TABLE III

Self-energies in the first Born approximation

$$\sigma_<^{(1)}(\omega) = M^2 \frac{2\Gamma}{(\omega+\Omega-E_r^0)^2+\Gamma^2} f_{eff}(\omega+\Omega), \qquad \sigma_>^{(1)}(\omega) = M^2 \frac{2\Gamma}{(\omega-\Omega-E_r^0)^2+\Gamma^2}[1-f_{eff}(\omega-\Omega)].$$

$$\mathrm{Im}\sigma_r^{(1)}(\omega) = -\frac{1}{2}\left(\sigma_<^{(1)}(\omega)+\sigma_>^{(1)}(\omega)\right),$$

$$\begin{aligned}\mathrm{Re}\sigma_r^{(1)}(\omega) = & -4\frac{M^2}{\Omega}P_{occ}^0 + M^2 P_{occ}^0 \frac{(\omega+\Omega-E_r^0)}{(\omega+\Omega-E_r^0)^2+\Gamma^2} + M^2 P_{emp}^0 \frac{(\omega-\Omega-E_r^0)}{(\omega-\Omega-E_r^0)^2+\Gamma^2} \\ & - M^2 \frac{1}{(\omega+\Omega-E_r^0)^2+\Gamma^2}\frac{\Gamma_L}{2\pi}\ln\left(\frac{(\omega+\Omega-\mu_L)^2}{(\mu_L-E_r^0)^2+\Gamma^2}\right) - M^2 \frac{1}{(\omega+\Omega-E_r^0)^2+\Gamma^2}\frac{\Gamma_R}{2\pi}\ln\left(\frac{(\omega+\Omega-\mu_R)^2}{(\mu_R-E_r^0)^2+\Gamma^2}\right) \\ & + M^2 \frac{1}{(\omega-\Omega-E_r^0)^2+\Gamma^2}\frac{\Gamma_L}{2\pi}\ln\left(\frac{(\omega-\Omega-\mu_L)^2}{(\mu_L-E_r^0)^2+\Gamma^2}\right) + M^2 \frac{1}{(\omega-\Omega-E_r^0)^2+\Gamma^2}\frac{\Gamma_R}{2\pi}\ln\left(\frac{(\omega-\Omega-\mu_R)^2}{(\mu_R-E_r^0)^2+\Gamma^2}\right).\end{aligned}$$

Using:

$$\Gamma_{L/R} \equiv \Gamma_{L/R}(E_r^0), \qquad \Gamma = \Gamma_L + \Gamma_R, \qquad f_{eff}(\omega) = \frac{\Gamma_L f_L(\omega) + \Gamma_R f_R(\omega)}{\Gamma},$$

$$\begin{aligned} P_{occ}^0 &\equiv \int \frac{d\omega}{2\pi} g_<^0(\omega) = \frac{\Gamma_L}{\Gamma}\left[\frac{1}{2}+\frac{1}{\pi}\arctan\left(\frac{\mu_L-E_r^0}{\Gamma}\right)\right] + \frac{\Gamma_R}{\Gamma}\left[\frac{1}{2}+\frac{1}{\pi}\arctan\left(\frac{\mu_R-E_r^0}{\Gamma}\right)\right], \\ P_{emp}^0 &\equiv \int \frac{d\omega}{2\pi} g_>^0(\omega) = \frac{\Gamma_L}{\Gamma}\left[\frac{1}{2}-\frac{1}{\pi}\arctan\left(\frac{\mu_L-E_r^0}{\Gamma}\right)\right] + \frac{\Gamma_R}{\Gamma}\left[\frac{1}{2}-\frac{1}{\pi}\arctan\left(\frac{\mu_R-E_r^0}{\Gamma}\right)\right]. \end{aligned}$$