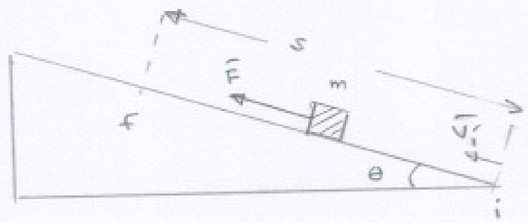
7.11.

Lösning:

givet: $s = 5{,}0\ m$ $\quad m = 10{,}0\ kg$

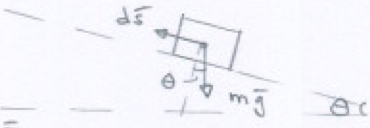
$\mu_k = 0{,}400$ $\quad v_i = 1{,}50\ m/s$

$F = 100\ N$ $\quad \theta = 20°$



a) Arbetet som gravitationskraften uträttar $i \to f$

$$dW_g = \bar{F}_g \cdot d\bar{s} = m\bar{g} \cdot d\bar{s} = -mg\sin\theta\, ds$$



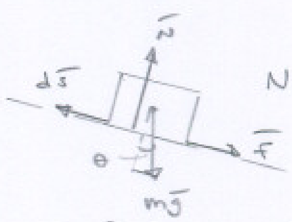
$$W_g = \int_i^f \bar{F}_g \cdot d\bar{s} = -\int_i^f mg\sin\theta\, ds = -mg\sin\theta \int_i^f ds =$$

$$= -mg\sin\theta \cdot s = -167{,}76\ J = \underline{\underline{-1{,}7 \cdot 10^2\ J}}$$

b) Ökningen av den inre energin hos lutande plan + låda under $i \to f$:

$\therefore$ beloppet av det arbete som friktionskraften $\bar{f}$ uträttar $i \to f$.

$$\left.\begin{array}{l} f = \mu_k N \\ N = mg\cos\theta \end{array}\right\} \Rightarrow f = \mu_k mg\cos\theta$$



$$W_f = \int_i^f \bar{f}\, d\bar{s} = -\mu_k mg\cos\theta \int_i^f ds =$$

$$= -\mu_k mg\cos\theta \cdot s = -0{,}400 \cdot 10{,}0 \cdot 9{,}81 \cdot \cos 20° \cdot 5{,}0 = -184{,}36\ J$$

$\Rightarrow$ $\underline{\underline{\text{svar: } 184\ J}}$. egentligen $1{,}8 \cdot 10^2\ J$.

c) Arbetet som $\bar{F}$ uträttar $i \to f$:

$$W_F = \int_i^f \bar{F} \cdot d\bar{s} = Fs = 100 \cdot 5{,}0 = \underline{\underline{5{,}0 \cdot 10^2\ N}}$$

d) Ändringen i kinetisk energi $i \to f$:

$$\Delta K = W_F + W_g + W_f = 500 - 167{,}76 - 184{,}36 = 147{,}8\ J = \underline{\underline{148\ J}}$$

e) Fart i punkten f:

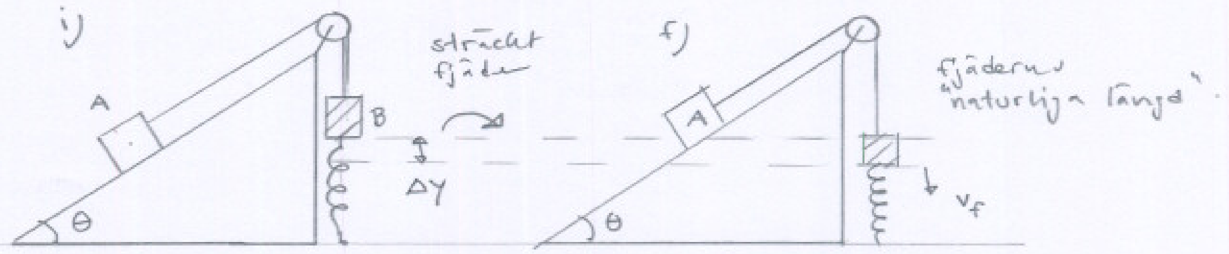
$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \Delta k \Rightarrow v_f = \sqrt{\frac{2\Delta K}{m} + v_i^2} =$$

$$\sqrt{\frac{2 \cdot 147{,}8}{10{,}0} + 1{,}50^2} = \underline{\underline{5{,}6\ m/s}}$$

7.59

Lösning:

friktionsfritt



Givet: $\theta = 40^\circ$ $m_A = 20{,}0$ kg $m_B = 30{,}0$ kg $k = 250$ N/m

$\Delta y = 0{,}20$ m $v_i = 0$ f: fjädern relaxerad.

Sökt: v_f

Den mekaniska energin bevaras eftersom de krafter som är relevanta är konservativa.

$\Rightarrow \Delta E_{mek} = E_{mek,f} - E_{mek,i}$ dvs "(efter) − (före)" $= 0$

$\Delta E_{mek} = \Delta E_{mek,A} + \Delta E_{mek,B} + \Delta E_{mek,fjäder} = 0$

$\Delta E_{mek,A} = \frac{1}{2} m_A \cdot v_f^2 + m_A g \Delta y \cdot \sin\theta$

$\Delta E_{mek,B} = \frac{1}{2} m_B v_f^2 - m_B g \Delta y - 0$

$\Delta E_{mek,fj.} = 0 - \frac{1}{2} k(\Delta y^2)$

$\Rightarrow \frac{1}{2} m_A v_f^2 + m_A g \Delta y \cdot \sin\theta + \frac{1}{2} m_B v_f^2 - m_B g \Delta y - \frac{1}{2} k(\Delta y)^2$

$$\Rightarrow v_f = \sqrt{\frac{2 g \Delta y\left(\frac{m_B}{m_A} - \sin\theta\right) + \frac{k}{m_A}(\Delta y)^2}{1 + \frac{m_B}{m_A}}} =$$

$$= \sqrt{\frac{2 \cdot 9{,}81 \cdot 0{,}20\left(\frac{30}{20} - \sin 40\right) + \frac{250}{20}(0{,}20)^2}{1 + \frac{30}{20}}} = \underline{\underline{1{,}24 \text{ m/s}}}$$

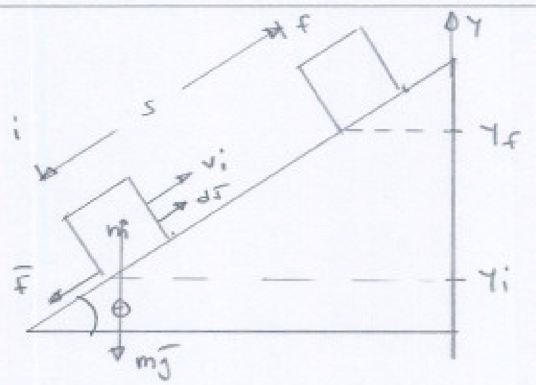
8.33

Lösning:

givet; $v_i = 8{,}00$ m/s $\quad v_f = 0$

$\theta = 30°$ $\quad m = 5{,}00$ kg

$s = 3{,}00$ m



a) $\underline{\Delta K}$:

$$\Delta K = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = 0 - \frac{1}{2} m v_i^2 = -\frac{1}{2}\, 5{,}0\,(8{,}00)^2 =$$
$$= \underline{-160\ \mathrm{J}}$$

---

b) $\underline{\Delta U \text{ (i grav.fältet)}}$

$$\Delta U = mg\,\Delta y = mg(y_f - y_i) = mg\, s\cdot \sin\theta =$$
$$= 5{,}00\cdot 9{,}81\cdot 3{,}00\cdot \sin 30° = \underline{73{,}5\ \mathrm{J}}$$

---

c) friktionskraften $|\bar f|$: $\qquad \bar F = \bar f + m\bar g$

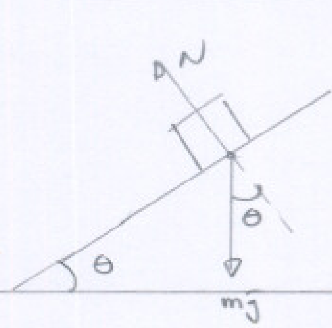
$$\int_i^f \bar F\cdot d\bar s = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \Delta K$$

$$\int_i^f (\bar f + m\bar g)\cdot d\bar s = \int_i^f \bar f\cdot d\bar s + \int_i^f m\bar g\cdot d\bar s = -fs + (-\Delta U)$$

$$\Rightarrow -fs - \Delta U = \Delta K \qquad \Rightarrow f = -\frac{\Delta K + \Delta U}{s} = -\frac{(-160) + 73{,}5}{3{,}00} =$$
$$= \underline{28{,}8\ \mathrm{N}}$$

---

d) $\underline{\mu_k}$: $\qquad \left.\begin{array}{l} f = \mu_k\cdot N \\ N = mg\cdot\cos\theta \end{array}\right\} \Rightarrow$



$$\Rightarrow f = \mu_k\, mg\cos\theta$$

$$\Rightarrow \mu_k = \frac{f}{mg\cdot\cos\theta} = \underline{0{,}678}$$

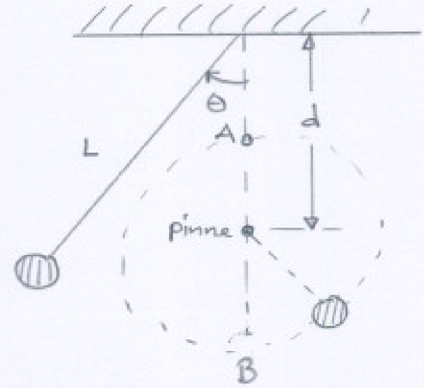
Obs! $\displaystyle\int_i^f \bar F\cdot d\bar s = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \Delta K$

Om endast konservativa krafter verkar: $\Delta K + \Delta U = 0 \quad \Rightarrow \Delta K = -\Delta U$

$$\therefore \int_i^f \bar F_c\cdot d\bar s = -\Delta U$$

8.72

Lösning:



Pendeln släpps från $\theta = 90^\circ$

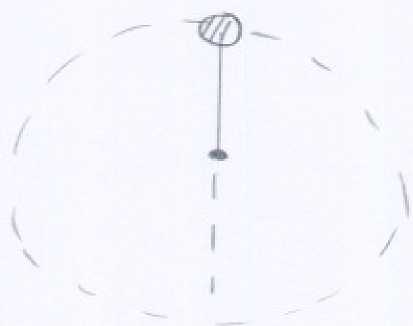
dvs  i)

Visa att $d_{min} = \frac{3}{5}L$ om pendeln ska kunna gå i en cirkulär bana runt pinnen.

Det kritiska läget är A:

radien i den cirkulära banan runt pinnen:

$$r = (L - d)$$



spännkraften T i snöret ges i A av

$$m\frac{v_A^2}{r} = mg + T$$

där $T \geq 0$

$$\Rightarrow m\frac{v_A^2}{r} > mg \Rightarrow \frac{v_A^2}{(L-d)} > g$$

sätt den potentielle energin = 0 i läge B.

i): $U_i = mgL \quad K_i = 0$

A): $U_A = mg2r = mg2(L-d)$, $K_A = \frac{1}{2}mv_A^2$

mek. energin bevarav: $mgL = mg2(L-d) + \frac{1}{2}mv_A^2$

$$\Rightarrow 2gL = 4gL - 4gd + v_A^2 \Rightarrow v_A^2 = 4gd - 2gL$$

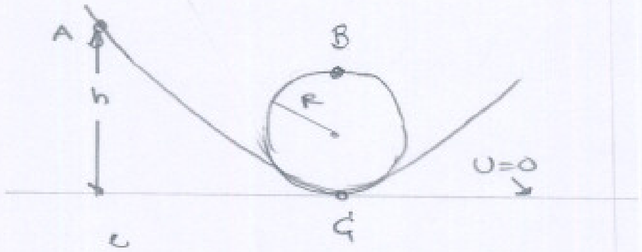
$$\Rightarrow \frac{v_A^2}{L-d} > g \Rightarrow \frac{4gd - 2gL}{L-d} > g \Rightarrow 4d - 2L > L - d \Rightarrow 5d > 3L$$

$$\Rightarrow \underline{\underline{d > \frac{3}{5}L}}$$ v.s.v.

8.73

Lösning:

a)



B: $m\frac{v_B^2}{R} = mg + N$

$N_{min} = 0$

$\Rightarrow m\frac{v_B^2}{R} > mg$

$\Rightarrow v_B^2 > gR$

A: mgh

B: $\frac{1}{2}mv_B^2 + 2Rgm$

∴ $mgh = \frac{1}{2}mv_B^2 + 2Rgm$ $\Rightarrow \cancel{m}gh > \frac{1}{2}\cancel{m}gR + 2Rg\cancel{m}$

$\Rightarrow \boxed{h_{min} = 2\frac{1}{2}R}$

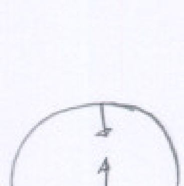
Skillnad i normalkraft $N_C - N_B$:

b) $h > 2\frac{1}{2}R$:

$$\left.\begin{aligned} B:&\quad m\frac{v_B^2}{R} = mg + N_B \\ C:&\quad m\frac{v_C^2}{R} = N_C - mg \end{aligned}\right\} \Rightarrow N_C - N_B = 2mg + \frac{m}{R}(v_C^2 - v_B^2)$$

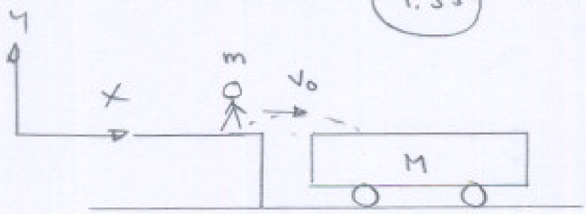
energiprincipen: $\frac{1}{2}mv_C^2 = \frac{1}{2}mv_B^2 + 2Rmg$

$\Rightarrow v_C^2 - v_B^2 = 4Rg$



$\Rightarrow N_C - N_B = 2mg + \frac{m}{R}\cdot 4Rg = \underline{6mg}$

9.55



Lösning:

$m = 60{,}0$ kg $\quad M = 120{,}0$ kg

$v_0 = 4{,}00$ m $\quad \mu_k = 0{,}400$

a) sluthastighet för $(m+M)$ relativt marken.

inga externa krafter i x-led

$\Rightarrow P_x$ bevaras

$m\bar{v}_0 + 0 = (m+M)\bar{v}_f$

$$\Rightarrow \bar{v}_f = \frac{m}{m+M}\bar{v}_0 = \frac{60}{60+120}4{,}00\hat{x} =$$

$$= 1{,}33\hat{x}\,\text{m/s}$$

b) friktionskraft på m under glidningsfasen:

$\Sigma \bar{F}_y = 0 \qquad N = mg \qquad f = \mu_k N = \mu_k mg = 0{,}400 \cdot 60 \cdot 9{,}81 = \underline{\underline{235\,N}}$

$\bar{f} = 235(-\hat{x})\,N$

c) Under hur lång tid glider m?

$$\frac{d\bar{p}}{dt} = \bar{F} = \bar{f} \Rightarrow d\bar{p} = \bar{F}\cdot dt \Rightarrow \left.\begin{array}{l}\Delta\bar{p} = \bar{F}\cdot\Delta t \\ \Delta\bar{p} = m\bar{v}_f - m\bar{v}_i \\ \bar{v}_f = 0\end{array}\right\} \Rightarrow$$

$$\Rightarrow m(\bar{v}_f - \bar{v}_i) = \bar{f}\,\Delta t \Rightarrow \Delta t = \frac{m(\bar{v}_f - \bar{v}_i)}{\bar{f}} = \frac{60{,}0(1{,}33-4{,}00)\hat{x}}{-235\hat{x}} = \underline{\underline{0{,}68\,s}}$$

alt. retardation $a = -\frac{f}{m} \qquad a\cdot\Delta t = v_f - v_i \Rightarrow \Delta t = \dfrac{4{,}00-1{,}33}{\frac{235}{60}} = 0{,}68\,s$

d) Δp_m & Δp_M:

$$\Delta\bar{p}_m = m\bar{v}_f - m\bar{v}_i = 60(1{,}33-4{,}00)\hat{x} =$$

$$= -160\,\text{kgm/s}\,\hat{x}$$

$$\Delta\bar{p}_M = M\bar{v}_f - 0 = 120\cdot 1{,}33\hat{x}\,\text{kgm/s} = +160\hat{x}\,\text{kgm/s}$$

e) Δx under glidningen. $\quad v_f^2 - v_i^2 = 2\cdot a\,\Delta x$

$$\Rightarrow \Delta x = \frac{v_f^2 - v_i^2}{-2\frac{f}{m}} = \frac{1{,}33^2 - 4{,}00^2}{-2\frac{235}{60}} = \underline{1{,}81\,m}$$

(9.35) forts.

f) Förflyttning av vagnen under glidningsfasen $\Delta x'$

På vagnen verkar $-\bar{f}$ dvs en accelererande kraft. $235\hat{x}$ N

$$\bar{a}_M = \frac{-\bar{f}}{M} = \frac{235}{120}\hat{x}$$

$$v_f^2 - v_i^2 = 2 \cdot a_M \cdot \Delta x' \Rightarrow \Delta x' = \frac{v_f^2 - v_i^2}{2 a_M} = \frac{1{,}33^2 - 0^0}{2\,\frac{235}{120}} \text{ m} = \underline{\underline{0{,}45 \text{ m}}}$$

---

g) ΔK_m:

$$\Delta K_m = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \frac{1}{2} 60 \cdot 1{,}33^2 - \frac{1}{2} 60 \cdot 4{,}00^2 =$$

$$= \underline{\underline{-426{,}7 \text{ J}}}$$

---

h) ΔK_M:

$$\Delta K_M = \frac{1}{2} M v_f^2 - 0 = \frac{1}{2} 120 \cdot 1{,}33^2 \text{ J} = \underline{\underline{106{,}7 \text{ J}}}$$

---

i) Skillnaden mellan ΔK_M och ΔK_m: $= 320$ J

Friktionsarbete under m:s glidning.

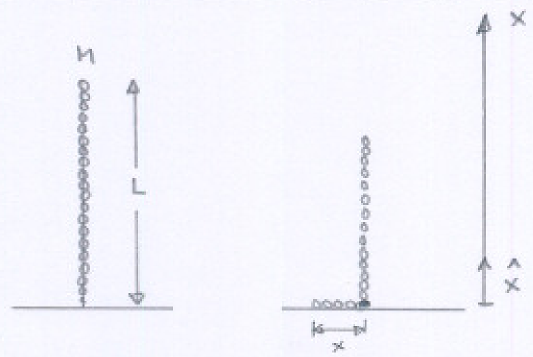
$\Delta x_1 = 1{,}81$ m glidning relativt marken

$\Delta x' = 0{,}45$ m vagnens rörelse rel. marken under glidningsfasen.

$\Delta s =$ m:s glidsträcka på vagnen $= \Delta x - \Delta x'$

$$W_f = f \cdot \Delta s = 235(1{,}81 - 0{,}45) \approx \underline{\underline{320 \text{ J}}}. \quad \text{ok!}$$

Lösning:



Det kedjeelement med bredden dx (massa dm) som når bordet har fallit från höjden x

$\therefore$ dess fart ges av $dm \cdot g \cdot x = \frac{1}{2} dm \cdot v^2$

$$\Rightarrow v^2 = 2gx \Rightarrow v = \sqrt{2gx}$$

$$\bar{v} = -v\hat{x}$$

När elementet med massan dm landar på bordet ändras dess rörelsemängd från $dm \cdot \bar{v}$ till 0 $\Rightarrow d\bar{p} = 0 - (dm\, v(-\hat{x}))$

$$\therefore\ d\bar{p} = dm \cdot v\, \hat{x}$$

På kedjeelementet måste en kraft $\bar{F}$ verka som ges av

$$\left.\begin{aligned} &\bar{F} = \frac{d\bar{p}}{dt} = \frac{dm \cdot v}{dt}\hat{x} \\ &\text{men} \quad v \cdot dt = dx \Rightarrow dt = \frac{dx}{v} \\ &\text{och} \quad dm = \frac{M}{L} \cdot dx \end{aligned}\right\} \Rightarrow \bar{F} = \frac{\frac{M}{L} \cdot dx \cdot v}{\frac{dx}{v}}\hat{x} = \frac{M}{L} v^2 \hat{x} = \frac{M}{L} 2gx \cdot \hat{x}$$

På bordet verkar reaktionskraften $-\bar{F} = \frac{M}{L} 2gx(-\hat{x})$

dessutom har vi tyngden från kedjan som redan ligger på bordet $\bar{F}_2 = \frac{M}{L} \cdot x\, g - \hat{x}$

Sammanlagd kraft: $-\bar{F} + \bar{F}_2 = \underline{\underline{3\frac{M}{L} gx}}$