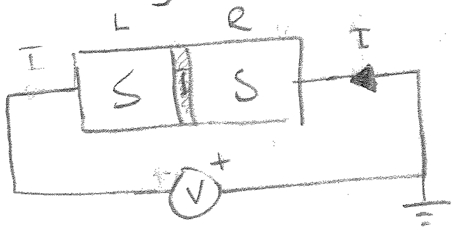
SIS Tunneling

Consider two superconductors separated by a insulating barrier.



The tunneling rate from L to R is proportional to the density of occupied states in metall L times the density of unoccupied states in in metall R

tunneling probability

$$\Gamma_{L\to R} = \frac{2\pi A}{\hbar}\int_{-\infty}^{\infty} |T_{LR}|^2 N_L(\varepsilon + eV)\, f(\varepsilon + eV)\, N_R(\varepsilon)\left(1 - f(\varepsilon)\right) d\varepsilon$$

ε energy counted from the fermi energy of metall R

Similarly the current from R to L is

$$\Gamma_{R\to L} = \frac{2\pi A}{\hbar}\int_{-\infty}^{\infty} |T_{RL}|^2 N_L(\varepsilon - eV)\left(1 - f(\varepsilon + eV)\right) N_R(\varepsilon)\, f(\varepsilon)\, d\varepsilon$$

Assume $|T_{LR}|^2 = |T_{RL}|^2 = |T|^2$ = indep of ε

Then

$$I = -e\left(\Gamma_{R\to L} - \Gamma_{L\to R}\right) = \frac{2\pi A |T|^2}{\hbar}\int_{-\infty}^{\infty} N_L(\varepsilon - eV)\, N_R(\varepsilon)\left(f(\varepsilon - eV) - f(\varepsilon)\right) d\varepsilon$$

In the normal state $N_L(\varepsilon) = N_R(\varepsilon) = N(0)$ and we get

$$I = \frac{2\pi e A |T|^2}{\hbar} N(0)^2 \underbrace{\int_{-\infty}^{\infty}\left(f(\varepsilon - eV) - f(\varepsilon)\right) d\varepsilon}_{\approx eV}$$

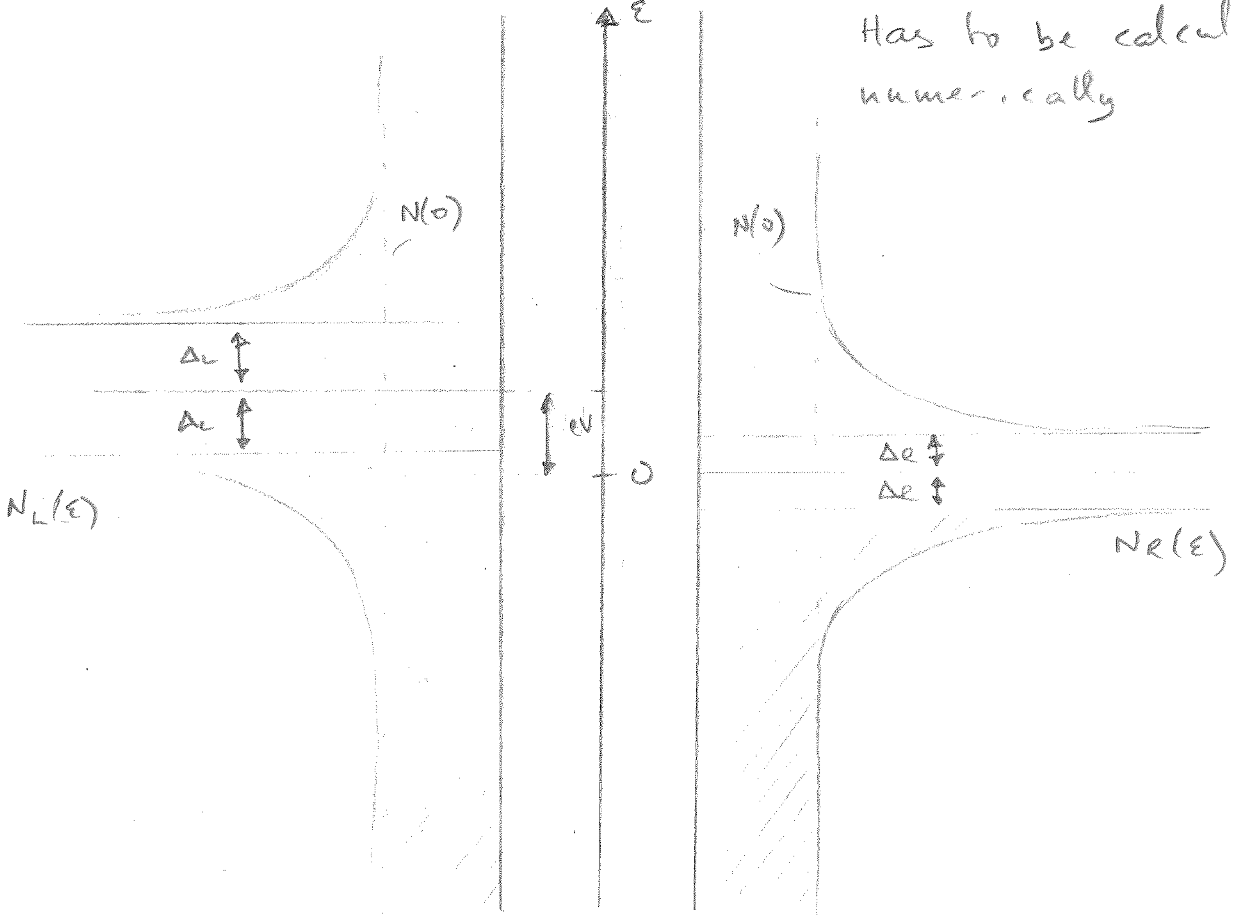
$$= \underbrace{\frac{2\pi e^2 A |T|^2}{\hbar} N(0)^2}_{= G_n = \frac{1}{R_n}} V$$

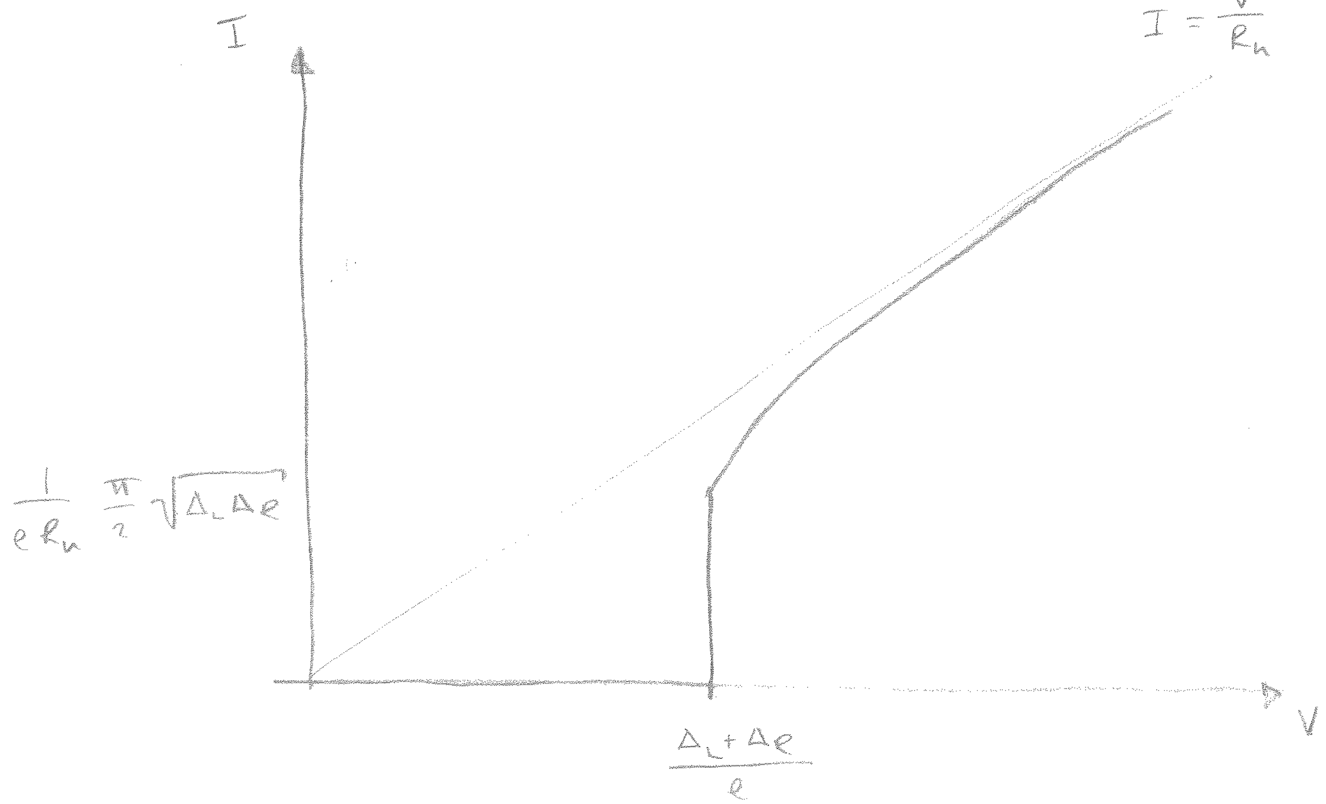
In the superconducting state $N(\varepsilon) = N(0)\dfrac{|\varepsilon|}{\sqrt{\varepsilon^2-\Delta^2}}$, $|\varepsilon|>\Delta$

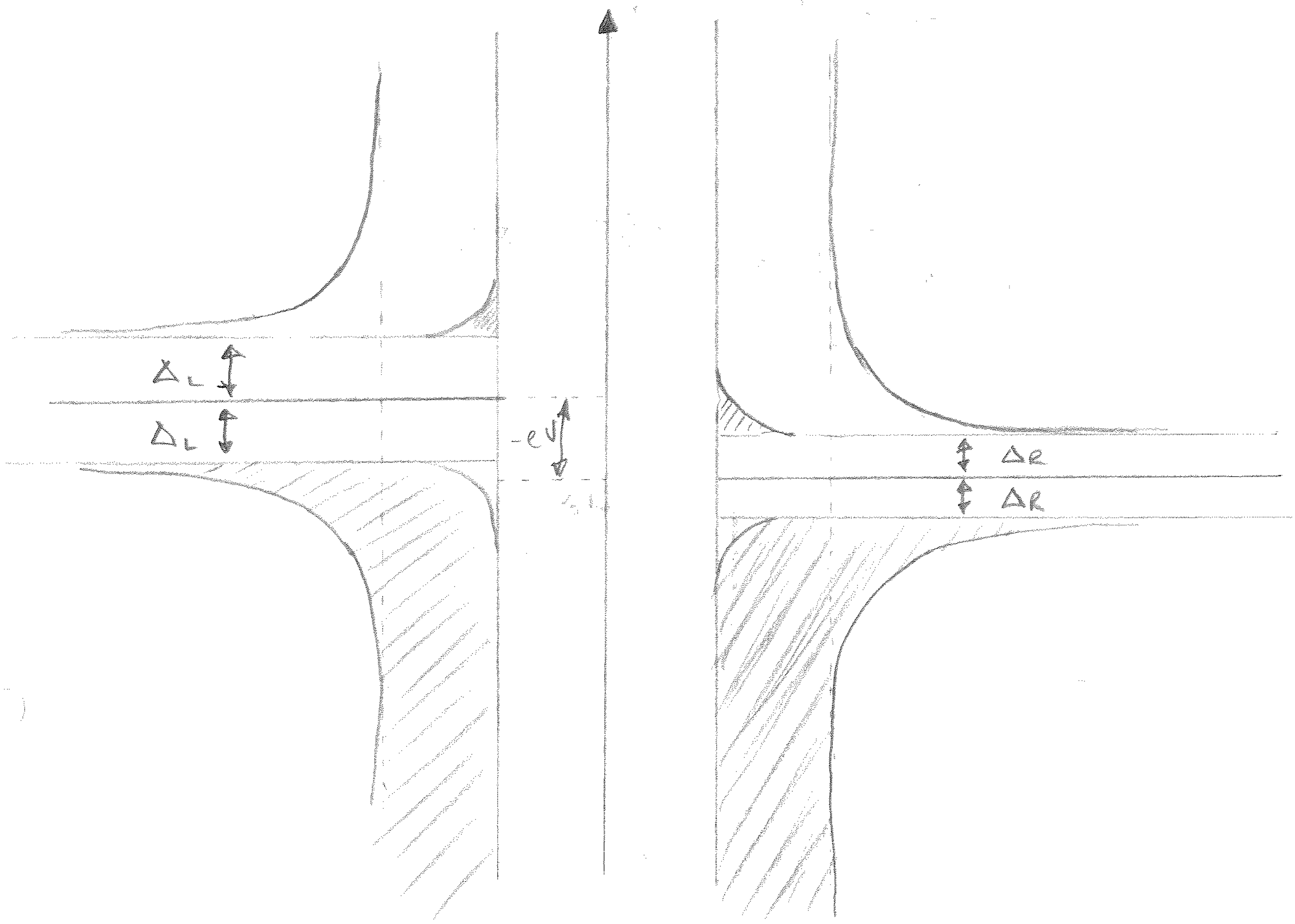
: $|\varepsilon - eV| < \Delta_L$ and $|\varepsilon| < \Delta_R$ has to be excluded from the integration

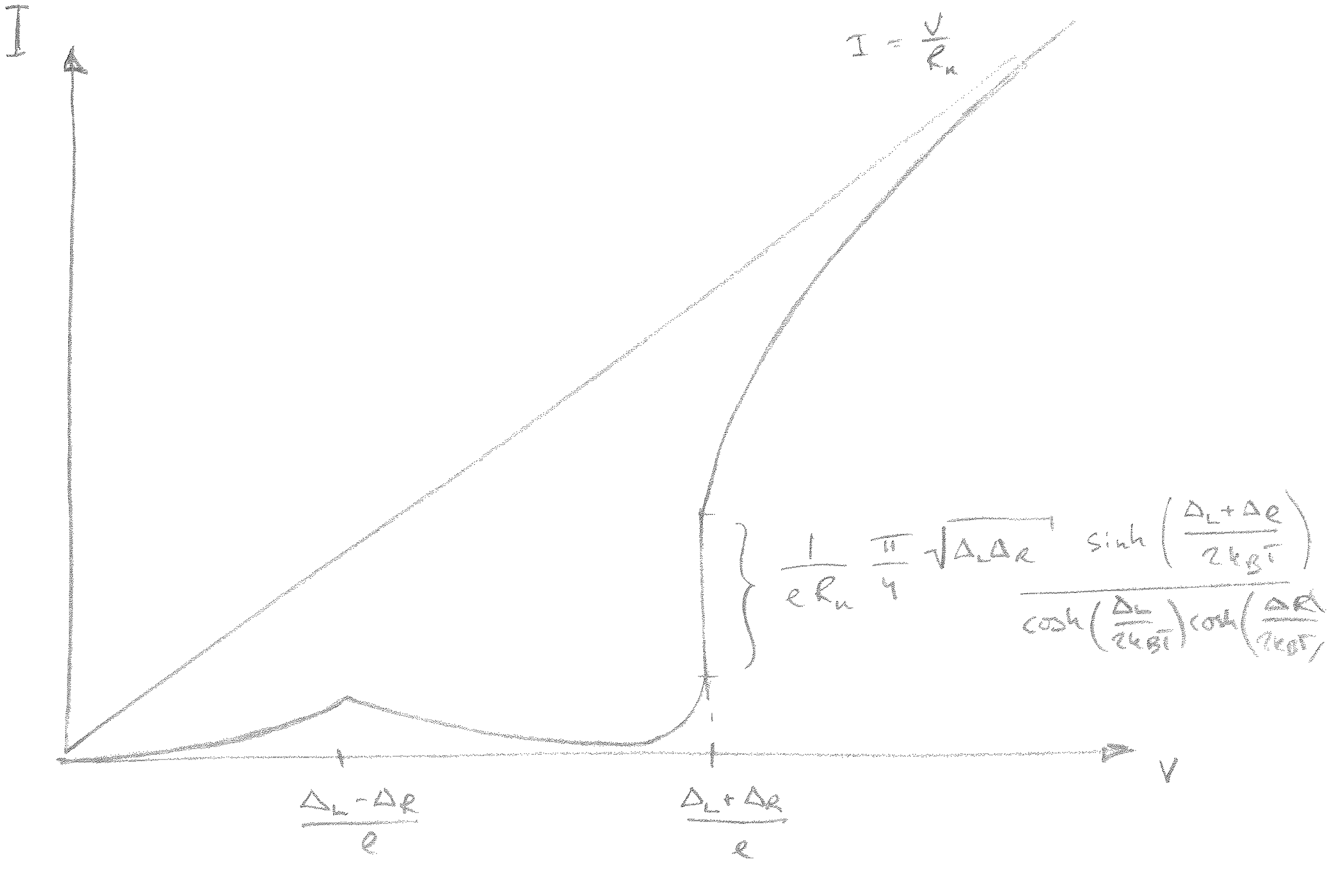
$$I = \frac{1}{eR_n}\int_{-\infty}^{\infty} \frac{|\varepsilon - eV|}{\sqrt{(\varepsilon-eV)^2-\Delta_L^2}}\,\frac{|\varepsilon|}{\sqrt{\varepsilon^2-\Delta_R^2}}\Big(f(\varepsilon-eV)-f(\varepsilon)\Big)\,d\varepsilon$$

Has to be calculated numerically

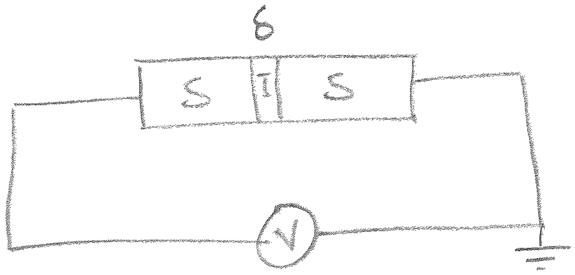





Δ_L
Δ_L
$-eV$
Δ_R
Δ_R


I
$I = \frac{V}{R_n}$
$\frac{1}{eR_n}\frac{\pi}{4}\sqrt{\Delta_L\Delta_R}\,\frac{\sinh\left(\frac{\Delta_L+\Delta_R}{2k_BT}\right)}{\cosh\left(\frac{\Delta_L}{2k_BT}\right)\cosh\left(\frac{\Delta_R}{2k_BT}\right)}$
$\frac{\Delta_L-\Delta_R}{e}$
$\frac{\Delta_L+\Delta_R}{e}$
V

Shapiro steps



Josephson equations:

$$\text{(DC)} \qquad I = I_c \sin \delta \qquad (1)$$

$$\text{(AC)} \qquad V = \frac{\hbar}{2e} \dot{\delta} \qquad (2)$$

Consider the case

$$V = V_0 + V_1 \cos \omega_1 t$$

From (2) we get

$$\dot{\delta} = \frac{2e}{\hbar} \left(V_0 + V_1 \cos \omega_1 t \right)$$

$$\delta = \delta_0 + \underbrace{\frac{2e}{\hbar} V_0}_{= \omega_0} t + \frac{2e}{\hbar \omega_1} V_1 \sin \omega_1 t$$

Inserted into (1) this gives

$$I = I_c \sin \left(\delta_0 + \omega_0 t + \frac{2eV_1}{\hbar \omega_1} \sin \omega_1 t \right) \qquad (3)$$

$$e^{x/2 (t - 1/t)} = \sum_{n=-\infty}^{\infty} J_n(x) t^n$$

$$t = e^{i\theta} \Rightarrow \quad e^{ix \sin \theta} = \sum_{n=-\infty}^{\infty} J_n(x) e^{in\theta}$$

$$e^{i(x_0 + x \sin \theta)} = \sum_{n=-\infty}^{\infty} J_n(x) e^{i(x_0 + n\theta)}$$

$$\sin(x_0 + x\sin\theta) = \frac{1}{2i}\sum_{n=-\infty}^{\infty}\Big(J_n(x)\,e^{i(x_0+n\theta)} - \underbrace{J_n(-x)}_{=(-1)^n J_n(x)}\,e^{i(-x_0+n\theta)}\Big)$$

$$= \frac{1}{2i}\sum_{n=-\infty}^{\infty}\Big\{\underbrace{J_{-n}(x)}_{=(-1)^n J_n(x)}\,e^{i(x_0-n\theta)} - (-1)J_n(x)\,e^{i(-x_0+n\theta)}\Big\}$$

$$= \sum_{n=-\infty}^{\infty}(-1)^n J_n(x)\,\frac{e^{i(x_0-n\theta)} - e^{-i(x_0-n\theta)}}{2i}$$

$$= \sum_{n=-\infty}^{\infty}(-1)^n J_n(x)\sin(x_0-n\theta)$$

The current can therefore be written

$$I = I_c\sum_{n=-\infty}^{\infty}(-1)^n J_n\left(\frac{2eV_1}{\hbar\omega_1}\right)\sin\big(\delta_0 + (\omega_0 - n\omega_1)t\big)$$

If $\omega_0 = n\omega_1 \iff \frac{2eV_0}{\hbar} = n\omega_1 \Rightarrow V_n = \frac{n\cdot\hbar\omega_1}{2e}$

the current has a dc component with maximum value

$$I_c\, J_n\left(\frac{2eV_1}{\hbar\omega_1}\right)$$

Photon-assisted tunneling

$$V(t) = V_0 + V_1 \cos\omega_1 t$$

$$\psi_i(x,t) = \psi_i(x,0)\, e^{-\frac{i}{\hbar}\int_0^t dt\,(\varepsilon_i + eV)} \qquad \Big/ \quad e^{ix\sin\theta} = \sum_{n=-\infty}^{\infty} J_n(x)\, e^{in\theta}$$

$$= \psi_i(x,0)\, e^{-\frac{i}{\hbar}(\varepsilon_i + eV_0)t} \sum_{n=-\infty}^{\infty} J_n\!\left(\frac{eV_1}{\hbar\omega_1}\right) e^{-in\omega_1 t}$$

$$= \psi_i(x,0) \sum_{n=-\infty}^{\infty} J_n\!\left(\frac{eV_1}{\hbar\omega_1}\right) e^{-\frac{i}{\hbar}(\varepsilon_i + eV_0 + n\hbar\omega_1)t}$$

Each energy level
is split into many levels
differing by $\hbar\omega_1$ in energy