Radio-Frequency Single-Electron Transistor as Readout Device for Qubits: Charge Sensitivity and Backaction

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We study the radio-frequency single-electron transistor (rf-SET) as a readout device for charge qubits. We measure the charge sensitivity of an rf-SET to be $6.3 \mu e/\sqrt{Hz}$ and evaluate the backaction of the rf-SET on a single Cooper-pair box. This allows us to compare the needed measurement time with the mixing time of the qubit imposed by the measurement. We find that the mixing time can be substantially longer than the measurement time, which would allow readout of the state of the qubit in a single-shot measurement.

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A large number of physical systems have been suggested as basic elements for qubits in quantum computers. In terms of scaling the system to a large number of qubits, the solid state systems have clear advantages [1–5]. The qubit system which we will discuss in this paper is the so-called single Cooper-pair box (SCB) [6] (see Fig. 1a), which operates based on the Coulomb blockade [7,8] and so-called single Cooper-pair box (SCB) [6] (see Fig. 1a), qubit system which we will discuss in this paper is the has been suggested as one of the qubit candidates [3].

The Cooper-pair box is a small superconducting island connected via a Josephson junction to a reservoir. It is its charging energy which operates based on the Coulomb blockade [7,8] and so-called single Cooper-pair box (SCB) [6] (see Fig. 1a), qubit system which we will discuss in this paper is the has been suggested as one of the qubit candidates [3].

From this data we can evaluate the measurement time $t_m$, needed to resolve the two states of the qubit. Second, using the experimentally measured quantities for the rf-SET, and assuming that the rf-SET is coupled to an aluminum SCB qubit via a coupling capacitance $C_r$, we calculate the backaction which the measurement would have on the qubit, and thus evaluate the mixing time, $t_{mix}$, in the qubit. This allows us to compare the needed measurement time with the mixing time.

Here we focus on the single Cooper-pair box, but it is important to note that fast detection of subelectron charge variations may be relevant also for other qubit schemes [16], and the results presented here are thus of more general interest.

The measured SET transistor was fabricated by electron-beam lithography and standard two-angle evaporation of aluminum, with oxidation after the first layer to create tunnel junctions. The total resistance of the SET was 44.1 kΩ, and the sum capacitance was $C_{SET} = 370$ aF, corresponding to $E_{SET} = e^2/(2C_{SET}) = 2.5$ K. The current-voltage (IV) characteristic of the SET could be modulated with a gate voltage as shown in Fig. 2. The gate voltage period was $\Delta V_g = 10$ mV corresponding to a gate capacitance $C_g = 16$ aF. The sample was mounted at the mixing chamber of a dilution refrigerator which had a base temperature of about 20 mK.
FIG. 1. (a) The measurement circuit of the radio-frequency single-electron transistor, and how it could be used to read out a qubit consisting of a single Cooper-pair box. (b) Energy levels for the single Cooper-pair box as a function of the external voltage, $V_{qb}$.

The drain of the SET was connected to a chip inductor, while the source was grounded. The value of the inductor (620 nH) was chosen so that it formed a resonant circuit with the capacitance $C_p$ of the substrate pad at a frequency $f_0 \approx 331$ MHz. Figure 1a shows the schematic of the measurement setup. The carrier rf signal was supplied by a network analyzer and launched to the tank circuit via a number of attenuators and a directional coupler.

To evaluate the tank circuit parameters, we applied a large dc current ($1\,\mu$A) through the SET and detected the resulting shot noise with a spectrum analyzer. Good agreement between the calculated and the measured power noise can be seen in the upper left inset of Fig. 2. We estimate the quality factor of the tank circuit to be $Q = 18$ [17], and the bandwidth of the system to be 9 MHz.

A cold amplifier with 24 dB gain was situated in the helium bath. Two more amplifiers with a total gain of 53 dB were placed at room temperature. The reflected and amplified signals could either be measured with a spectrum analyzer, or could be detected with a diode detector and recorded using a sampling oscilloscope.

To evaluate the noise temperature of the whole system, we measured the noise power as a function of the current through the SET (see Fig. 2). From the slope of the linear parts, originating from the shot noise in the SET, we extract the total gain of the system, which was found to be 77.2 dB. From the crossing point of the linear asymptotes, we extract the noise temperature of the amplifier system, which was 10.3 K. Note that when the SET is not loading the tank circuit, i.e., when the SET is in the Coulomb blockade state, the amplifier experiences a different source impedance, and thus the noise contribution from the amplifier is higher, causing the peak at zero bias in the lower right inset of Fig. 2. In this case we get a noise temperature of 12.0 K. The warm amplifiers contribute to the noise temperature with $\approx 1$ K.

For a given carrier signal at the resonance frequency $f_0$, the reflected power from the SET was amplified and detected as a function of $V_g$. The reflected signal showed the typical periodic response as a function of the gate voltage corresponding to the addition of individual electrons to the island [15].

Figure 3 shows the two sidebands of the amplitude-modulated carrier for a gate signal with an amplitude of $1e_{rms}$ and at 2.1 MHz. The noise floor within $\pm 1$ MHz of the main peak is high due to the relatively high phase noise of the carrier source.

Measuring the signal-to-noise ratio (SNR) of the side peak and changing the rf amplitude, we can optimize the rf amplitude. The right inset of Fig. 3 shows the response to a $0.0085e_{rms}$ gate signal at 2 MHz as a function of rf amplitude. The best SNR of 21.7 dB, obtained with a
The reflected power as a function of frequency. The carrier is amplitude modulated by the SET, generating two sidebands, when a signal of 0.1 \( e_{\text{rms}} \) and 2.1 MHz is applied to the gate. Note the very large signal-to-noise ratio, and that the phase noise of the carrier source dominates within \( \pm 1 \) MHz around the main peak. The right inset shows the charge sensitivity as a function of the carrier amplitude with a gate signal corresponding to 0.0085\( e_{\text{rms}} \). The left inset shows the rf-SET response to a 0.2\( e \) step function at the gate, with no averaging.

The ultimate sensitivity should be limited by shot noise, and it has been estimated for a dc operated SET in the normal state to \( \delta \varepsilon = 0.7\hbar \) [18]. For an rf-SET the sensitivity is expected to be a factor 1.4 worse [19]. Assuming that our superconducting rf-SET would have a sensitivity similar to a normal state device, we could thus hope to reach \( \delta \varepsilon = \hbar \).

From our noise measurements in the normal state (see the insets of Fig. 2) we can estimate the shot noise and the amplifier contributions. At the rms current 6.7 nA, which is where we find the optimum sensitivity, we find a shot noise addition of approximately 93 pW (referred to the output of the system, and using a resolution bandwidth of 100 kHz). However, because of the correlated tunneling in the superconducting state, we expect twice as much noise compared to the normal state that is 186 pW. This should now be compared to the noise from the amplifier, which varies from 750 to 880 pW depending on the state of the SET. Summarizing, we estimate that only 20\%–25\% of the noise comes from the shot noise and that the rest comes from the amplifier. This indicates that the shot noise contributes \( \sim 3\hbar \), and that the amplifier contributes \( \sim 10\hbar \) to our \( \delta \varepsilon \).

A factor which further limits the sensitivity of our system is the asymmetric rf bias of the SET. When the rf signal is positive, an extra gate charge of \( C_p V_p \) is added, and for negative rf bias the same gate charge is subtracted. This leads to an effective gate charge difference between the positive and the negative rf bias of the order of 0.2\( e \) (using \( V = 1 \) mV), which leads to a reduction in sensitivity.

The left inset of Fig. 3 shows the time domain response of the rf-SET to a step signal corresponding to 0.2\( e \) at the gate. The bandwidth was limited to 1 MHz and no averaging was done. The noise level is about 0.007\( e_{\text{rms}} \). This very high charge sensitivity and fast response demonstrate the potential of the rf-SET as a readout device of the quantum state in the SCB.

For a given sensitivity and measurement time \( t_m \), the uncertainty in charge is given by \( \Delta q = \delta q / \sqrt{t_m} \). To separate the two qubit states in a real measurement, we need the two intervals \( 0 \pm \Delta q \) and \( 2e \pm \Delta q \) not to overlap. We define the coupling \( \kappa = C_p/C_{qb} \) and we can thus write the needed measuring time \( t_m \) as

\[
\frac{2e}{C_p} = \frac{2\delta q}{\sqrt{t_m}} \Rightarrow t_m = \left( \frac{\delta q}{\kappa e} \right)^2.
\]

During the measurement, the measurement itself and the presence of the rf-SET can cause transitions between the two states. There are a number of different processes that contribute to this mixing [20]. Here we evaluate the effect of the two processes which we consider to be the most important, namely, the shot noise in the SET and the quantum fluctuations of the qubit’s environment. Both of these processes can be described in terms of voltage fluctuations of the SET island. The rate \( \Gamma_1 = 1/t_{\text{mix}} \) for the transitions is proportional to the spectral density of the voltage noise on the SET island \( S_V(\omega) \) at the frequency corresponding to the qubit transition \( \Delta E/\hbar \) (see Fig. 1b), and may be evaluated using standard methods [14,18]

\[
\Gamma_1 = \frac{1}{t_{\text{mix}}} = \frac{e^2}{\hbar^2} \kappa^2 \frac{E_1^2}{\Delta E^2} S_V(\Delta E/\hbar).
\]

Equation (2) is valid in the limit used below, \( \Delta E \gg E_1 \), i.e., away from the degeneracy point of the qubit. Note that the rf carrier has a frequency which is much smaller than the transition frequency and will thus not contribute to the transitions.

The shot noise in the SET may, for low transition frequencies, \( \Delta E/\hbar \ll 1/e = 40 \) GHz, be evaluated within “orthodox” SET theory [21]

\[
S_V(\omega, \omega_t) = 4 \frac{E_{\text{SET}}^2}{e^2} \frac{4\omega_t}{\omega^2 + 16\omega_t^2}.
\]

Here \( \omega_t = 1/e \) is the tunneling rate through the SET. We have assumed a symmetric SET and that only single particle tunneling events are relevant.

In addition to the above fluctuations, the qubit couples to electromagnetic modes in the rf-SET. For high frequencies, \( \Delta E \gg E_{\text{SET}} \), this relaxation process dominates, as can be seen in the inset of Fig. 4. At these frequencies
Making the qubit from niobium instead of aluminum in the qubit can improve SNR by more than an order of magnitude. We would like to acknowledge fruitful discussions with M. Devoret, D. Gunnarsson, K. Bladh, P. Wahlgren, T. Claeson, V. Shumeiko, D. Vion, and K. Lennert, with special thanks to D. Esteve for many enlightening discussions. Samples were made at the Swedish Nanometer Laboratory. We were supported by the Swedish NFR, the Wallenberg and Göran Gustafsson foundations, as well as the European Union under the IST and TMR programmes.

[17] Here we refer to the so-called loaded Q value (Z_LC/R + Z_0/Z_LC)^{-1}, where Z_LC = \sqrt{E/F_p}.