Josephson-junction qubits with controlled couplings

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Quantum computers, if available, could perform certain tasks much more efficiently than classical computers by exploiting different physical principles[1–3]. A quantum computer would be comprised of coupled, two-state quantum systems or qubits, whose coherent time evolution must be controlled in a computation. Experimentally, trapped ions[4,5], nuclear magnetic resonance[6–8] in molecules, and quantum optical systems[9] have been investigated for embodying quantum computation. But solid-state implementations[10–14] would be more practical, particularly nanometre-scale electronic devices: these could be easily embedded in electronic circuitry and scaled up to provide the large numbers of qubits required for useful computations. Here we present a proposal for solid-state qubits that utilizes controllable, low-capacitance Josephson junctions. The design exploits coherent tunnelling of Cooper pairs in the superconducting state, while employing the control mechanisms of single-charge devices: single- and two-bit operations can be controlled by gate voltages. The advantages of using tunable Josephson couplings include the simplification of the operation and the reduction of errors associated with permanent couplings.

Two versions of Josephson-junction qubits are shown in Fig. 1. The simpler one (Fig. 1a), proposed earlier[10], consists of a superconducting electron box, that is, a low-capacitance island coupled via a Josephson tunnel junction to a lead. The Coulomb interaction (charging energy) restricts the number, n, of Cooper-pair charges, $Q = 2ne$ (where e is the charge on an electron), on the island. If biased near a degeneracy point the system constitutes a qubit with two states differing by one Cooper-pair charge. Quantum logic operations can be performed by switching the gate voltage. Before describing the systems in detail we will first present an ideal model. This puts in perspective the possibilities and drawbacks of the simple design, as well as the advantages of the new design with Josephson coupling controlled by a superconducting quantum interference device (SQUID: Fig. 1b). These are, first, during idle periods between operations the energy splitting between logical states is tuned to zero, thus avoiding an undesired phase evolution. With this drawback of most proposals overcome, the requirement on the precision of time control is substantially reduced; second, the 2-bit couplings can be switched on and off avoiding errors associated with permanent couplings.

To realize a quantum computer we search for a system with the following "ideal" model hamiltonian:

$$\hat{H} = -\sum_{i=1}^{N}[H_z^i(t)\hat{\sigma}_z^i + H_x^i(t)\hat{\sigma}_x^i] + \sum_{i\neq j} J^{ij}(t)\hat{\sigma}_+^i\hat{\sigma}_-^i \qquad (1)$$

A spin notation is used for the qubits with Pauli matrices $\hat{\sigma}_z$, $\hat{\sigma}_x$, $\hat{\sigma}_\pm = \frac{1}{2}(\hat{\sigma}_x \pm i\hat{\sigma}_y)$. Ideally, each energy $H_z^i(t)$, $H_x^i(t)$ and the (real symmetric) couplings $J^{ij}(t)$ can be switched separately for controlled times between zero and finite values. We assume that H_z^i is the largest energy, suggesting the choice of basis states $|\uparrow_i\rangle$ and $|\downarrow_i\rangle$ aligned along the z-axis. Residual inelastic interactions (which destroy the coherence), and the measurement device (when turned on) should be accounted for by extra terms $\hat{H}_{\rm res}$ and $\hat{H}_{\rm meas}(t)$, respectively.

Quantum computation requires four elementary steps.
(1) The system has to be prepared in a well defined initial state. For this we turn on at low temperature all $H_z^i \gg k_{\rm B}T$, while $H_x^i = J^{ij} = 0$. After sufficient time the residual interaction $\hat{H}_{\rm res}$ relaxes all spins to the ground state, $|\uparrow\uparrow\ldots\rangle$. Then $H_z^i(t)$ is set back to zero.
(2) Single-bit operations (gates) have to be performed. They are controlled by turning on one of the fields. If H_x^i is switched on, the spin i evolves according to the unitary transformation $U_{\rm 1b}^i(\tau) = \exp(iH_x^i\tau\hat{\sigma}_x^i/\hbar)$. Depending on the time span τ, a π- or $\pi/2$-rotation is performed, producing a spin flip or an equal-weight superposition of spin states. Switching on one H_z^i produces another needed operation: a phase shift between $|\uparrow_i\rangle$ and $|\downarrow_i\rangle$. Back in the idle state, where $\hat{H} = 0$, the relative phase shift of the states does not evolve further.
(3) A two-bit operation on qubits i and j is achieved by turning on the corresponding J^{ij}. In the basis $|\uparrow_i\downarrow_j\rangle$, $|\downarrow_i\uparrow_j\rangle$, the result is described by $U_{\rm 2b}^{ij}(\tau) = \begin{pmatrix} \cos\alpha & i\sin\alpha \\ i\sin\alpha & \cos\alpha \end{pmatrix}$, with $\alpha = J^{ij}\tau/\hbar$, while the states $|\uparrow_i\uparrow_j\rangle$,$|\downarrow_i\downarrow_j\rangle$ are not affected. For $\alpha = \pi/2$ the result is a spin-swap operation, while $\alpha = \pi/4$ yields a 'square-root swap'. The latter transforms the state $|\uparrow_i\downarrow_j\rangle$ into the entangled state $(|\uparrow_i\downarrow_j\rangle + i|\downarrow_i\uparrow_j\rangle)/\sqrt{2}$. The combination with single-bit operations allows us to perform the 'controlled-not' gate; in fact, they provide a universal set, sufficient for all logic gates of quantum computations[15].
(4) The final state has to be read out, which constitutes a quantum measurement process[16].

Searching for nanometre-scale electronic realizations of qubits, one might consider normal-metal single-electron devices. But they are ruled out, because in the normal state different tunnelling processes are incoherent. Ultrasmall quantum dots with discrete levels or spin degrees of freedom in nanostructured circuits[13,14] are candidates, but are difficult to fabricate in a controlled way. More

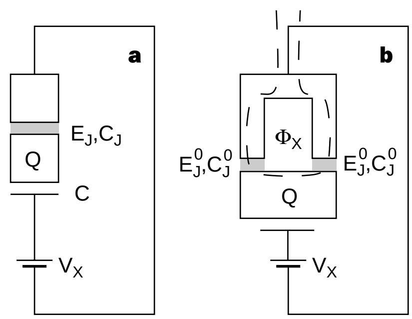

Figure 1 Josephson junction qubits. **a**, A simple realization of a qubit is provided by the superconducting electron box. A superconducting metallic island is coupled by a Josephson tunnel barrier (with capacitance C_J and Josephson coupling energy E_J; grey area) to a superconducting lead and through a gate capacitor C to a voltage source. The important degree of freedom is the Cooper-pair charge $Q = 2ne$ on the island. **b**, The improved design of the qubit. The island is coupled to the circuit via two Josephson junctions with parameters C_J^0 and E_J^0. This d.c.-SQUID can be tuned by the external flux Φ_x which is controlled by the current through the inductor loop (dashed line). If the self-inductance L_Φ of the SQUID is low, $\Phi_0^2/L_\Phi \gg 4\pi^2E_J^0$, e^2/C_J^0, fluctuations of the flux from Φ_x are weak. Furthermore, if the frequency of flux oscillations is high, $\hbar\omega_\Phi = \hbar(L_\Phi C_J^0/2)^{-1/2} \gg E_J^0$, $E_{\rm ch}$, $k_{\rm B}T$, the Φ-degree of freedom is in the ground state. In this case, the set-up allows switching the effective Josephson coupling to zero. ($E_J = 0$ requires the Josephson energies of two junctions in the loop to be equal. This has been reached with a precision of 1% in quantum tunnelling experiments[17]. Even with this precision, taking into account[10] the finite value of E_J one can perform a large number of logical gates. On the other hand, by replacing one junction in **b** by another SQUID, one can tune the Josephson couplings to be equal.) The effective junction capacitance is $C_J = 2C_J^0$.

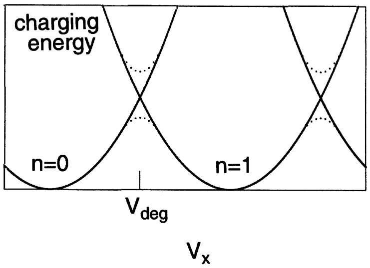

Figure 2 Spectrum of a superconducting electron box. The charging energy, $(Q - CV_x)^2/(2(C + C_J))$, of the superconducting electron box is shown (solid lines) as a function of the applied gate voltage V_x for different numbers n of extra Cooper pairs on the island. Near degeneracy points, the weaker Josephson coupling energy mixes the charge states and modifies the energy of the eigenstates (dotted line). In this regime, the system effectively reduces to a 2-state quantum system.

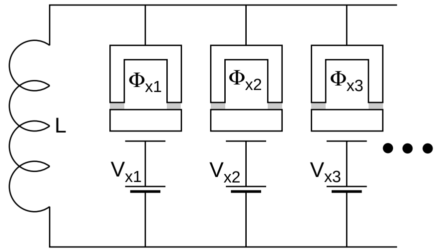

Figure 3 Design of a quantum computer. The coupling of the qubits is provided by the LC-oscillator mode in circuit shown. If the frequency of the LC-mode in the resulting circuit is large, $\hbar\omega_{LC} = \hbar(NC_{qb}L)^{-1/2} \gg E_J, E_{ch}, k_BT$, the fast oscillations produce an effective coupling of the qubits. We note that the system can be scaled to large numbers of qubits. In the idle state all effective Josephson couplings are tuned to zero, and the voltages are chosen such that the charge states are degenerate. Single-bit operations are performed by changing the gate voltage or flux of one qubit at a time. Two-bit operations between any two qubits are triggered by turning on the corresponding two Josephson couplings. The two lowest states of the qubit are separated from higher states, which exist in the real system, by the energies E_C, $\hbar\omega_{LC}$, $\hbar\omega_{\Phi}$. These should be larger than the energy scales of the qubit, E_J, E_{ch}, k_BT. If, in addition, switching processes of V_x and Φ_x are slow on the corresponding timescales, the requirements presented above also ensure that the higher states are not excited. Alternatively, instead of sudden switching, one can change the biases adiabatically. Another advantage of the presented design is that the result of a single-bit operation depends only on the time integral of energies $E_J(t)$ and $E_{ch}(t)$ over the operation period, but not on the profile of their time dependences.

promising are systems built from Josephson junctions, where the coherence of the superconducting state can be exploited. Quantum extension of elements based on single-flux logic have been considered (ref. 17, and J. E. Mooij, personal communication). Encouraged by successful experiments that demonstrated the superposition of charge states[12,18,19], we suggest here the use of superconducting electron boxes with low-capacitance Josephson junctions as qubits.

In the system of Fig. 1a Cooper pairs tunnel coherently, while Coulomb blockade effects allow the control of the charge. The relevant conjugate variables are the phase difference γ across the junction and the charge $Q = 2ne$ on the island. If quasiparticle tunnelling is suppressed by the superconducting gap and only 'even-parity' states are involved[20], the circuit dynamics is governed by the hamiltonian:

$$\hat{H} = \frac{(Q - CV_x)^2}{2(C + C_J)} - E_J \cos\gamma; \quad Q = \frac{\hbar}{i}\frac{\partial}{\partial(\hbar\gamma/2e)} \tag{2}$$

For the junctions considered, the charging energy with scale $E_C \equiv e^2/2(C + C_J)$ dominates over the Josephson coupling E_J. It is plotted in Fig. 2 as a function of the external voltage V_x for different n. In equilibrium at $k_BT \ll E_C$, the system is in the state corresponding to the lowest parabola. But, near the voltages $V_{deg} = (2n+1)e/C$, the states n and $n+1$ are near-degenerate, and E_J mixes them strongly. Here, in the basis of charge states $|\uparrow\rangle = |n\rangle$ and $|\downarrow\rangle = |n+1\rangle$, the hamiltonian reduces to a two-state model

$$\hat{H} = E_{ch}(V_x)\hat{\sigma}_z - \frac{1}{2}E_J\hat{\sigma}_x \tag{3}$$

where $E_{ch}(V_x) = e(V_x - V_{deg})C_{qb}/C_J$, and the capacitance of the qubit in the circuit is $C_{qb}^{-1} = C_J^{-1} + C^{-1}$.

On the way towards the model of equation (1), we achieved a tunable $H_z^i(t)$; but the Josephson coupling is fixed, $H_x^i(t) = E_J/2$. Still, single-bit operations can be performed by controlling the bias voltage V_x (ref. 10). Furthermore, when the qubits are connected in parallel with an inductor (as in Fig. 3), the common LC-oscillator mode provides a two-bit coupling with weak, but constant $J^{ij} \approx (C^2/C_J^2)(E_J^2L/\Phi_0^2)$, where $\Phi_0 = h/2e$. This coupling provides a two-bit gate if two qubits, i and j, are brought into resonance by biasing them with the same gate voltage $V_{xi} = V_{xj}$. Out of resonance, the two-bit coupling provides only a weak perturbation.

The external voltage source is part of a dissipative circuit with effective resistance R_V. Its Johnson–Nyquist voltage fluctuations destroy the phase coherence. The dephasing rate varies slightly during manipulations[21,22]. At the degeneracy point, the decoherence time is:

$$\tau_V = \frac{1}{4\pi}\frac{R_K}{R_V}\left(\frac{C_J}{C_{qb}}\right)^2 \frac{\hbar}{E_J}\tanh\left(\frac{E_J}{2k_BT}\right) \tag{4}$$

Here R_V is compared to the quantum resistance $R_K = h/e^2 \approx 26\,\mathrm{k}\Omega$. A small gate capacitance $C \approx C_{qb} \ll C_J$ helps further decoupling of the qubit from the environment. Both can be optimized to yield a phase coherence time that is long compared to typical operation times $\hbar/E_J$.

A problem with the simple design is that the eigenstates of the hamiltonian shown in equation (3) are non-degenerate at all V_x. Therefore, the relative phase of two logical states evolves even during idle periods. We can still store quantum information in the qubit, as becomes apparent after a transformation to the interaction representation. But this introduces an explicit time dependence in the operators, with the result that the unitary transformations not only depend on the time span τ of the operations but also on the time t_0 when they start. Hence the time elapsed since the beginning of the computation, multiplied by the energy spacing between the logical states should be controlled with high accuracy. A second problem of the simple design is the non-vanishing two-bit coupling, even out of resonance. It introduces an error in the computation. The design discussed below overcomes both these problems.

A crucial step towards the ideal model (equation (1)) is to tune the Josephson coupling. This is achieved in the design of Fig. 1b, where each Josephson junction is replaced by a d.c.-SQUID (see, for example, ref. 20). The SQUID is biased by an external flux Φ_x, coupled into the system through an inductor loop. If the loop self-inductance L_Φ is low the SQUID-controlled qubit is described by a hamiltonian of the form of equation (2), but with potential energy $2E_J^0\cos(\pi\Phi_x/\Phi_0)\cos\gamma$. Hence, the effective Josephson coupling is tunable by the external flux Φ_x between $2E_J^0$ and zero:

$$E_J(\Phi_x) = 2E_J^0\cos(\pi\Phi_x/\Phi_0) \tag{5}$$

The SQUID-controlled qubit is described by the first two terms of the model hamiltonian shown in equation (1), with z- and x-components controlled independently by the gate voltage and the flux. In the idle state we keep $V_x = V_{deg}$ and $\Phi_x = \Phi_0/2$, so that the hamiltonian $\hat{H} = 0$. Changing one of them generates z- or x-rotations, respectively, that is, the elementary one-qubit operations.

With the improved design there is no need to control the total operation time t_0, while the time dependence of the voltage and flux can be optimized such that the time span of the manipulations τ is long enough to simplify time control and short enough to speed up the computation.

Also, the circuit of the current source, with resistance R_I, which couples the flux Φ_x to the SQUID by the mutual inductance M, introduces fluctuations and may destroy the coherence of the qubit dynamics. At the degeneracy point, the decoherence time is[21,22] $\tau_I = (1/\pi^3)(R_I/R_K)[\Phi_0^2/(E_J^0 M)]^2(\hbar/k_B T)$. This dephasing is slow if the current source is coupled weakly to the qubit (small M) and its resistance is high.

The control of the Josephson energies $E_J(\Phi_{xi})$ provides the possibility of coupling each selected pair of qubits, while keeping all the other ones uncoupled, bringing us close to the ideal model of equation (1). The simplest implementation of the coupling is to connect all N qubits in parallel with each other, and with inductor L (Fig. 3). Fast oscillations in the resulting LC-circuit produce an effective coupling of the qubits

$$\hat{H}_{\text{int}} = -\sum_{i<j} \frac{E_J(\Phi_{xi})E_J(\Phi_{xj})}{E_L}\hat{\sigma}_y^i\hat{\sigma}_y^i \qquad (6)$$

where $E_L = [\Phi_0^2/(\pi^2 L)](C_J/C_{qb})^2$. The coupling shown in equation (6) can be understood as the magnetic energy of the inductor which is biased by a current composed of contributions from all qubits, $I^i \propto E_J^i\hat{\sigma}_y^i$.

With this design we can perform all gate operations. In the idle state the interaction hamiltonian of equation (6) is zero as all the Josephson couplings are turned off. The same is true during a one-qubit operation, as long as we perform one such operation at a time that is, only one $E_J^i \neq 0$. To perform a two-qubit operation with any given pair of qubits, say 1 and 2, E_J^1 and E_J^2 are switched on simultaneously, yielding the total hamiltonian $\hat{H} = -(E_J^1/2)\hat{\sigma}_x^i - (E_J^2/2)\sigma_x^2 - (E_J^1 E_J^2/E_L)\hat{\sigma}_y^1\hat{\sigma}_y^2$. Although not identical to equation (1), these two-bit gates, in combination with the single-bit operations discussed above, also provide a complete set of gates required for quantum computation.

To demonstrate that the constraints on the set of system parameters can be met by available technology, we suggest a suitable set. We choose junctions with capacitance $C_J = 300$ aF, corresponding to a charging energy (in temperature units) $E_C \approx 3$ K, and a smaller gate capacitance $C = 30$ aF to reduce the coupling to the environment (even lower C are available and improve the performance further). The superconducting gap has to be slightly larger, $\Delta > E_C$. Thus at a working temperature of the order of $T = 50$ mK, the initial thermalization is assured. We further choose $E_J^0 = 50$ mK; so the timescale of one-qubit operations is $\tau_{op} = \hbar/E_J \approx 70$ ps. Fluctuations associated with the gate voltages (equation (4)), with resistance $R_V \approx 50\,\Omega$, limit the coherence time to $\tau_V/\tau_{op} \approx 4{,}000$ operations. With the parameters of the flux-circuit $L_\Phi = 0.1$ nH, $M = 1$ nH and $R_I = 10^2$–$10^6\,\Omega$, current fluctuations have a weak dephasing effect. To assure fast two-bit operations, we choose the energy scale E_L to be of the order of $10E_J$, which is achieved for $L \approx 3\,\mu$H. With these parameters, the number of qubits in the circuit can be chosen in the range of 10–50, of course at the expense of shorter coherence times $\tau_{V,I}/N$.

Some further remarks are in order.

(1) After the gate operations, the resulting quantum state has to be read out. This can be achieved by coupling a normal-state single-electron transistor capacitively to a qubit. The important aspect is that during computation the transistor is kept in a zero-current state and adds only to the total capacitance. When the transport voltage is turned on, the phase coherence of the qubit is destroyed, and the dissipative current in the transistor, which depends on the state of the qubit, can be read out. This quantum measurement process has been described explicitly in ref. 16 by an analysis of the time-evolution of the density matrix of the coupled system.

(2) Inaccuracy in the control of fluxes, voltages and the time-span of operations leads to diffusion of the actual quantum state from the one that exists in the absence of errors[23]. A random error of order ϵ per gate limits the number of operations to a value which is of order ϵ^{-2}. For the circuit parameters above, $\epsilon = 1\%$ would lead to smaller effects than those produced by environment.

(3) Many powerful quantum algorithms make use of parallel operations on different qubits. Although this is not possible with the present system, it may be achievable by a more advanced design, making use of further tunable SQUIDs decoupling different parts of the circuit. Such modifications, as well as the further progress of nanotechnology, should provide longer coherence times and allow scaling to larger numbers of qubits. □

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Evidence against 'ultrahard' thermal turbulence at very high Rayleigh numbers

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Several theories[1–5] predict that a limiting and universal turbulent regime—'ultrahard' turbulence—should occur at large Rayleigh numbers (Ra, the ratio between thermal driving and viscous dissipative forces) in Rayleigh–Bénard thermal convection in a closed, rigid-walled cell. In this regime, viscosity becomes negligible, gravitationally driven buoyant plumes transport the heat