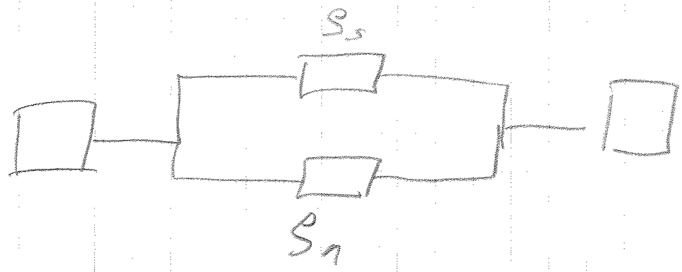
The two fluid model

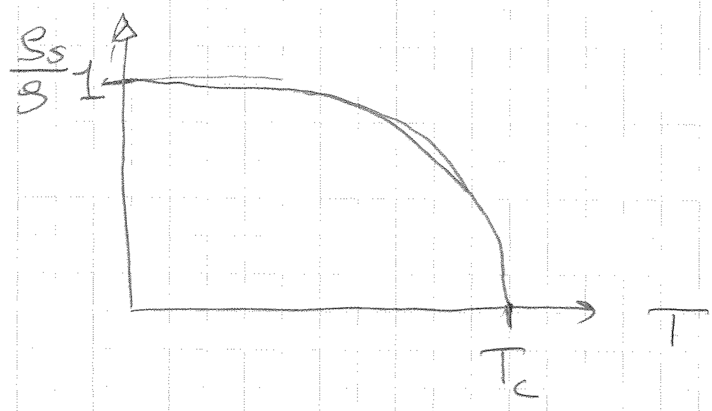
$$\rho = \rho_s + \rho_n$$

ρ_s	ρ_n
"Superelectrons"	Normal electrons
No scattering	scattering
No entropy	Entropy
Infinite conductance	Normal conductance

BCS: Cooper pairs



$R \Leftarrow 0 \qquad T < T_c$



$T = 0 \qquad \mathcal{S} \propto \rho_n$ ie very small

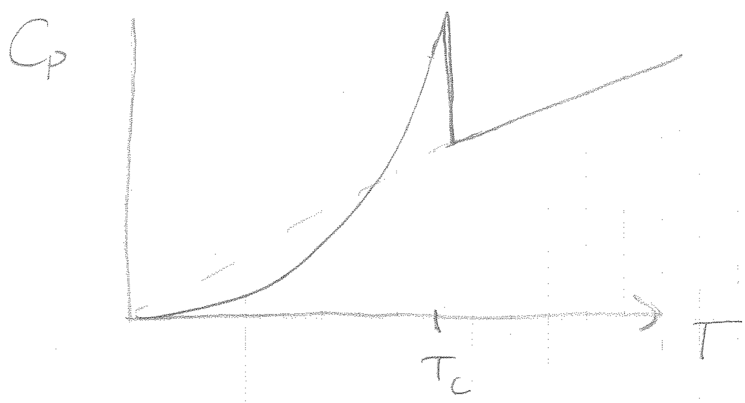
since ρ_s carries no entropy

$$|\Psi\rangle = \sqrt{n_s} \cdot e^{i\theta}$$

n_s can be thought of as the electrons that form Cooper-pairs

The super current is given by:

$$\bar{J}_s = -n_s e \bar{V}_s$$



From the thermal properties

$$n_s \sim n \left(1 - \left(\frac{T}{T_c}\right)^4\right)$$

High frequency conductivity of SC

$$\rho = \rho_n + \rho_s$$

$$J_s = -\rho_s e \bar{v}_s$$

$$J_n = -\rho_n \cdot e \langle \bar{v}_n \rangle$$

$\boxed{\text{Normal electrons}}$

$$m \cdot \frac{d\langle v_n \rangle}{dt} + m \cdot \frac{\langle v_n \rangle}{\tau} = \bar{F} = -e\bar{E}$$

$\tau \sim 10^{-13}\,s$

if variations in $\bar{E}$ are slow on this time scale

$$\frac{m \langle \bar{v}_n \rangle}{\tau} = -e\bar{E}$$

$$\bar{J}_n = -\rho_n e \langle v_n \rangle = -\rho e \left(-\frac{e\tau}{m}\right) \bar{E}$$

$$\bar{J}_n = \sigma_n \cdot \bar{E} \Rightarrow$$

$$\sigma_n = \frac{\rho_n e^2 \tau}{m}$$

$\boxed{\text{Superelectrons}}$

$$m \frac{d\bar{v}_s}{dt} = -e\bar{E}$$

assume $\quad \bar{E} = \bar{E}_0 e^{j\omega t} \Rightarrow$

$$\bar{v}_s = -\frac{e}{m} \bar{E}_0 \frac{1}{j\omega} e^{j\omega t}$$

$$\bar{J}_s = -\rho_s e \bar{v}_s = -\rho_s e \frac{e}{m}\frac{1}{j\omega}\bar{E} = \sigma_s \bar{E}$$

$$\sigma_s = \rho_s \frac{e^2}{m}\frac{1}{j\omega} = -j\,\rho_s \frac{e^2\tau}{m}\frac{1}{\omega\tau}$$

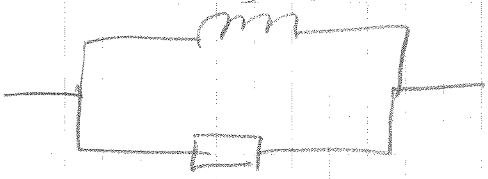
$$\bar{J} = \bar{J}_n + \bar{J}_s = (\sigma_n - j\sigma_s)\,\bar{E}$$

$$\sigma_n = \rho_n \frac{e^2\tau}{m} \qquad \sigma_s = \rho_s \frac{e^2}{m}\frac{1}{\omega}$$

i.e. real part dep. on ρ_n $\qquad$ $\omega\tau \ll 1$

imagin. part dep on ρ_s $\qquad$ $f < 10\,\mathrm{GHz}$

$$\sigma_2 = \frac{1}{\omega L} \Rightarrow L = \frac{m\rho_s}{e^2}$$



ρ_n

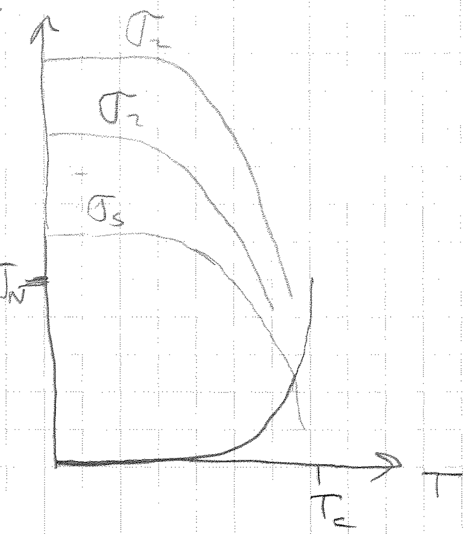
$$\sigma_1 = \rho_n \frac{e^2\tau}{m}$$

$$\sigma_{tot} = \rho_n \frac{e^2\tau}{m} - j\,\rho_s \frac{e^2\tau}{m}\frac{1}{\omega\tau}$$

$$\sigma_N \equiv \rho \frac{e^2\tau}{m}$$

$$\sigma_{tot} = \left(\frac{\rho_n}{\rho} - j\frac{\rho_s}{\rho}\frac{1}{\omega\tau}\right)\sigma_N$$

$$\frac{\sigma_1}{\sigma_2} = \frac{\rho_n}{\rho_s}\,\omega\tau \quad (\ll 1)$$



Dissipated power per unit volume

$$\frac{P}{V} = \rho \cdot J^2 = \mathrm{Re}\left(\frac{1}{\sigma}\right) J^2 = \frac{\sigma_1}{\sigma_1^2+\sigma_2^2} J^2 \simeq \frac{\sigma_1}{\sigma_2^2} J^2$$

$$\left[\frac{1}{\sigma_1 - j\sigma_2} = \frac{\sigma_1 + j\sigma_2}{\sigma_1^2+\sigma_2^2} \right]$$

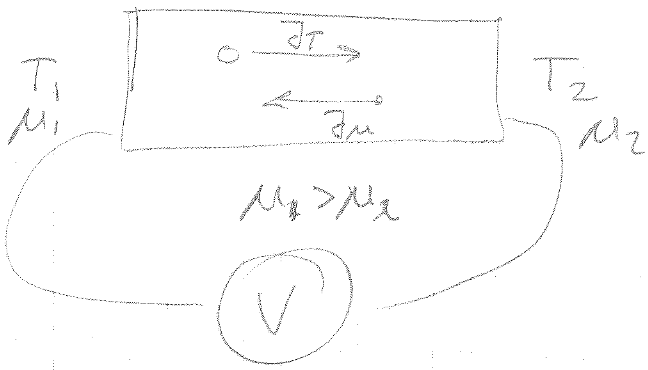
$$\frac{\sigma_1}{\sigma_2^2} \propto \omega^2$$

at 100 GHz SC no better than N

The thermoelectric effect

Normal Metal

$T_1 > T_2$

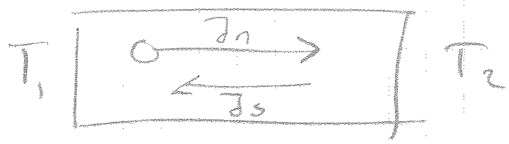


Flow of e from hot to cold gives rise to an electro chemical potential which drives curent in the opposite direction leading to a net zero current

$J = J_T + J_\mu = 0$

Seebeck effect

SC



$J = J_n + J_s = 0$

In a SC no potential difference can develop

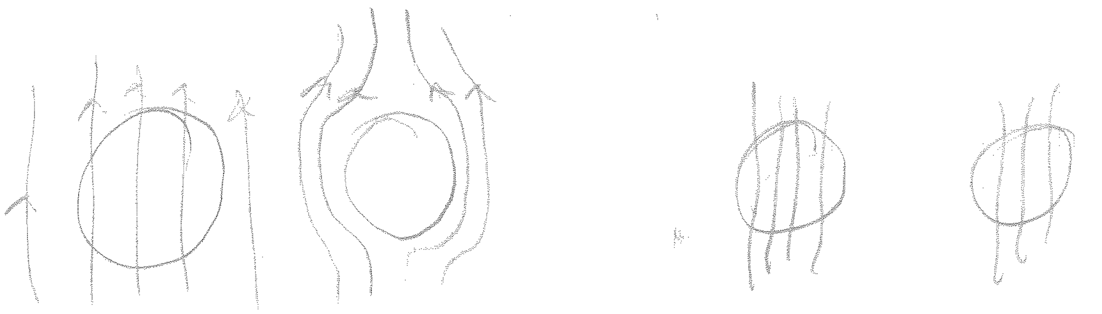
J_s exactly cancels J_n

$\bar{J} = \bar{J}_n + \bar{J}_s = 0$

No Seebeck effect

This can be used to measure the seebeck coefficient of a single metal

The Meissner effect



$T > T_c$ $T < T_c$ $T > T_c$ $T < T_c$

Superconductor $\neq$ Perfect conductor

Something more

$R = 0$ implies that $\bar{E} = 0$ inside SC

Faradays Law

$$\oint \bar{E}\, dl = -\frac{d\Phi}{dt}$$

Well inside SC $\bar{E} = 0 \Rightarrow$

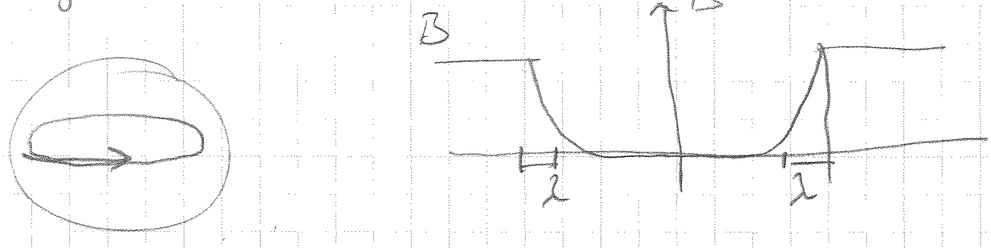
$$0 = -A \cdot \frac{dB}{dt} \Rightarrow \frac{dB}{dt} = 0$$

Infact $\bar{B} \equiv 0$ inside for SC

$$\bar{B} = \mu_0 (\bar{H} + \bar{M}) = \mu_0 (1 + \chi) H$$

$\bar{B} = 0 \Rightarrow \chi = -1$ ie perfect diamagnet

Screening currents at the surface



$$B = B_0 \cdot e^{-\frac{x}{\lambda}}$$

$$J_s = J_{s0}\, e^{-\frac{x}{\lambda}}$$

λ = Londons penetration depth

$\sim 50\,nm$

DEMONSTRATION

The thermodynamic critical field

The Helmholtz Free energy (per unit Volume)

$$F = E - TS - \mu_0 \bar{M} \cdot \bar{H}$$

$$dF = -S\,dT - \mu_0 \bar{M}\, d\bar{H}$$

$$F(H,T) = F(0,T) - \mu_0 \int_0^H \bar{M}\, d\bar{H}$$

Normal Metal, negligable magnetization

$$F(H,T) = F(0,T) = F_N(T)$$

Superconductor $\chi = -1 \Rightarrow \bar{M} = -\bar{H}$ (Type I)

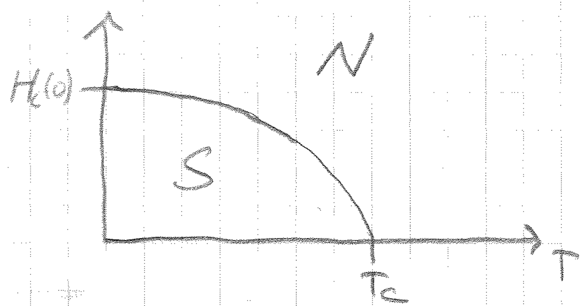
$$F_S(H,T) = F_S(0,T) + \int_0^H H' dH'$$

$$= F_S(0,T) + \frac{H^2}{2\mu_0}$$

At the critical field H_c $F_N(T) = F_S(H_c, T)$

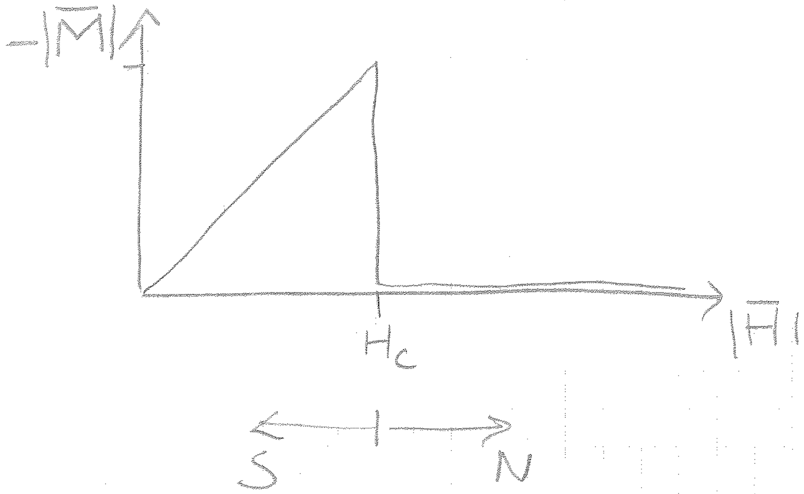
$$\boxed{\Delta F = F_N(T) - F_S(H_c, T) = \frac{\mu_0}{2} H_c^2(T)}$$

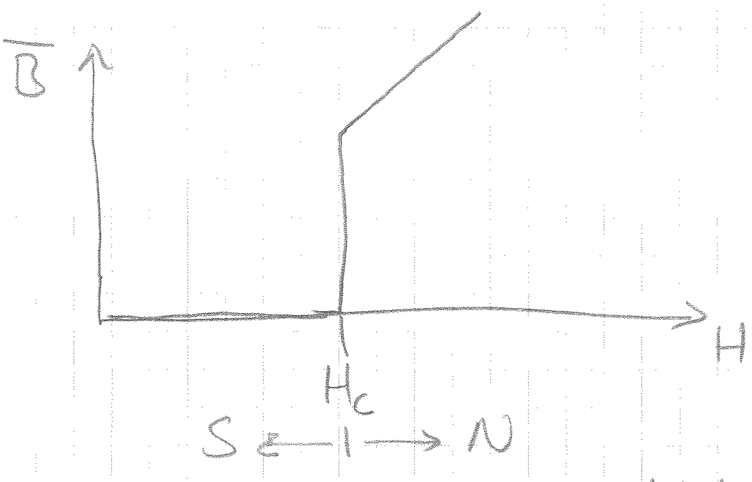
$$H_c(T) = H_c(0)\left[1 - \frac{T^2}{T_c^2}\right]$$

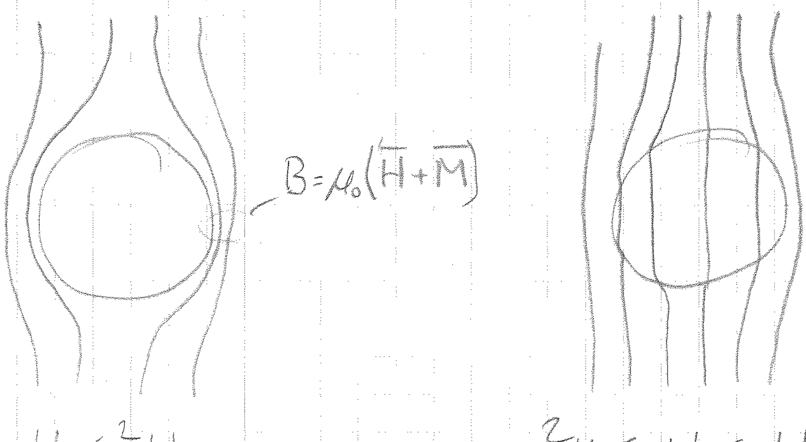


Type I SC in a magnetic field

Diamagnetism $\quad \bar{M} = -\bar{H}$







$H < \frac{2}{3} H_c$ $\qquad$ $\frac{2}{3} H_c < H < H_c$

If $H > \frac{2}{3} H_c$ than $H + M$ at the surface will be larger tha H_c whic will turn the SC normal and flux will enter

Maxwells Equations

$$\bar{\nabla} \cdot \bar{B} = 0$$

$$\bar{\nabla} \times \bar{B} = \mu_0 j + \frac{1}{c^2} \frac{\partial E}{\partial t} = 0$$

outside

$B = \mu_0 H \quad r \to \infty$

Meissner no field lines enter the SC

$$\Rightarrow \quad \bar{B}_n = 0 \quad r = a$$

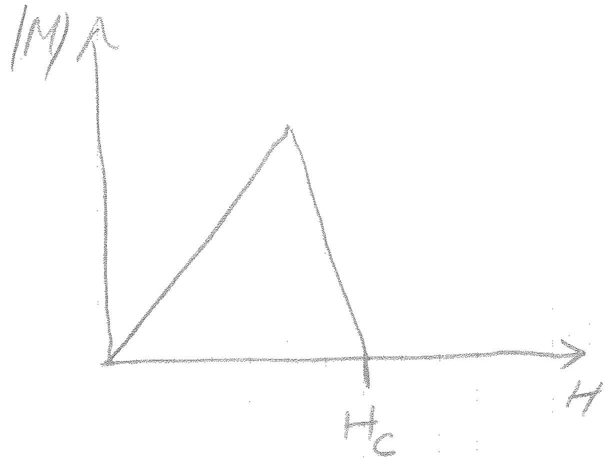
Solution (using spherical coordinates

$$\bar{B} = \mu_0 H + \mu_0 H \frac{a^3}{2} \nabla \left(\frac{\cos \theta}{r^2} \right)$$

$$\Rightarrow \quad B_{\parallel} = \frac{3}{2} \mu_0 H \cdot \sin \theta$$

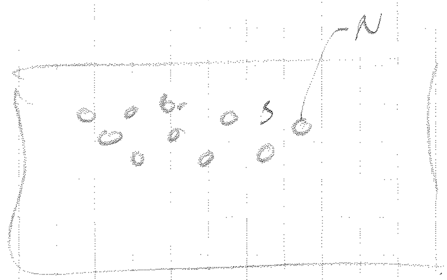
$$B_{\parallel \, max} = \frac{3}{2} \mu_0 H$$

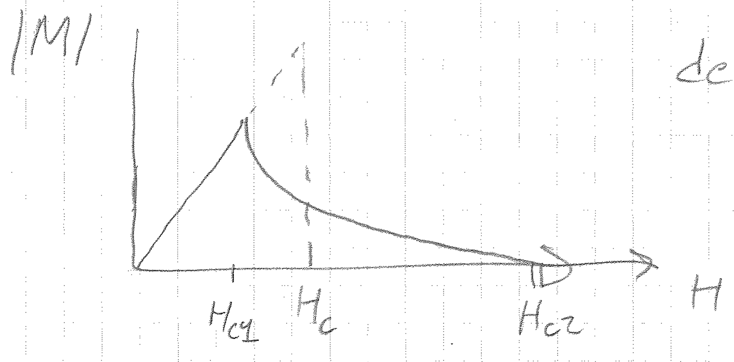
$H = \frac{2}{3} H_c$ something will happen the inter mediate state



For type I SC the interface energy is positive, i.e. the SC tries to minimize the number of

For Type II the interface energy is negativ an the SC tries to maximize the numperoff interfaces



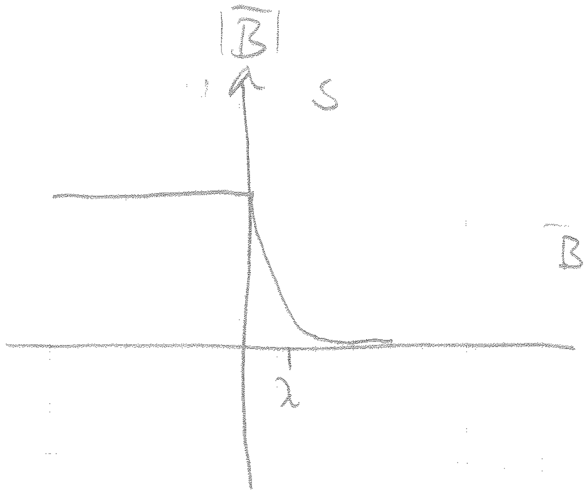


depends on $\varkappa$

Enhancement of H_c in thin films

Type I

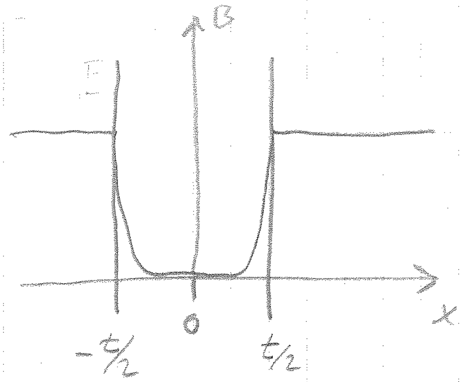
Thick film



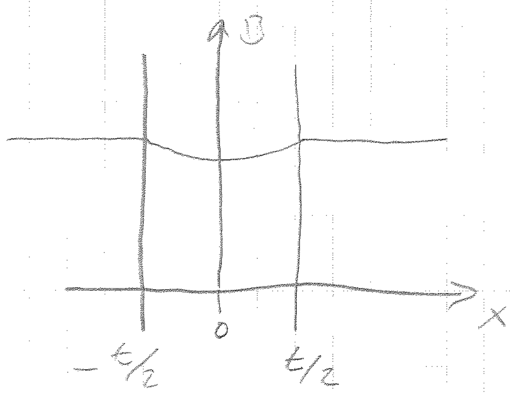
$$\bar{B} = \mu_0 \bar{H} e^{-x/\lambda}$$

Thin film

$\lambda << t$



$\lambda >> t$



$$\bar{B} = \mu_0 \bar{H} \cdot \frac{\cosh \frac{x}{\lambda}}{\cosh \frac{t}{\lambda}}$$

$$\bar{M} = \frac{\bar{B}}{\mu_0} - \bar{H} = \bar{H}\left(\frac{\cosh \frac{x}{\lambda}}{\cosh \frac{t}{\lambda}} - 1\right)$$

$$M_{av} = \frac{1}{V}\int dV \bar{M} = \frac{1}{t}\int_{-\frac{t}{2}}^{\frac{t}{2}} dx\, M(x) =$$

$$= H\frac{1}{t}\int_{-\frac{t}{2}}^{\frac{t}{2}}\left(\frac{\cosh\frac{x}{\lambda}}{\cosh\frac{t}{2\lambda}} - 1\right)dx = H\cdot\frac{1}{t}\left[\lambda\frac{\sinh\frac{x}{\lambda}}{\cosh\frac{t}{2\lambda}} - x\right]_{-\frac{t}{2}}^{\frac{t}{2}}$$

$$= H\frac{1}{t}\left[\lambda \tanh\frac{t}{2\lambda} - \frac{t}{2} - \tanh\left(-\frac{t}{2\lambda}\right) + \left(-\frac{t}{2}\right)\right]$$

$$\tanh(-x) = -\tanh(x)$$

$$M_{av} = H\left[\frac{2\lambda}{t}\tanh\frac{t}{2\lambda} - 1\right]$$

$$\Delta F = -\mu_0 \int_0^H M_{av} \cdot dH'$$

$$= \frac{\mu_0}{2} H^2 \left[1 - \frac{2\lambda}{t}\tanh\left(\frac{t}{2\lambda}\right)\right]$$

$$\Delta F_{TF} = \Delta F_{Bulk} \Rightarrow$$

$$\frac{\mu_0}{2} {H_c^{TF}}^2 \left[1 - \frac{2\lambda}{t}\tanh\left(\frac{t}{2\lambda}\right)\right] = \frac{\mu_0}{2} H_c^2$$

$$\frac{1}{x}\tanh x \approx \frac{1}{x}\left(x - \frac{x^3}{3}\cdots\right) = 1 - \frac{x^2}{3}$$

$$H_c^{TF} = H_c \frac{1}{\sqrt{1 - \frac{2\lambda}{t}\tanh\left(\frac{t}{2\lambda}\right)}} \approx H_c \frac{1}{\sqrt{\frac{1}{3}\left(\frac{t}{2\lambda}\right)^2}}$$

$$H_c^{TF} = H_c^{Bulk} \sqrt{12}\,\frac{\lambda}{t}$$

$$GL \Rightarrow H_c^{TF} = H_c \sqrt{24} \cdot \frac{\lambda}{t}$$

Example Al 20nm thick

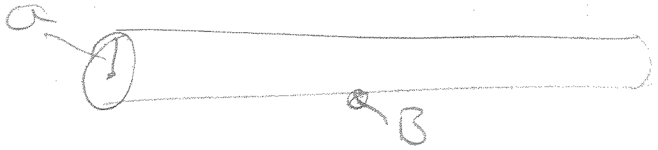
$\lambda = 50\,nm$

$$H_c^{TF} = H_c \cdot \sqrt{24}\,\frac{2}{5} = H_c \cdot 12{,}2 \approx$$

$$108 \cdot 12.2 \approx 1320\,G = 0{,}13\,T$$

Critical current of a SC wire

The Silsbee criterion



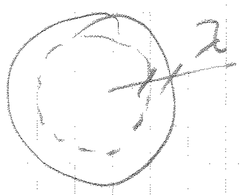
Biot-Sawart

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r} = \frac{\mu_0}{2\pi}\frac{I}{a}$$

$$B_c = \frac{\mu_0}{2\pi}\frac{I_c}{a} = \mu_0 H_c$$

$$I_c = \underbrace{2\pi a}_{\text{circumference}} \cdot H_c$$

"thick wire"
$a \gg \lambda$



$$J_c = \frac{I_c}{A} = \frac{I_c}{2\pi a \cdot \lambda} = \frac{H_c}{\lambda}$$

$$\boxed{J_c = \frac{H_c}{\lambda}}$$

$$\boxed{I_c = 2\pi a H_c}$$