HELP TO SOME OF THE QUESTIONS

1 Noise Reduction

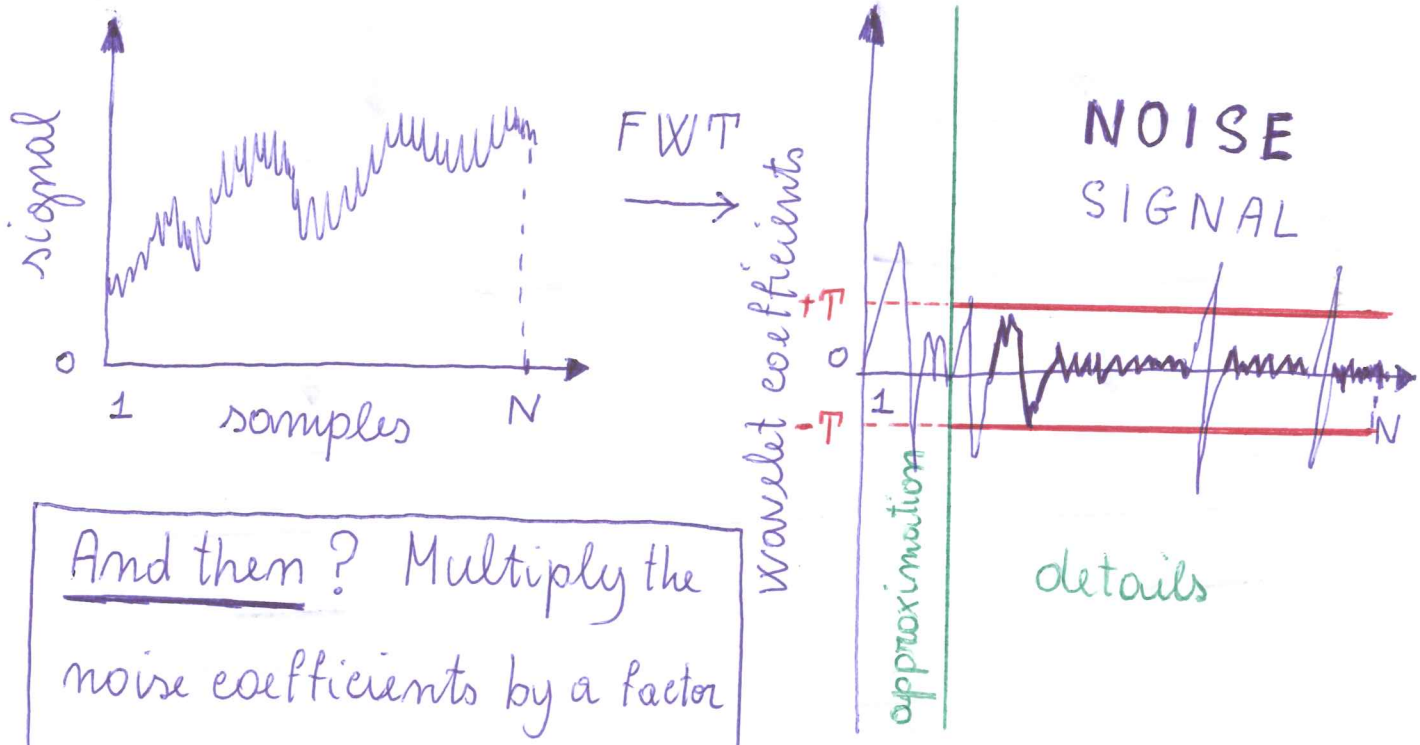
(c) ... assuming that the image has **NO** distortions!

(d) First of all, we should separate signal from noise.

In image space ?... No !

In Fourier space ?... Possible, but not so good ...

In wavelet space ?... YES, by thresholding !

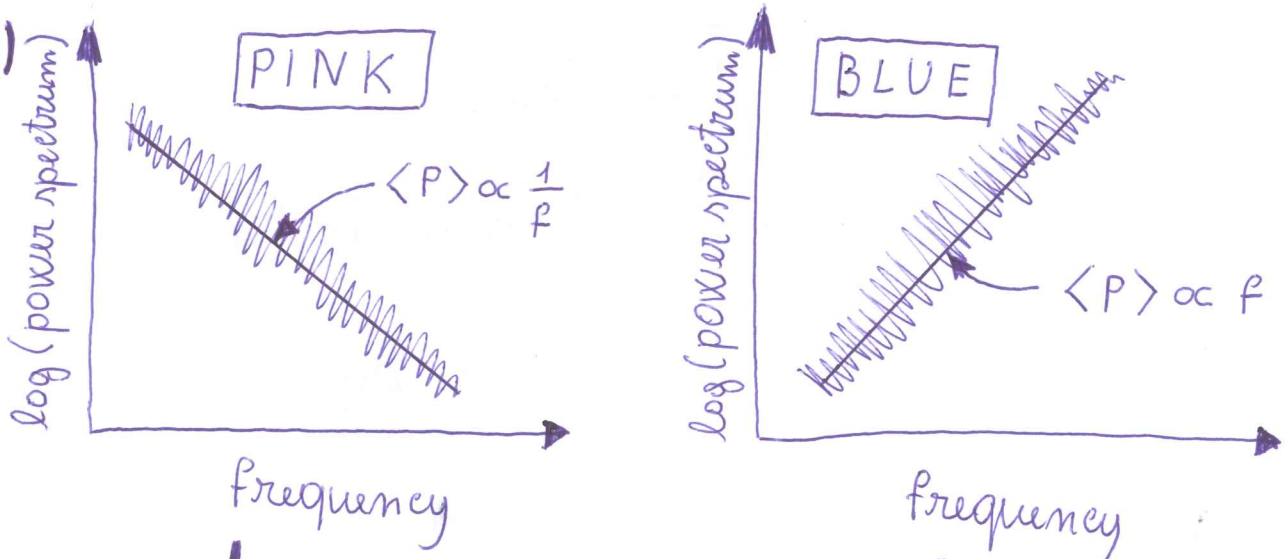


And then ? Multiply the noise coefficients by a factor 1/2 and IFWT !

To think about:

- How can we determine the threshold?
  (The noise is Gaussian, white and additive ...)
- Why threshold only the detail coefficients?
  (Because the approximation ...)
- At which level should we FWT?
  (Size of the wavelet vs. size of the coarsest detail ...)

(e)



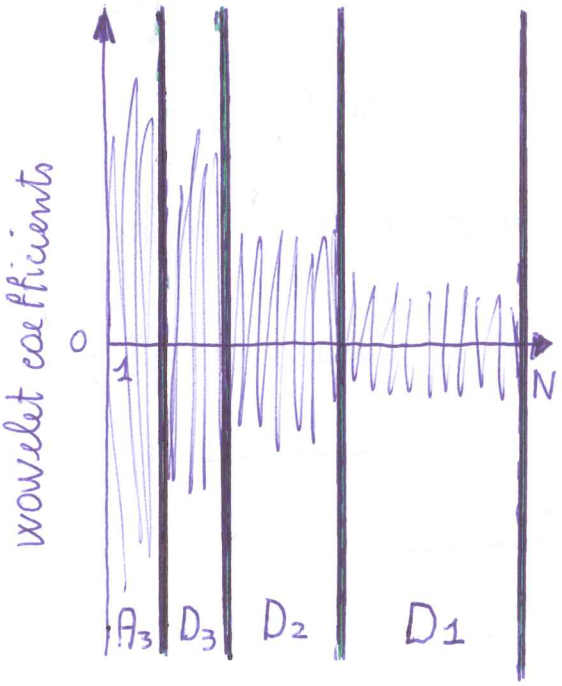
more power at low frequencies

more power at high frequencies

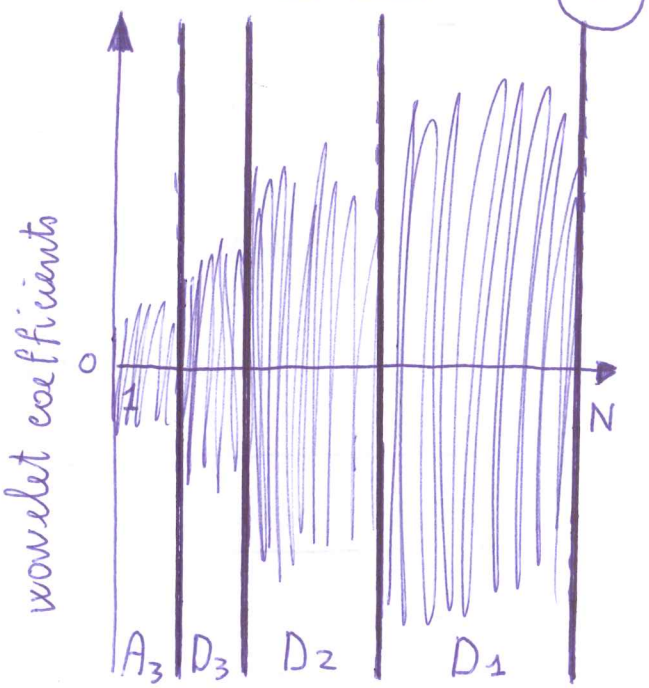
Pink noise: strongly coupled to the signal	The signal, in general, has more power at low frequencies	Blue noise: weakly coupled to the signal

Pink noise is more difficult to remove properly, in general.

PINK



BLUE



Reflect now!

- $\langle$ Fourier power spectrum $\rangle$ $\propto$

  mean square amplitude of the noise at <u>frequency</u> f

  $\propto 1/f^2$

- σ_n = standard deviation of D_n $\propto$

  <u>root</u> mean square amplitude of the noise out <u>scale</u>

  s_n $(= 2^{n-1} \times$ sampling scale$)$

- scale $\propto$ 1 / frequency

→ **<u>Pink noise</u>:**

$\sigma_{n+1} = \underbrace{\sqrt{2}}\ \sigma_n$

<u>Blue noise</u>: ←

$\sigma_{n+1} = \underbrace{1/\sqrt{2}}\ \sigma_n$

If the noise is Gaussian and additive, then we can remove it from an image by thresholding the detail coefficients. In contrast to the case of white noise, now the threshold is scale-dependent:

Pink noise:

$$T_{n+1} = \underbrace{\sqrt{2}}\ T_n$$

Blue noise:

$$T_{n+1} = \underbrace{1/\sqrt{2}}\ T_n$$

In both cases, T_1 can be determined as for white noise:

$$T_1 = \sqrt{2 \ln N}\ \underbrace{\sigma_1}$$

can be robustly estimated through the median absolute deviation of D_1

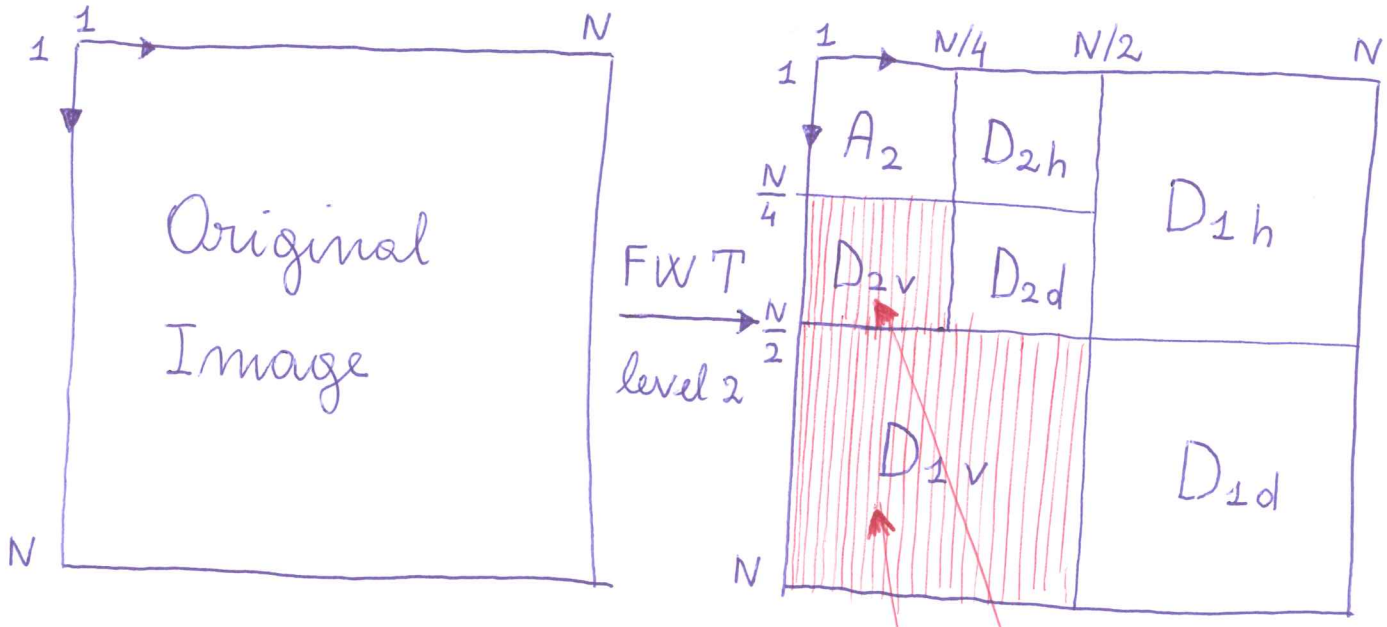
Further thinking:

- Pink vs. Blue ...
- And if the noise is not Gaussian? ...
- And if the noise is not additive? ...
- And if the colour of noise is not known? ...

2 Miscellanea

(a) Which transform is able to decompose an image at various scales and separate vertical features (from horizontal features, etc) ? The fast wavelet transform!

REMEMBER the FWT at level 2 of a house ...



- D_1 = detail coefficients at the sampling scale
- D_2 = detail coefficients at a scale twice as large
  - * h = horizontal
  - * v = vertical
  - * d = diagonal
- A_2 = approximation coefficients

→ The artifacts will appear here and here

How to pre-compress and get rid of the artifacts at the same time: set D_{1v} and D_{2v} to zero!

$$\underbrace{CF}_{\text{compression factor}} = \frac{\text{total number of wavelet coefficients}}{\text{number of wavelet coefficients that are not set to zero}}$$

$$= \frac{N^2}{N^2 - \left(\frac{N}{2}\right)^2 - \left(\frac{N}{4}\right)^2} = \frac{16}{11} \simeq 1.45$$