Gothenburg University

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Optical Tweezers

Construction of an optical tweezers and derivation of the electromagnetic forces on a sphere.

Group Members:
Lisa Hamberg
Terese Olsson
Matilda Sjöholm

Supervisors:
Dag Hanstorp
Tobias Ambjörnsson
Mattias Ericsson

Abstract

An optical twezeers is an instrument with which small particles such as cells can be caught with help of sharp focused laser light. During the the early 80's optical tweezers were developed in order to catch and hold microsize particles. In a tweezers continuous laser light is used, but by sending laser light out in short pulses it is possible to cut the particles. This is called an optical scissors.

The purpose of this report is to inform other people how to build an optical tweezers and scissors with help of our own experiences, but also to understand the electromagnetic forces on a sphere.

We will describe how to build an optical scissors using our own experiences and deriving the forces in an optical trap using geometrical optics approach.

Contents

1 Introduction

An optical twezeers is an instrument with which small particles such as cells can be caught with help of sharp focused laser light. A tweezers uses continuous laser light, but by sending laser light by special wavelength out in short pulses it is possible to cut the particles. The latter method is used in optical scissors. During the early 80's optical tweezers were developed in order to catch and hold microsize particles. In the beginning tweezers were used in single atom studies [1]. Today, optical tweezers are used in several different areas, such as micromanipulation of biologic materials and single cell studies. The reason why the method is so useful is that it is possible to work in a sterile environment.

The purpose of this report is to inform other people how to build an optical tweezers and scissors with help of our own experiences, but also to understand the forces on a sphere caught in an optical trap.

At first we built an optical tweezers using an Ar^+-laser in order to learn and understand the function and the forces. With that knowledge we later built an optical scissors.

This report starts with descriptions of optical tweezers and scissors and how to build them. After that the electromagnetic forces on a sphere is derived, using a geometrical optics approach.

2 Optical tweezers and scissors

Optical tweezers are used to capture small particles, such as bacteria and cells, with focused light. Optical tweezers and scissors are built up by several mirrors, lenses and a microscope. An optical tweezer works due to the fact that focused light from a laser creates a so called optical trap in the sample. The main difference between a tweezers and a scissors is the wavelength of the laser light. In a tweezers the wavelength of the laser light must not be absorbed in water since that would cause heating. For a scissors a wavelength that is absorbed by water is used so a hole can be "drilled" with the heat. In the scissors pulsed laser light is used. The N_2-laser we used had a maximum frequence of 20 pulses per second. The scissors and the tweezers can be combined so that it is posible to both hold and drill in the sample at the same time.

2.1 Use of optical tweezers

Optical tweezers are mostly used in microbiology. A tweezers can for example hold a sperm while drilling a tiny hole in the *zona pellucida*[1] with an optical scissors [2]. To that hole the sperm can be moved and enter the egg cell and the sperm insertion is fulfilled. When something is caught in an optical trap it takes a minimum force, F_{trap}, to take it out of the trap.

[1] Zona pellucida is the egg cell membrane in a human egg.

That means a particle can be caught and moved so slow that the viscous drag force,

$$F_{viscous} = 6\pi\eta S v$$

is smaller than the trapping force. The viscosity of the sample is η, S the radii of the caught particle and v the velocity of the movement.

2.2 Building an optical tweezers

-An optical tweezers is built up by a laser, several mirrors, lenses and a microscope, see figure 1. To be able to catch particles with light you need to get light with high intensity, relative to the sample, focused. This is achieved by sending laser light through a microscope objective, but to get the focus right you need to put a lens, L_3, in the beam trace on a certain distance from the objective. The lens is placed at the dictance, d, plus the focal length of the lens, see figure 1. A dichroic mirror which only allows light of a special wavelength to

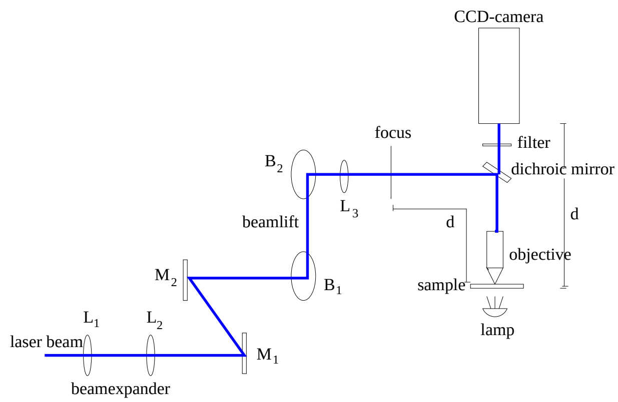

Figure 1: *An Optical tweezers, d are the distance and L_1, L_2, L_3 are lenses, M_1 and M_2 are mirrors that are used when linearize the tweezers. The beamlift are two mirrors B_1 and B_2 that lift the beam from the table and up into the microscope.*

pass through while reflecting the rest is installed. The light from the lamp that passes through the mirror goes into the CCD-camera[2] and the rest of the light passes on towards the laser. It is necessary to have at least two mirrors, M_1 and M_2, and one beamlift. The mirrors are used when linearizing the tweezers. The linearization will be described in 2.2.1. It is desireable to keep the laser beam below eye-height, but to get it into the microscope objective it is necessary to lift it up. It is achieved with the beamlift, B_1 and B_2, see figure 1.

To get as large force in the trap as possible it is desireable to overfill the entrance. That is to have laser light as near the edge of the lenses inside the objective as possible. The reason is that more rays from the edges of the lensens inside the microscope objective increase the force that pull the particle sidewards. Since most laser beams are not wide enough to overfill the entrance you need to expand them. This is made with a beamexpander, see figure 2. When

[2]CCD-camera acronym for charge coupled device camera.

moving L_1 the focus of L_3 is moved, because after the lens L_2 the beam is insignificant spread. The expander can be placed anywhere where there is enough space for it.

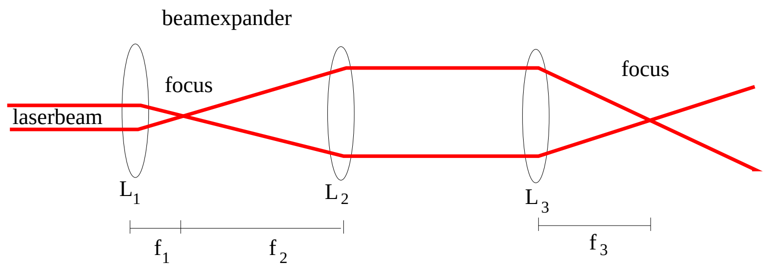

Figure 2: *The lenses, L_1 and L_2, functions as a beamexpander and when moving L_1 the focus for L_3 is moved. The focal length of the lenses are f_1, f_2 and f_3. The beam expansion is depending on the focal length by $M = \frac{f_2}{f_1}$, M is the beam expansion.*

2.2.1 Linearization and finding focus

When linearizing the tweezer the light from the lamp is followed through the microscope objective, all the way back to the laser via the dichroic mirror, the lenses and the mirrors, see figure 1. As fixed light-point we used the center of a cross, which was carved into a black object-glass. This way, light can pass through the cross and the point can be seen with the CCD-camera. To be able to see the cross all the way in the construction it is easiest to use a $10\times$-objective so the cross can be seen in a handable size. The point shall then be seen in the center of the dichroic mirror, the other mirrors and the lenses. When you have followed the cross all the way to the laser and seen that it has been in the center all the way it is time to turn the laser on and follow the laser beam all the way back to the sample. At this point it is very important to block where reflexes might occur. Now, the laser beam shall be in the center of the cross all the way to the object-glass. To get the beam where we wanted the mirrors, M_1 and M_2 see figure 1, were used. While doing all this it is important to wear goggles and to keep danger away it is also recommended to keep the power of the laser as low as possible. When we linearized our tweezers we used about 10 mW.

When the laser and the cross are in the center all the way it is time to change from $10\times$- to $100\times$-objective, to make sure there is focus in the test. To find focus with an Ar^+-laser the lamp is turned off and the filter in front of the CCD-camera is removed. If there is a focus it shall be a sharp point with light circles around it. They are caused by interference [3]. By moving L_3, see figure 1, the rings can be found and then also the focus. When focus is found it is time to see if the tweezers really works. This is for example done with latex spheres in a solution which is put on an object-glass. To capture them it might be necessary to increase the power of the laser. The spheres will then hopefully be captured in the optical trap.

2.3 Our scissors

An optical scissors is constructed practically just like an optical tweezers, but is used to other things. The main difference is that the scissors can not catch or hold anything, another difference is that the laser light is not continuous. Instead, the laser is sent out in light pulses. These pulses can make marks, or even holes, on the surface they hit [4].

To our scissors we used a N_2-laser with light of wavelength 337 nm. The reason for using this kind of laser is that blue light can be absorbed in water. Since this light is in the UV-section we can not use ordinary mirrors and lenses. The mirrors and lenses that were used was made especially for UV-optics.

The main difference in construction between the tweezers and the scissors is that we did not use a beamexpander in the tweezers. Since the N_2-laser can not be seen the with the CCD-camera focus must be found in another way. To do this we came up with an alternative method. This method was to paint an objective glass black and then put it under the microscope objective. If there is a focus the laser would make marks on the black surface. By moving L_1, see figure 2, with a micrometer screw, focus in the sample can be found.

3 Force on a sphere

An optical tweezers will exert a force on the particle in the sample, a so called trapping force. In this section we will derive an expression for that force on a spheric particle.

To decrease the problem we choose to look at the sphere in two dimensions, like a coin. Because of the symmetry of the sphere, we can still see the total force. We have also assumed that there is no absorption.

We wish to calculate the force on spheres with radies greater then the wavelenght of the incoming beams. To do this we can use geometrical optics. The intensity of the reflected fields and the transmitted fields are given by the Fresnel equations, see section 3.4.

Through geometrical optics we can see how the beam reflects and transmits at every surface. To see how the beam acts in a sphere, see figure 3. To calculate the trapping force we take the incoming momentum flux minus the outgoing momentum flux. The momentum flux of a beam is

$$nP/c \tag{1}$$

where n is the refractive index of the surrounding medium, P is the power of the beam and c is the speed of light in vacuum. The incoming momentum flux to the sphere is like equation 1. To calculate the outgoing momentum flux we need to sum all the beams that are directed away from the sphere. If we want the trapping force to be as large as possible we want the force to be large in the x-direction and small in the y-direction. That is why it is convenient to split the force into x -and y-direction.

3.1 The force in x-directon

At first we derive the force in x-direction. By looking at figure 3, we see that the force can be written as

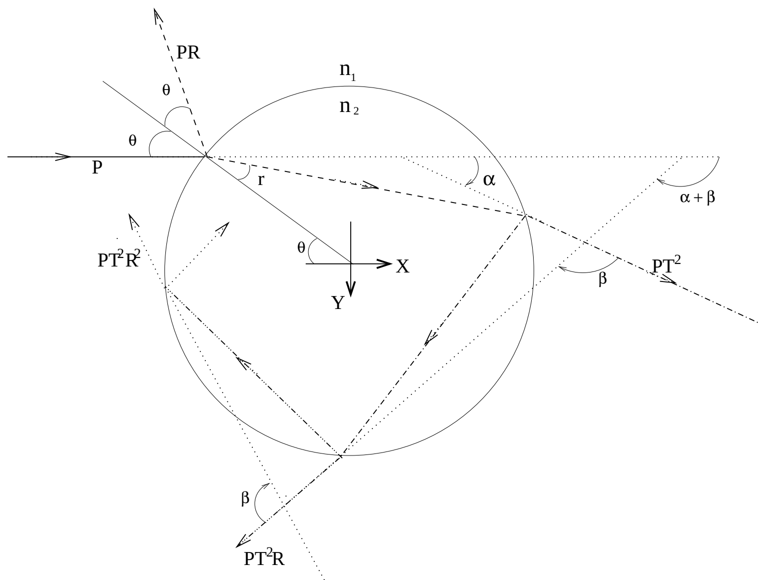

Figure 3: *The beams in the sphere. The power of the incoming beam is P, for the reflected beam it is PR and for the other transmitted beams are the power PT^2R^k, where R is the power reflection coefficient and T is the power transmission coefficient see section 3.3. As you can see, $\theta > r$ and according to Snell's law it means that the refractive coefficient, n, is greater for medium two than for medium one.*

$$
\begin{aligned}
F_x &= \overbrace{\frac{nP}{c}}^{\text{incoming}} - \left[\overbrace{\underbrace{\frac{nPR}{c}(-\cos 2\theta)}_{\text{reflected beam}} + \underbrace{\frac{nP}{c}\sum_{k=0}^{\infty} T^2 R^k (\cos(\alpha + k\beta)}_{\text{transmitted beams}}}^{\text{outgoing}}\right] \\
&= \frac{nP}{c}\left[1 + R\cos 2\theta - Re\left[T^2 \exp(i\alpha)\sum_{k=0}^{\infty} R\exp(i\beta)^k\right]\right],
\end{aligned}
$$

where R is the part of the incoming power that reflect and T is the part of the incoming power that transmits. These are called the reflectance and the transmittance coefficients. It is the same R and T if the beam reflects or transmits at the first or the second boundary, because of the reciprocity relation $R(\theta_i, m) = R(\theta_t, \frac{1}{m})$, where $m = \frac{n_2}{n_1}$. These two quantities will be derived in terms of the refractiv coefficient, n_1 and n_2, in section 3.3. The sum, see equation above, is a geometric series. We will then have

$$
F_x = \frac{nP}{c}\left[1 + R\cos 2\theta - Re\left[T^2 \frac{\exp(i\alpha)}{1 - R\exp(i\beta)}\right]\right].
$$

The angles α and β can be written as $\alpha = 2\theta - 2r$ and $\beta = \pi - 2r$[5]. If we use this and multiply and divide with the complex conjugate the equation becomes

$$F_x = \frac{nP}{c}\left[1 + R\cos 2\theta - \frac{T^2(\cos(2\theta - 2r) + R\cos(2r))}{1 + R^2 + 2R\cos 2\theta}\right]$$

The sphere always wishes to be in its state of equilibrium, which means that the net force is zero. This is the reason why the sphere follows the beam when it moves.

3.2 The force in y-directon

For the trapping force in y-direction we have

$$F_y = 0 - \left[\frac{nPR}{c}(-\sin 2\theta) + \frac{nP}{c}\sum_{k=0}^{\infty} T^2 R^k \sin(\alpha + k\beta)\right],$$

which can be written as

$$\begin{aligned} F_y &= \frac{nP}{c}\left[R(\sin 2\theta - Im\left[T^2 \frac{\exp(i\alpha)}{1 - R\exp(i\beta)}\right]\right] \\ &= \frac{nP}{c}\left[R\sin 2\theta - \frac{T^2(\sin(2\theta - 2r) + R\sin(2r))}{1 + R^2 + 2R\cos 2\theta}\right]. \end{aligned}$$

If all the power of the beam reflects ($R = 1$ and $T = 0$), then the force is

$$F_y = \frac{nP}{c}\sin 2\theta.$$

The opposite, when the whole beam passes through the sphere, ($T = 1$ and $R = 0$), the force is

$$F_y = \frac{nP}{c}\sin(2\theta - 2r).$$

If $\theta = r$ the force is zero as expected.

3.3 Derivation of R and T

The definitions of the power reflection coefficient, R, and the power transmission coefficient, T, are

$$R \equiv \frac{|E_r^0|^2}{|E_i^0|^2}$$

and

$$T \equiv \frac{n_2}{n_1} \frac{|E_t^0|^2}{|E_i^0|^2},$$

[6] where E_i^0, E_r^0 and E_t^0 are the amplitudes of the incoming, reflected and the transmitted beams. $|E_i^0|^2$ is proportional to the energy of the beam, so R and T must show how the incoming beam gets distributed.

To get expressions for R and T some boundary conditions are needed. At first we have that the normal component of the displacement field, $\mathbf{D}$, that is continuous. If we look at the general boundary equation, which is obtained from the Maxwell's equation [6],

$$(\mathbf{D_2} - \mathbf{D_1}) \cdot \mathbf{n} = 4\pi M \tag{2}$$

we can see that the surface density of free charge [charges/area]), M, is zero and $\mathbf{n}$ is the unit vector normal to the plane of the interface. The next boundary condition is that the tangential component of the electric field, $\mathbf{E}$, which is continuous across the boundary. This is written as

$$(\mathbf{E_2} - \mathbf{E_1}) \times \mathbf{n} = 0. \tag{3}$$

The third equation is

$$(\mathbf{B_2} - \mathbf{B_1}) \cdot \mathbf{n} = 0, \tag{4}$$

which means that the normal component of the magnetic field, $\mathbf{B}$, is continuous across the boundary. The fourth and last condition is that the tangential component of the magnetic intensity field, $\mathbf{H}$, is continuous. As equation

$$(\mathbf{H_2} - \mathbf{H_1}) \times \mathbf{n} = \frac{4\mathbf{K_f}\pi}{c} \tag{5}$$

tells us, this can only occur when $\mathbf{K_f} = 0$, where $\mathbf{K_f}$ is the current density at the boundary [current/width]. We assume that both surrounding materials are non-magneticall which means that their relative permabilities may be replaced by unity.

3.4 Calculations for a beam

Let us look how the beam acts at a boundary. The proper plane-wave solutions to Maxwells equations are [6]

$$\mathbf{E_i} = \mathbf{E_i^0} \exp i(\mathbf{k_i r} - \omega t) \tag{6}$$

$$\mathbf{H_i} = \frac{n_1}{k_i} \mathbf{k_i} \times \mathbf{E_i}, \tag{7}$$

where $\mathbf{k}$ is the propagation vector, $\mathbf{r}$ is the placement vector, ω is the angle velocity and t is the time. The reflected wave can be written as

$$\mathbf{E_r} = \mathbf{E_r^0} \exp i(\mathbf{k_r r} - \omega t) \tag{8}$$

$$\mathbf{H_r} = \frac{n_1}{k_r}\mathbf{k_r} \times \mathbf{E_r} \tag{9}$$

and the transmitted wave as

$$\mathbf{E_t} = \mathbf{E_t^0} \exp\left(i(\mathbf{k_t r} - \omega t)\right) \tag{10}$$

$$\mathbf{H_t} = \frac{n_2}{k_t}\mathbf{k_t} \times \mathbf{E_t}. \tag{11}$$

We can rewrite equation 3 as

$$(\mathbf{E_i} + \mathbf{E_r}) \times \mathbf{n} = \mathbf{E_t} \times \mathbf{n} \tag{12}$$

and if we do the same thing with equation 5 and put $K_f = 0$ we find

$$(\mathbf{H_i} + \mathbf{H_r}) \times \mathbf{n} = \mathbf{H_t} \times \mathbf{n}. \tag{13}$$

If we use the plane-wave equations, and the connection $\frac{k_t}{k_i} = \frac{n_2}{n_1}$ [6], equation 13 can be written as

$$(\mathbf{k_i} \times \mathbf{E_i} + \mathbf{k_r} \times \mathbf{E_r}) \times \mathbf{n} = (\mathbf{k_t} \times \mathbf{E_t}) \times \mathbf{n}. \tag{14}$$

To make the analysis even easier, we split every wave into two waves. One with the electricfield vector polarized parallel to the plane of incidence and one with electricfield vector perpendicular to the plane. We start by looking at the one that is polarized perpendicular to the plane, see figure 4.

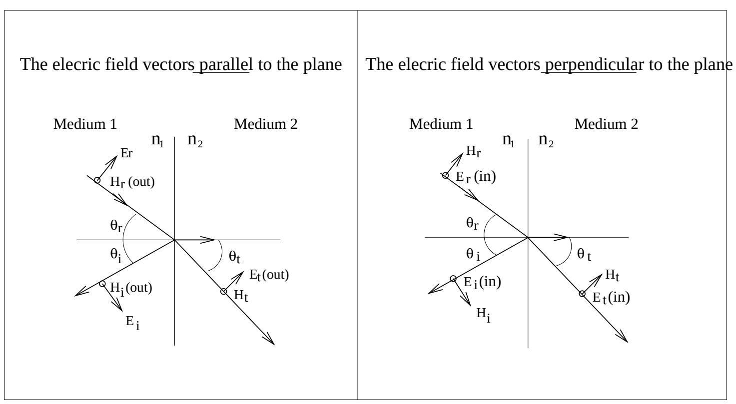

Figure 4: *How the wave acts when it enters a new media. In the first square the electric field vectors are polorized parallel to the plane of incident. In the second square these vectors are polorized perpendicular. E is the electic field, H is the magnetic intensity field, n is the refractiv index and the indexes i, r and t stands for incoming, reflected and transmitted.*

We can write equation 14 as

$$[(\mathbf{n} \cdot \mathbf{k_i})\mathbf{E_i}] - [(\mathbf{n} \cdot \mathbf{E_i})\mathbf{k_i}] + [(\mathbf{n} \cdot \mathbf{k_r})\mathbf{E_r}] - [(\mathbf{n} \cdot \mathbf{E_r})\mathbf{k_r}] = [(\mathbf{n} \cdot \mathbf{k_t})\mathbf{E_t}] - [(\mathbf{n} \cdot \mathbf{E_t})\mathbf{k_t}].$$

Because the electric vectors are all directed into the plane, all the $\mathbf{n} \cdot \mathbf{E}$ will vanish and $\mathbf{n} \cdot \mathbf{k} = \mathbf{k}_\eta \cos\theta_\eta$, η= i, r, t. This gives us

$$E_i^0 \cos\theta_i - E_r^0 \cos\theta_r = \frac{k_t}{k_r} E_t^0 \cos\theta_t$$

which can be written as

$$(E_i^0 - E_r^0)\cos\theta_i = \frac{n_2}{n_1} E_t^0 \cos\theta_t, \tag{15}$$

because $\cos\theta_i = \cos\theta_r$ and $\frac{k_t}{k_i} = \frac{n_2}{n_1}$. Since the electric field vectors are parallel to the boundary surface, we can write equation 12 as

$$E_t^0 = E_i^0 + E_r^0. \tag{16}$$

If we combine equation 15 and 16 we find the equations

$$E_r^0 = \frac{\cos\theta_i - (n_2/n_1)\cos\theta_t}{\cos\theta_i + (n_2/n_1)\cos\theta_t} E_i^0 \tag{17}$$

and

$$E_t^0 = \frac{2\cos\theta_i}{\cos\theta_i + (n_2/n_1)\cos\theta_t} E_i^0. \tag{18}$$

We shall now look at the wave with the electric field parallel to the plane. This time the magnetic field vectors pointing are out of the plane. By the same way as for the parallel wave we can derive the equations

$$(E_i^0 - E_r^0)\cos\theta_i = E_t^0 \cos\theta_t \tag{19}$$

and

$$E_i^0 + E_r^0 = \frac{n_2}{n_1} E_t^0. \tag{20}$$

If we combine these equations we can get

$$E_r^0 = \frac{\cos\theta_i - (n_1/n_2)\cos\theta_t}{\cos\theta_i + (n_1/n_2)\cos\theta_t} E_i^0 \tag{21}$$

$$E_t^0 = \frac{2\cos\theta_i}{\cos\theta_t + (n_2/n_1)\cos\theta_i} E_i^0. \tag{22}$$

These four equations that we have written for E_r^0 and E_r^0 (equations 17, 18, 21 and 22) are called the Fresnel equations and it is those we shall use to calculate R and T. Earlier we wrote the definitions of R and T. Let us now deifine the mean reflection coefficient, which is appropriate for unpolarized light

$$R_m \equiv \left(\frac{(E^0_{r\|})^2}{(E^0_{i\|})^2} + \frac{(E^0_{r\perp})^2}{(E^0_{i\perp})^2} \right) /2.$$

If we use the Fresnel equations in this definition we get

$$R_m = \left(\left(\frac{\cos\theta_i - (n_1/n_2)\cos\theta_t}{\cos\theta_i + (n_1/n_2)\cos\theta_t} E^0_i \right)^2 + \left(\frac{\cos\theta_i - (n_2/n_1)\cos\theta_t}{\cos\theta_i + (n_2/n_1)\cos\theta_t} E^0_i \right)^2 \right) /2. \quad (23)$$

The mean transmitted coefficent is defined in the same way. The equation is

$$T_m \equiv \left(\frac{n_2}{n_1} \frac{(E^0_{t\|})^2}{(E^0_{i\|})^2} + \frac{n_2}{n_1} \frac{(E^0_{t\perp})^2}{(E^0_{i\perp})^2} \right) /2$$

and with the Fresnel equations

$$T_m = \left(\left(\frac{2\cos\theta_i}{\cos\theta_t + (n_2/n_1)\cos\theta_i} E^0_i \right)^2 + \left(\frac{2\cos\theta_i}{\cos\theta_i + (n_2/n_1)\cos\theta_t} E^0_i \right)^2 \right) /2.$$

From equation 23 we can se that $R = 1$ only occures when $\frac{n_1}{n_2} = 0$, which means that $n_2 \to \infty$. An example for this situation is when a light beam in air hits a metal like for example gold. The opposite, when everything transmits, occurs when $n_1 = n_2$.

4 Discussion

When building an optical scissors it is important to use optics that are specially made for the specific wave-length used. Otherwise the laser light might be absorbed insted of reflected and so on. You should also remember that there is a loss of power in every lens and mirror. If the mirrors are missadjusted the loss can be much greater.

The goggles used are depending on the wavelength of the laser. Even reflexes can injure your eyes to be sure always control there are no reflexes.

The tweezers and scissors are fragile since a minor shake can ruin the linearization. To avoid shaking there is good to have a table placed on airbags. You should also use stabile mirrors and stands or such a simple thing as gravity can deplace the mirrors. If there were no vibrations our work would have been a lot easier.

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5 Acknowledgements

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