Conceptual Questions

1. denotes answer available in Student Solutions Monuol/Study Gui

- 1. Can a normal force do work? If not, why not? If so, give an example.
- 2. Object 1 pushes on object 2 as the objects move together, like a bulldozer pushing a stone. Assume object 1 does 15.0 J of work on object 2. Does object 2 do work on object 1? Explain your answer. If possible, determine how much work and explain your reasoning.
- 3. A student has the idea that the total work done on an object is equal to its final kinetic energy. Is this idea true always, sometimes, or never? If it is sometimes true, under what circumstances? If it is always or never true, explain why.
- **4.** (a) For what values of the angle θ between two vectors is their scalar product positive? (b) For what values of θ is their scalar product negative?
- 5. Can kinetic energy be negative? Explain.
- 6. Discuss the work done by a pitcher throwing a baseball. What is the approximate distance through which the force acts as the ball is thrown?
- 7. Discuss whether any work is being done by each of the following agents and, if so, whether the work is positive or negative. (a) a chicken scratching the ground (b) a person studying (c) a crane lifting a bucket of concrete (d) the gravitational force on the bucket in part (c) (e) the leg muscles of a person in the act of sitting down

- 8. If only one external force acts on a particle, does it nec essarily change the particle's (a) kinetic energy? (b) Its velocity?
- 9. Preparing to clean them, you pop all the removable keys off a computer keyboard. Each key has the shape of a tiny box with one side open. By accident, you spill the keys onto the floor. Explain why many more keys land letter-side down than land open-side down.
- 10. You are reshelving books in a library. You lift a book from the floor to the top shelf. The kinetic energy of the book on the floor was zero and the kinetic energy of the book on the top shelf is zero, so no change occurs in the kinetic energy, yet you did some work in lifting the book. Is the work-kinetic energy theorem violated? Explain.
- 11. A certain uniform spring has spring constant k. Now the spring is cut in half. What is the relationship between k and the spring constant k' of each resulting smaller spring? Explain your reasoning.
- 12. What shape would the graph of U versus x have if a particle were in a region of neutral equilibrium?
- 13. Does the kinetic energy of an object depend on the frame of reference in which its motion is measured? Provide an example to prove this point.
- 14. Cite two examples in which a force is exerted on an object without doing any work on the object.

Problems



The problems found in this chapter may be assigned online in Enhanced WebAssign

- 1. straightforward; 2. intermediate;
- 3. challenging
- 1. full solution available in the Student Solutions Manual/Study Guide
- AMT Analysis Model tutorial available in Enhanced WebAssign
 - GP Guided Problem
 - Master It tutorial available in Enhanced WebAssign
 - W Watch It video solution available in Enhanced WebAssign

Section 7.2 Work Done by a Constant Force

- 1. A shopper in a supermarket pushes a cart with a force of 35.0 N directed at an angle of 25.0° below the horizontal. The force is just sufficient to balance various friction forces, so the cart moves at constant speed. (a) Find the work done by the shopper on the cart as she moves down a 50.0-m-long aisle. (b) The shopper goes down the next aisle, pushing horizontally and maintaining the same speed as before. If the friction force doesn't change, would the shopper's applied force be larger, smaller, or the same? (c) What about the work done on the cart by the shopper?
- 2. A raindrop of mass 3.35×10^{-5} kg falls vertically at W constant speed under the influence of gravity and air resistance. Model the drop as a particle. As it falls

- 100 m, what is the work done on the raindrop (a) by the gravitational force and (b) by air resistance?
- 3. In 1990, Walter Arfeuille of Belgium lifted a 281.5-kg object through a distance of 17.1 cm using only his teeth. (a) How much work was done on the object by Arfeuille in this lift, assuming the object was lifted at constant speed? (b) What total force was exerted on Arfeuille's teeth during the lift?
- 4. The record number of boat lifts, including the boat and its ten crew members, was achieved by Sami Heinonen and Juha Räsänen of Sweden in 2000. They lifted a total mass of 653.2 kg approximately 4 in. off the ground a total of 24 times. Estimate the total work done by the two men on the boat in this record lift, ignoring the negative work done by the men when they lowered the boat back to the ground.

5. A block of mass m = 2.50 kg is pushed a distance d = 2.20 m along a frictionless, horizontal table by a constant applied force of magnitude F = 16.0 N directed at an angle $\theta = 25.0^{\circ}$ below the hori-

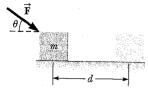


Figure P7.5

zontal as shown in Figure P7.5. Determine the work done on the block by (a) the applied force, (b) the normal force exerted by the table, (c) the gravitational force, and (d) the net force on the block.

6. Spiderman, whose mass is 80.0 kg, is dangling on the free end of a 12.0-m-long rope, the other end of which is fixed to a tree limb above. By repeatedly bending at the waist, he is able to get the rope in motion, eventually getting it to swing enough that he can reach a ledge when the rope makes a 60.0° angle with the vertical. How much work was done by the gravitational force on Spiderman in this maneuver?

Section 7.3 The Scalar Product of Two Vectors

- 7. For any two vectors $\overrightarrow{\mathbf{A}}$ and $\overrightarrow{\mathbf{B}}$, show that $\overrightarrow{\mathbf{A}} \cdot \overrightarrow{\mathbf{B}} = A_x B_x + A_y B_y + A_z B_z$. Suggestions: Write $\overrightarrow{\mathbf{A}}$ and $\overrightarrow{\mathbf{B}}$ in unit-vector form and use Equations 7.4 and 7.5.
- 8. Vector \overrightarrow{A} has a magnitude of 5.00 units, and vector \overrightarrow{B} has a magnitude of 9.00 units. The two vectors make an angle of 50.0° with each other. Find $\overrightarrow{A} \cdot \overrightarrow{B}$.

Note: In Problems 9 through 12, calculate numerical answers to three significant figures as usual.

- 9. For $\vec{A} = 3\hat{i} + \hat{j} \hat{k}$, $\vec{B} = -\hat{i} + 2\hat{j} + 5\hat{k}$, and $\vec{C} = \mathbf{W} 2\hat{j} 3\hat{k}$, find $\vec{C} \cdot (\vec{A} \vec{B})$.
- 10. Find the scalar product of the vectors in Figure P7.10.

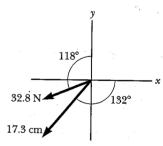


Figure P7.10

- 11. A force $\vec{F} = (6\hat{i} 2\hat{j})$ N acts on a particle that undergoes a displacement $\Delta \vec{r} = (3\hat{i} + \hat{j})$ m. Find (a) the work done by the force on the particle and (b) the angle between \vec{F} and $\Delta \vec{r}$.
- 12. Using the definition of the scalar product, find the angles between (a) $\vec{A} = 3\hat{i} 2\hat{j}$ and $\vec{B} = 4\hat{i} 4\hat{j}$, (b) $\vec{A} = -2\hat{i} + 4\hat{j}$ and $\vec{B} = 3\hat{i} 4\hat{j} + 2\hat{k}$, and (c) $\vec{A} = \hat{i} 2\hat{j} + 2\hat{k}$ and $\vec{B} = 3\hat{j} + 4\hat{k}$.
- 13. Let $\vec{B} = 5.00$ m at 60.0°. Let the vector \vec{C} have the same magnitude as \vec{A} and a direction angle greater than that of \vec{A} by 25.0°. Let $\vec{A} \cdot \vec{B} = 30.0$ m² and $\vec{B} \cdot \vec{C} = 35.0$ m². Find the magnitude and direction of \vec{A} .

Section 7.4 Work Done by a Varying Force

14. The force acting on a particle varies as shown in Figure P7.14. Find the work done by the force on the particle w as it moves (a) from x = 0 to x = 8.00 m, (b) from x = 8.00 m to x = 10.0 m, and (c) from x = 0 to x = 10.0 m.

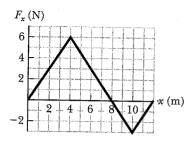


Figure P7.14

15. A particle is subject to a force F_x that varies with position as shown in Figure P7.15. Find the work done by the force on the particle as it moves (a) from x = 0 to x = 5.00 m, (b) from x = 5.00 m to x = 10.0 m, and (c) from x = 10.0 m to x = 15.0 m. (d) What is the total work done by the force over the distance x = 0 to x = 15.0 m?

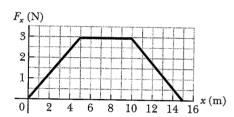


Figure P7.15 Problems 15 and 34.

- 16. In a control system, an accelerometer consists of a 4.70-g object sliding on a calibrated horizontal rail. A low-mass spring attaches the object to a flange at one end of the rail. Grease on the rail makes static friction negligible, but rapidly damps out vibrations of the sliding object. When subject to a steady acceleration of 0.800g, the object should be at a location 0.500 cm away from its equilibrium position. Find the force constant of the spring required for the calibration to be correct.
- I7. When a 4.00-kg object is hung vertically on a ceramit tain light spring that obeys Hooke's law, the spring stretches 2.50 cm. If the 4.00-kg object is removed, (a) how far will the spring stretch if a 1.50-kg block is hung on it? (b) How much work must an external agent do to stretch the same spring 4.00 cm from its unstretched position?
- 18. Hooke's law describes a certain light spring of unstretched length 35.0 cm. When one end is attached to the top of a doorframe and a 7.50-kg object is hung from the other end, the length of the spring is 41.5 cm. (a) Find its spring constant. (b) The load and the spring are taken down. Two people pull in opposite directions on the ends of the spring, each with a force of 190 N. Find the length of the spring in this situation.
- 19. An archer pulls her bowstring back 0.400 m by exerting a force that increases uniformly from zero to 230 N.(a) What is the equivalent spring constant of the bow?

- (b) How much work does the archer do on the string in drawing the bow?
- 20. A light spring with spring constant 1 200 N/m is hung from an elevated support. From its lower end hangs a second light spring, which has spring constant 1 800 N/m. An object of mass 1.50 kg is hung at rest from the lower end of the second spring. (a) Find the total extension distance of the pair of springs. (b) Find the effective spring constant of the pair of springs as a system. We describe these springs as in series.
- 21. A light spring with spring constant k_1 is hung from an elevated support. From its lower end a second light spring is hung, which has spring constant k_2 . An object of mass m is hung at rest from the lower end of the second spring. (a) Find the total extension distance of the pair of springs. (b) Find the effective spring constant of the pair of springs as a system.
- 22. Express the units of the force constant of a spring in SI fundamental units.
- 23. A cafeteria tray dispenser supports a stack of trays on a shelf that hangs from four identical spiral springs under tension, one near each corner of the shelf. Each tray is rectangular, 45.3 cm by 35.6 cm, 0.450 cm thick, and with mass 580 g. (a) Demonstrate that the top tray in the stack can always be at the same height above the floor, however many trays are in the dispenser. (b) Find the spring constant each spring should have for the dispenser to function in this convenient way. (c) Is any piece of data unnecessary for this determination?
- 24. A light spring with force constant 3.85 N/m is compressed by 8.00 cm as it is held between a 0.250-kg block on the left and a 0.500-kg block on the right, both resting on a horizontal surface. The spring exerts a force on each block, tending to push the blocks apart. The blocks are simultaneously released from rest. Find the acceleration with which each block starts to move, given that the coefficient of kinetic friction between each block and the surface is (a) 0, (b) 0.100, and (c) 0.462.
- 25. A small particle of mass *m* is pulled to the top of a frictionless half-cylinder (of radius *R*) by a light cord that passes over the top of the cylinder as illustrated in Figure P7.25. (a) Assuming the particle moves at

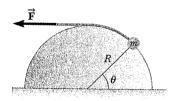


Figure P7.25

- a constant speed, show that $F = mg \cos \theta$. Note: If the particle moves at constant speed, the component of its acceleration tangent to the cylinder must be zero at all times. (b) By directly integrating $W = \int \vec{F} \cdot d\vec{r}$, find the work done in moving the particle at constant speed from the bottom to the top of the half-cylinder.
- 26. The force acting on a particle is $F_x = (8x 16)$, where F is in newtons and x is in meters. (a) Make a plot of this force versus x from x = 0 to x = 3.00 m. (b) From your graph, find the net work done by this force on the particle as it moves from x = 0 to x = 3.00 m.

27. When different loads hang on a spring, the spring stretches to different lengths as shown in the following table. (a) Make a graph of the applied force versus the extension of the spring. (b) By least-squares fitting, determine the straight line that best fits the data (c) To complete part (b), do you want to use all the data points, or should you ignore some of them Explain. (d) From the slope of the best-fit line, find the spring constant k. (e) If the spring is extended to 105 mm, what force does it exert on the suspended object?

F(N)2.0 4.0 6.0 8.0 10 16 18 20 L (mm) 15 32 49 64 79 98 112 126 149 175 190

- 28. A 100-g bullet is fired from a rifle having a barrel 0.600 m long. Choose the origin to be at the location where the bullet begins to move. Then the force (in newtons) exerted by the expanding gas on the bullet is 15 000 + 10 000x 25 000x², where x is in meters (a) Determine the work done by the gas on the bullet as the bullet travels the length of the barrel. (b) What If? If the barrel is 1.00 m long, how much work is done and (c) how does this value compare with the work calculated in part (a)?
- 29. A force $\vec{F} = (4x\hat{i} + 3y\hat{j})$, where \vec{F} is in newtons and \vec{w} and \vec{y} are in meters, acts on an object as the object moves in the x direction from the origin to x = 0

5.00 m. Find the work $W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ done by the force on the object.

30. **Review.** The graph in Figure P7.30 specifies a functional relationship between the two variables u and v. (a) Find $\int_a^b u \, dv$. (b) Find $\int_b^a u \, dv$. (c) Find $\int_a^b v \, du$.

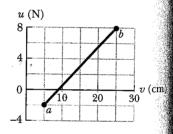


Figure P7.30

Section 7.5 Kinetic Energy and the Work-Kinetic Energy Theorem

- 31. A 3.00-kg object has a velocity $(6.00\,\hat{\mathbf{i}} 2.00\,\hat{\mathbf{j}})$ m/s \mathbf{W} (a) What is its kinetic energy at this moment? (b) What is the net work done on the object if its velocity changes to $(8.00\,\hat{\mathbf{i}} + 4.00\,\hat{\mathbf{j}})$ m/s? (*Note:* From the definition of the dot product, $v^2 = \vec{\mathbf{v}} \cdot \vec{\mathbf{v}}$.)
- 32. A worker pushing a 35.0-kg wooden crate at a constant speed for 12.0 m along a wood floor does 350 J of work by applying a constant horizontal force of magnitude F on the crate. (a) Determine the value of F. (b) If the worker now applies a force greater than F, describe the subsequent motion of the crate. (c) Describe what would happen to the crate if the applied force is less than F.
- 33. A 0.600-kg particle has a speed of 2.00 m/s at point (a) and kinetic energy of 7.50 J at point (b). What is (a) its kinetic energy at (a), (b) its speed at (b), and (c) the net work done on the particle by external forces as it moves from (a) to (b)?

- 34. A 4.00-kg particle is subject to a net force that varies with position as shown in Figure P7.15. The particle starts from rest at x = 0. What is its speed at (a) x = 5.00 m, (b) x = 10.0 m, and (c) x = 15.0 m?
- 35. A 2 100-kg pile driver is used to drive a steel I-beam into the ground. The pile driver falls 5.00 m before coming into contact with the top of the beam, and it drives the beam 12.0 cm farther into the ground before coming to rest. Using energy considerations, calculate the average force the beam exerts on the pile driver while the pile driver is brought to rest.
- 36. Review. In an electron microscope, there is an electron all gun that contains two charged metallic plates 2.80 cm apart. An electric force accelerates each electron in the beam from rest to 9.60% of the speed of light over this distance. (a) Determine the kinetic energy of the electron as it leaves the electron gun. Electrons carry this energy to a phosphorescent viewing screen where the microscope's image is formed, making it glow. For an electron passing between the plates in the electron gun, determine (b) the magnitude of the constant electric force acting on the electron, (c) the acceleration of the electron, and (d) the time interval the electron spends between the plates.
- 37. Review. You can think of the work-kinetic energy theorem as a second theory of motion, parallel to Newton's laws in describing how outside influences affect the motion of an object. In this problem, solve parts (a), (b), and (c) separately from parts (d) and (e) so you can compare the predictions of the two theories. A 15.0-g bullet is accelerated from rest to a speed of 780 m/s in a rifle barrel of length 72.0 cm. (a) Find the kinetic energy of the bullet as it leaves the barrel. (b) Use the work-kinetic energy theorem to find the net work that is done on the bullet. (c) Use your result to part (b) to find the magnitude of the average net force that acted on the bullet while it was in the barrel. (d) Now model the bullet as a particle under constant acceleration. Find the constant acceleration of a bullet that starts from rest and gains a speed of 780 m/s over a distance of 72.0 cm. (e) Modeling the bullet as a particle under a net force, find the net force that acted on it during its acceleration. (f) What conclusion can you draw from comparing your results of parts (c) and (e)?
- 38. Review. A 7.80-g bullet moving at 575 m/s strikes the hand of a superhero, causing the hand to move 5.50 cm in the direction of the bullet's velocity before stopping.

 (a) Use work and energy considerations to find the average force that stops the bullet. (b) Assuming the force is constant, determine how much time elapses between the moment the bullet strikes the hand and the moment it stops moving.
- 39. Review. A 5.75-kg object passes through the origin at time t=0 such that its x component of velocity is 5.00 m/s and its y component of velocity is -3.00 m/s. (a) What is the kinetic energy of the object at this time? (b) At a later time t=2.00 s, the particle is located at x=8.50 m and y=5.00 m. What constant force acted

on the object during this time interval? (c) What is the speed of the particle at t = 2.00 s?

Section 7.6 Potential Energy of a System

- 40. A 1 000-kg roller coaster car is initially at the top of a rise, at point . It then moves 135 ft, at an angle of 40.0° below the horizontal, to a lower point . (a) Choose the car at point . to be the zero configuration for gravitational potential energy of the roller coaster—Earth system. Find the potential energy of the system when the car is at points . and . and the change in potential energy as the car moves between these points. (b) Repeat part (a), setting the zero configuration with the car at point .
- 41. A 0.20-kg stone is held 1.3 m above the top edge of a water well and then dropped into it. The well has a depth of 5.0 m. Relative to the configuration with the stone at the top edge of the well, what is the gravitational potential energy of the stone-Earth system (a) before the stone is released and (b) when it reaches the bottom of the well? (c) What is the change in gravitational potential energy of the system from release to reaching the bottom of the well?
- 42. A 400-N child is in a swing that is attached to a pair of ropes 2.00 m long. Find the gravitational potential energy of the child-Earth system relative to the child's lowest position when (a) the ropes are horizontal, (b) the ropes make a 30.0° angle with the vertical, and (c) the child is at the bottom of the circular arc.

Section 7.7 Conservative and Nonconservative Forces

43. A 4.00-kg particle moves from the origin to position ©, having coordinates x = 5.00 m and y = 5.00 m (Fig. P7.43). One force on the particle is the gravitational force acting in the negative y direction. Using Equation 7.3, calculate the work done by the gravitational force on the particle as it goes from O

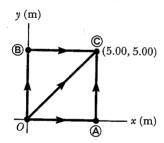


Figure P7.43 Problems 43 through 46.

- to © along (a) the purple path, (b) the red path, and (c) the blue path. (d) Your results should all be identical. Why?
- 44. (a) Suppose a constant force acts on an object. The force does not vary with time or with the position or the velocity of the object. Start with the general definition for work done by a force

$$W = \int_{i}^{f} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$$

and show that the force is conservative. (b) As a special case, suppose the force $\vec{F} = (3\hat{i} + 4\hat{j})$ N acts on a particle that moves from O to O in Figure P7.43. Calculate the work done by \overrightarrow{F} on the particle as it moves along each one of the three paths shown in the figure

and show that the work done along the three paths is identical.

- 45. A force acting on a particle moving in the xy plane is given by $\vec{F} = (2y\hat{i} + x^2\hat{j})$, where \vec{F} is in newtons and x and y are in meters. The particle moves from the origin to a final position having coordinates x = 5.00 m and y = 5.00 m as shown in Figure P7.43. Calculate the work done by \vec{F} on the particle as it moves along (a) the purple path, (b) the red path, and (c) the blue path. (d) Is \vec{F} conservative or nonconservative? (e) Explain your answer to part (d).
- 46. An object moves in the xy plane in Figure P7.43 and experiences a friction force with constant magnitude 3.00 N, always acting in the direction opposite the object's velocity. Calculate the work that you must do to slide the object at constant speed against the friction force as the object moves along (a) the purple path O to © followed by a return purple path to O, (b) the purple path O to © followed by a return blue path to O, and (c) the blue path O to © followed by a return blue path to O. (d) Each of your three answers should be nonzero. What is the significance of this observation?

Section 7.8 Relationship Between Conservative Forces and Potential Energy

- 47. The potential energy of a system of two particles separated by a distance r is given by $\underline{U}(r) = A/r$, where A is a constant. Find the radial force $\overline{\mathbf{F}}_r$ that each particle exerts on the other.
- 48. Why is the following situation impossible? A librarian lifts a book from the ground to a high shelf, doing 20.0 J of work in the lifting process. As he turns his back, the book falls off the shelf back to the ground. The gravitational force from the Earth on the book does 20.0 J of work on the book while it falls. Because the work done was 20.0 J + 20.0 J = 40.0 J, the book hits the ground with 40.0 J of kinetic energy.
- 49. A potential energy function for a system in which a two-dimensional force acts is of the form $U = 3x^3y 7x$. Find the force that acts at the point (x, y).
- 50. A single conservative force acting on a particle within a system varies as $\vec{F} = (-Ax + Bx^2)\hat{i}$, where A and B are constants, \vec{F} is in newtons, and x is in meters. (a) Calculate the potential energy function U(x) associated with this force for the system, taking U = 0 at x = 0. Find (b) the change in potential energy and (c) the change in kinetic energy of the system as the particle moves from x = 2.00 m to x = 3.00 m.
- A single conservative force acts on a 5.00-kg particle within a system due to its interaction with the rest of the system. The equation $F_x = 2x + 4$ describes the force, where F_x is in newtons and x is in meters. As the particle moves along the x axis from x = 1.00 m to x = 5.00 m, calculate (a) the work done by this force on the particle, (b) the change in the potential energy of the system, and (c) the kinetic energy the particle has at x = 5.00 m if its speed is 3.00 m/s at x = 1.00 m.

Section 7.9 Energy Diagrams and Equilibrium of a System

52. For the potential energy curve shown in Figure P7.52, (a) determine whether the force F_x is positive, negative, or zero at the five points indicated. (b) Indicate points of stable, unstable, and neutral equilibrium. (c) Sketch the

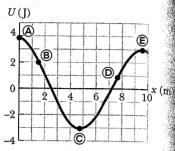


Figure P7.52

curve for F_x versus x from x = 0 to x = 9.5 m.

53. A right circular cone can theoretically be balanced on a horizontal surface in three different ways. Sketch these three equilibrium configurations and identify them as positions of stable, unstable, or neutral equilibrium.

Additional Problems

- 54. The potential energy function for a system of particles is given by $U(x) = -x^3 + 2x^2 + 3x$, where x is the position of one particle in the system. (a) Determine the force F_x on the particle as a function of x. (b) For what values of x is the force equal to zero? (c) Plot U(x) versus x and F_x versus x and indicate points of stable and unstable equilibrium.
- 55. Review. A baseball outfielder throws a 0.150-kg baseball at a speed of 40.0 m/s and an initial angle of 30.0° to the horizontal. What is the kinetic energy of the baseball at the highest point of its trajectory?
- 56. A particle moves along the x axis from x = 12.8 m to x = 23.7 m under the influence of a force

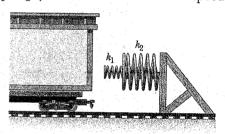
$$F = \frac{375}{x^3 + 3.75x}$$

where F is in newtons and x is in meters. Using numerical integration, determine the work done by this force on the particle during this displacement. Your result should be accurate to within 2%.

57. Two identical steel balls, each of diameter 25.4 mm and moving in opposite directions at 5 m/s, run into each other head-on and bounce apart. Prior to the col lision, one of the balls is squeezed in a vise while precise measurements are made of the resulting amount of compression. The results show that Hooke's law is a fair model of the ball's elastic behavior. For one datum a force of 16 kN exerted by each jaw of the vise results in a 0.2-mm reduction in the diameter. The diameter returns to its original value when the force is removed. (a) Modeling the ball as a spring, find its spring constant. (b) Does the interaction of the balls during the collision last only for an instant or for a nonzero time interval? State your evidence. (c) Compute an estimate for the kinetic energy of each of the balls before they collide. (d) Compute an estimate for the maximum amount of compression each ball undergoes when the balls collide. (e) Compute an order-of-magnitude estimate for the time interval for which the balls are in

contact. (In Chapter 15, you will learn to calculate the contact time interval precisely.)

- 58. When an object is displaced by an amount x from stable equilibrium, a restoring force acts on it, tending to return the object to its equilibrium position. The magnitude of the restoring force can be a complicated function of x. In such cases, we can generally imagine the force function F(x) to be expressed as a power series in x as $F(x) = -(k_1x + k_2x^2 + k_3x^3 + \cdots)$. The first term here is just Hooke's law, which describes the force exerted by a simple spring for small displacements. For small excursions from equilibrium, we generally ignore the higher-order terms, but in some cases it may be desirable to keep the second term as well. If we model the restoring force as $F = -(k_1x + k_2x^2)$, how much work is done on an object in displacing it from x = 0 to $x = x_{\text{max}}$ by an applied force -F?
- 59. A 6 000-kg freight car rolls along rails with negligible friction. The car is brought to rest by a combination of two coiled springs as illustrated in Figure P7.59. Both springs are described by Hooke's law and have spring constants $k_1 = 1\,600\,\text{N/m}$ and $k_2 = 3\,400\,\text{N/m}$. After the first spring compresses a distance of 30.0 cm, the second spring acts with the first to increase the force as additional compression occurs as shown in the graph. The car comes to rest 50.0 cm after first contacting the two-spring system. Find the car's initial speed.



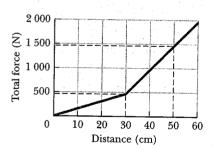


Figure P7.59

60. Why is the following situation impossible? In a new casino, a supersized pinball machine is introduced. Casino advertising boasts that a professional basketball player can lie on top of the machine and his head and feet will not hang off the edge! The ball launcher in the machine sends metal balls up one side of the machine and then into play. The spring in the launcher (Fig. P7.60) has a force constant of 1.20 N/cm. The surface on which the ball moves is inclined $\theta = 10.0^{\circ}$ with respect to the horizontal. The spring is initially compressed its maximum distance d = 5.00 cm. A

ball of mass 100 g is projected into play by releasing the plunger. Casino visitors find the play of the giant machine quite exciting.



Figure P7.60

61. Review. Two constant forces act on an object of mass $m=5.00~{\rm kg}$ moving in the xy plane as shown in Figure P7.61. Force $\vec{\mathbf{F}}_1$ is 25.0 N at 35.0°, and force $\vec{\mathbf{F}}_2$ is 42.0 N at 150°. At time t=0, the object is at the origin and has velocity $(4.00\,\hat{\mathbf{i}}+2.50\,\hat{\mathbf{j}})$ m/s. (a) Express the two forces in unit-vector notation. Use unit-vector notation for your other answers. (b) Find the total force exerted on the object. (c) Find the object's acceleration. Now, considering the instant t=3.00 s, find (d) the

object's velocity, (e) its position, (f) its kinetic energy from $\frac{1}{2}mv_f^2$, and (g) its kinetic energy from $\frac{1}{2}mv_i^2 + \sum \vec{\mathbf{F}} \cdot \Delta \vec{\mathbf{r}}$. (h) What conclusion can you draw by comparing the answers to parts (f) and (g)?

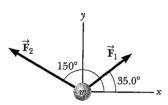


Figure P7.61

- 62. The spring constant of an automotive suspension spring increases with increasing load due to a spring coil that is widest at the bottom, smoothly tapering to a smaller diameter near the top. The result is a softer ride on normal road surfaces from the wider coils, but the car does not bottom out on bumps because when the lower coils collapse, the stiffer coils near the top absorb the load. For such springs, the force exerted by the spring can be empirically found to be given by $F = ax^b$. For a tapered spiral spring that compresses 12.9 cm with a 1 000-N load and 31.5 cm with a 5 000-N load, (a) evaluate the constants a and b in the empirical equation for F and (b) find the work needed to compress the spring 25.0 cm.
- 63. An inclined plane of angle $\theta = 20.0^{\circ}$ has a spring of force constant k = 500 N/m fastened securely at the bottom so that the spring is parallel to the surface as shown in Figure P7.63. A block of mass m = 2.50 kg is placed on the

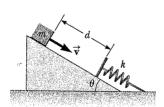


Figure P7.63 Problems 63 and 64.

plane at a distance d = 0.300 m from the spring. From this position, the block is projected downward toward the spring with speed v = 0.750 m/s. By what distance is the spring compressed when the block momentarily comes to rest?

64. An inclined plane of angle θ has a spring of force constant k fastened securely at the bottom so that the

spring is parallel to the surface. A block of mass m is placed on the plane at a distance d from the spring. From this position, the block is projected downward toward the spring with speed v as shown in Figure P7.63. By what distance is the spring compressed when the block momentarily comes to rest?

65. (a) Take U=5 for a system with a particle at position x=0 and calculate the potential energy of the system as a function of the particle position x. The force on the particle is given by $(8e^{-2x})\hat{i}$. (b) Explain whether the force is conservative or nonconservative and how you can tell.

Challenge Problems

66. A particle of mass m = 1.18 kg is attached between two identical springs on a frictionless, horizontal tabletop. Both springs have spring constant k and are initially unstressed, and the particle is at x = 0.
(a) The particle is pulled a distance x along a direction perpendicular to the initial configuration of the springs as shown in Figure P7.66. Show that the force exerted by the springs on the particle is

$$\vec{\mathbf{F}} = -2kx \left(1 - \frac{L}{\sqrt{x^2 + L^2}}\right)\hat{\mathbf{i}}$$

(b) Show that the potential energy of the system is

$$U(x) = kx^{2} + 2kL(L - \sqrt{x^{2} + L^{2}})$$

(c) Make a plot of U(x) versus x and identify all equilibrium points. Assume L = 1.20 m and k = 40.0 N/m. (d) If the particle is pulled 0.500 m to the right and then released, what is its speed when it reaches x = 0?

Overhead view

Figure P7.66

67. Review. A light spring has unstressed length

15.5 cm. It is described by Hooke's law with spring constant 4.30 N/m. One end of the horizontal spring is held on a fixed vertical axle, and the other end is attached to a puck of mass m that can move without friction over a horizontal surface. The puck is set into motion in a circle with a period of 1.30 s. (a) Find the extension of the spring x as it depends on m. Evaluate x for (b) m = 0.070 0 kg, (c) m = 0.140 kg, (d) m = 0.180 kg, and (e) m = 0.190 kg. (f) Describe the pattern of variation of x as it depends on m.