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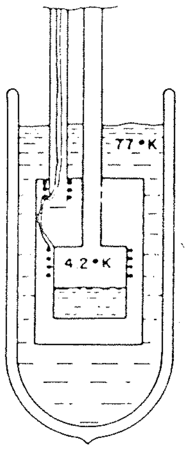
Low Temperature Physics: Calculations of Heat Leaks

The purpose of this exercise is to give you a feeling of the magnitudes of the heat leaks of a cryostat and to estimate which parameters that are the most important ones to consider in the design of low temperature apparatus. Hence, we will estimate the main contributions of the heat transport into the (simplified) cryostat which is specified below.



Inner part: polished Cu surface 500 cm^2
$\varepsilon = 0{,}02$ volume 800 cm^3

Outer part: tarnished brass
$\varepsilon = 0.6$

Support: stainless steel tube with
diameter Ø=2.0 cm
thickness t=0.3 mm
length (77 to 4.2 K) l=6.0 cm

Pressure (He gas) in vacuum space:
p=10^{-5} mmHg

Electrical leads: 8 Cu wires, Ø=0.1 mm
4 constantan, Ø=0.2 mm

[illegible]
I = 2mA

Total length of cryostat, about 50 cm

Rough scale of figure: 1:10

What are the different contributions to the heat transport into the inner part? You should find at least 6 ones. Estimate these heat leaks with the aid of the specifications above and the tables that are enclosed.

Investigate if the Joule heating in connecting leads and a Ge thermometer attached to the inner space will give any problems. (Try different values of the measuring current.) The thermometer has a resistance of about 1 kΩ in the temperature range in question.

In order to get a feeling of the effectiveness of the liquid nitrogen cooled shield, you should make the same calculations for an outer space temperature of 295 K, inner one 4.2 K.

Would the heat leaks change drastically if the outer container were at 4.2 K and the inner one at 1.2 K? (This would correspond to the case of a pumped ^{4}He bath surrounded by an unpumped one.) Any other issues to be considered in this case?

Starting from the calculated values (which can be presented in the form of a table to get an overview) we will discuss possible improvements of the cryostat construction (the elements of which are rather antiquated, but is treated for pedagogic reasons).

The experimental figures suggest that for a metal surface in the

TABLE XII

Accommodation coefficient for helium gas

Metal	
Platinum	0·49 (90° K, 153° K); 0·38 (34°–264° C)
Bright platinum	0·44 (50°–150° C)
Blackened platinum	0·91 (50°–150° C)
Clean fresh tungsten	0·025 (79° K); 0·046 (195° K); 0·057 (22° C)
Gas-filled tungsten	0·19 to 0·82
Gas-free nickel	0·048 (90° K); 0·060 (195° K); 0·071 (273° K)
Nickel (gas layer adsorbed)	0·413 (90° K); 0·423 (195° K); 0·360 (273° K)
Glass	0·67 (12° K); 0·38 (77° K); 0·34 (273° K)

usual condition encountered in a cryostat, exposed to helium gas at low pressure, a value

$$a \leqslant 0{\cdot}5$$

should provide a useful upper limit for calculating heat transfer.

TABLE XIII

Mean values of thermal conductivity expressed in W/cm degK

	λ $T_2 = 300°$ K $T_1 = 77°$ K	λ $T_2 = 300°$ K $T_1 = 20°$ K	λ $T_2 = 300°$ K $T_1 = 4°$ K	λ $T_2 = 77°$ K $T_1 = 20°$ K	λ $T_2 = 77°$ K $T_1 = 4°$ K	λ $T_2 = 20°$ K $T_1 = 4°$ K	λ $T_2 = 4°$ K $T_1 = 2°$ K
Pyrex glass	0·0082	0·0071	0·0068	0·0028	0·0025	0·0012	0·0007
Stainless steel†	0·123	0·109	0·103	0·055	0·045	$0{\cdot}009_7$	0·0022
Inconel (*ca.* 72 Ni, 14–17 Cr, 6–10 Fe, 0·1 C) hard-drawn	0·125	0·111	0·106	0·061	0·051	0·012	0·003
Monel (c. 66 Ni, 2 Fe, 2 Mn, 30 Cu) annealed	0·207	0·192	0·183	0·133	0·113	0·040	0·007
German silver (47 Cu, 41 Zn, 9 Ni, 2 Pb) as received	0·20	0·19	0·18	0·14	0·12	0·039	0·005
Constantan (60 Cu, 40 Ni) wire as received	0·22	0·21	0·20	0·16	0·14	$0{\cdot}04_6$	0·006
Brass (30 Zn, 70 Cu) as received	0·81	0·70	0·67	0·31	0·26	0·078	0·015
Copper (phosphorus deoxidized) as received	1·91	1·71	1·63	0·95	0·80	0·25	0·07
Copper (electrolytic tough pitch) as received	4·1	5·4	5·7	9·7	9·8	10	4

† These figures for stainless steel are calculated from thermal conductivity data which are representative of the behaviour of types 303, 304, 347, and therefore an appropriate composition could be 18% Cr, 9% Ni, traces of Mn, Nb, Si, Ti totalling 2·3 with remainder Fe.

TABLE XV

Experimental values of emissivity

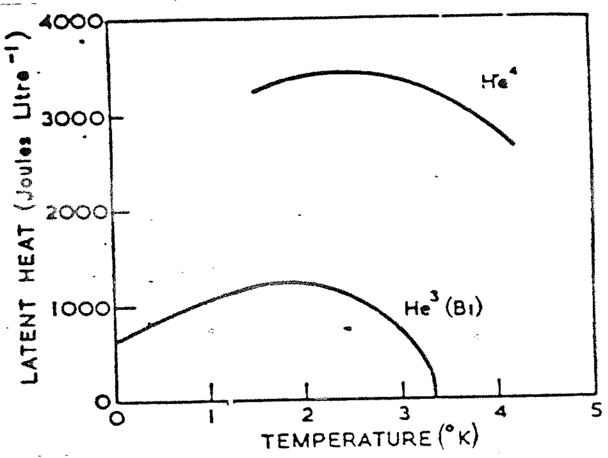
Material	*Fulk, Reynolds, and Park* (1955), 300° K *radiation on* 78° K *surface*	*McAdams* (1954), *room temperature*	*Ramanathan* (1952), 14 μm *radiation on* 2° K *surface*	*Blackman, Egerton, and Truter* (1945), 293° K *radiation on* 90° K *surface*	*Ziegler and Cheung* (1957), 273° K *radiation on* 77° K *surface*
Al, clean polished foil	0·02	0·04	0·011†	0·055	0·043‡
Al, plate	0·03	—	—	—	—
Al, highly oxidized	—	0·31	—	—	—
Brass, clean polished	0·029	0·03	0·018†	0·046	0·10‡
Brass, highly oxidized	—	0·6	—	—	—
Cu, clean polished	0·015–0·019	0·02	0·0062–0·015†	0·01[illegible]–0·035	—
Cu, highly oxidized	—	0·6	—	—	—
Cr, plate	0·03	0·03	—	0·065	0·084‡
Au, foil	0·010–0·023	0·02–0·03	—	0·026	—
Au, plate	0·026	—	—	—	—
Monel	—	0·2	—	—	0·11‡
Ni, polished	—	0·045	—	—	—
Rh, plate	0·078	—	—	—	—
Ag, plate	0·008	0·02–0·03	—	0·023–0·036	—
Stainless steel	0·048	0·074	—	—	—
Sn, clean foil	0·013	0·06	0·013†	0·033	—
Soft solder	0·03	—	—	—	0·047‡
Glass	—	0·9	—	0·87	—
Wood's metal	—	—	—	—	0·16

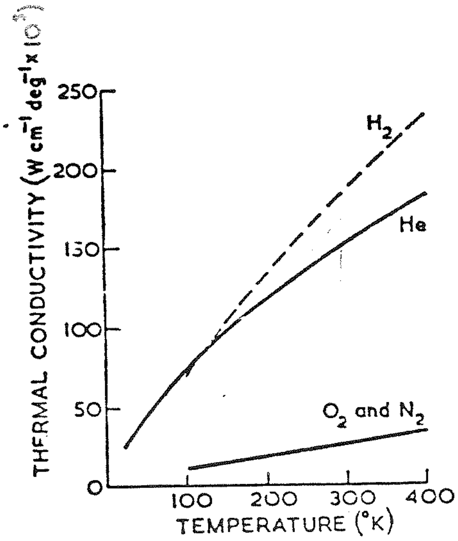
† These surfaces were electro-polished (Ramanathan).

‡ These surfaces were neither highly polished nor heavily oxidized, but as encountered in normal practice (Ziegler). Ziegler observed that a thin layer of oil or Apiezon grease on a low emissivity surface raised the emissivity to 0·2 or 0·3. He also found that varnishes such as GEC adhesive No. 7031 and bakelite lacquer gave an emissivity $\epsilon \simeq 0{\cdot}87$; similarly, Scotch tape (Sellotape) had an emissivity of about 0·88.

Latent heat of vaporization and boiling-point at standard atmospheric pressure of some common gases

Gas	CO_2†	O_2	A	N_2	Ne	H_2	^{4}He
Latent heat (cal/gm) .	137	51·0	37·9	47·8	20·8	106·8	5·2
Latent heat (cal/cm^3) .	223	58·1	53·5	38·6	25	7·56	0·65
Boiling-point (° K) .	194·6	90·1	87·4	77·3	27·2	20·4	4·2





Thermal Conductivities of Gases at Atmospheric Pressure

Summary of heat leaks into a cryostat

February 28, 2007

Radiation

For two plane parallel surfaces with emissivities ε_1 and ε_2,

$$\dot{Q} = \sigma A \varepsilon_{eff} \left(T_1^4 - T_2^4\right)$$

where

$$\varepsilon_{eff} = \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}$$

Heat conduction through gas

For low pressures such that the mean free path is larger then the separation, the molecules will go from one side to the the other without colliding with each other. The conduction through the gas will proportional to the pressure (i.e. the number of molcules).

$$\dot{Q} = \mathrm{const} \cdot a_0 \cdot P_{mm} \left(T_1 - T_2\right) \qquad \left[\frac{\mathrm{W}}{\mathrm{cm}^2}\right]$$

where const = 0.028 for He, P_{mm} is the pressure in units of mmHg and

$$a_0 = \frac{a_1 a_2}{a_1 + (A_1/A_2)(1 - a_1)a_2}$$

a_1 and a_2 are the accommodation coefficients for the two surfaces and A_1 and A_2 are the corresponding areas.

Heat conduction through support and leads

The heat conduction can be calulated by using Fourier's law, $\dot{Q} = -\lambda \nabla T$. We consider a volume with cross-sectional area A and temperature dependence only in the x-direction.

$$\dot{Q} = -A\lambda(T)\frac{\partial T}{\partial x} = -\frac{A}{l}\int\limits_{T_1}^{T_2} \lambda(T)dt = \frac{A}{l}\bar{\lambda}\left(T_1 - T_2\right)$$

$\bar{\lambda}$ is the mean value of the thermal conductivity between the two temperatures

$$\bar{\lambda} = \frac{1}{T_2 - T_1}\int\limits_{T_1}^{T_2} \lambda(T)dt$$

and is tabulated for different materials and temperatures.

Experiment

Joule heating

$$\dot{Q} = RI^2$$

Summary

Type	77 to 4.2K	295 to 4.2K	4.2 to 1.2K
1. Radiation from warm to cold vessel	2	430	0.000018
2. Radiation in pumping tube	0.075[a] - 130[b]	0.075 - 130	0.075 - 130
3. Conduction through pumping (SS) tube	100	190[c]	0.2
4. Conduction through leads	38	31[d]	0.63
5. Conduction through vaccum space	3.4	13.5	0.14
6. Conduction through gas in pump tube	9.5	23[e]	0
7. Joule heating in thermometer	1[f]	1	1
Total heat [mW]	154 - 284	689 - 819	2 - 130
He boil off [l/h]	0.21 - 0.38	0.92 -1.09	-

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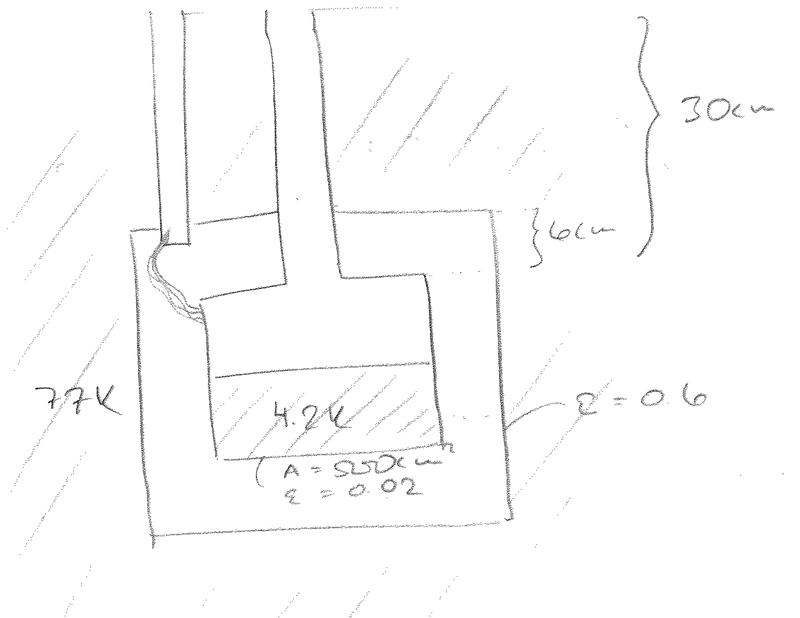
[a] Blackened tube
[b] Total reflection
[c] $l = 30$ cm
[d] $l = 36$ cm
[e] $l = 30$ cm
[f] $I = 1$ mA

Table 1: Contributions of heat leaks in units of mW.

Calculations of heat leaks



Contributions to heat leaks

A Radiation

1) From 77K to 4.2 K (alt 295 K to 4.2K)
2) Through pumping tube from RT to 4.2K

B Heat through support and leads

3) stainless steel tube
4) Cu and constantan leads

C Heat conduction through gas

5) Through vaccum space
6) Through He tube

D Experiment

7) Ge themometer $R \sim 1k\Omega$

A Radiation

$$\dot{Q} = \sigma A \, \varepsilon_{eff} \left(T_1^4 - T_2^4 \right)$$

$$\sigma = 5.67 \cdot 10^{-12} \; W/cm^2 K^4$$

$$\varepsilon_{eff} = \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2 - \varepsilon_1 \varepsilon_2}$$

1) From brass to Cu cans

$$\varepsilon_{eff} = \frac{0.02 \cdot 0.6}{0.02 + 0.6 - 0.02 \cdot 0.6} \approx 0.02$$

$$A = 500 \, cm^2$$

a) $77 \to 4.2 K$

$$\dot{Q} = \sigma \cdot A \cdot \varepsilon_{eff} \left(77^4 - 4.2 K^4 \right) = 2 mW$$

b) $295 \to 4\,2K$

$$\dot{Q} = 0\,43 W$$

c) $4.2 \to 12 K$

$$\dot{Q} = 18 nW$$

2) Through tube

a) Perfect reflection

$A = \pi \, 1^2 \, cm^2$. Black body $\varepsilon = 1$

$$\dot{Q} = \sigma A \, 295^4 = 0\,13 W$$

b) Blackened tube

$$\dot{Q} = \sigma \cdot A \cdot 295^4 \frac{\cancel{\pi}}{2 \cancel{\pi} \, 30^2} = 75 \mu W$$

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c) Radiation trap at 77K

$\dot{Q} = \sigma A \; 77^4 = 06\,mW$ (Black body)

$\dot{Q} = \sigma A \cdot 0.048 \; 77^4 = 30\,\mu W$ (Polished SS)

B Through solids

$$\dot{Q} = \frac{A}{l} \cdot \overline{\lambda} \cdot \Delta T$$

3) Stainless steel tube

$\overline{\lambda} = 0.045$ W/cmK   77 → 4K

$\overline{\lambda} = 0.103$ W/cmK   300 → 4K

$\overline{\lambda} = 0.0022$ W/cmK   4 → 2K

$A = 2\pi \cdot 0.03\ cm^2$

$l = 6\ cm$

a) 77 → 4.2K

$\dot{Q} = \frac{A}{l} \cdot 0045\,(77 - 4.2) = 0.10\ W$

b) 295 → 4.2K

$\dot{Q} = \frac{A}{l} \cdot 0.103\,(295 - 4.2) = 0.94\ W$

0.19 W ($l = 30cm$)

c) 4.2 → 1.2K

$\dot{Q} = \frac{A}{l} \cdot 0.022 \cdot (4.2 - 1.2) = 0.2\ mW$

4) Throug leads

$$\left.\begin{aligned}\bar{\lambda}_{Cu} &= 9.8 \ W/cmK \\ \bar{\lambda}_{Cu} &= 0\,14 \ W/cmK\end{aligned}\right\} \quad 77 \to 4\,K$$

$$\left.\begin{aligned}\bar{\lambda}_{Cu} &= 5.7 \ W/cmK \\ \bar{\lambda}_{Cu} &= 0.20 \ W/cmK\end{aligned}\right\} \quad 300 \to 4K$$

$$\left.\begin{aligned}\bar{\lambda}_{Cu} &= 4 \ W/cmK \\ \bar{\lambda}_{Cu} &= 0.006 \ W/cmK\end{aligned}\right\} \quad 4 \to 2\,K$$

$$A_{Cu} = 8\,\pi\,\frac{001^2}{4}\ cm^2$$

$$A_{Cu} = 4 \cdot \pi \ 001^2 \ cm^2$$

$$l = 12\,cm$$

a) $77 \to 4.2k$

$$\dot{Q} = 0\,037 + 0.001 = 38mW$$

b) $295 \to 4.2\,k$

$$\dot{Q} = 0.087 + 0.006 = 93mW$$
$$0.029 + 0\,002 - 31\,mW \quad (l = 36\,cm)$$

c $42 \to 1.2K$

$$\dot{Q} = 000063 + 0.0000019 = 063mW$$

c Through gas

3/4

5) Vaccum space

$P = 10^{-5}$ torr

$\dot{Q} = \text{const} \cdot a_0 \, P_{mm} \left(T_1 - T_2\right) \; W/cm^2$

where const = 0.028 for He and

$$a_0 = \frac{a_1 a_2}{a_1 + \left(\frac{A_1}{A_2}\right)\left(1 - a_1\right) a_2}$$

$A_1 \approx A_2$, $a_1 = a_2 = 0.5$ (upper limit)

$a_0 = \frac{1}{3}$

a) $77 \to 4.2K$

$\dot{Q} = 0.028 \cdot \frac{1}{3} \cdot 10^{-5} \left(77 - 4.2\right) \cdot 500 = 3.4 mW$

b) $295 \to 4.2K$

$\dot{Q} = 13.5 mW$

c) $4.2 \to 1.2K$

$\dot{Q} = 0.14 mW$

6) Through pump tube

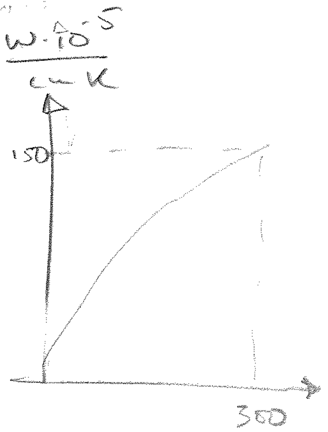
$\dot{Q} = \frac{A}{l} \cdot \bar{\lambda} \left(T_1 - T_2\right)$

$A = \pi r^2$, $l = 6 cm$,

From graph

$\bar{\lambda} = 25\ 10^{-5}$ W/cm K $77 \to 4$ K

$\bar{\lambda} = 75 \cdot 10^{-5}$ W/cm K $300 \to 4$ K



a) $77 \to 4.2$ K

$$\dot{Q} = \frac{\pi}{6} \cdot 25 \cdot 10^{-5} \cdot (77 - 4.2) = 9.5\,\text{mW}$$

b) $295 \to 4.2$ K

$$\dot{Q} = \frac{\pi}{6} \cdot 75 \cdot 10^{-5} (295 - 4.2) = 0.11\,\text{W}$$

23 mW ($l = 30$ cm)

c) $4.2 \to 1.2$ K

Neglectable contribution

Pumping on He ⇒ flow > diffusion velocity

D Experiment

7) Joule heating

Themometer 1 kΩ

$$\dot{Q} = RI^2 = \begin{cases} 0.1\,\text{W} & I = 10\,\text{mA} \\ 1\,\text{mW} & I = 1\,\text{mA} \end{cases}$$

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		77 → 4.2K	295 → 4.2K	4.2 → 12K
A	1	2	430	0.000018
	2	total reflection ↓ 0.075-130 ↑ Blackened tube	0.075-130	0.075-130
B	3	100	190 (l=30cm)	0.2
	4	38	31 (l=36)	0.63
C	5	3.4	13.5	0.14
	6	9.5	23	0
D	7	1	1	1
Total		154-284 mW	689-819 mW	2(-132) mW
Boil off		0.21-0.38 l/h	0.92-1,09 l/h	-