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Large Thermoelectric Effects in Unconventional Superconductors

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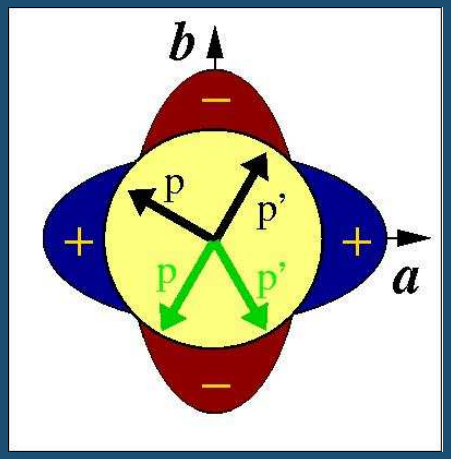
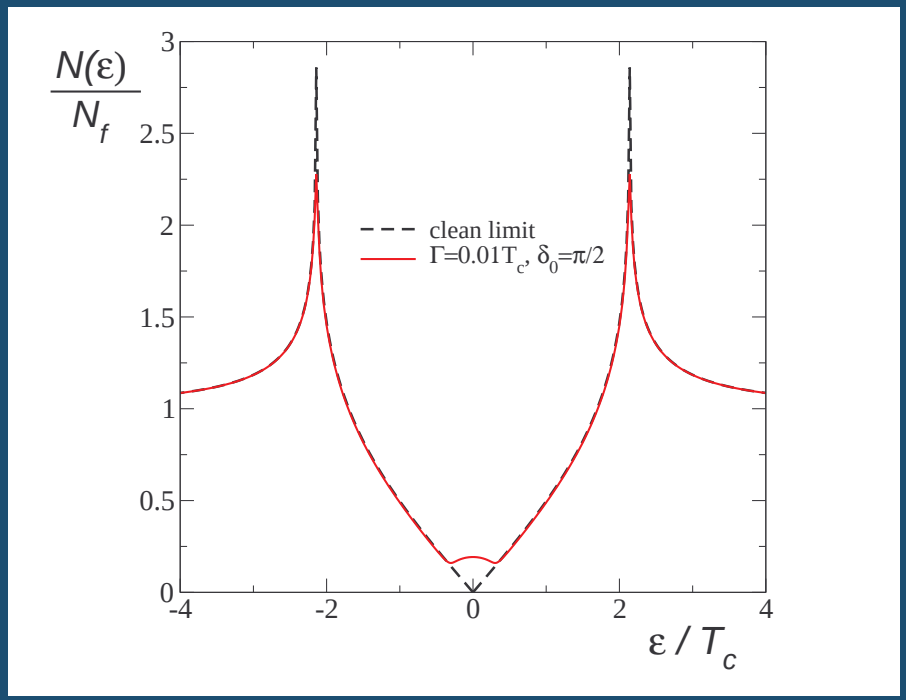
Low energy properties



d-wave symmetry – line nodes
linear low-energy DOS: $N(\varepsilon) \sim N_f\ \varepsilon$
nodal quasiparticles dominate at low temperatures



Impurity scattering is pair breaking
⇨
finite (constant) low energy DOS:

$$N(\varepsilon) \sim N_f\, \gamma(n_i, \delta_0)$$

hierarchy: $T \ll \gamma \ll T_c \ll T_f$

Linear response and transport coefficients

$$\begin{pmatrix} \delta \vec{j}_e \\ \delta \vec{j}_\varepsilon \end{pmatrix} = \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} \dfrac{\vec{E}}{T} \\ \nabla \dfrac{1}{T} \end{pmatrix}$$

ß Electric field in the zero-frequency limit

ß Thermal gradient

Charge conductance: $\sigma = \dfrac{L_{11}}{T}$

Heat conductance: $\kappa = \dfrac{L_{22}}{T^2}$

Thermoelectric coefficient: $\eta = \dfrac{L_{12}}{T^2}$

For a d-wave superconductor the charge and heat conductivities are *universal*, i.e. independent of:

(i) the density of impurities

(ii) the impurity scattering phase shift

The thermoelectric coefficient is zero in conventional superconductors in leading order theory ~T_c/T_f due to electron-hole symmetry

σ: P.A. Lee, Phys. Rev. Lett. 71, 1887 (1993)

κ: M.J. Graf, S.-K. Yip, J.A. Sauls, D. Rainer, Phys. Rev. B 53, 15147 (1996)

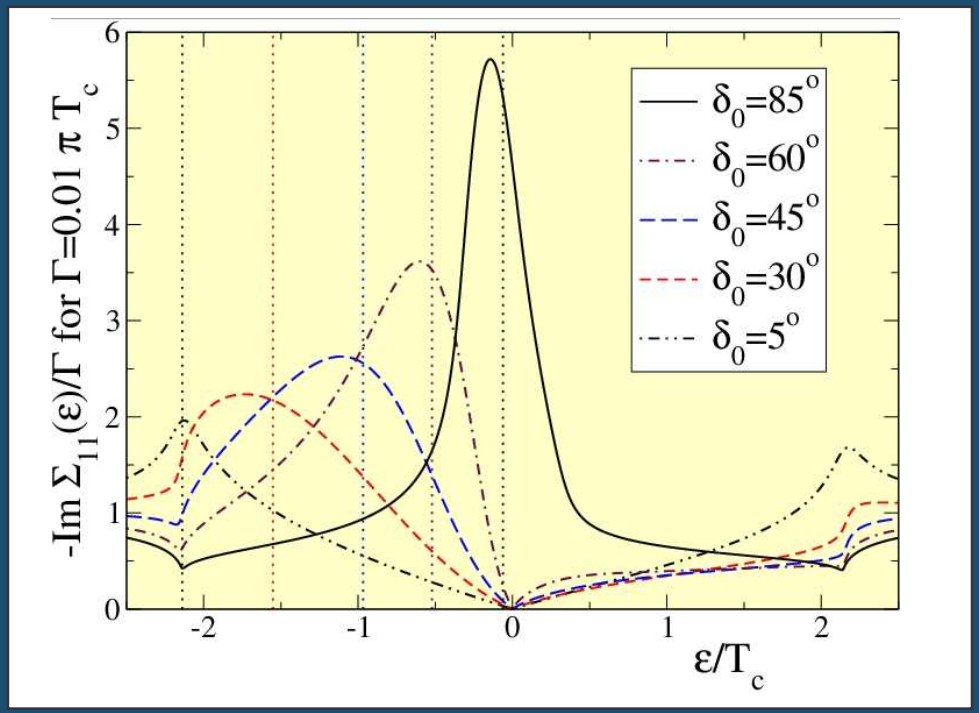
Impurity induced electron-hole *asymmetry*

$$\hat{\Sigma}^R_{imp} = \begin{pmatrix} \Sigma^R_{11} & 0 \\ 0 & \Sigma^R_{22} \end{pmatrix} = \Sigma^R_0 \hat{1} + \Sigma^R_3 \hat{\tau}_3$$

energy renormalization $\tilde{\varepsilon}^R = \varepsilon - \Sigma^R_3$

- *e-h* symmetry breaking unit part
- *leading order*
- vanishes in the normal state
- vanishes in the strict Born and Unitary limits

Imaginary part of electron-like component of impurity self energy

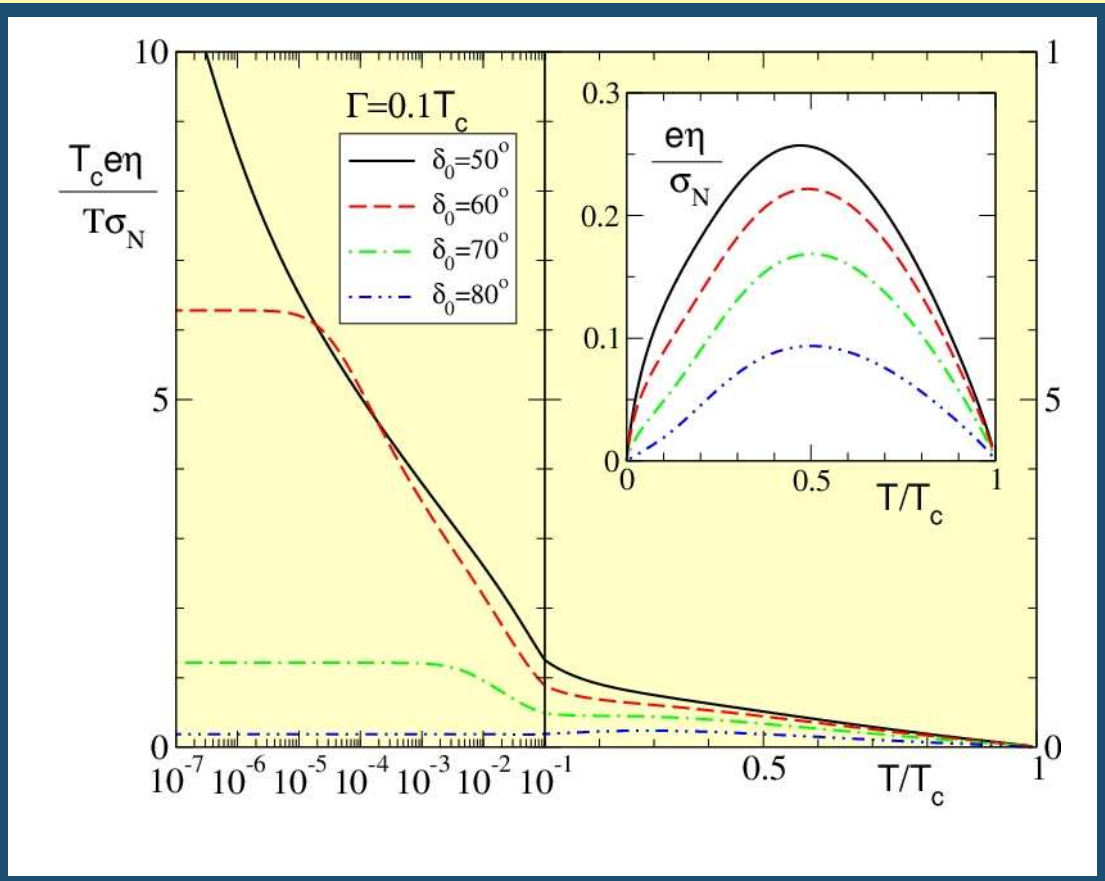


***We assume <u>s-wave</u> scattering off impurities only**

***This mode of breaking e-h symmetry was first discussed by Monien *et al*. Solid State Commun. 63, 263 (1987)**

Thermoelectric response function

$$\eta(T) = -\frac{e}{4T^2}\int d\varepsilon\, \varepsilon\, \mathrm{sech}\frac{\varepsilon}{2T}\int d\,\vec{p}_f\left[v_{f,i}v_{f,j}\right]\frac{N(\vec{p}_f,\varepsilon)\,\Im\Sigma_0^R(\varepsilon)}{\left[\Re\sqrt{\left|\Delta(\vec{p}_f)\right|-\left(\tilde{\varepsilon}^R\right)^2}\right]^2-\left[\Im\Sigma_0^R(\varepsilon)\right]^2}$$



η/T has a non-universal low-T behaviour

Effects of impurity induced e-h asymmetry previously noted in connection to the heavy fermion systems by: B. Arfi, H. Bahlouli, C.J. Pethick, & D. Pines, Phys. Rev. Lett. **60**, 2206 (1988); B. Arfi, H. Bahlouli, & C.J. Pethick, Phys. Rev. B **39**, 8959 (1989)

Their study was only valid in the high-T region assuming

$\gamma << T < T_c$ and $\Im\Sigma_3^R(\varepsilon) << \varepsilon$

At lower temperatures, their theory breaks down since:

$$\Im\Sigma_3^R(\varepsilon) \approx \gamma >> \varepsilon$$

Low-T behavior

Low energy ansatz for T << γ:

$$\tilde{\varepsilon}^R(\varepsilon) = a\varepsilon + i(\gamma + b\varepsilon^2) + d\varepsilon^3 + O(\varepsilon^4), \quad \Im\Sigma_0^R(\varepsilon) = c\varepsilon + h\varepsilon^3 + O(\varepsilon^5)$$

$$\frac{\eta(T << \gamma)}{T} = e\frac{\pi^2}{3}\frac{2N_f v_f^2}{\pi\mu\Delta_0}\left(\frac{c}{\gamma}\right)\left(1 + \frac{7\pi^2}{15}\frac{a_{12}^2 T^2}{\gamma^2}\right) + O(T^4)$$

$$\mu = \frac{1}{\Delta_0}\left|\frac{d\Delta(\phi_{p_f})}{d\phi_{p_f}}\right|_{p_f = p_{f,node}}$$

maximum gap Δ_0

direct dependence on
(i) impurity concentration n_i
(ii) phase shift δ_0

Charge conductance:

$$\sigma(T << \gamma, \omega = 0) = e^2\frac{2N_f v_f^2}{\pi\mu\Delta_0}\left(1 + \frac{\pi^2}{3}\frac{a_{11}^2 T^2}{\gamma^2}\right) + O(T^4)$$

Heat conductance:

$$\frac{\kappa(T << \gamma)}{T} = \frac{\pi^2}{3}\frac{2N_f v_f^2}{\pi\mu\Delta_0}\left(1 + \frac{7\pi^2}{15}\frac{a_{22}^2 T^2}{\gamma^2}\right) + O(T^4)$$

$\delta_0 < \pi/2$ -> Σ_0 modifies T^2-coeff.

$$a_{11}^2 = a^2 + c^2$$

$$a_{22}^2 = a^2 + 2c^2$$

$$a_{12}^2 = a^2 + 2c^2 + \frac{3h\gamma^2}{c} - 3b\gamma$$

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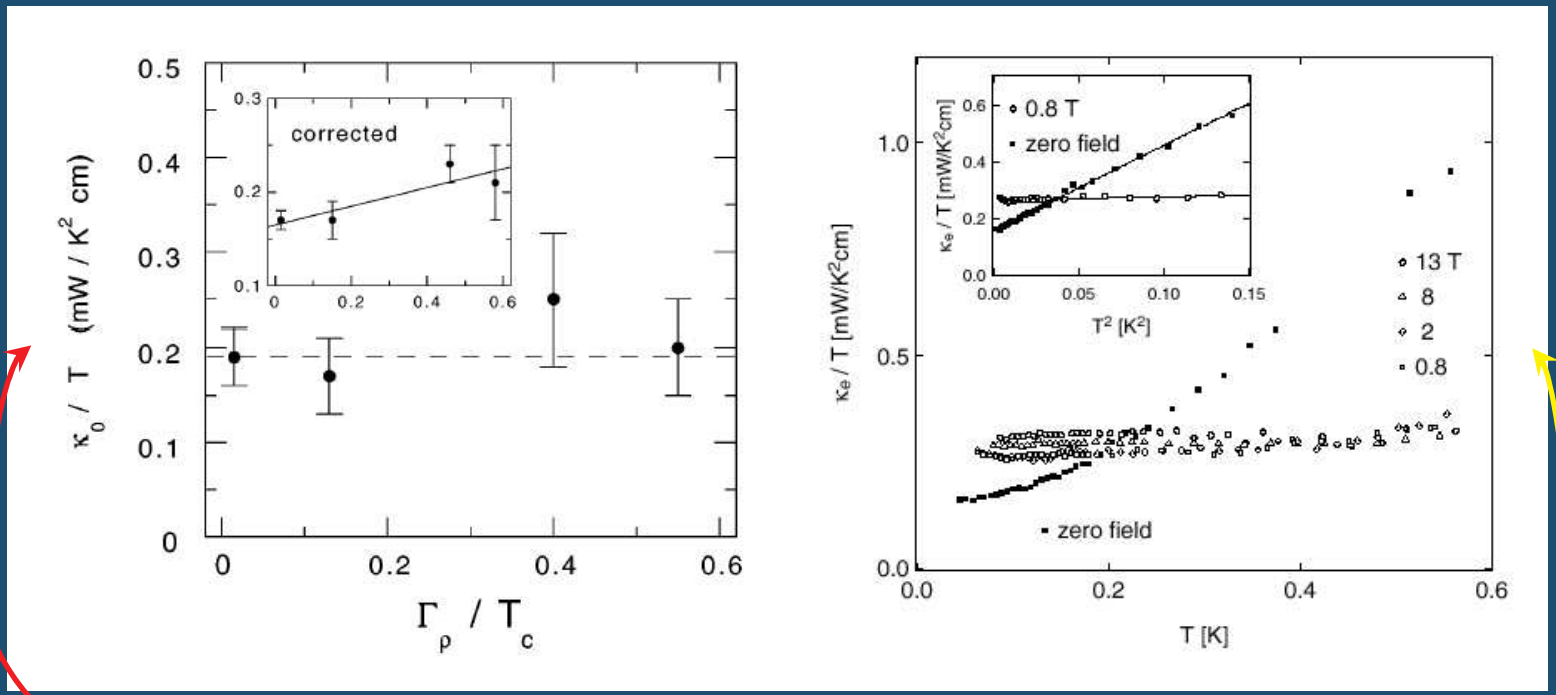
Experiments at low temperatures



L. Taillefer, et al. *Phys. Rev. Lett.* **79**, 483 (1997):
Universal heat conductivity in Zn-doped YBCO single crystals

R. W. Hill, et al. *Phys. Rev. Lett.* **92**, 027001 (2004):
Ultra clean YBCO → intermediate scattering phase shifts

$$\frac{\kappa(T<<\gamma)}{T}=\frac{\pi^2}{3}\frac{2N_f v_f^2}{\pi\mu\Delta_0}\left(1+\frac{7\pi^2}{15}\frac{a_{22}^2T^2}{\gamma^2}\right)+O(T^4)$$

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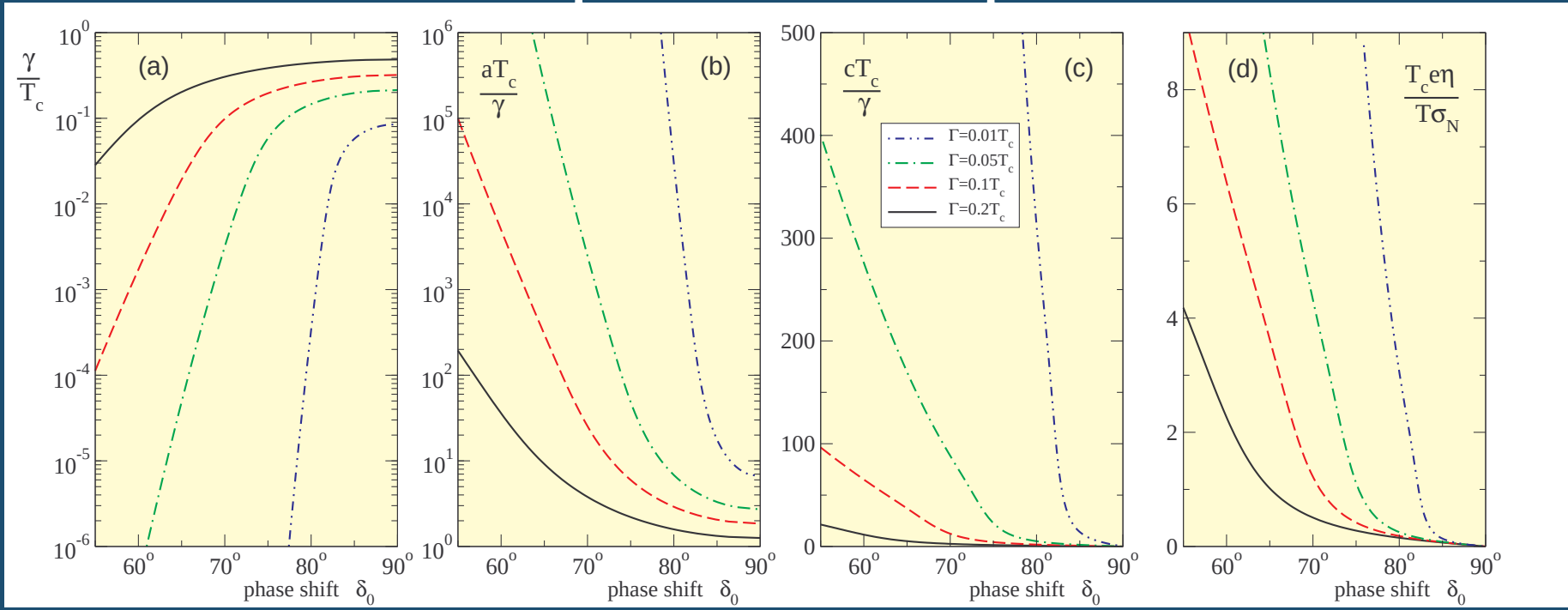
Low-T expansion vs phase shift



$$\sigma(T \ll \gamma, \omega = 0) = e^2 \frac{2N_f v_f^2}{\pi\mu\Delta_0}\left(1 + \frac{\pi^2}{3}\frac{a_{11}^2 T^2}{\gamma^2}\right) + O(T^4)$$

$$\frac{\kappa(T \ll \gamma)}{T} = \frac{\pi^2}{3}\frac{2N_f v_f^2}{\pi\mu\Delta_0}\left(1 + \frac{7\pi^2}{15}\frac{a_{22}^2 T^2}{\gamma^2}\right) + O(T^4)$$

$$\frac{\eta(T \ll \gamma)}{T} = e\frac{\pi^2}{3}\frac{2N_f v_f^2}{\pi\mu\Delta_0}\left(\frac{c}{\gamma}\right)\left(1 + \frac{7\pi^2}{15}\frac{a_{12}^2 T^2}{\gamma^2}\right) + O(T^4)$$

T^2-coefficients depend in general **both** on e-h symmetric and e-h asymmetric parts of the impurity self energy eg: $a_{22}^2 = a^2 + 2c^2$

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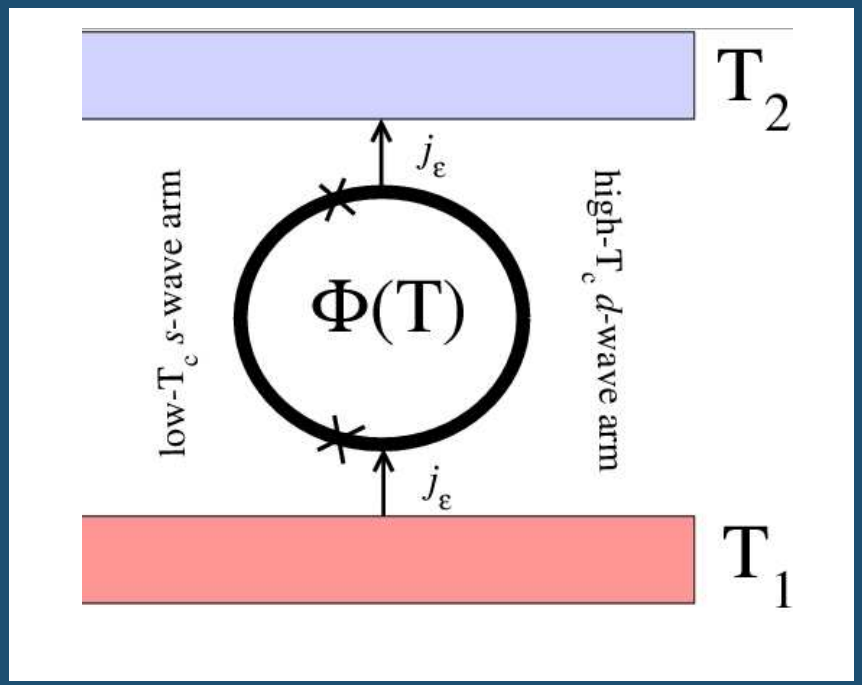
Thermally induced magnetic flux

thermoelectrically induced quasiparticle current: $\delta j_e = -\eta \nabla T$

counterflowing supercurrent: $j_s = (e/m) n_s p_s, \quad p_s = \frac{1}{2}\left(\nabla\chi - \frac{2e}{c}A\right)$



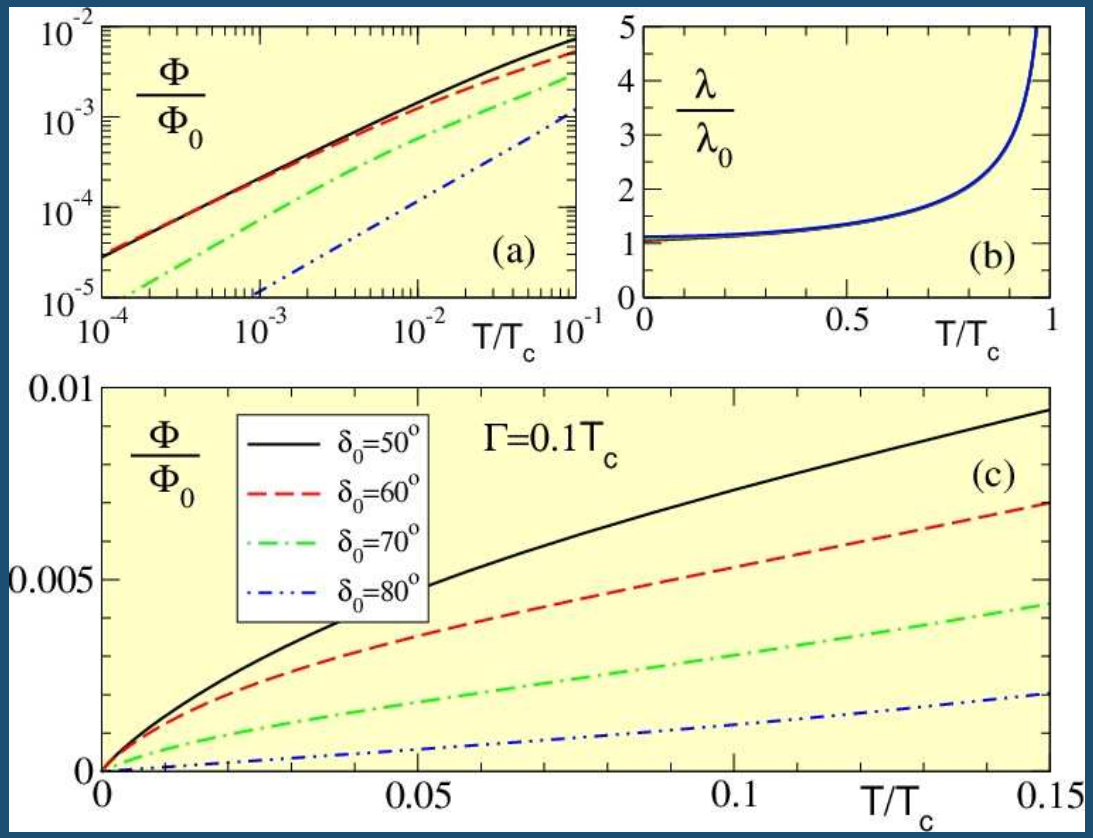
$$\vec{E} = 0 \quad \Rightarrow \quad j_s + \delta j_e = 0$$

$$\boxed{\frac{\Phi(T)}{\Phi_0} = n2\pi + \frac{\lambda(T)^2}{\lambda_0^2}\frac{e\eta(T)}{\sigma_N}\frac{\delta T}{\Gamma}}$$

Estimate: $\Phi \sim 10^{-4}\,\Phi_0$ to $\Phi > \Phi_0$

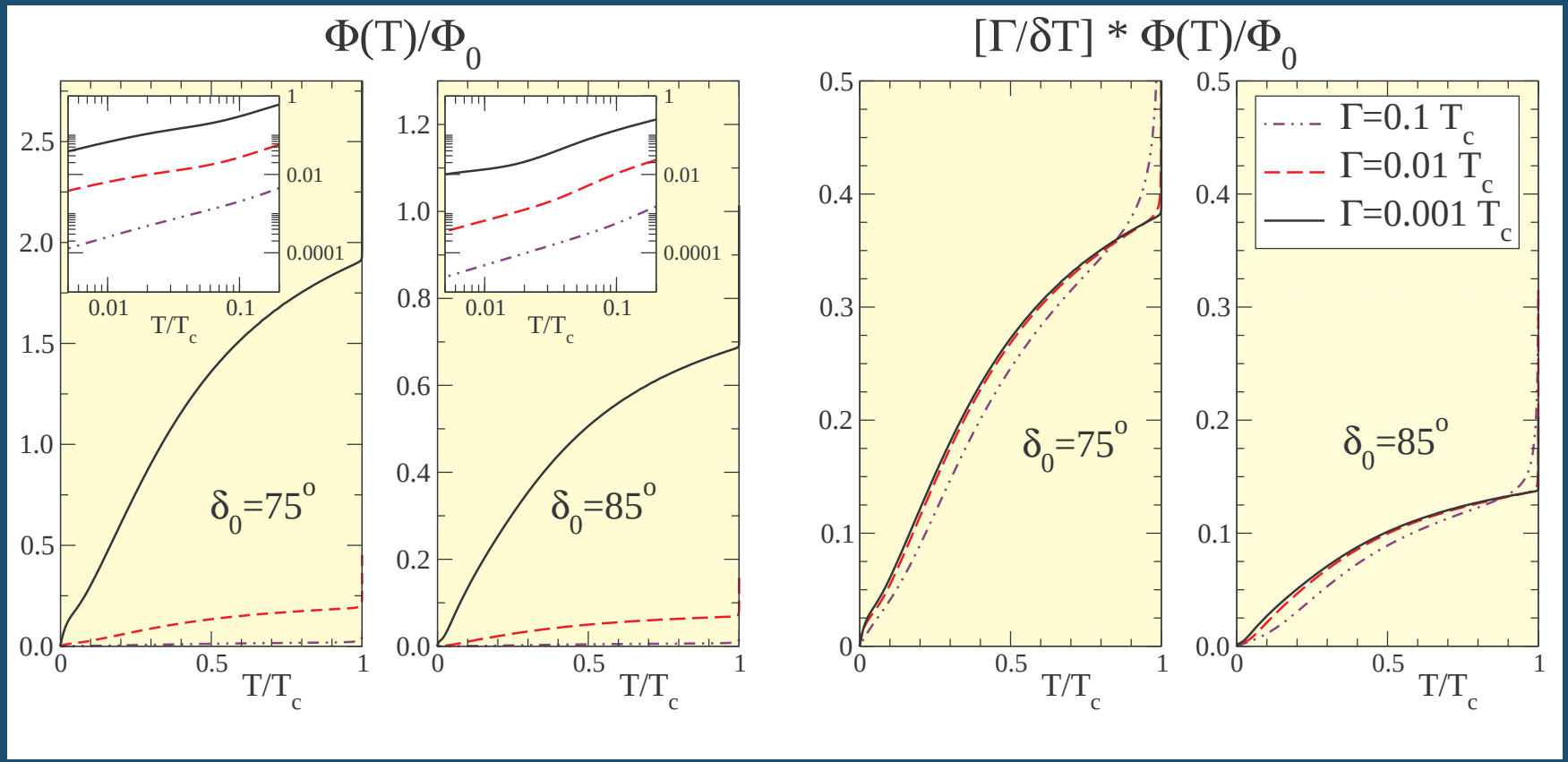
Also: $\frac{v_f p_s}{T_c} = \frac{\xi_0}{L}\frac{\Phi}{\Phi_0}$

Magnetic flux with $\delta T=0.005T_c$



$$\frac{\Phi(T)}{\Phi_0} = n2\pi + \frac{\lambda(T)^2}{\lambda_0^2}\frac{e\eta(T)}{\sigma_N}\frac{\delta T}{\Gamma}$$

Magnetic flux in cleaner systems



$$\frac{\Phi(T)}{\Phi_0} = n2\pi + \frac{\lambda(T)^2}{\lambda_0^2}\frac{e\eta(T)}{\sigma_N}\frac{\delta T}{\Gamma}$$

Summarizing....

- Impurity scattering + unconventional superconductivity lead to non-trivial thermoelectric effects that are large (leading order in small parameters)
  - in particular at low T in clean systems for $\delta_0 < \pi/2$
  - detectable magnetic flux in a hybrid SQUID setup generated by δT

- Analytic expressions for response functions at low temperatures makes it possible to extract material parameters of interest:
  - Gap size, Δ_0
  - Slope of the gap at the gap node, μ
  - Impurity band width, γ
  - Impurity scattering rate, Γ, and phase shift, δ_0

Preprint: cond-mat/0403195

Universal limit: $T << \gamma$ I

- Low energy density of states ~γ
- Reduced phase space for scattering ~$1/\gamma$

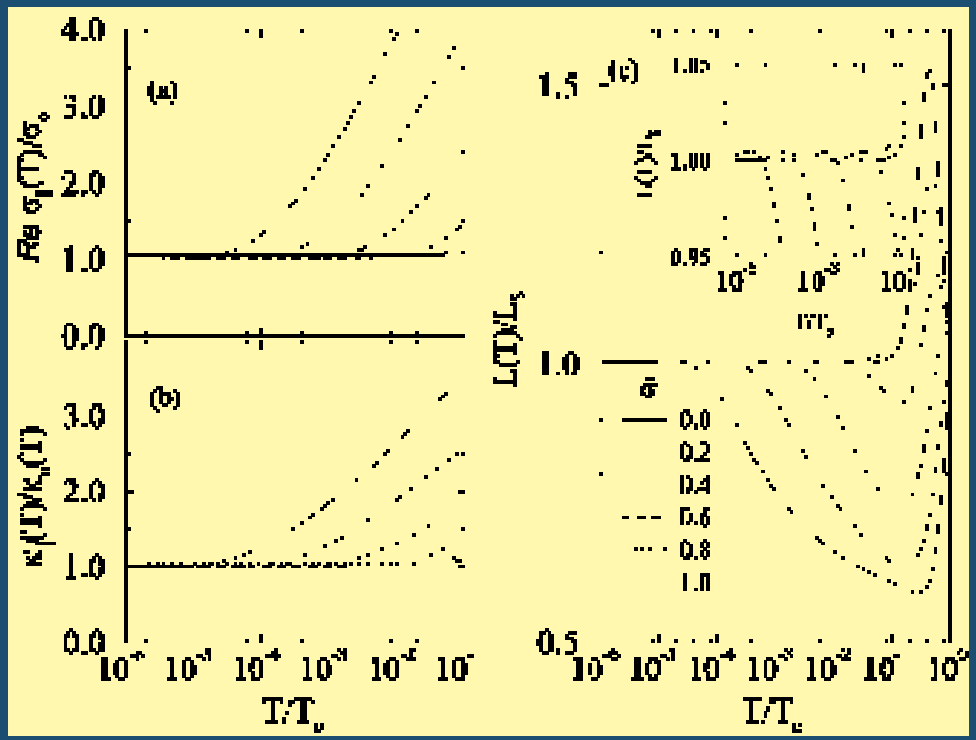
$$\kappa \sim N_f \frac{\cancel{\gamma}}{\Delta_0} k_B^2 T v_f^2 \frac{\hbar}{\cancel{\gamma}} \sim N_f v_f^2 k_B^2 T \frac{\hbar}{\Delta_0}$$

- The charge and heat conductivities are independent of
  - (i) the density of impurities
  - (ii) the impurity scattering phase shift

Charge conductance: P.A. Lee, Phys. Rev. Lett. **71**, 1887 (1993)
Thermal conductance: M.J. Graf, S.-K. Yip, J.A. Sauls, D. Rainer, Phys. Rev. B **53**, 15147 (1996)

Universal limit: $T << \gamma$ II

recovering Wiedermann-Franz law at T<<γ



$$\mathrm{Re}\,\sigma(T,\omega\to 0) = e^2\,\frac{2N_f v_f^2}{\pi\mu\Delta_0}\left(1+\frac{\pi^2}{3}\frac{a_{11}^2T^2}{\gamma^2}\right)$$

$$\kappa(T) = T\,\frac{\pi^2}{3}\,\frac{2N_f v_f^2}{\pi\mu\Delta_0}\left(1+\frac{7\pi^2}{15}\frac{a_{22}^2T^2}{\gamma^2}\right)$$

$$L = \frac{\kappa}{T\,\mathrm{Re}\,\sigma} = \frac{\pi^2}{3e^2}$$

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