Quasi particles in Fermi systems

Kristian Berland

Chalmers, MC2

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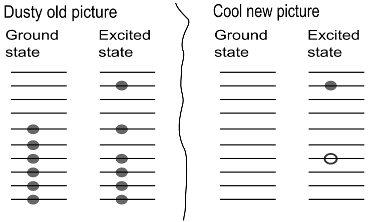
Particle-hole Picture (noninteracting)

- 1: GS: Particle fill up the states below Fermi-level
- 1: ES: Some particles in states above Fermi-level



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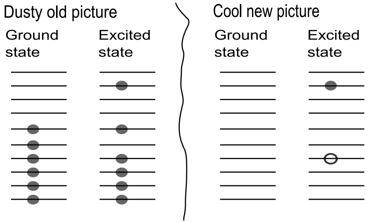

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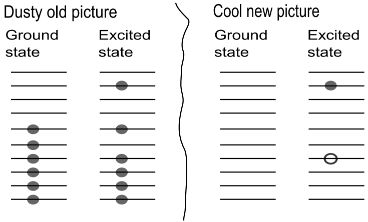
- GS Wave - 1 particle $\rightarrow$ 'Vacuum' $\times \phi_k e^{-i\epsilon_k(-t)}$



Occupation number formalism

States

- Wave-function formalism: LC of $|k_1 k_3 .... k_x\rangle$
- Occupation number formalism: LC of $|n_1 n_2 n_3 ...\rangle$

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- The previous ES:
  $|k_1 k_2 k_3 k_4 k_6 k_9\rangle_{WF} = |1_1 1_2 1_3 1_4 0_5 1_6 0_7 0_8 1_9 0_{10} ....\rangle_{ON}$
- Fancy hole-particle picture: $|1_5^h 1_9^p\rangle$

Creation/destruction operators

Creation/destruction operators:

$$c_i|n_1 n_2....n_i...\rangle = n_i|n_1 n_2...., n_i - 1, ...\rangle$$
$$c_i^\dagger|n_1 n_2....n_i...\rangle = (1 - n_i)|n_1 n_2...., n_i + 1, ...\rangle$$

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- $k_i < k_F : b_i^\dagger = c_i$ Hole-creation operator
- $k_i < k_F : b_i = c_i^\dagger$ Hole-destruction operator

Matrix elements

One particle case:

- WF: $\langle i|\hat{O}|j\rangle = \int \mathrm{d}x \int \mathrm{d}x' \phi_i^*(x) O(x,x') \phi_j(x') = O_{ij}$
- ON: $\langle 0...01_i 0...|\hat{O}^{occ}|0...01_j 0...\rangle = O_{ij} \implies \hat{O}^{occ} = O_{ij} c_i^\dagger c_j + \mathrm{more}$

Peace of mind $\implies$

$$\hat{O}^{occ} = \sum_{mn} O_{mn} c_m^\dagger c_n$$

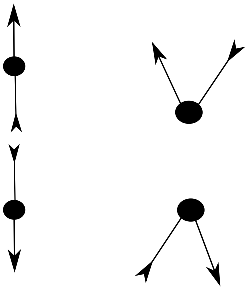
External perturbing potential

$$\hat{H} = \underbrace{\sum_i h_0(\hat{p}_i, \mathbf{r}_i)}_{H_0} + \underbrace{\sum_i V(\mathbf{r}_i)}_{H_1}$$

$$H_0 = \sum_{k>k_f} \epsilon_k a_k^\dagger a_k + \sum_{k<k_f} \epsilon_k b_k b_k^\dagger$$

$$H_1 = \sum_{m,n>k_f} V_{mn} a_m^\dagger a_n + \sum_{m>k_f, n<k_f} V_m n a_m^\dagger b_n^\dagger \quad (1)$$

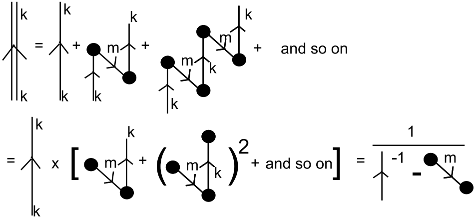
$$+ \sum_{m,n<k_f} V_{mn} b_m b_n^\dagger + \sum_{m>k_f, n<k_f} V_{mn} a_m b_n$$



Propagator in external potential

- Free particle: $\mathrm{i}G_0^+(k, t_2 - t_1) = \theta(t_2 - t_1)\theta(\epsilon_k - \epsilon_F)e^{-\mathrm{i}\epsilon_k(t_2-t_1)}$
- Free hole: $\mathrm{i}G_0^-(k, t_2 - t_1) = -\theta(t_1 - t_2)\theta(\epsilon_F - \epsilon_k)e^{\mathrm{i}\epsilon_k(t_2-t_1)}$
- Free hole in Fourier space (different δ -sign for convergence): $\mathrm{i}G_0^-(k, \omega) = \frac{\mathrm{i}\theta(\epsilon_F-\epsilon_k)}{\omega-\epsilon_k-\mathrm{i}\delta}$

Example: V_{mk} is much larger than the other perturbations

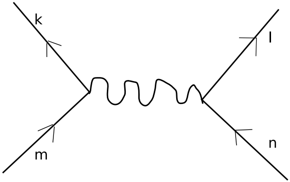


$$G^+(k,\omega) = \frac{1}{\omega - \epsilon_k - \frac{|V_{mk}|^2}{\omega-\epsilon_m} + \mathrm{i}\delta}$$

Self-interacting fermi-system

Two particles in,two goes out. Transition amplitude:

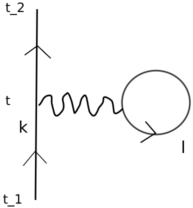
$$V_{klmn} = \int_{\mathbf{r}} \int_{\mathbf{r}'} \phi_k^*(\mathbf{r}) \phi_l^*(\mathbf{r}') V(\mathbf{r}, \mathbf{r}') \phi_m(\mathbf{r}) \phi_n(\mathbf{r}')$$



$$= (-\mathrm{i}) \frac{1}{2} V_{klmn}$$

The bubble diagram

Interpretation: A hole is excited, which lasts 0 seconds.



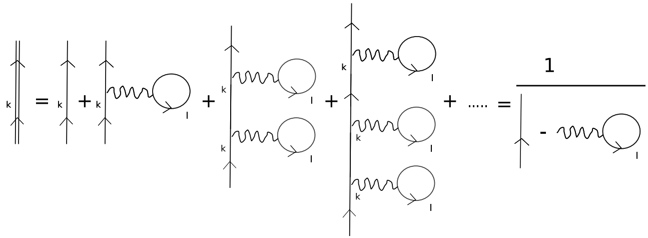
$$=(-1)\int_{-\infty}^{\infty} \mathrm{d}t \sum_{l<k_f} [\mathrm{i}G^+(k,t-t_1)] \left[-\frac{\mathrm{i}}{2} V_{klkl}\right]$$

$$[\mathrm{i}G^-(l,t-t)][\mathrm{i}G^+(k,t_2-t)] = \mathcal{B}.$$

The circle is $\mathrm{i}G^-(l,t-t) = -1$. In Fourier space:

$$\mathcal{B} = [iG_0^+(k,\omega)]^2 \sum_{l<k_f} \left[-\frac{1}{2} V_{klkl}\right] . \tag{2}$$

Hartree -approximation



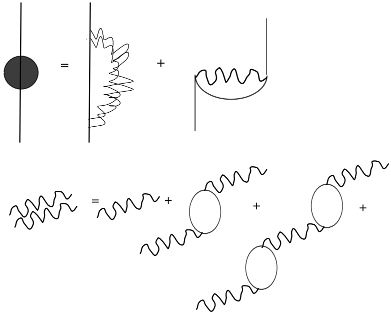
$$G^{+}(k,\omega) = \frac{1}{\omega - \epsilon_k - \sum_{l<k_f} V_{klkl} + \mathrm{i}\delta}$$

$$V_{klkl} = \int_{\mathbf{r}} \phi_k^*(\mathbf{r}) \underbrace{\int_{\mathbf{r}'} \sum_{l<k_f} |\phi_l^*(\mathbf{r}')|^2 V(\mathbf{r}-\mathbf{r}')}_{v_{eff}(\mathbf{r})} \phi_k(\mathbf{r})$$

New effective one-particle problem!

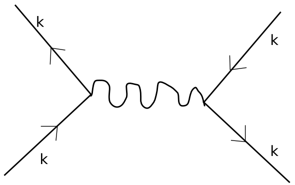
Electron gas: Random phase approximation

Hartree-Fock gives $m^* = 0$. (because of long range Coloumb interaction), another ressummation:



Interprentation: screening!

- Why is the following diagram impossible:



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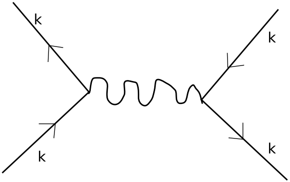

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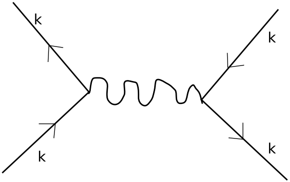

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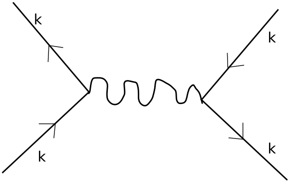

- Why is impulse consvered in vertices and not energy?
- Are particles more real than holes?
- If holes can be interprented as particles going backwards in time: Why is $m_h \neq m_p$?