

11. If you see an object rotating, is there necessarily a net torque acting on it?
12. If a small sphere of mass  $M$  were placed at the end of the rod in Figure 10.21, would the result for  $\omega$  be greater than, less than, or equal to the value obtained in Example 10.11?
13. Three objects of uniform density—a solid sphere, a solid cylinder, and a hollow cylinder—are placed at the top of an incline (Fig. CQ10.13). They are all released from rest at the same elevation and roll without slipping. (a) Which object reaches the bottom first? (b) Which reaches it last? *Note:* The result is independent of the masses and the radii of the objects. (Try this activity at home!)

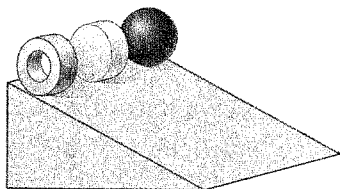


Figure CQ10.13

14. Which of the entries in Table 10.2 applies to finding the moment of inertia (a) of a long, straight sewer pipe rotating about its axis of symmetry? (b) Of an embroidery hoop rotating about an axis through its center and perpendicular to its plane? (c) Of a uniform door turning on its hinges? (d) Of a coin turning about an axis through its center and perpendicular to its faces?
15. Figure CQ10.15 shows a side view of a child's tricycle with rubber tires on a horizontal concrete sidewalk. If a string were attached to the upper pedal on the

far side and pulled forward horizontally, the tricycle would start to roll forward. (a) Instead, assume a string is attached to the lower pedal on the near side and pulled forward horizontally as shown by A. Will the tricycle start to roll? If so, which way? Answer the same questions if (b) the string is pulled forward and upward as shown by B, (c) if the string is pulled straight down as shown by C, and (d) if the string is pulled forward and downward as shown by D. (e) **What If?** Suppose the string is instead attached to the rim of the front wheel and pulled upward and backward as shown by E. Which way does the tricycle roll? (f) Explain a pattern of reasoning, based on the figure, that makes it easy to answer questions such as these. What physical quantities must you evaluate?

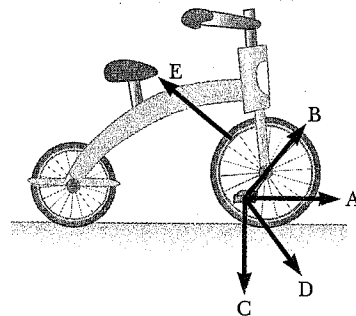


Figure CQ10.15

16. A person balances a meterstick in a horizontal position on the extended index fingers of her right and left hands. She slowly brings the two fingers together. The stick remains balanced, and the two fingers always meet at the 50-cm mark regardless of their original positions. (Try it!) Explain why that occurs.

## Problems

### WebAssign

The problems found in this chapter may be assigned online in Enhanced WebAssign

1. straightforward; 2. intermediate; 3. challenging

1. full solution available in the *Student Solutions Manual/Study Guide*

**AMT** Analysis Model tutorial available in Enhanced WebAssign

**GP** Guided Problem

**MT** Master It tutorial available in Enhanced WebAssign

**W** Watch It video solution available in Enhanced WebAssign

## Section 10.1 Angular Position, Velocity, and Acceleration

1. (a) Find the angular speed of the Earth's rotation about its axis. (b) How does this rotation affect the shape of the Earth?
2. A potter's wheel moves uniformly from rest to an angular speed of 1.00 rev/s in 30.0 s. (a) Find its average angular acceleration in radians per second per second. (b) Would doubling the angular acceleration during the given period have doubled the final angular speed?
3. During a certain time interval, the angular position **W** of a swinging door is described by  $\theta = 5.00 + 10.0t + 2.00t^2$ , where  $\theta$  is in radians and  $t$  is in seconds. Deter-

mine the angular position, angular speed, and angular acceleration of the door (a) at  $t = 0$  and (b) at  $t = 3.00$  s.

4. A bar on a hinge starts from rest and rotates with an angular acceleration  $\alpha = 10 + 6t$ , where  $\alpha$  is in rad/s<sup>2</sup> and  $t$  is in seconds. Determine the angle in radians through which the bar turns in the first 4.00 s.

## Section 10.2 Analysis Model: Rigid Object Under Constant Angular Acceleration

5. A wheel starts from rest and rotates with constant **W** angular acceleration to reach an angular speed of 12.0 rad/s in 3.00 s. Find (a) the magnitude of the angu-

lar acceleration of the wheel and (b) the angle in radians through which it rotates in this time interval.

6. A centrifuge in a medical laboratory rotates at an angular speed of 3 600 rev/min. When switched off, it rotates through 50.0 revolutions before coming to rest. Find the constant angular acceleration of the centrifuge.
7. An electric motor rotating a workshop grinding wheel at  $1.00 \times 10^2$  rev/min is switched off. Assume the wheel has a constant negative angular acceleration of magnitude  $2.00 \text{ rad/s}^2$ . (a) How long does it take the grinding wheel to stop? (b) Through how many radians has the wheel turned during the time interval found in part (a)?
8. A machine part rotates at an angular speed of  $0.060 \text{ rad/s}$ ; its speed is then increased to  $2.2 \text{ rad/s}$  at an angular acceleration of  $0.70 \text{ rad/s}^2$ . (a) Find the angle through which the part rotates before reaching this final speed. (b) If both the initial and final angular speeds are doubled and the angular acceleration remains the same, by what factor is the angular displacement changed? Why?
9. A dentist's drill starts from rest. After 3.20 s of constant angular acceleration, it turns at a rate of  $2.51 \times 10^4 \text{ rev/min}$ . (a) Find the drill's angular acceleration. (b) Determine the angle (in radians) through which the drill rotates during this period.
10. Why is the following situation impossible? Starting from rest, a disk rotates around a fixed axis through an angle of  $50.0 \text{ rad}$  in a time interval of 10.0 s. The angular acceleration of the disk is constant during the entire motion, and its final angular speed is  $8.00 \text{ rad/s}$ .
11. A rotating wheel requires 3.00 s to rotate through 37.0 revolutions. Its angular speed at the end of the 3.00-s interval is  $98.0 \text{ rad/s}$ . What is the constant angular acceleration of the wheel?
12. The tub of a washer goes into its spin cycle, starting from rest and gaining angular speed steadily for 8.00 s, at which time it is turning at  $5.00 \text{ rev/s}$ . At this point, the person doing the laundry opens the lid, and a safety switch turns off the washer. The tub smoothly slows to rest in 12.0 s. Through how many revolutions does the tub turn while it is in motion?
13. A spinning wheel is slowed down by a brake, giving it a constant angular acceleration of  $-5.60 \text{ rad/s}^2$ . During a 4.20-s time interval, the wheel rotates through 62.4 rad. What is the angular speed of the wheel at the end of the 4.20-s interval?
14. **Review.** Consider a tall building located on the Earth's equator. As the Earth rotates, a person on the top floor of the building moves faster than someone on the ground with respect to an inertial reference frame because the person on the ground is closer to the Earth's axis. Consequently, if an object is dropped from the top floor to the ground a distance  $h$  below, it lands east of the point vertically below where it was dropped. (a) How far to the east will the object land? Express your answer in terms of  $h$ ,  $g$ , and the angular speed  $\omega$  of the Earth. Ignore air resistance and assume the free-fall acceleration is constant over this range of heights. (b) Evaluate the eastward displacement for  $h = 50.0 \text{ m}$ . (c) In your judgment,

were we justified in ignoring this aspect of the *Coriolis effect* in our previous study of free fall? (d) Suppose the angular speed of the Earth were to decrease due to tidal friction with constant angular acceleration. Would the eastward displacement of the dropped object increase or decrease compared with that in part (b)?

### Section 10.3 Angular and Translational Quantities

15. A racing car travels on a circular track of radius 250 m. Assuming the car moves with a constant speed of  $45.0 \text{ m/s}$ , find (a) its angular speed and (b) the magnitude and direction of its acceleration.
16. Make an order-of-magnitude estimate of the number of revolutions through which a typical automobile tire turns in one year. State the quantities you measure or estimate and their values.
17. A discus thrower (Fig. P4.33, page 104) accelerates a discus from rest to a speed of  $25.0 \text{ m/s}$  by whirling it through 1.25 rev. Assume the discus moves on the arc of a circle 1.00 m in radius. (a) Calculate the final angular speed of the discus. (b) Determine the magnitude of the angular acceleration of the discus, assuming it to be constant. (c) Calculate the time interval required for the discus to accelerate from rest to  $25.0 \text{ m/s}$ .
18. Figure P10.18 shows the drive train of a bicycle that has wheels 67.3 cm in diameter and pedal cranks 17.5 cm long. The cyclist pedals at a steady cadence of 76.0 rev/min. The chain engages with a front sprocket 15.2 cm in diameter and a rear sprocket 7.00 cm in diameter. Calculate (a) the speed of a link of the chain relative to the bicycle frame, (b) the angular speed of the bicycle wheels, and (c) the speed of the bicycle relative to the road. (d) What pieces of data, if any, are not necessary for the calculations?

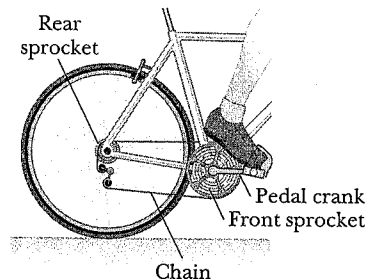


Figure P10.18

19. A wheel 2.00 m in diameter lies in a vertical plane and rotates about its central axis with a constant angular acceleration of  $4.00 \text{ rad/s}^2$ . The wheel starts at rest at  $t = 0$ , and the radius vector of a certain point  $P$  on the rim makes an angle of  $57.3^\circ$  with the horizontal at this time. At  $t = 2.00 \text{ s}$ , find (a) the angular speed of the wheel and, for point  $P$ , (b) the tangential speed, (c) the total acceleration, and (d) the angular position.
20. A car accelerates uniformly from rest and reaches a speed of  $22.0 \text{ m/s}$  in 9.00 s. Assuming the diameter of a tire is 58.0 cm, (a) find the number of revolutions the tire makes during this motion, assuming that no slipping occurs. (b) What is the final angular speed of a tire in revolutions per second?

- 21.** A disk 8.00 cm in radius rotates at a constant rate of **1** 200 rev/min about its central axis. Determine (a) its angular speed in radians per second, (b) the tangential speed at a point 3.00 cm from its center, (c) the radial acceleration of a point on the rim, and (d) the total distance a point on the rim moves in 2.00 s.

- 22.** A straight ladder is leaning against the wall of a house. The ladder has rails 4.90 m long, joined by rungs 0.410 m long. Its bottom end is on solid but sloping ground so that the top of the ladder is 0.690 m to the left of where it should be, and the ladder is unsafe to climb. You want to put a flat rock under one foot of the ladder to compensate for the slope of the ground. (a) What should be the thickness of the rock? (b) Does using ideas from this chapter make it easier to explain the solution to part (a)? Explain your answer.

- 23.** A car traveling on a flat (unbanked), circular track **W** accelerates uniformly from rest with a tangential acceleration of  $1.70 \text{ m/s}^2$ . The car makes it one-quarter of the way around the circle before it skids off the track. From these data, determine the coefficient of static friction between the car and the track.

- 24.** A car traveling on a flat (unbanked), circular track accelerates uniformly from rest with a tangential acceleration of  $a$ . The car makes it one-quarter of the way around the circle before it skids off the track. From these data, determine the coefficient of static friction between the car and the track.

- 25.** In a manufacturing process, a large, cylindrical roller is used to flatten material fed beneath it. The diameter of the roller is 1.00 m, and, while being driven into rotation around a fixed axis, its angular position is expressed as

$$\theta = 2.50t^2 - 0.600t^3$$

where  $\theta$  is in radians and  $t$  is in seconds. (a) Find the maximum angular speed of the roller. (b) What is the maximum tangential speed of a point on the rim of the roller? (c) At what time  $t$  should the driving force be removed from the roller so that the roller does not reverse its direction of rotation? (d) Through how many rotations has the roller turned between  $t = 0$  and the time found in part (c)?

- 26. Review.** A small object with mass 4.00 kg moves counterclockwise with constant angular speed  $1.50 \text{ rad/s}$  in a circle of radius 3.00 m centered at the origin. It starts at the point with position vector  $3.00\hat{i} \text{ m}$ . It then undergoes an angular displacement of 9.00 rad. (a) What is its new position vector? Use unit-vector notation for all vector answers. (b) In what quadrant is the particle located, and what angle does its position vector make with the positive  $x$  axis? (c) What is its velocity? (d) In what direction is it moving? (e) What is its acceleration? (f) Make a sketch of its position, velocity, and acceleration vectors. (g) What total force is exerted on the object?

### Section 10.4 Torque

- 27.** Find the net torque on the wheel in Figure P10.27 about **1** the axle through  $O$ , taking  $a = 10.0 \text{ cm}$  and  $b = 25.0 \text{ cm}$ .

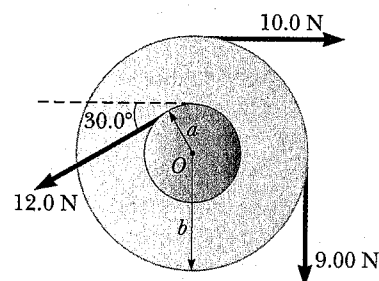


Figure P10.27

- 28.** The fishing pole in Figure P10.28 makes an angle **W** of  $20.0^\circ$  with the horizontal. What is the torque exerted by the fish about an axis perpendicular to the page and passing through the angler's hand if the fish pulls on the fishing line with a force  $\vec{F} = 100 \text{ N}$  at an angle  $37.0^\circ$  below the horizontal? The force is applied at a point 2.00 m from the angler's hands.

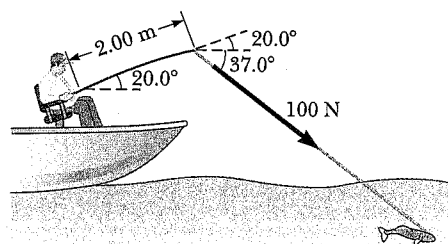


Figure P10.28

### Section 10.5 Analysis Model: Rigid Object Under a Net Torque

- 29.** An electric motor turns a flywheel through a drive belt that joins a pulley on the motor and a pulley that is rigidly attached to the flywheel as shown in Figure P10.29. The flywheel is a solid disk with a mass of 80.0 kg and a radius  $R = 0.625 \text{ m}$ . It turns on a frictionless axle. Its pulley has much smaller mass and a radius of  $r = 0.230 \text{ m}$ . The tension  $T_u$  in the upper (taut) segment of the belt is 135 N, and the flywheel has a clockwise angular acceleration of  $1.67 \text{ rad/s}^2$ . Find the tension in the lower (slack) segment of the belt.

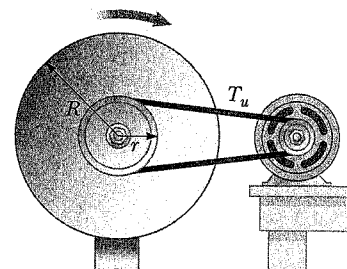


Figure P10.29

- 30.** A grinding wheel is in the form of a uniform solid disk **AMT** of radius 7.00 cm and mass 2.00 kg. It starts from rest **W** and accelerates uniformly under the action of the constant torque of  $0.600 \text{ N} \cdot \text{m}$  that the motor exerts on the wheel. (a) How long does the wheel take to reach its final operating speed of 1 200 rev/min? (b) Through how many revolutions does it turn while accelerating?

11. A 150-kg merry-go-round in the shape of a uniform, solid, horizontal disk of radius 1.50 m is set in motion by wrapping a rope about the rim of the disk and pulling on the rope. What constant force must be exerted on the rope to bring the merry-go-round from rest to an angular speed of 0.500 rev/s in 2.00 s?

32. **Review.** A block of mass  $m_1 = 2.00$  kg and a block of mass  $m_2 = 6.00$  kg are connected by a massless string over a pulley in the shape of a solid disk having radius  $R = 0.250$  m and mass  $M = 10.0$  kg. The fixed, wedge-shaped ramp makes an angle of  $\theta = 30.0^\circ$  as shown in Figure P10.32. The coefficient of kinetic friction is 0.360 for both blocks. (a) Draw force diagrams of both blocks and of the pulley. Determine (b) the acceleration of the two blocks and (c) the tensions in the string on both sides of the pulley.

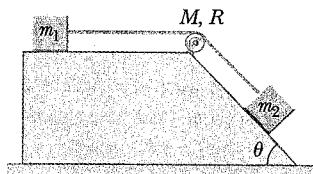


Figure P10.32

33. A model airplane with mass 0.750 kg is tethered to the ground by a wire so that it flies in a horizontal circle 30.0 m in radius. The airplane engine provides a net thrust of 0.800 N perpendicular to the tethering wire. (a) Find the torque the net thrust produces about the center of the circle. (b) Find the angular acceleration of the airplane. (c) Find the translational acceleration of the airplane tangent to its flight path.

34. A disk having moment of inertia  $100 \text{ kg} \cdot \text{m}^2$  is free to rotate without friction, starting from rest, about a fixed axis through its center. A tangential force whose magnitude can range from  $F = 0$  to  $F = 50.0$  N can be applied at any distance ranging from  $R = 0$  to  $R = 3.00$  m from the axis of rotation. (a) Find a pair of values of  $F$  and  $R$  that cause the disk to complete 2.00 rev in 10.0 s. (b) Is your answer for part (a) a unique answer? How many answers exist?

35. The combination of an applied force and a friction force produces a constant total torque of  $36.0 \text{ N} \cdot \text{m}$  on a wheel rotating about a fixed axis. The applied force acts for 6.00 s. During this time, the angular speed of the wheel increases from 0 to 10.0 rad/s. The applied force is then removed, and the wheel comes to rest in 60.0 s. Find (a) the moment of inertia of the wheel, (b) the magnitude of the torque due to friction, and (c) the total number of revolutions of the wheel during the entire interval of 66.0 s.

36. **Review.** Consider the system shown in Figure P10.36 with  $m_1 = 20.0$  kg,  $m_2 = 12.5$  kg,  $R = 0.200$  m, and the mass of the pulley  $M = 5.00$  kg.

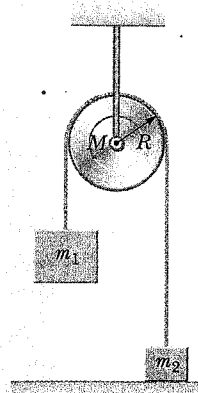


Figure P10.36

Object  $m_2$  is resting on the floor, and object  $m_1$  is 4.00 m above the floor when it is released from rest. The pulley axis is frictionless. The cord is light, does not stretch, and does not slip on the pulley. (a) Calculate the time interval required for  $m_1$  to hit the floor. (b) How would your answer change if the pulley were massless?

37. A potter's wheel—a thick stone disk of radius 0.500 m and mass 100 kg—is freely rotating at 50.0 rev/min. The potter can stop the wheel in 6.00 s by pressing a wet rag against the rim and exerting a radially inward force of 70.0 N. Find the effective coefficient of kinetic friction between wheel and rag.

### Section 10.6 Calculation of Moments of Inertia

38. Imagine that you stand tall and turn about a vertical axis through the top of your head and the point halfway between your ankles. Compute an order-of-magnitude estimate for the moment of inertia of your body for this rotation. In your solution, state the quantities you measure or estimate and their values.

39. A uniform, thin, solid door has height 2.20 m, width 0.870 m, and mass 23.0 kg. (a) Find its moment of inertia for rotation on its hinges. (b) Is any piece of data unnecessary?

40. Two balls with masses  $M$  and  $m$  are connected by a rigid rod of length  $L$  and negligible mass as shown in Figure P10.40. For an axis perpendicular to the rod, (a) show that the system has the minimum moment of inertia when the axis passes through the center of mass. (b) Show that this moment of inertia is  $I = \mu L^2$ , where  $\mu = mM/(m + M)$ .

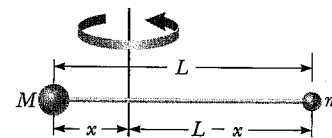


Figure P10.40

41. Figure P10.41 shows a side view of a car tire before it is mounted on a wheel. Model it as having two sidewalls of uniform thickness 0.635 cm and a tread wall of uniform thickness 2.50 cm and width 20.0 cm. Assume the rubber has uniform density  $1.10 \times 10^3 \text{ kg/m}^3$ . Find its moment of inertia about an axis perpendicular to the page through its center.

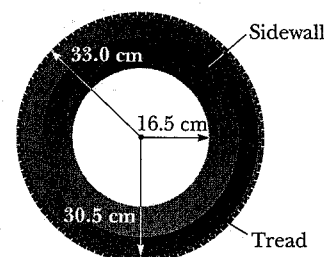


Figure P10.41

42. Following the procedure used in Example 10.7, prove that the moment of inertia about the  $y$  axis of the rigid rod in Figure 10.15 is  $\frac{1}{3}ML^2$ .

43. Three identical thin rods, each of length  $L$  and mass  $m$ , are welded perpendicular to one another as shown in Figure P10.43. The assembly is rotated about an axis that passes through the end of one rod and is parallel to another. Determine the moment of inertia of this structure about this axis.

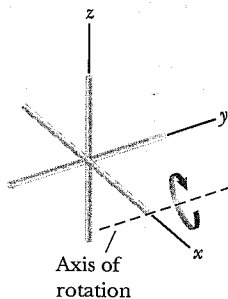


Figure P10.43

### Section 10.7 Rotational Kinetic Energy

44. Rigid rods of negligible mass lying along the  $y$  axis connect three particles (Fig. P10.44). The system rotates about the  $x$  axis with an angular speed of  $2.00 \text{ rad/s}$ . Find (a) the moment of inertia about the  $x$  axis, (b) the total rotational kinetic energy evaluated from  $\frac{1}{2}I\omega^2$ , (c) the tangential speed of each particle, and (d) the total kinetic energy evaluated from  $\sum \frac{1}{2}m_i v_i^2$ . (e) Compare the answers for kinetic energy in parts (a) and (b).

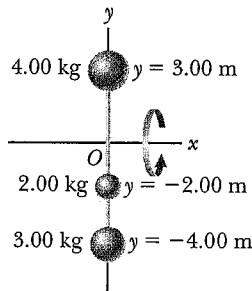


Figure P10.44

45. The four particles in Figure P10.45 are connected by rigid rods of negligible mass. The origin is at the center of the rectangle. The system rotates in the  $xy$  plane about the  $z$  axis with an angular speed of  $6.00 \text{ rad/s}$ . Calculate (a) the moment of inertia of the system about the  $z$  axis and (b) the rotational kinetic energy of the system.

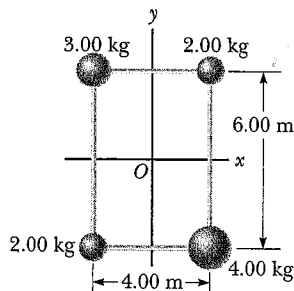


Figure P10.45

46. Many machines employ cams for various purposes, such as opening and closing valves. In Figure P10.46, the cam is a circular disk of radius  $R$  with a hole of diameter  $R$  cut through it. As shown in the figure, the

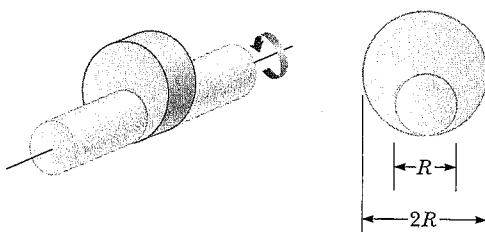


Figure P10.46

hole does not pass through the center of the disk. The cam with the hole cut out has mass  $M$ . The cam is mounted on a uniform, solid, cylindrical shaft of diameter  $R$  and also of mass  $M$ . What is the kinetic energy of the cam-shaft combination when it is rotating with angular speed  $\omega$  about the shaft's axis?

47. A *war-wolf* or *trebuchet* is a device used during the Middle Ages to throw rocks at castles and now sometimes used to fling large vegetables and pianos as a sport. A simple trebuchet is shown in Figure P10.47. Model it as a stiff rod of negligible mass,  $3.00 \text{ m}$  long, joining particles of mass  $m_1 = 0.120 \text{ kg}$  and  $m_2 = 60.0 \text{ kg}$  at its ends. It can turn on a frictionless, horizontal axle perpendicular to the rod and  $14.0 \text{ cm}$  from the large-mass particle. The operator releases the trebuchet from rest in a horizontal orientation. (a) Find the maximum speed that the small-mass object attains. (b) While the small-mass object is gaining speed, does it move with constant acceleration? (c) Does it move with constant tangential acceleration? (d) Does the trebuchet move with constant angular acceleration? (e) Does it have constant momentum? (f) Does the trebuchet-Earth system have constant mechanical energy?

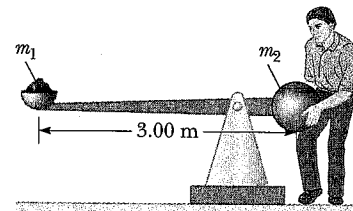


Figure P10.47

### Section 10.8 Energy Considerations in Rotational Motion

48. A horizontal  $800\text{-N}$  merry-go-round is a solid disk of radius  $1.50 \text{ m}$  and is started from rest by a constant horizontal force of  $50.0 \text{ N}$  applied tangentially to the edge of the disk. Find the kinetic energy of the disk after  $3.00 \text{ s}$ .
49. Big Ben, the nickname for the clock in Elizabeth Tower (named after the Queen in 2012) in London, has an hour hand  $2.70 \text{ m}$  long with a mass of  $60.0 \text{ kg}$  and a minute hand  $4.50 \text{ m}$  long with a mass of  $100 \text{ kg}$  (Fig. P10.49). Calculate the total rotational kinetic energy of the two hands about the axis of rotation. (You may

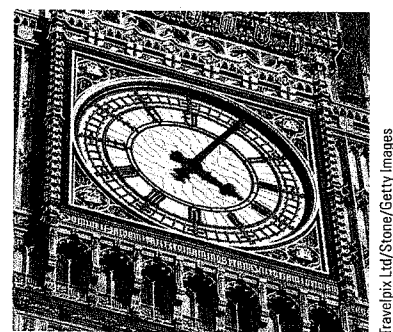


Figure P10.49 Problems 49 and 72.

model the hands as long, thin rods rotated about one end. Assume the hour and minute hands are rotating at a constant rate of one revolution per 12 hours and 60 minutes, respectively.)

50. Consider two objects with  $m_1 > m_2$  connected by a light string that passes over a pulley having a moment of inertia of  $I$  about its axis of rotation as shown in Figure P10.50. The string does not slip on the pulley or stretch. The pulley turns without friction. The two objects are released from rest separated by a vertical distance  $2h$ . (a) Use the principle of conservation of energy to find the translational speeds of the objects as they pass each other. (b) Find the angular speed of the pulley at this time.

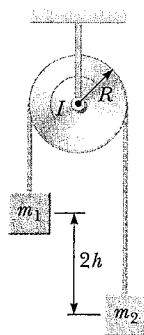


Figure P10.50

51. The top in Figure P10.51 has a moment of inertia of  $4.00 \times 10^{-4} \text{ kg} \cdot \text{m}^2$  and is initially at rest. It is free to rotate about the stationary axis  $AA'$ . A string, wrapped around a peg along the axis of the top, is pulled in such a manner as to maintain a constant tension of 5.57 N. If the string does not slip while it is unwound from the peg, what is the angular speed of the top after 80.0 cm of string has been pulled off the peg?

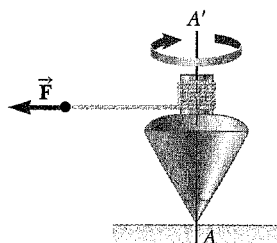


Figure P10.51

52. Why is the following situation impossible? In a large city with an air-pollution problem, a bus has no combustion engine. It runs over its citywide route on energy drawn from a large, rapidly rotating flywheel under the floor of the bus. The flywheel is spun up to its maximum rotation rate of 3 000 rev/min by an electric motor at the bus terminal. Every time the bus speeds up, the flywheel slows down slightly. The bus is equipped with regenerative braking so that the flywheel can speed up when the bus slows down. The flywheel is a uniform solid cylinder with mass 1 200 kg and radius 0.500 m. The bus body does work against air resistance and rolling resistance at the average rate of 25.0 hp as it travels its route with an average speed of 35.0 km/h.
53. In Figure P10.53, the hanging object has a mass of  $m_1 = 0.420 \text{ kg}$ ; the sliding block has a mass of  $m_2 = 0.850 \text{ kg}$ ;

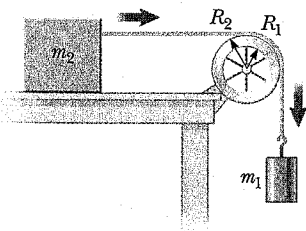


Figure P10.53

and the pulley is a hollow cylinder with a mass of  $M = 0.350 \text{ kg}$ , an inner radius of  $R_1 = 0.0200 \text{ m}$ , and an outer radius of  $R_2 = 0.0300 \text{ m}$ . Assume the mass of the spokes is negligible. The coefficient of kinetic friction between the block and the horizontal surface is  $\mu_k = 0.250$ . The pulley turns without friction on its axle. The light cord does not stretch and does not slip on the pulley. The block has a velocity of  $v_i = 0.820 \text{ m/s}$  toward the pulley when it passes a reference point on the table. (a) Use energy methods to predict its speed after it has moved to a second point, 0.700 m away. (b) Find the angular speed of the pulley at the same moment.

54. **Review.** A thin, cylindrical rod  $\ell = 24.0 \text{ cm}$  long with mass  $m = 1.20 \text{ kg}$  has a ball of diameter  $d = 8.00 \text{ cm}$  and mass  $M = 2.00 \text{ kg}$  attached to one end. The arrangement is originally vertical and stationary, with the ball at the top as shown in Figure P10.54. The combination is free to pivot about the bottom end of the rod after being given a slight nudge. (a) After the combination rotates through 90 degrees, what is its rotational kinetic energy? (b) What is the angular speed of the rod and ball? (c) What is the linear speed of the center of mass of the ball? (d) How does it compare with the speed had the ball fallen freely through the same distance of 28 cm?

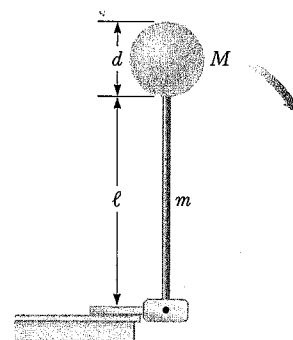


Figure P10.54

55. **Review.** An object with a mass of  $m = 5.10 \text{ kg}$  is attached to the free end of a light string wrapped around a reel of radius  $R = 0.250 \text{ m}$  and mass  $M = 3.00 \text{ kg}$ . The reel is a solid disk, free to rotate in a vertical plane about the horizontal axis passing through its center as shown in Figure P10.55. The suspended object is released from rest 6.00 m above the floor. Determine (a) the tension in the string, (b) the acceleration of the object, and (c) the speed with which the object hits the floor. (d) Verify your answer to part (c) by using the isolated system (energy) model.

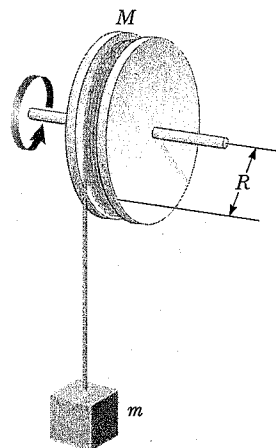


Figure P10.55



56. This problem describes one experimental method for determining the moment of inertia of an irregularly shaped object such as the payload for a satellite. Figure P10.56 shows a counterweight of mass  $m$  suspended by a cord wound around a spool of radius  $r$ , forming part of a turntable supporting the object. The turntable can rotate without friction. When the counterweight is released from rest, it descends through a distance  $h$ , acquiring a speed  $v$ . Show that the moment of inertia  $I$  of the rotating apparatus (including the turntable) is  $mr^2(2gh/v^2 - 1)$ .

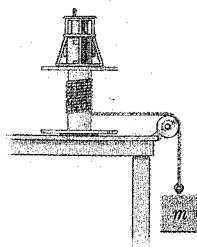


Figure P10.56

57. A uniform solid disk of radius  $R$  and mass  $M$  is free to rotate on a frictionless pivot through a point on its rim (Fig. P10.57). If the disk is released from rest in the position shown by the copper-colored circle, (a) what is the speed of its center of mass when the disk reaches the position indicated by the dashed circle? (b) What is the speed of the lowest point on the disk in the dashed position? (c) **What If?** Repeat part (a) using a uniform hoop.
58. The head of a grass string trimmer has 100 g of cord wound in a light, cylindrical spool with inside diameter 3.00 cm and outside diameter 18.0 cm as shown in Figure P10.58. The cord has a linear density of 10.0 g/m. A single strand of the cord extends 16.0 cm from the outer edge of the spool. (a) When switched on, the trimmer speeds up from 0 to 2 500 rev/min in 0.215 s. What average power is delivered to the head by the trimmer motor while it is accelerating? (b) When the trimmer is cutting grass, it spins at 2 000 rev/min and the grass exerts an average tangential force of 7.65 N on the outer end of the cord, which is still at a radial distance of 16.0 cm from the outer edge of the spool. What is the power delivered to the head under load?

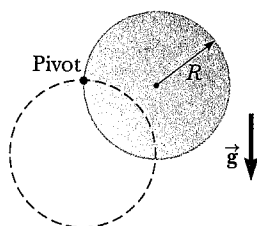


Figure P10.57

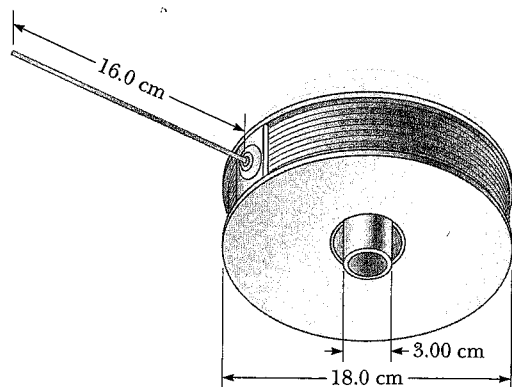


Figure P10.58

## Section 10.9 Rolling Motion of a Rigid Object

59. A cylinder of mass 10.0 kg rolls without slipping on a horizontal surface. At a certain instant, its center of mass has a speed of 10.0 m/s. Determine (a) the translational kinetic energy of its center of mass, (b) the rotational kinetic energy about its center of mass, and (c) its total energy.
60. A solid sphere is released from height  $h$  from the top of an incline making an angle  $\theta$  with the horizontal. Calculate the speed of the sphere when it reaches the bottom of the incline (a) in the case that it rolls without slipping and (b) in the case that it slides frictionlessly without rolling. (c) Compare the time interval required to reach the bottom in cases (a) and (b).
61. (a) Determine the acceleration of the center of mass of a uniform solid disk rolling down an incline making angle  $\theta$  with the horizontal. (b) Compare the acceleration found in part (a) with that of a uniform hoop. (c) What is the minimum coefficient of friction required to maintain pure rolling motion for the disk?
62. A smooth cube of mass  $m$  and edge length  $r$  slides with speed  $v$  on a horizontal surface with negligible friction. The cube then moves up a smooth incline that makes an angle  $\theta$  with the horizontal. A cylinder of mass  $m$  and radius  $r$  rolls without slipping with its center of mass moving with speed  $v$  and encounters an incline of the same angle of inclination but with sufficient friction that the cylinder continues to roll without slipping. (a) Which object will go the greater distance up the incline? (b) Find the difference between the maximum distances the objects travel up the incline. (c) Explain what accounts for this difference in distances traveled.
63. A uniform solid disk and a uniform hoop are placed side by side at the top of an incline of height  $h$ . (a) If they are released from rest and roll without slipping, which object reaches the bottom first? (b) Verify your answer by calculating their speeds when they reach the bottom in terms of  $h$ .
64. A tennis ball is a hollow sphere with a thin wall. It is set rolling without slipping at 4.03 m/s on a horizontal section of a track as shown in Figure P10.64. It rolls around the inside of a vertical circular loop of radius  $r = 45.0$  cm. As the ball nears the bottom of the loop, the shape of the track deviates from a perfect circle so that the ball leaves the track at a point  $h = 20.0$  cm below the horizontal section. (a) Find the ball's speed at the top of the loop. (b) Demonstrate that the ball will not fall from the track at the top of the loop. (c) Find the ball's speed as it leaves the track at the bottom. (d) **What If?** Suppose that static friction between ball and track were

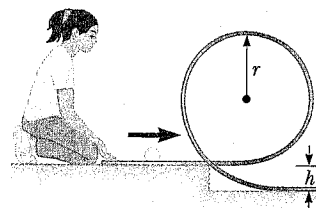


Figure P10.64

negligible so that the ball slid instead of rolling. Would its speed then be higher, lower, or the same at the top of the loop? (e) Explain your answer to part (d).

65. A metal can containing condensed mushroom soup has mass 215 g, height 10.8 cm, and diameter 6.38 cm. It is placed at rest on its side at the top of a 3.00-m-long incline that is at  $25.0^\circ$  to the horizontal and is then released to roll straight down. It reaches the bottom of the incline after 1.50 s. (a) Assuming mechanical energy conservation, calculate the moment of inertia of the can. (b) Which pieces of data, if any, are unnecessary for calculating the solution? (c) Why can't the moment of inertia be calculated from  $I = \frac{1}{2}mr^2$  for the cylindrical can?

### Additional Problems

66. As shown in Figure 10.13 on page 306, toppling chimneys often break apart in midfall because the mortar between the bricks cannot withstand much shear stress. As the chimney begins to fall, shear forces must act on the topmost sections to accelerate them tangentially so that they can keep up with the rotation of the lower part of the stack. For simplicity, let us model the chimney as a uniform rod of length  $\ell$  pivoted at the lower end. The rod starts at rest in a vertical position (with the frictionless pivot at the bottom) and falls over under the influence of gravity. What fraction of the length of the rod has a tangential acceleration greater than  $g \sin \theta$ , where  $\theta$  is the angle the chimney makes with the vertical axis?

67. **Review.** A 4.00-m length of light nylon cord is wound around a uniform cylindrical spool of radius 0.500 m and mass 1.00 kg. The spool is mounted on a frictionless axle and is initially at rest. The cord is pulled from the spool with a constant acceleration of magnitude  $2.50 \text{ m/s}^2$ . (a) How much work has been done on the spool when it reaches an angular speed of  $8.00 \text{ rad/s}$ ? (b) How long does it take the spool to reach this angular speed? (c) How much cord is left on the spool when it reaches this angular speed?

68. An elevator system in a tall building consists of a 800-kg car and a 950-kg counterweight joined by a light cable of constant length that passes over a pulley of mass 280 kg. The pulley, called a sheave, is a solid cylinder of radius 0.700 m turning on a horizontal axle. The cable does not slip on the sheave. A number  $n$  of people, each of mass 80.0 kg, are riding in the elevator car, moving upward at 3.00 m/s and approaching the floor where the car should stop. As an energy-conservation measure, a computer disconnects the elevator motor at just the right moment so that the sheave-car-counterweight system then coasts freely without friction and comes to rest at the floor desired. There it is caught by a simple latch rather than by a massive brake. (a) Determine the distance  $d$  the car coasts upward as a function of  $n$ . Evaluate the distance for (b)  $n = 2$ , (c)  $n = 12$ , and (d)  $n = 0$ . (e) For what integer values of  $n$  does the expression in part (a) apply? (f) Explain your answer to part (e). (g) If an infinite number of people could fit on the elevator, what is the value of  $d$ ?

69. A shaft is turning at  $65.0 \text{ rad/s}$  at time  $t = 0$ . Thereafter, its angular acceleration is given by

$$\alpha = -10.0 - 5.00t$$

where  $\alpha$  is in  $\text{rad/s}^2$  and  $t$  is in seconds. (a) Find the angular speed of the shaft at  $t = 3.00 \text{ s}$ . (b) Through what angle does it turn between  $t = 0$  and  $t = 3.00 \text{ s}$ ?

70. A shaft is turning at angular speed  $\omega$  at time  $t = 0$ . Thereafter, its angular acceleration is given by

$$\alpha = A + Bt$$

(a) Find the angular speed of the shaft at time  $t$ . (b) Through what angle does it turn between  $t = 0$  and  $t$ ?

71. **Review.** A mixing beater consists of three thin rods, each 10.0 cm long. The rods diverge from a central hub, separated from each other by  $120^\circ$ , and all turn in the same plane. A ball is attached to the end of each rod. Each ball has cross-sectional area  $4.00 \text{ cm}^2$  and is so shaped that it has a drag coefficient of 0.600. Calculate the power input required to spin the beater at 1 000 rev/min (a) in air and (b) in water.

72. The hour hand and the minute hand of Big Ben, the Elizabeth Tower clock in London, are 2.70 m and 4.50 m long and have masses of 60.0 kg and 100 kg, respectively (see Fig. P10.49). (a) Determine the total torque due to the weight of these hands about the axis of rotation when the time reads (i) 3:00, (ii) 5:15, (iii) 6:00, (iv) 8:20, and (v) 9:45. (You may model the hands as long, thin, uniform rods.) (b) Determine all times when the total torque about the axis of rotation is zero. Determine the times to the nearest second, solving a transcendental equation numerically.

73. A long, uniform rod of length  $L$  and mass  $M$  is pivoted about a frictionless, horizontal pin through one end. The rod is nudged from rest in a vertical position as shown in Figure P10.73. At the instant the rod is horizontal, find (a) its angular speed, (b) the magnitude of its angular acceleration, (c) the  $x$  and  $y$  components of the acceleration of its center of mass, and (d) the components of the reaction force at the pivot.

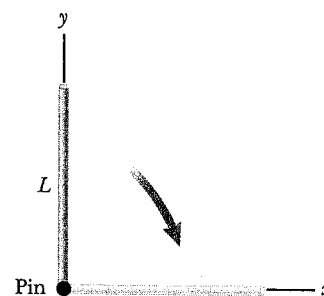


Figure P10.73

74. A bicycle is turned upside down while its owner repairs a flat tire on the rear wheel. A friend spins the front wheel, of radius 0.381 m, and observes that drops of water fly off tangentially in an upward direction when the drops are at the same level as the center of the wheel. She measures the height reached by drops moving vertically (Fig. P10.74 on page 332). A drop



that breaks loose from the tire on one turn rises  $h = 54.0$  cm above the tangent point. A drop that breaks loose on the next turn rises 51.0 cm above the tangent point. The height to which the drops rise decreases because the angular speed of the wheel decreases. From this information, determine the magnitude of the average angular acceleration of the wheel.

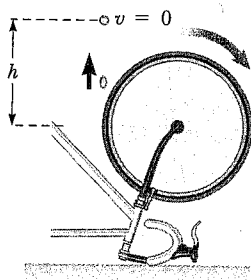


Figure P10.74 Problems 74 and 75.

75. A bicycle is turned upside down while its owner repairs a flat tire on the rear wheel. A friend spins the front wheel, of radius  $R$ , and observes that drops of water fly off tangentially in an upward direction when the drops are at the same level as the center of the wheel. She measures the height reached by drops moving vertically (Fig. P10.74). A drop that breaks loose from the tire on one turn rises a distance  $h_1$  above the tangent point. A drop that breaks loose on the next turn rises a distance  $h_2 < h_1$  above the tangent point. The height to which the drops rise decreases because the angular speed of the wheel decreases. From this information, determine the magnitude of the average angular acceleration of the wheel.

76. (a) What is the rotational kinetic energy of the Earth about its spin axis? Model the Earth as a uniform sphere and use data from the endpapers of this book. (b) The rotational kinetic energy of the Earth is decreasing steadily because of tidal friction. Assuming the rotational period decreases by  $10.0 \mu\text{s}$  each year, find the change in one day.

- 77. Review.** As shown in Figure P10.77, two blocks are connected by a string of negligible mass passing over a pulley of radius  $r = 0.250$  m and moment of inertia  $I$ . The block on the frictionless incline is moving with a constant acceleration of magnitude  $a = 2.00$  m/s<sup>2</sup>. From this information, we wish to find the moment of inertia of the pulley. (a) What analysis model is appropriate for the blocks? (b) What analysis model is appropriate

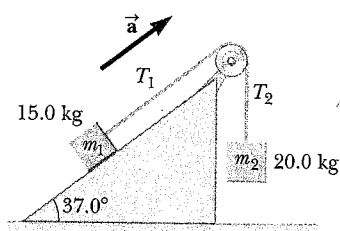


Figure P10.77

for the pulley? (c) From the analysis model in part (b), find the tension  $T_1$ . (d) Similarly, find the tension  $T_2$ . (e) From the analysis model in part (b), find a symbolic expression for the moment of inertia of the pulley in terms of the tensions  $T_1$  and  $T_2$ , the pulley radius  $r$ , and the acceleration  $a$ . (f) Find the numerical value of the moment of inertia of the pulley.

- 78. Review.** A string is wound around a uniform disk of radius  $R$  and mass  $M$ . The disk is released from rest with the string vertical and its top end tied to a fixed bar (Fig. P10.78). Show that (a) the tension in the string is one third of the weight of the disk, (b) the magnitude of the acceleration of the center of mass is  $2g/3$ , and (c) the speed of the center of mass is  $(4gh/3)^{1/2}$  after the disk has descended through distance  $h$ . (d) Verify your answer to part (c) using the energy approach.

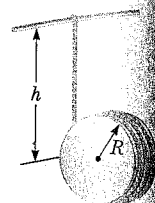


Figure P10.78

79. The reel shown in Figure P10.79 has radius  $R$  and moment of inertia  $I$ . One end of the block of mass  $m$  is connected to a spring of force constant  $k$ , and the other end is fastened to a cord wrapped around the reel. The reel axle and the incline are frictionless. The reel is wound counterclockwise so that the spring stretches a distance  $d$  from its unstretched position and the reel is then released from rest. Find the angular speed of the reel when the spring is again unstretched.

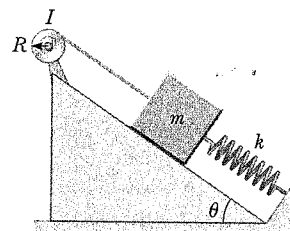


Figure P10.79

80. A common demonstration, illustrated in Figure P10.80, consists of a ball resting at one end of a uniform board of length  $\ell$  that is hinged at the other end and elevated at an angle  $\theta$ . A light cup is attached to the board at  $r_c$  so that it will catch the ball when the support stick is removed suddenly. (a) Show that the ball will lag behind the falling board when  $\theta$  is less than  $35.3^\circ$ .

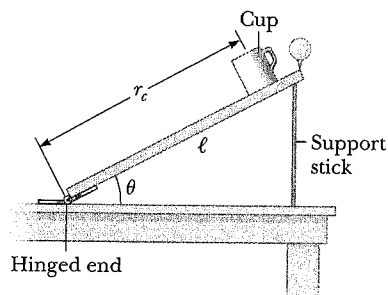


Figure P10.80

(b) Assuming the board is 1.00 m long and is supported at this limiting angle, show that the cup must be 18.4 cm from the moving end.

81. A uniform solid sphere of radius  $r$  is placed on the inside surface of a hemispherical bowl with radius  $R$ . The sphere is released from rest at an angle  $\theta$  to the vertical and rolls without slipping (Fig. P10.81). Determine the angular speed of the sphere when it reaches the bottom of the bowl.

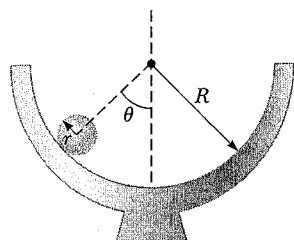


Figure P10.81

82. **Review.** A spool of wire of mass  $M$  and radius  $R$  is unwound under a constant force  $\vec{F}$  (Fig. P10.82). Assuming the spool is a uniform, solid cylinder that doesn't slip, show that (a) the acceleration of the center of mass is  $4\vec{F}/3M$  and (b) the force of friction is to the right and equal in magnitude to  $F/3$ . (c) If the cylinder starts from rest and rolls without slipping, what is the speed of its center of mass after it has rolled through a distance  $d$ ?

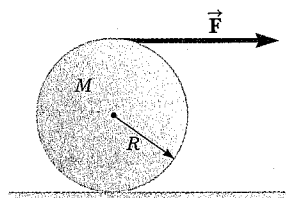


Figure P10.82

83. A solid sphere of mass  $m$  and radius  $r$  rolls without slipping along the track shown in Figure P10.83. It starts from rest with the lowest point of the sphere at height  $h$  above the bottom of the loop of radius  $R$ , much larger than  $r$ . (a) What is the minimum value of  $h$  (in terms of  $R$ ) such that the sphere completes the loop? (b) What are the force components on the sphere at the point  $P$  if  $h = 3R$ ?

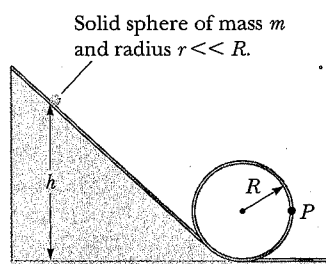


Figure P10.83

84. A thin rod of mass 0.630 kg and length 1.24 m is at rest, hanging vertically from a strong, fixed hinge at its

top end. Suddenly, a horizontal impulsive force  $14.7\hat{i}$  N is applied to it. (a) Suppose the force acts at the bottom end of the rod. Find the acceleration of its center of mass and (b) the horizontal force the hinge exerts. (c) Suppose the force acts at the midpoint of the rod. Find the acceleration of this point and (d) the horizontal hinge reaction force. (e) Where can the impulse be applied so that the hinge will exert no horizontal force? This point is called the *center of percussion*.

85. A thin rod of length  $h$  and mass  $M$  is held vertically with its lower end resting on a frictionless, horizontal surface. The rod is then released to fall freely. (a) Determine the speed of its center of mass just before it hits the horizontal surface. (b) **What If?** Now suppose the rod has a fixed pivot at its lower end. Determine the speed of the rod's center of mass just before it hits the surface.

86. **Review.** A clown balances a small spherical grape at the top of his bald head, which also has the shape of a sphere. After drawing sufficient applause, the grape starts from rest and rolls down without slipping. It will leave contact with the clown's scalp when the radial line joining it to the center of curvature makes what angle with the vertical?

### Challenge Problems

87. A plank with a mass  $M = 6.00$  kg rests on top of two identical, solid, cylindrical rollers that have  $R = 5.00$  cm and  $m = 2.00$  kg (Fig. P10.87). The plank is pulled by a constant horizontal force  $\vec{F}$  of magnitude 6.00 N applied to the end of the plank and perpendicular to the axes of the cylinders (which are parallel). The cylinders roll without slipping on a flat surface. There is also no slipping between the cylinders and the plank. (a) Find the initial acceleration of the plank at the moment the rollers are equidistant from the ends of the plank. (b) Find the acceleration of the rollers at this moment. (c) What friction forces are acting at this moment?

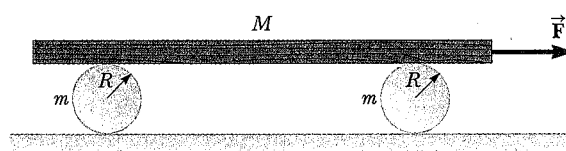


Figure P10.87

88. As a gasoline engine operates, a flywheel turning with the crankshaft stores energy after each fuel explosion, providing the energy required to compress the next charge of fuel and air. For the engine of a certain lawn tractor, suppose a flywheel must be no more than 18.0 cm in diameter. Its thickness, measured along its axis of rotation, must be no larger than 8.00 cm. The flywheel must release energy 60.0 J when its angular speed drops from 800 rev/min to 600 rev/min. Design a sturdy steel (density  $7.85 \times 10^3$  kg/m<sup>3</sup>) flywheel to meet these requirements with the smallest mass you can reasonably attain. Specify the shape and mass of the flywheel.

89. As a result of friction, the angular speed of a wheel changes with time according to

$$\frac{d\theta}{dt} = \omega_0 e^{-\sigma t}$$

where  $\omega_0$  and  $\sigma$  are constants. The angular speed changes from 3.50 rad/s at  $t = 0$  to 2.00 rad/s at  $t = 9.30$  s. (a) Use this information to determine  $\sigma$  and  $\omega_0$ . Then determine (b) the magnitude of the angular acceleration at  $t = 3.00$  s, (c) the number of revolutions the wheel makes in the first 2.50 s, and (d) the number of revolutions it makes before coming to rest.

90. To find the total angular displacement during the playing time of the compact disc in part (B) of Example 10.2, the disc was modeled as a rigid object under constant angular acceleration. In reality, the angular acceleration of a disc is not constant. In this problem, let us explore the actual time dependence of the angular acceleration. (a) Assume the track on the disc is a spiral such that adjacent loops of the track are separated by a small distance  $h$ . Show that the radius  $r$  of a given portion of the track is given by

$$r = r_i + \frac{h\theta}{2\pi}$$

where  $r_i$  is the radius of the innermost portion of the track and  $\theta$  is the angle through which the disc turns to arrive at the location of the track of radius  $r$ . (b) Show that the rate of change of the angle  $\theta$  is given by

$$\frac{d\theta}{dt} = \frac{v}{r_i + (h\theta/2\pi)}$$

where  $v$  is the constant speed with which the disc surface passes the laser. (c) From the result in part (b), use integration to find an expression for the angle  $\theta$  as a function of time. (d) From the result in part (c), use differentiation to find the angular acceleration of the disc as a function of time.

91. A spool of thread consists of a cylinder of radius  $R_1$  with end caps of radius  $R_2$  as depicted in the end view shown in Figure P10.91. The mass of the spool, including the thread, is  $m$ , and its moment of inertia about an axis through its center is  $I$ . The spool is placed on a rough, horizontal surface so that it rolls without slipping when a force  $\vec{T}$  acting to the right is applied to the free end of the thread. (a) Show that the magnitude of the friction force exerted by the surface on the spool is given by

$$f = \left( \frac{I + mR_1R_2}{I + mR_2^2} \right) T$$

- (b) Determine the direction of the force of friction.

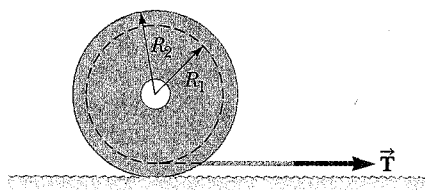


Figure P10.91

92. A cord is wrapped around a pulley that is shaped like a disk of mass  $m$  and radius  $r$ . The cord's free end is connected to a block of mass  $M$ . The block starts from rest and then slides down an incline that makes an angle  $\theta$  with the horizontal as shown in Figure P10.92. The coefficient of kinetic friction between block and incline is  $\mu$ . (a) Use energy methods to show that the block's speed as a function of position  $d$  down the incline is

$$v = \sqrt{\frac{4Mgd(\sin \theta - \mu \cos \theta)}{m + 2M}}$$

- (b) Find the magnitude of the acceleration of the block in terms of  $\mu$ ,  $m$ ,  $M$ ,  $g$ , and  $\theta$ .

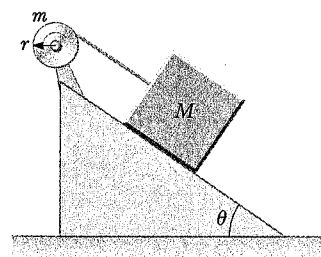


Figure P10.92

93. A merry-go-round is stationary. A dog is running around the merry-go-round on the ground just outside its circumference, moving with a constant angular speed of 0.750 rad/s. The dog does not change his pace when he sees what he has been looking for: a bone resting on the edge of the merry-go-round one third of a revolution in front of him. At the instant the dog sees the bone ( $t = 0$ ), the merry-go-round begins to move in the direction the dog is running, with a constant angular acceleration of 0.0150 rad/s<sup>2</sup>. (a) At what time will the dog first reach the bone? (b) The confused dog keeps running and passes the bone. How long after the merry-go-round starts to turn do the dog and the bone draw even with each other for the second time?

94. A uniform, hollow, cylindrical spool has inside radius  $R/2$ , outside radius  $R$ , and mass  $M$  (Fig. P10.94). It is mounted so that it rotates on a fixed, horizontal axle. A counterweight of mass  $m$  is connected to the end of a string wound around the spool. The counterweight falls from rest at  $t = 0$  to a position  $y$  at time  $t$ . Show that the torque due to the friction forces between spool and axle is

$$\tau_f = R \left[ m \left( g - \frac{2y}{t^2} \right) - M \frac{5y}{4t^2} \right]$$

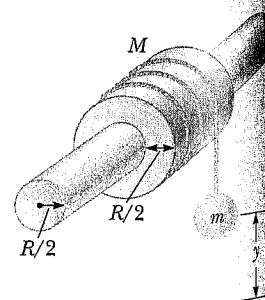


Figure P10.94