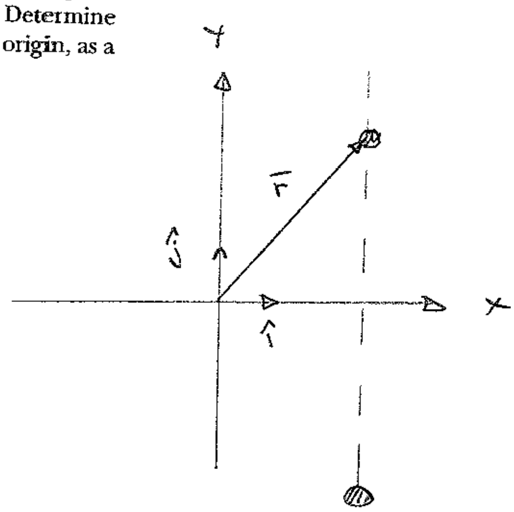
10.13

The position vector of a particle of mass 2.00 kg is given as a function of time by $\mathbf{r} = (6.00\hat{\mathbf{i}} + 5.00t\hat{\mathbf{j}})$ m. Determine the angular momentum of the particle about the origin, as a function of time.



$$\bar{r} = 6{,}00\,\hat{i} + 5{,}00\,t\,\hat{j} =$$

$$= x_1\hat{i} + y_1 t\,\hat{j}$$

$$\hat{i} = \hat{x}$$

$$\hat{j} = \hat{y}$$

$$\hat{k} = \hat{z}$$

$$\bar{L} = \bar{r} \times \bar{p}$$

$$\bar{p} = m\bar{v} = m\frac{d\bar{r}}{dt} = m\frac{d}{dt}\left(x_1\hat{i} + y_1 t\,\hat{j}\right) =$$

$$= m(0 + y_1\hat{j})$$

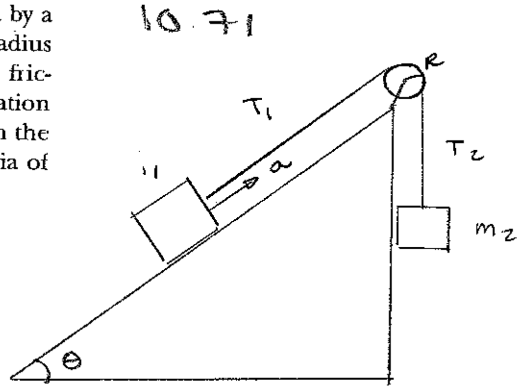
$$\Rightarrow \bar{L} = (x_1\hat{i} + y_1 t\,\hat{j}) \times m y_1\hat{j} =$$

$$= m x_1 y_1 (\hat{i} \times \hat{j}) = m_1 x_1 y_1 \hat{k} =$$

$$= 2{,}00 \cdot 6{,}00 \cdot 5{,}00 \quad \hat{k} = \underline{\underline{60{,}00}} \quad kg\,m/s\;\hat{k}$$

71. Two blocks, as shown in Figure P10.71, are connected by a string of negligible mass passing over a pulley of radius 0.250 m and moment of inertia I. The block on the frictionless incline is moving up with a constant acceleration of 2.00 m/s^2. (a) Determine T_1 and T_2, the tensions in the two parts of the string. (b) Find the moment of inertia of the pulley.

10.71



Givet: $\theta = 37{,}0^\circ$

$m_1 = 15{,}0$ kg

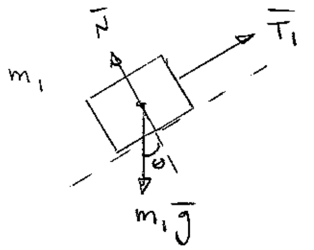
$m_2 = 20{,}0$ kg

$a = 2{,}00$ m/s^2

$R = 0{,}250$ m

Sökt: T_1, T_2 samt trissans tröghetsmoment.

Friläggning

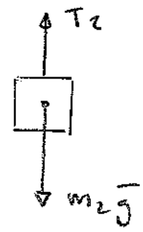


$$\boxed{T_1 - m_1 g \cdot \sin\theta = m_1 a} \quad (1)$$

$$\Rightarrow T_1 = m_1(a + g\sin\theta)$$

$$\Rightarrow T_1 = 15{,}0(2{,}00 + 9{,}81 \cdot \sin 37^\circ) = $$

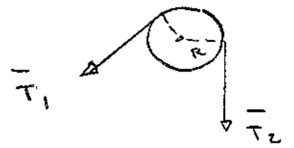
$$= \underline{118\text{ N}}$$



$$\boxed{m_2 g - T_2 = m_2 a} \quad (2)$$

$$\Rightarrow T_2 = m_2(g - a)$$

$$\Rightarrow T_2 = 20{,}0(9{,}81 - 2{,}00) = \underline{\underline{156\text{ N}}}$$



$$\tau_{netto} = T_2 R - T_1 R = I \cdot \alpha = I \frac{a}{R}$$

$$\Rightarrow \boxed{(T_2 - T_1) R = I \frac{a}{R}} \quad (3)$$

$$\Rightarrow \left[m_2(g - a) - m_1(a + g \cdot \sin\theta)\right] R = I \frac{a}{R}$$

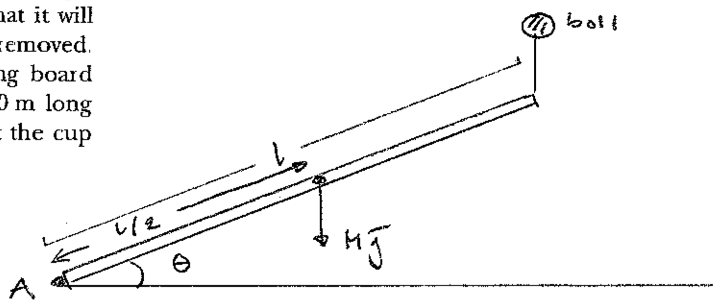
$$\Rightarrow I = \left[m_2\left(\frac{g}{a} - 1\right) - m_1\left(1 + \frac{g}{a}\sin\theta\right)\right] R^2 =$$

$$= \Big[\underbrace{20{,}0\left(\frac{9{,}81}{2{,}00} - 1\right)}_{78{,}1} - \underbrace{15{,}0\left(1 + \frac{9{,}81}{2{,}00} \cdot \sin 37^\circ\right)}_{59{,}2}\Big] 0{,}250^2 =$$

$$= \underline{1{,}18\text{ kg m}^2}$$

A common demonstration, illustrated in Figure P10.72, consists of a ball resting at one end of a uniform board of length ℓ, hinged at the other end, and elevated at an angle θ. A light cup is attached to the board at r_c so that it will catch the ball when the support stick is suddenly removed. (a) Show that the ball will lag behind the falling board when θ is less than 35.3°. (b) If the board is 1.00 m long and is supported at this limiting angle, show that the cup must be 18.4 cm from the moving end.

10.72



Hur stor får θ högst vara för att bollen ska falla långsammare än änden på plankan?

Vridande moment m.a.p. A : $\tau_A = Mg\,\frac{L}{2}\cdot\cos\theta$

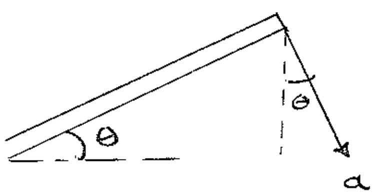
Plankans tröghetsmoment m.a.p. A : $I_A = \frac{1}{3}ML^2$

$$\tau_A = I_A\cdot\alpha$$

$$\Rightarrow \cancel{M}g\frac{\cancel{L}}{2}\cos\theta = \frac{1}{3}\cancel{M}L^{\cancel{2}}\cdot\alpha \qquad \Rightarrow \alpha = \frac{3}{2}\frac{\cos\theta}{L}g$$

$$\Rightarrow a = \frac{3}{2}\cos\theta\cdot g$$

men plankänden rör sig inte rakt mot marken



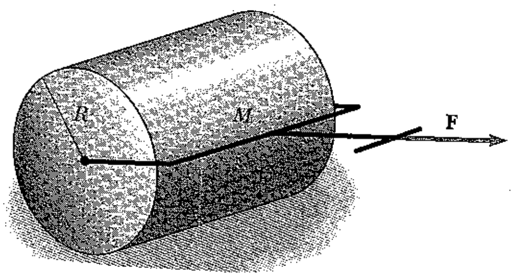
Det gäller att $a\cdot\cos\theta > g$ för att plankan ska hinna före bollen

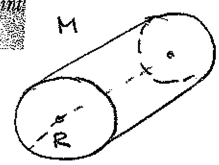
$$\Rightarrow \left(\frac{3}{2}\cos\theta\cdot g\right)\cdot\cos\theta > g$$

$$\Rightarrow \cos^2\theta > \frac{2}{3} \qquad \Rightarrow \underline{\underline{\theta < 35{,}26^\circ}}$$

A constant horizontal force **F** is applied to a lawn roller in the form of a uniform solid cylinder of radius R and mass M (Fig. P10.78). If the roller rolls without slipping on the horizontal surface, show that (a) the acceleration of the center of mass is $2\mathbf{F}/3M$ and (b) the minimum coefficient of friction necessary to prevent slipping is $F/3Mg$. (*Hint*: Take the torque with respect to the center of mass.)

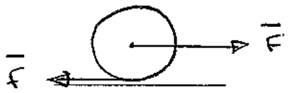
10.78





Homogen cylinder :

$$I = \frac{1}{2} M R^2$$



a) Tyngdpunktens acceleration :

ges av summan av externa krafter dvs $\bar{F} + \bar{f}$

$$F - f = M \cdot a_{CM}$$

men friktionskraften f möjliggör rotations-rörelsen ;

$$\left.\begin{aligned} \tau &= I \cdot \alpha \\ \tau &= f \cdot R \\ \alpha &= \frac{a_{CM}}{R} \end{aligned}\right\} \Rightarrow f = \frac{\tau}{R} = \frac{I\alpha}{R} = \frac{I a_{CM}}{R^2} = \frac{\frac{1}{2} M R^2 \cdot a_{CM}}{R^2} = \frac{1}{2} M a_{CM}$$

$$\therefore \; F - f = F - \frac{1}{2} M a_{CM} = M a_{CM} \Rightarrow F = \frac{3}{2} M a_{CM}$$

$$\Rightarrow \boxed{a_{CM} = \frac{2F}{3M}} \qquad \Rightarrow \alpha = \frac{2F}{3MR}$$

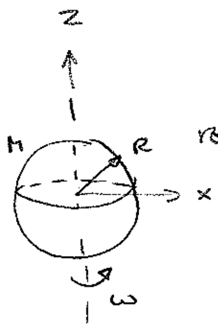
b)

$$\left.\begin{aligned} f_{max} &= \mu_s M g \\ f_{max} R &= I\alpha \end{aligned}\right\} \Rightarrow \mu_s \cancel{M} g \cancel{R} = \frac{1}{\cancel{2}} \cancel{M} R^{\cancel{2}} \frac{\cancel{2} F}{3 M \cancel{R}}$$

$$\Rightarrow \boxed{\mu_s = \frac{F}{3Mg}}$$

11.22

A uniform solid sphere of radius 0.500 m and mass 15.0 kg turns counterclockwise about a vertical axis through its center. Find its vector angular momentum when its angular speed is 3.00 rad/s.



roterande sfär : $I_{CM} = \frac{2}{5} MR^2$

Givet : $R = 0{,}500$ m
$M = 15{,}0$ kg
$\omega = +3{,}00$ rad/s (rot. moturs)

$$\bar{L} = I\bar{\omega} = \frac{2}{5} MR^2 \cdot \omega \hat{z} =$$

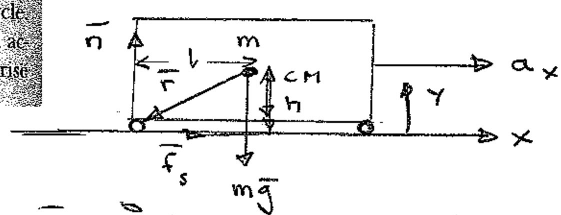
$$= \frac{2}{5} 15{,}0 \cdot 0{,}500^2 \cdot 3{,}00 \hat{z} =$$

$$= 4{,}5 \text{ kgm}^2/\text{s} \; \hat{z}$$

11.26

The distance between the centers of the wheels of a motorcycle is 155 cm. The center of mass of the motorcycle, including the biker, is 88.0 cm above the ground and halfway between the wheels. Assume the mass of each wheel is small compared to the body of the motorcycle. The engine drives the rear wheel only. What horizontal acceleration of the motorcycle will make the front wheel rise off the ground?

Schematisk bild av en motorcykel



Givet : $2l = 155$ cm
$h = 88$ cm

Sökt : $a_{x,max}$ utan stegring

Friktionskraften $\bar{f}_s$ åstadkommer ett vridande moment som vill vrida ekipaget moturs.

När a_x är liten är normalkraften fördelad mellan fram- och bakhjul, men när vi är på gränsen till stegring ligger normalkraften endast på bakhjulet. $\bar{n}$ åstadkommer ett vridande moment som vrider medurs.

ekvationer : $\Sigma F_x = f_s \Rightarrow \boxed{f_s = ma_x} \Rightarrow a_x = \frac{f_s}{m}$

$\Sigma F_y = n - mg = 0$

$\Rightarrow \boxed{n = mg}$

momenten map CM :

$\bar{f}_s$: $\bar{\tau}_f = \bar{r} \times \bar{f}_s = hf_s \hat{z}$

$\bar{\tau}_n = \bar{r} \times \bar{n} = nl(-\hat{z})$

$\bar{\tau}_{tot} = I\bar{\alpha}$ ingen stegring innebär : $\bar{\alpha} = 0$
$\Rightarrow \bar{\tau}_{tot} = 0$

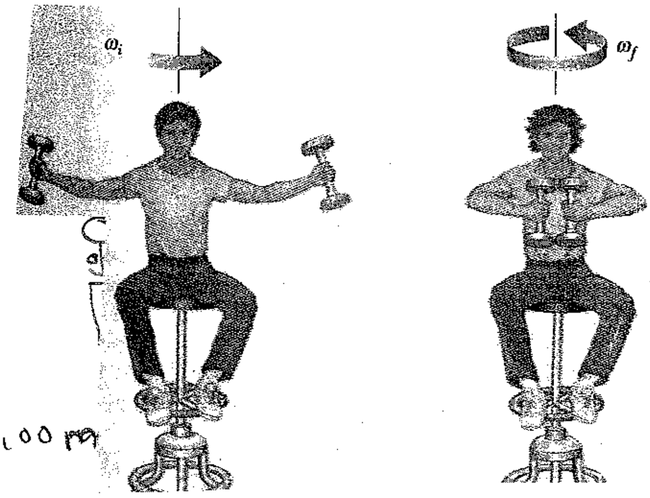
$\therefore hf_s - nl = 0$

$\Rightarrow hf_s = mgl \Rightarrow f_s = \frac{mgl}{h} \Rightarrow a_x = \frac{mgl}{mh} = \frac{gl}{h} =$

$= \frac{9{,}81 \cdot 0{,}775}{0{,}88}$ m/s^2 $= \underline{8{,}63 \text{ m/s}^2}$

(11.30)

A student sits on a freely rotating stool holding two weights, each of mass 3.00 kg (Figure P11.30). When his arms are extended horizontally, the weights are 1.00 m from the axis of rotation and he rotates with an angular speed of 0.750 rad/s. The moment of inertia of the student plus stool is 3.00 kg · m² and is assumed to be con-



Givet: $m = 3{,}00$ kg $\quad r_1 = 1{,}00$ m

$\omega_i = 0{,}750$ rad/s

$I_0 = 3{,}00$ kgm² (student + stol)

$r_2 = 0{,}300$ m

Sökt: ω_f

Inga yttre vridande moment på systemet

$\Rightarrow L_i = L_f$

$L_i = I_0 \omega_i + 2 m r_1^2 \cdot \omega_i$

$L_f = I_0 \omega_f + 2 m r_2^2 \cdot \omega_f$

$$\therefore \quad \omega_f = \frac{I_0 + 2mr_1^2}{I_0 + 2mr_2^2}\,\omega_i = \frac{3{,}00 + 2 \cdot 3{,}00 \cdot 1{,}00^2}{3{,}00 + 2 \cdot 3{,}00 \cdot 0{,}300^2}\,0{,}75 =$$

$$= \underline{\underline{1{,}91 \text{ rad/s}}}$$

Rot. energier:

$K_i = \frac{1}{2}(I_0 + 2mr_1^2) \cdot \omega_i^2 = 2{,}53$ J

$K_f = \frac{1}{2}(I_0 + 2mr_2^2) \cdot \omega_f^2 = 6{,}44$ J

$\therefore$ $6{,}44 - 2{,}53 = 3{,}91$ J utträttas av muskelarbetet

A puck of mass 80.0 g and radius 4.00 cm slides along an air table at a speed of 1.50 m/s as shown in Figure P11.34a. It makes a glancing collision with a second puck of radius 6.00 cm and mass 120 g (initially at rest) such that their rims just touch. Because their rims are coated with instant-acting glue, the pucks stick together and spin after the collision (Fig. P11.34b). (a) What is the angular momentum of the system relative to the center of mass? (b) What is the angular speed about the center of mass?

10 34

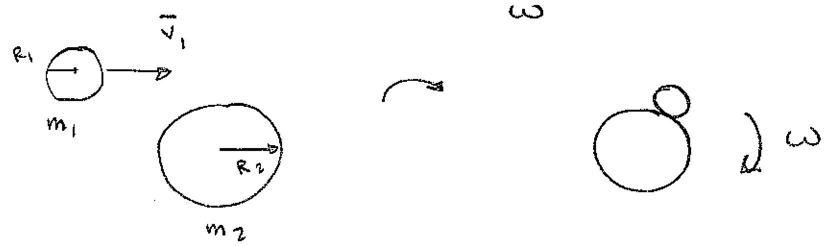
1

Givet: $m_1 = 80{,}0\ g$ $\quad R_1 = 4{,}00\ cm$ $\quad v_1 = 1{,}50\ m/s$

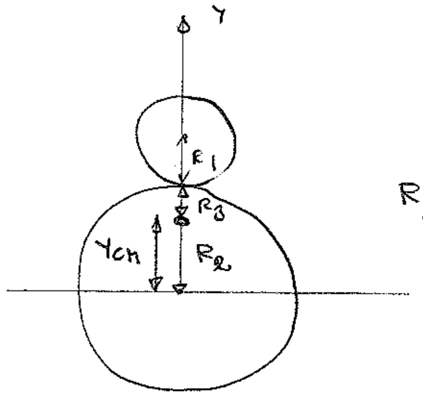
$m_2 = 120\ g$ $\quad R_2 = 6{,}00\ cm$

Sökt: L relativt masscentrum hos 

ω



a) Bestäm masscentrum för det sammanslagna systemet.



$$\bar{Y}_{CM} = \frac{m_1(R_2+R_1)+0}{m_1+m_2}\hat{y} =$$

$$= \frac{80{,}0\,(0{,}06+0{,}04)}{80+120}\hat{y} = 0{,}04$$

$$= \underline{4\ cm}$$

$R_2 - Y_{CM} = R_3$

$R_3 = 2{,}00\ cm$

b) L map $\bar{Y}_{CM}$

Omedelbart före kollision: $V_{stor} = 0$

$$\bar{L}_i = -m_1 v_1 (R_1+R_3)\hat{z} = -0{,}080\cdot 1{,}5\,(4{,}00+2{,}00)\hat{z} =$$

$$= \underline{\underline{7{,}20\cdot 10^{-3}\ kgm/s}}$$

c) ω: L bevaras $\quad L_f = I_1\omega + I_2\omega$

I_1 = tröghetsmomentet m.a.p. $\bar{Y}_{CM}$ (liten)

I_2 = —— " —— (stor)

$$I_1 = \frac{1}{2}m_1R_1^2 + m_1(R_1+R_3)^2 = \frac{1}{2}0{,}080\cdot 0{,}04^2 + 0{,}080\,(0{,}04+0{,}02)^2 =$$

$$= 3{,}52\cdot 10^{-4}\ kgm^2$$

$$I_2 = \frac{1}{2}m_2R_2^2 + m_2Y_{CM}^2 =$$

$$= \frac{1}{2}0{,}120\ 0{,}06^2 + 0{,}120\cdot 0{,}04^2 = 4{,}08\cdot 10^{-4}\ kgm^2$$

$$\Rightarrow I_{tot} = I_1 + I_2 = \underline{7{,}60\ kgm^2}$$

11.34 forts.

$$L_i = L_f = I_{tot} \cdot \omega$$

$$\Rightarrow \omega = \frac{L_i}{I_{tot}} = \frac{7{,}20 \cdot 10^{-3}}{7{,}60 \cdot 10^{-4}} \text{ rad/s} =$$

$$= \underline{\underline{9{,}47 \text{ rad/s}}}$$