Wavelet Multiresolution Analysis of Spray Images from a Diesel Injector

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Abstract

Wavelet multiresolution analysis has been applied to digital images of ethanol sprays injected into air at atmospheric pressure and at room temperature. The imaging was performed with laser-sheet imaging by using the 308 nm light from a XeCl excimer laser. Image capturing was performed with a CCD-camera. The importance of different scales in the turbulent structure of the sprays was easily recognized when using wavelet multiresolution analysis. It was observed that the intermediate and small-scale levels contained information of such important phenomena as spray break-up. This was not easily recognized in the original images, due to averaging over all scales.

Keywords: spray, combustion, image processing, wavelet multiresolution analysis

1. Introduction

The interest in spray characterization is mainly due to the many applications of sprays in aerosols and combustion systems, and the need to understand the spray behavior in order to improve the performance of these differing systems. In combustion engines, there is currently also a trend towards increased use of fuel injection based on high-pressure spray injectors, being used in direct-injected Otto and two-stroke engines and in Diesel engines (Heywood, 1988).

In this paper, laser sheet imaging with Mie-scattered light is used to image cross-sections through the spray in order to reveal the internal structure of the spray. It is a technically simple method. In a single image, it gives a 2-D cross-section through the spray, yielding both radial and axial information. The time evolution of the spray is covered by recording a sequence of images from different times in the spray cycle.

In order to reveal information about different scale behavior of the spray, as a function of both time and position, wavelet multiresolution analysis is employed (Mallat, 1989). This technique was successfully used by (Li et al., 1999) in the analysis of a jet-fluid concentration in the far field of round, liquid-phase, turbulent jets. In the present paper, the importance of different scales in the turbulent structure of the sprays is recognized when using wavelet multiresolution analysis. For the analysis, Daubechies wavelet filter coefficients were used (Daubechies, 1992). These coefficients give an orthogonal decomposition of the image, which makes it possible to reconstruct the original image from the sum of all different multiresolution images. This latter feature of discrete wavelet transforms, DWT, is necessary for a meaningful comparison of the importance of different scale levels in the image.

2. Experimental Method

An overview of the experimental set-up is shown in Fig. 1. The light source was a XeCl excimer laser with wavelength 308 nm, pulse energy 80 mJ, and a pulse length of about 20 ns. The beam had a rectangular shape with sides 3 cm (height) by 1.5 cm (width). Excimer lasers have a relatively low
directionality, which is a problem when a tight focus is necessary. Since we ideally want to study a two-dimensional cross-section of the spray, it is important that the laser sheet is as narrow as possible. For this reason the beam was filtered as discussed next. The beam is focused by a cylindrical \( f=150 \) mm quartz lens, which focuses it in the horizontal plane but leaves its height unaffected. A slit, 0.02 mm in width, is placed in the focus of the beam. Diverging parts of the beam will not focus correctly, and will be rejected by this spatial filter. Next, the beam passes a second cylindrical \( f=150 \) mm lens and is focused to a light sheet passing through the center line of the spray. At the focus of this light sheet, we get an image of the spatial filter described above, with a beam waist of 30 \( \mu \)m.

The spray equipment consisted of an in-line diesel pump, with a peak pressure of approximately 350 bars, and a Bosch fuel injector with a specially made, single-hole nozzle on axis. The bore of the nozzle had a diameter of approximately 0.2 mm. The opening pressure of the injector (spring-activated) was adjusted to 245 bars. The liquid injected was ethanol, chosen because it is transparent to 308 nm light. The ethanol spray was injected into air at atmospheric pressure and at temperature 295K, which was the operating condition in this study. The duration of the injection was 1 ms, measured with a needle lift gauge, which was also used to synchronize the laser with the spray start.

Light scattered at 90° from the direction of the laser beam was collected with a quartz telephoto lens (Nikon) and imaged onto a CCD camera (Princeton Instruments). The CCD array was 578 by 384 pixels in size. The dynamic range was 14 bits. Also, a band-pass filter (Schott UG11) centered on the laser wavelength, 308 nm, was mounted on the telephoto lens to suppress stray light and, to some extent, diverse fluorescence light. The FWHM bandwidth of the filter was 110 nm. With this experimental set-up we could record one image per spray cycle. The long readout time (1-2 seconds) of the CCD chip did not allow multiple exposures during a single spray cycle (which lasts only 1ms). An example of image recorded with this set-up is shown in Fig. 2. The laser sheet is impinging into the spray from the left side, giving rise to a higher intensity on the left side of the sprays (Abu-Gharbieh et al., 2000). Figure 3 shows the 256 by 256 sub-image of Fig. 2 which is studied in this paper. The size of this image is 1.8 by 1.8 mm.

Figure 1. A schematic overview of the experimental set-up.

3. Wavelet Multiresolution Analysis

Wavelet multiresolution analysis can process small amounts of signal data by selecting the relevant details that are necessary to perform a particular recognition task. The first step in wavelet multiresolution analysis is to calculate the discrete wavelet transform, DWT, of the studied signal. There is no loss of generality in discussing one-dimensional signals instead of two-dimensional ones.
The numerical implementation of the DWT is considerably simpler than that for the continuous wavelet transform (Press et al., 1992). The technique is largely similar to that for the fast Fourier transform, FFT. The difference lies in the base functions upon which the studied signal is projected. For FFT the base functions are sines and cosines with different frequencies. For the DWT the base functions are not uniquely defined but are rather loosely termed “mother functions”, or “wavelets”.

The difference between sines and cosines on the one hand, and wavelets on the other, is that the choice of wavelets is quite free. In particular, the wavelets can be chosen such that they are localized both in frequency and in space, in contrast to sines and cosines, which are perfectly located in frequency but are non-localized in space. One especially useful set of wavelets was identified by Daubechies (Daubechies, 1992). They are defined in terms of wavelet filter coefficients. The signal to be analyzed is multiplied with a matrix containing the wavelet filter coefficients in different configurations. For the simplest example, using a set of filter coefficients called DAUB4, the matrix has the following appearance:

\[
\begin{bmatrix}
  c_0 & c_1 & c_2 & c_3 \\
  c_3 & -c_2 & c_1 & -c_0 \\
  c_0 & c_1 & c_2 & c_3 \\
  c_3 & -c_2 & c_1 & -c_0 \\
  \vdots & \vdots & \vdots & \vdots \\
  c_2 & c_3 \\
  c_1 & -c_0 \\
\end{bmatrix}
\]

DAUB4 contains only four coefficients, which is the minimum number of coefficients if both orthogonality and at least two vanishing moments for the difference filtering are required. DAUB4 is also the set of filter coefficients that gives the most spatially localized wavelets, and therefore also the least spectrally localized wavelets. In the matrix in (1) the odd rows consist of an average filter, while the even rows are difference filters. The first time the matrix is multiplied with the signal, the elements in the even positions in the resulting vector have been difference-filtered. These are the wavelet coefficients of the highest level, containing the most detailed information about the studied signal. After the multiplication, the even elements in the resulting vector are permuted to the end of the vector and the elements at the odd, average-filtered, positions are placed in the beginning. The next step is to multiply the first half of this new permuted vector, that is, the elements that have been average-filtered, with the matrix in (1), although now the size of this vector is halved. This process is repeated until the wavelet matrix (1) cannot be made smaller due to the number of filter coefficients. In this way the final vector will contain wavelet coefficients corresponding to different levels of average and difference filtering. This is the origin of the term “multiresolution”; the different levels correspond to different spectral resolution, and therefore also to different spatial resolution.

The extension of the above-discussed methods to two dimensions is straightforward. First, all columns in the image are processed with the one-dimensional method. Secondly, the resulting matrix is processed again in the horizontal direction. The two-dimensional DWT of the image in Fig. 3 is shown in Fig. 4. The set of wavelet filter coefficients was DAUB20 which was found most appropriate to use for all images in this study. One immediately sees that almost all wavelet coefficients are very close to zero. This means that most of the energy in the image is concentrated to the first, large-scale, wavelet level. This is the basis for the very effective image compression techniques that have been made possible since the introduction of DWT into image processing.

The second step of the wavelet multiresolution analysis is to reconstruct the original signal from the different wavelet levels. The analogy for FFT would be to plot the different sines and cosines with the amplitude corresponding to their respective strength in the FFT spectrum. However, since the sines and cosines are non-localized in space this would be quite a meaningless set of curves, or images. On the contrary, since the DWT are localized in both frequency and space, the reconstructed signal...
corresponding to a given wavelet level would show interesting features about how the periodicity of the signal changes in space.

The reconstruction procedure is to a large extent simply the reverse of the calculation of the DWT as discussed above. The difference is that instead of keeping the full information about the wavelet coefficients, only those corresponding to one particular level are retained, the rest being set to zero. It was shown by (Li et al., 1998) that orthonormal wavelet families with index less than 10 may be inappropriate to use in analysis of turbulent flow. This was also seen in the analysis of the spray images used in this paper. Therefore, DAUB20 (Daubechies, 1992) were chosen as the wavelet filter coefficients here. Figures 5 to 8 show the different reconstructed levels of the DWT shown in Fig. 4. It is recognized that most of the energy in the original Fig. 3 is contained in level 1. The higher levels contribute smaller energies containing information about the higher frequencies, or smaller scales, of the original image.

For a vector of length 256 pixels, and using DAUB20, there will be four levels in the DWT: level 4 containing 128 points, level 3 containing 64 points, level 2 containing 32 points, and level 1 containing 16 points. When each level is reconstructed to the full length of 256 pixels, they will correspond to different scales. Looking at the highest, most detailed, level 4, the 128 wavelet coefficients describe the 1.8 mm side of the image. Thus, each wavelet coefficient corresponds to 1.8/128=14 µm. This is called the scale of level 4. All in all, levels 1-4 correspond to the scales 112 µm, 55 µm, 28 µm, and 14 µm, respectively.

The time consumption for performing the full analysis of an image with 256x256 pixels, and with 14-bit dynamic range, was around 20 seconds on a Toshiba Satellite Pro 4300, with 600 MHz processor and 196 MB memory. The approximate number of floating point operations during the Matlab execution was about 9·10^8.

4. Results and Discussion

Figure 2 shows a typical image acquired with the experimental set-up described above. The front of the injector is seen as weakly illuminated at the top, ending at around 1.1 mm. Since the numerical implementation of the wavelet multiresolution analysis is best suited for images with pixel size equal to an integral power of two, the 256x256-pixel part of Fig. 2 is cut out for the analysis. This sub-image is shown in Fig. 3. The colorbar to the right gives the mapping of intensity into color. Since the dynamic range of the CCD array was 14 bits, the maximum intensity was 16384. The beginning of the spray in Fig. 3, y=0 mm, corresponds to the beginning of the spray, or tip of the injector, in Fig. 2, y=1.1 mm.

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1 In fact, the lowest level 1 contains 32 elements: 16 wavelet coefficients and 16 “mother-function coefficients”. The latter are the remainder that have never been difference-filtered but only average-filtered (Press et al., 1992).
Figure 2. A full original spray image. The injector is seen weakly illuminated at the top.

Figure 3. A 256x256-pixel sub-image of Fig. 2.

Figure 4 shows the two-dimensional DWT of the image in Fig. 3. The wavelet filter coefficients used were DAUB20. The little square in the upper left corner, which is 32 by 32 pixels in size, contains a little “mini-spray” as can be seen if studied carefully. Almost all energy, that is, intensity squared, is contained in this part of the wavelet transform, which corresponds to the first level in the wavelet multiresolution. The inverse wavelet transform of the first multiresolution level is shown in Fig. 5. This level corresponds to the scale 112 µm. The image looks like the original image in Fig. 3, but blurred.
Figure 4. The two-dimensional wavelet transform of the image in Fig. 3. The wavelet filter coefficients are DAUB20 (Daubechies, 1992).

Figure 5. Level 1, scale 112 µm, of the spray in Fig. 3.

Level 2, with scale 55 µm, in the wavelet multiresolution analysis of the image in Fig. 3 is shown in Fig. 6. This level looks quite different as compared to the original image and the first multiresolution image. Most of the image is zero except for some well-defined positions in the spray. This feature is even more emphasized for levels 3 and 4, scales 28 µm and 14 µm, shown in Figs. 7 and 8, respectively. Figure 5 shows very low intensities in the center of the spray, as can also be discerned in the original Fig. 3. This is not due to laser attenuation, which can be ruled out since the intensity increases again on the right side of the spray, although it never regains the high intensities of the left side, where the laser sheet impinges on the spray. Rather, the center of the spray is believed to be more transparent, and thereby less scattering, than the edges of the spray. The probable explanation for this is that
cavitation bubbles created inside the nozzle follow the fuel on its way out into the air. Bubbles obviously possess a high scattering efficiency since the refraction index is abruptly changed at their surface. The cavitation bubbles are preferably created close to the nozzle wall.

In Fig. 6 streakiness in the image appears. This is typical for higher-level wavelet multiresolution images. The amplitude of this periodicity is a measure of how important the periodicity is at the given level, or scale, and at different positions in the image. The streakiness is most clear in the horizontal direction, that is, vertical streaking. This comes from the change in scattered light when the laser sheet passes the spray, because of attenuation but also because of different degrees of inhomogeneites, as discussed above. Comparing Fig. 6, showing a scale of 55 µm with Figs. 7 and 8, which exhibit smaller scales, it is seen that the periodicity is biased towards the larger scales, since the amplitude is greatest and most easily seen in Fig. 6.

Figure 6. Level 2, scale 55 µm, of the spray in Fig. 3.
The image in Fig. 9 shows a spray that has developed for 10 μs longer than the spray in Fig. 2, or 3. The end of the spray is out of the image, the focus in this paper being on the behaviour of the fuel close to the injector. The attenuation of the laser light that enters into the spray from the left is even more evident than in Fig. 3. Figures 10-13 show level 1-4 of the wavelet multiresolution of the spray in Fig. 9.

Again, it is seen in Figs. 9 and 10 that almost no light is scattered from the center of the spray close to the nozzle, where the fuel has not had much time to reorganize from its state inside the injector.
Interestingly enough, the highest level in Fig. 13 displays a feature which is less clearly seen in the lower levels. At \((x,y) = (0.8, 0.5-0.7)\) the amplitude of the multiresolution image for level 4, scale 14 \(\mu\text{m}\), is relatively high. This is less true for the lower levels, especially level 1, where the highest intensities are further down the spray. Therefore some process is going on which exhibits fine scale behaviour. One speculation could be that this is the precursor of the break-up that occurs at \(y=1.2-1.5, 20\ \mu\text{s}\) later, see Fig. 14 below.

![Figure 13](image13.png)  
**Figure 9.** Development of a spray. There is a 10 \(\mu\text{s}\) delay between this image and the image in Fig. 2.

![Figure 14](image14.png)
Figure 10. Level 1, scale 112 $\mu$m, of the spray in Fig. 9.

Figure 11. Level 2, scale 55 $\mu$m, of the spray in Fig. 9.

Figure 12. Level 3, scale 28 $\mu$m, of the spray in Fig. 9.
Finally, Fig. 14 shows a spray that has developed 30 µs longer than the spray in Fig. 2. It seems that at 1.2 mm from the injector the spray starts to break up into ligaments. The different wavelet multiresolution levels for this spray are shown in Figs. 15-18.

An interesting feature is seen at \((x,y)=(0.6, 1.6)\). In the original image, Fig. 14, all that is seen is intense scattering and detaching droplets, or ligaments. This is quite obscured in the first multiresolution level in Fig. 15. However, level 2 in Fig. 16 clearly shows that for this scale the spray is very “active” at the given position. Interestingly, the high amplitude is retained also for level 3 in Fig. 17. This corresponds to the scale 28 µm. For the highest level 4, with scale 14 µm, the amplitude has decreased again and this break-up region do not show any important periodicity for this smallest scale. Accordingly, the break-up into droplets typically involves scales around 55 and 28 µm. This type of analysis would not have been possible using Fourier transform methods, since these methods measure the periodicity of the full image as a whole, and not spatially resolved periodicity.
Figure 14. Further development of a spray. There is a 30 µs delay between this image and the image in Fig. 2.

Figure 15. Level 1, scale 112 µm, of the spray in Fig. 14.
Figure 16. Level 2, scale 55 µm, of the spray in Fig. 14.

Figure 17. Level 3, scale 28 µm, of the spray in Fig. 14.
6. Conclusions and Outlook

The wavelet multiresolution analysis seems to give useful information about the importance of different length scales for a fuel spray. Droplet/ligament detachment typically involves length scales of around 24 and 55 µm. There are indications, although not conclusive, that the initiation of such break-up behaviour involves even smaller length-scales.

This work will be extended to include spray images for higher pressures, 10 and 20 bars. Also low-magnification overview images of the spray should be studied. This could possibly give valuable information about how the pressure in the reaction chamber, such as an internal combustion engine’s cylinder, affects the behaviour of the fuel spray.

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