delta f})^2 + 1\Big)(\frac{f_{1/f}(G)}{\Delta f} + 1)\frac{FkT}{P} \tag{1}$$

Here, Δf is the offset from the carrier frequency (f_0), $f_{1/f}$ is the frequency where the flicker noise of the active element equals the thermal noise and F, k, T, P are the noise figure, Boltzmann's constant, the temperature, and the signal power, respectively. This model is based on an assumption of a linear time invariant response around the loop. More sophisticated models incorporating for instance the time dependency exist [3, 5], however, in our experiments so far the measured phase noise has been in very good accordance with Leeson's model. The model shows the importance of having a resonator with a high Q_L and an active element with low flicker noise. This noise typically has a strong dependence on the level of compression, or equivalently the resulting gain factor in the oscillating circuit. Therefore a combined optimization of Q_L and $f_{1/f}(G)$ should be made. The correlation between the two parameters comes through the fact that an increase in Q_L is made by a reduction in the coupling strength and thus, a higher gain in the amplifier will be required for the oscillation to start.

The method to realize a resonator with a very high unloaded Q, is by exciting a Whispering Gallery Mode in a sapphire loaded cavity (SLC). The SLC consists of a mono crystalline sapphire pill inserted in an evacuated cavity made from a good conducting material (e.g. Cu, Ag or a superconducting material if in a cryogenic environment) see figure 2 (left). The WGMs are modes that primarily reside inside the periphery of the sapphire crystal. The power density in the electromagnetic field drops polynomially or exponentially as we move towards the cylinder axis ($r \rightarrow 0$) or towards the cavity boundary, respectively. Thus, the dominating factor in the damping of the resonance is the dielectric loss in the sapphire crystal. The reason why we can get extremely high quality factors is that this loss is the lowest for any known material and furthermore, the loss has a T^5 temperature dependence in a range from around 70K-300K. Therefore the resonator is highly improved if operated under cryogenic conditions.

The paper is organized as follows. Firstly, the resonator is treated. To be able to find appropriate designs and dimensions of the resonator, we need to be able to compute the resonances in an inhomogeneous and anisotropic media with cylindrical symmetry. A simple numerical procedure is sketched and the specific class of resonances - the Whispering Gallery Modes - are described. Based on such calculations we have made a resonator with a sharp resonance at 10.4 GHz, and we compare experimental and numerical results. Secondly, we describe the active element of our choice, and last, a section will be devoted to the phase noise measurement techniques.

2 The Sapphire Loaded Cavity Resonator

Sapphire mono crystal is an anisotropic material. Therefore, the SLC resonator involves both an anisotropic media and inhomogeneities due to the presence of both vacuum and sapphire in the cavity. Therefore, we cannot calculate the electromagnetic eigen modes in the resonator analytically. Thus, a numerical calculation is required to be able to design the resonator to have a high Q resonance at a desired frequency. The initial design was done with a commercial program (CST Microwave Studio) which solves Maxwell's equations both in a time and frequency domain involving 3D geometries. In the frequency domain the program computes all the resonances or eigen modes up to a predefined frequency.

The resonances that we seek are the Whispering Gallery Modes that as mentioned have most of their energy confined close inside the periphery of the sapphire cylinder. Just as in the case of an empty cavity supporting transverse electric or magnetic (TE or TM) modes the WGMs are grouped in the classes WGH (for a predominantly transverse magnetic mode) and WGE (for a predominantly transverse electric mode). Furthermore, the modes are indexed according to the number of maxima in the azimuthal, the radial, and the z-direction. The mode that we are currently employing in our resonator is a WGH_{511}, which indicates that there are 5 waves in the azimuthal direction and 1 maxima in both the radial direction and the z-direction. The electric field of this mode is depicted in figure 2 (right). The WGH_{511} is the 60'th mode found by CST after the fundamental $TE_{01\delta}$ mode used in ordinary dielectric resonators. As such, the post processing of the data produced by CST is prolonged by the search for the desired mode. Furthermore, the resonances gets increasingly confined in the sapphire cylinder with increasing azimuthal index, and thus in later designs we may consider employing a higher order WGM, which will require too much computation time, storage capacity and postprocessing for CST to be a useful tool in the design process. Therefore we were motivated to make our own program able to find only the relevant resonances.

Firstly, the geometry is cylindrically symmetric and thus we can assume a $e^{im\phi}$ azimuthal dependence of all field components, and furthermore we only need to compute the eigen modes in the (r, z)-plane. From the inspection of the solutions found by CST, we notice that especially in the WGH class of modes that there is a very obvious decoupling of the z component of the electric field, E_z, and H_z, i.e. $E_z \neq 0$ and $H_z \approx 0$. This is the motivation to search for a scalar partial differential equation to find E_z. This scalar equation will be an approximation since the dielectric discontinuity at the boundary between the sapphire crystal and the vacuum gives rise to a coupling between E_z, E_r, H_z and H_r.

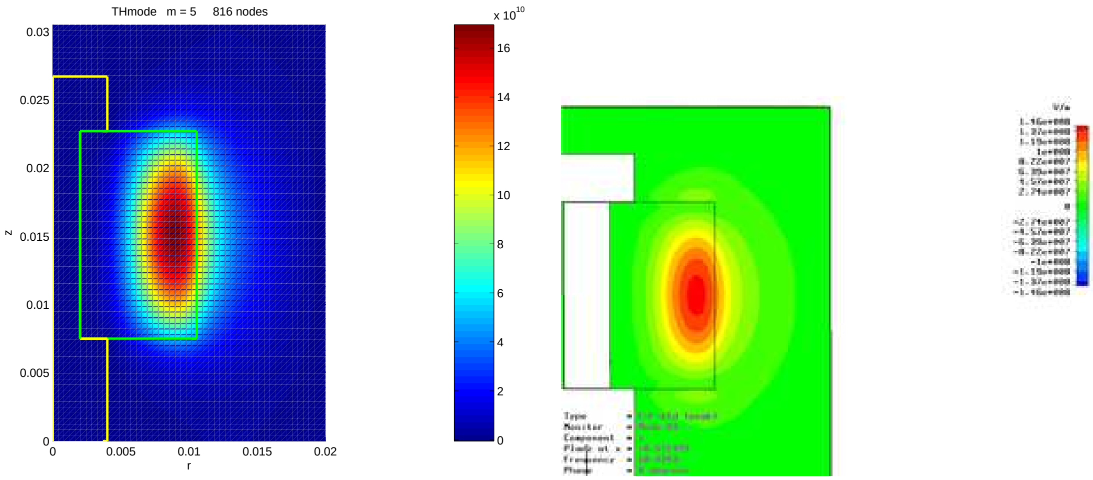
Using the four Maxwell equations, the $e^{im\phi}$ azimuthal dependence and neglecting the discontinuities of the dielectric constant we arrive at the following set of decoupled equations for the z-component of the electric and magnetic field giving rise to either a genuine TE or a genuine TM class of modes:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \tilde{E}_z}{\partial r}\right) + \frac{\epsilon_\parallel}{\epsilon_\perp}\frac{\partial^2 \tilde{E}_z}{\partial z^2} + \left(k_\parallel^2 - \frac{n^2}{r^2}\right)\tilde{E}_z = 0 \tag{2}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \tilde{H}_z}{\partial r}\right) + \frac{\partial^2 \tilde{H}_z}{\partial z^2} + (k_\perp - \frac{m^2}{r^2})\tilde{H}_z = 0, \tag{3}$$

where $E_z = \tilde{E}_z(r, z)e^{im\phi}$ and $H_z = \tilde{H}_z(r, z)e^{im\phi}$, and $k_\parallel = \omega^2\epsilon_\parallel(r, z)\epsilon_0\mu_0$ and $k_\perp = \omega^2\epsilon_\perp(r, z)\epsilon_0\mu_0$. The (r, z) dependency of the dielectric constant is of course only present at the boundary between the sapphire and the vacuum and only in the sapphire crystal will $\epsilon_\perp \neq \epsilon_\parallel$ due to the anisotropy. The scalar equations have been solved numerically by the Finite Element Method (FEM). A good description of this method is given by Wait and Mitchell [8]. If the fully consistent set of coupled (vectorial) equations are to be solved a so-called Mixed-FEM method could be used. This method is described for waveguide geometries in [1], but can be rather easily adapted to be able to deal with the SLC. We used a triangular mesh arranged so that no element would cover a region both in the sapphire and in the vacuum (see figure), and therefore a fixed dielectric constant could be used in the integration over each element. The basis function employed was the Lagrangian 1.st order function and thus the solution was approximated by a continuous function made of piecewise linear planes. The Galerkin method was employed to obtain a set of algebraic equations. These were arranged in a general matrix equation, $\mathbf{Ax} = \lambda\mathbf{Bx}$, where $\mathbf{A}$ and $\mathbf{B}$ are $N \times N$ matrices with known coefficients, λ is the eigen value (related to the eigen frequency) and $\mathbf{x}$ is the corresponding eigen vector yielding either the values of $\tilde{E}_z$ or $\tilde{H}_z$ at the nodes of the mesh. The implementation of the numerical scheme was done in `MatLab 6.5`, so the solution to the general matrix problem was found using the `MatLab` function `eigs`.

In figure 3 a comparison between the WGH_{511} $\tilde{E}_z$ component computed in `MatLab` and by CST has been shown at the right half of the (r, z)-cross section. A rather good correspondence is seen between the two results although our model gives rise to discrepancies at the top and bottom boundary between the sapphire crystal and vacuum. In table 1 a comparison between the CST and our results for the WGH modes is shown. The deviation between the two methods is seen to be below 4% and it decreases for increasing azimuthal



a **b**

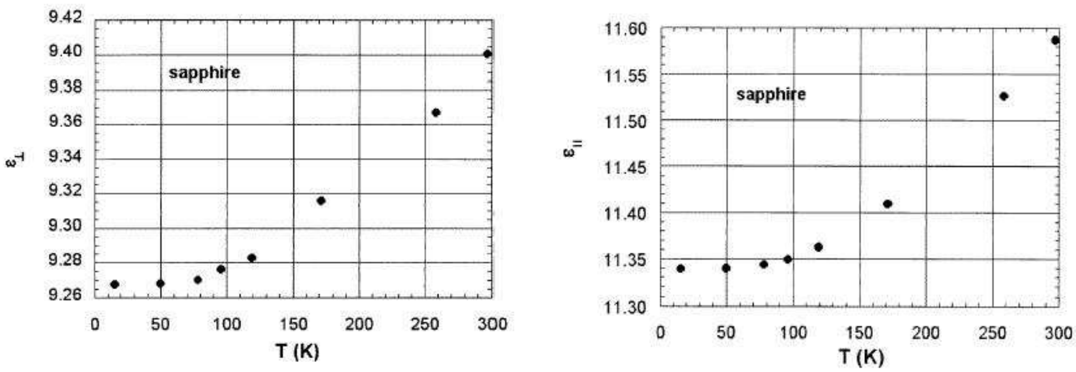
Figure 3: $\tilde{E}_z$ component of the WGH_{511} mode. **a**: Computation in Matlab. **b**: Computation by CST.

index. This behavior is expected since the dielectric discontinuity becomes less and less important due to the increased confinedness of the electromagnetic power in the sapphire with increased azimuthal index. The pseudo TE modes found by CST also displayed a significant E_z component. Therefore, the correlation between our program and CST was not very good. Currently we are working on implementing the Mixed-FEM scheme, so that we can both have the accuracy of CST for all modes and the short computation and post-processing time of our scalar implementation.

The new program is very easy to use. Input data consist of a description of the geometry and the desired azimuthal index m. Like CST it will compute as many modes as the user likes. The difference is, however, that all these modes will have the desired azimuthal index and the first mode (with lowest eigen frequency) will be the WGH_{m11} mode. The program therefore considerably speeds up the design process. The final design uses a sapphire cylinder of height 15.2mm and diameter of 21.2mm. All surfaces are polished. The sapphire crystal

Mode	`MatLab`	CST	% dev.
WGH_{211}	5.702 GHz	5.894 GHz	3.26
WGH_{311}	7.234 GHz	7.405 GHz	2.31
WGH_{411}	8.796 GHz	8.908 GHz	1.26
WGH_{511}	10.347 GHz	10.401 GHz	0.52
WGH_{611}	11.882 GHz	11.864 GHz	0.15

Table 1: Calculation of the eigen frequencies of various WGH modes by CST and `MatLab`.



a **b**

Figure 4: Permittivity of crystalline sapphire (from [4]). **a**: In the (a, b) plane of the crystal corresponding to the (r, ϕ)-plane of the cylinder in the SLC and **b**: Along the crystal c-axis which is aligned with the z-axis of the SLC.

c axis is oriented along the symmetry axis (the z-direction). Centrally, there is a 4mm hole to facilitate a mounting of the crystal onto a 7.5mm high stag using a Cu screw. The Cu cavity has a height of 30.5mm and diameter of 40mm. This design should have a WGH_{511} at 10.40GHz. To couple power into and out of the cavity two small antennas have been inserted in the cavity. One in the top plate and one in the bottom plate. Using a Network Analyzer a very sharp resonance has been detected at 10.41GHz (unloaded Q of around 120,000 at 300K). It has not been experimentally verified that this is in fact the WGH_{511} mode we are probing. Such an experiment could for instance be made by having an antenna that could be moved along the periphery of the sapphire which could facilitate a counting of the number of standing waves.

As mentioned in the introduction, the dielectric properties of sapphire are highly dependent on temperature. In figure 4 the dielectric constant vs. temperature has been shown (from [4]). The almost linear dependence on temperature down to around 100K facilitates a slow tuning of the resonance frequencies by changing the temperature of the resonator. By cooling our resonator down to 240K the frequency of the WGH_{511} resonance shifted to around 10.44GHz. In figure 5 the dielectric loss or $\tan\delta$ of the sapphire has been shown (also from [4]). When the WGMs are employed the loss of energy primarily takes place in the sapphire and thus, the Q of the resonance is determined by the loss in the sapphire, i.e. $Q \approx 1/\tan\delta$. Therefore, Qs in the excess of several billions can be achieved by cooling the resonator down to say 4.2K.

The simple oscillator based on figure 1 has been analyzed analytically and experimentally by Galani *et al* [9]. Here, it is found that in order to minimize the phase noise, the coupling coefficient at the two ports should be 0.5, thus giving the resonator a loaded Q of half the value of the unloaded Q. To maintain the coupling coefficients at 0.5 during the process of cooling it is necessary to be able to change the length of the small antennas in the cavity. This is done by a mechanical actuator attached to each antenna.

3 The active element

Due to the up-conversion of the 1/f noise in the amplifier giving rise to the $1/f^3$ phase noise around the oscillator center frequency it is important to design the amplifier not only to have a good noise figure F, but also to have the minimum $1/f$ noise. However, the noise of the amplifier is of course limited by the properties of the applied transistor. Here, a compromise has to be made. Field effect transistors (FET)generally give good noise figures and can be used at high frequencies, however, their 1/f noise is high. High electron mobility transistors (HEMT) and MESFETs have shown better 1/f performance. Currently, however, the best choice of transistor for use in up to 10GHz oscillators are the bipolar transistors and the heterojunction bipolar transistor (HBT). Disadvantages are the rapid drop in the gain of these transistors at increasing frequencies and the higher noise figure as compared to the FETs. The transistor that we have chosen for our oscillator is a Silicon Germanium (SiGe) HBT, LPT16ED, delivered by SiGe Semiconductor. Measured at 10GHz, $I_c = 5mA$ and $V_{CE} = 1V$ this device has a residual phase noise below -160dBc/Hz at 10kHz offset. There are numerous references to this device in the literature, for instance in [2] a 4.85 GHz oscillator has been built based on a sapphire resonator and the SiGe HBT and they measure the lowest phase noise ever achieved for a single loop, free running microwave oscillator (-133dBc/Hz at 1kHz offset).

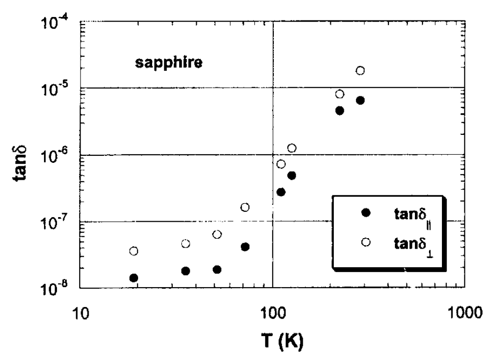


Figure 5: The loss-tangent of sapphire vs. temperature (from [4])

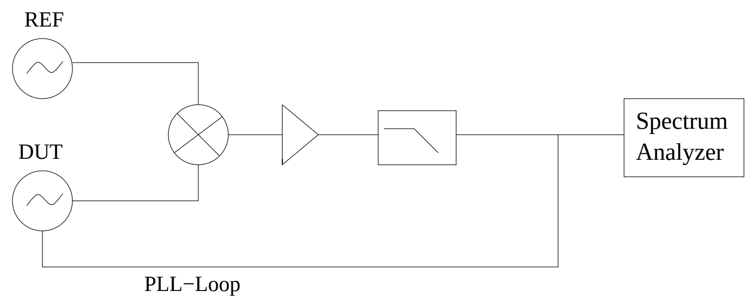


Figure 6: Measurement of phase-noise

4 Phase noise measurements

In figure 6, a schematic diagram of the phase noise measurement procedure has been shown. To measure the phase noise of an arbitrary oscillator another oscillator operating at the same frequency with at least the same or lower noise is required. In the diagram the two oscillators are phase locked to each other by a standard Phase Locked Loop circuit. The lock is maintained using a very narrowband lowpass filter, so effectively we only have a frequency lock. The beat signal therefore contains the sum of the noise around the carrier frequency of the two oscillators. To find the noise produced by the state-of-the-art oscillators we therefore need to have two of them. The baseband signal will then consist of approximately twice the noise of one of the oscillators. This means that we just have to subtract 3dBc/Hz from the baseband spectrum of the beat signal to obtain the phase noise of one oscillator.

As yet we have not measured the phase noise of the oscillator based on the cryogenic SLC and the SiGe HBT transistor. We have, however, measured various room temperature oscillators based on both low and high Q resonators and amplifiers using different transistors. Here, we found good accordance with the results predicted by Leeson's model in equation 1. We expect to reach lower than -140dBc/Hz at 1kHz offset from the carrier with the improved resonator and amplifier.

This work was partially sponsored by the Hartman Foundation and INTAS 2001-0809,-0686.

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