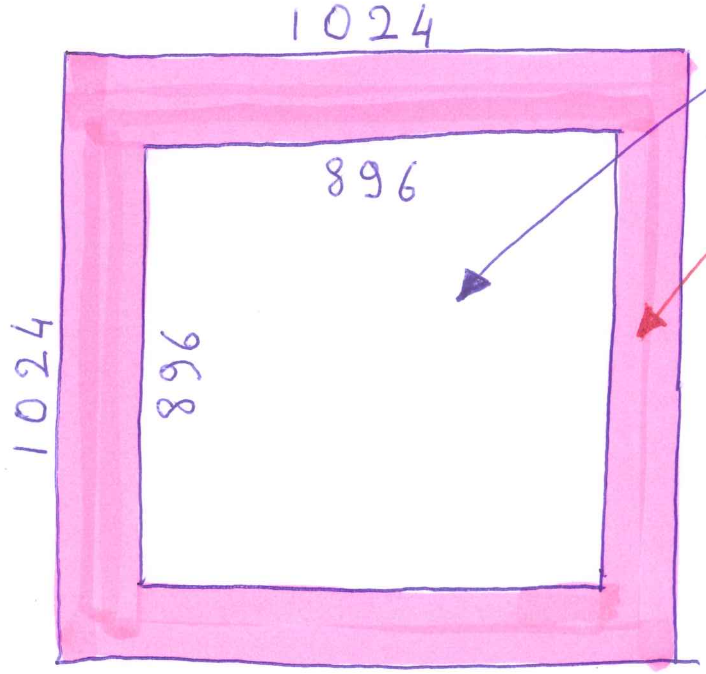
HELP TO SOME OF THE QUESTIONS ①

1 Image Enhancement / Restoration

(e)

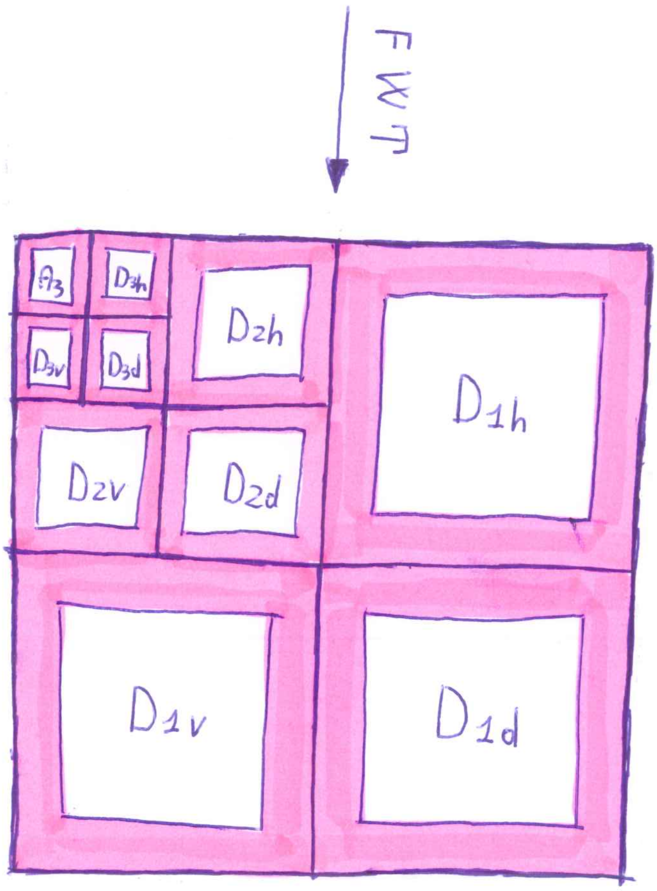


high-quality

polluted by noise:

- additive
- Gaussian
- white
- standard deviation?

→ ... and LOCALIZED!



De-noise the image using the Fast Wavelet Transform

- Choose the wavelet:

bi-orthogonal & quasi-orthogonal

→ bior4.4 or rbio6.8

- Choose the level:

$$2^{l-1} \times \underbrace{\text{wavelet size}}_{\substack{*12 \text{ pixels for bior4.4} \\ *20 \text{ pixels for rbio6.8}}} \approx \underbrace{\text{'frame' thickness}}_{64 \text{ pixels}}$$

→ $l = 3$ in both cases

- FWT the original image at level 3

- Compute the standard deviation of noise:

$$\sigma = \frac{1}{0.6745} \times \text{Median Absolute Deviation} \{D_1\}_{frame}$$

- Compute the threshold:

$$T = \sqrt{2 \ln \underbrace{N_{frame}}_{1024^2 - 896^2}} \; \sigma$$

- Threshold $\{D_1\}_{frame}$, $\{D_2\}_{frame}$ and $\{D_3\}_{frame}$

- IFWT 

2 Miscellanea

(a) We know that:

- Given an image of $M \times N$ pixels, the FFT and the FWT are best computed if $M = 2^m$ and $N = 2^n$, where m and n are positive integers.

- If the size of the image is not a power of two, then the usual recipe is to (zero-) pad:

  * $1010 \times 1020 \rightarrow 1024 \times 1024$
  * $1020 \times 1030 \rightarrow 1024 \times 2048$
  * $1030 \times 1040 \rightarrow 2048 \times 2048$
  * $1025 \times 1025 \rightarrow 2048 \times 2048$

BUT:

- When you want to transform (not to convolve) an image, whatever type of padding you use, it will always produce artifacts.
- Padding 'slows down' the transform.
- Usually, the information contained near the boundaries of an image is irrelevant.

→ 1025 x 1025 crop to 1024 x 1024.

Why? This is like cutting off the outer ≈ 0.1 millimeters from a square image of ≈ 10 centimeters!

→ 1030 x 1040 crop to 1024 x 1024.

Do you think that the information contained in the outer 1–1.5 mm of a 10 cm image is significant?!

... 1020 x 1030 → 1024 x 1024

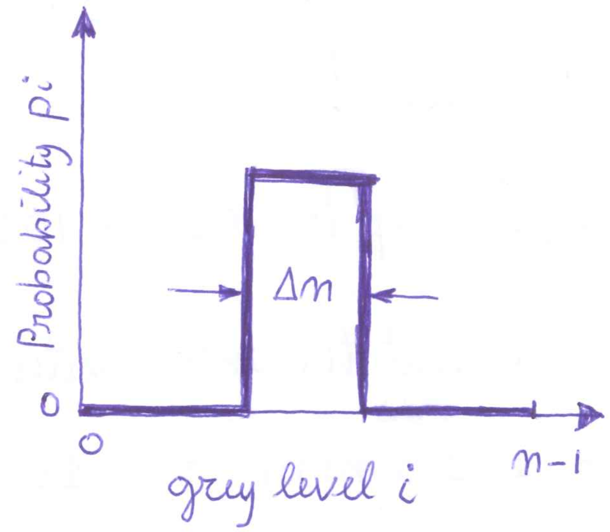
1010 x 1020 → 1024 x 1024

(b)

Low-contrast image → narrow histogram → low single-pixel entropy
Histogram-equalised image → flat histogram → high single-pixel entropy

→ The original image can be compressed more than the enhanced one.

For example, consider the following two 'toy models':

• 

Low-contrast image

* Single-pixel entropy

$$H_1 = -\sum_{i=0}^{n-1} p_i \log_2 p_i$$

$$= -\Delta n \left(\frac{1}{\Delta n} \log_2 \frac{1}{\Delta n} \right)$$

$$= \log_2 \Delta n$$

* Theoretical maximum compression =

$$\frac{\text{\# bits / pixel in the image}}{\text{single-pixel entropy}} = \frac{\log_2 n}{\log_2 \Delta n} > 1$$

→ The smaller Δn, the more the image can be compressed!

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histogram-equalized image

* Single-pixel entropy

$$H_1 = - \sum_{i=0}^{n-1} p_i \log_2 p_i$$

$$= - n \left(\frac{1}{n} \log_2 \frac{1}{n} \right)$$

$$= \log_2 n$$

* Theoretical maximum compression ... =

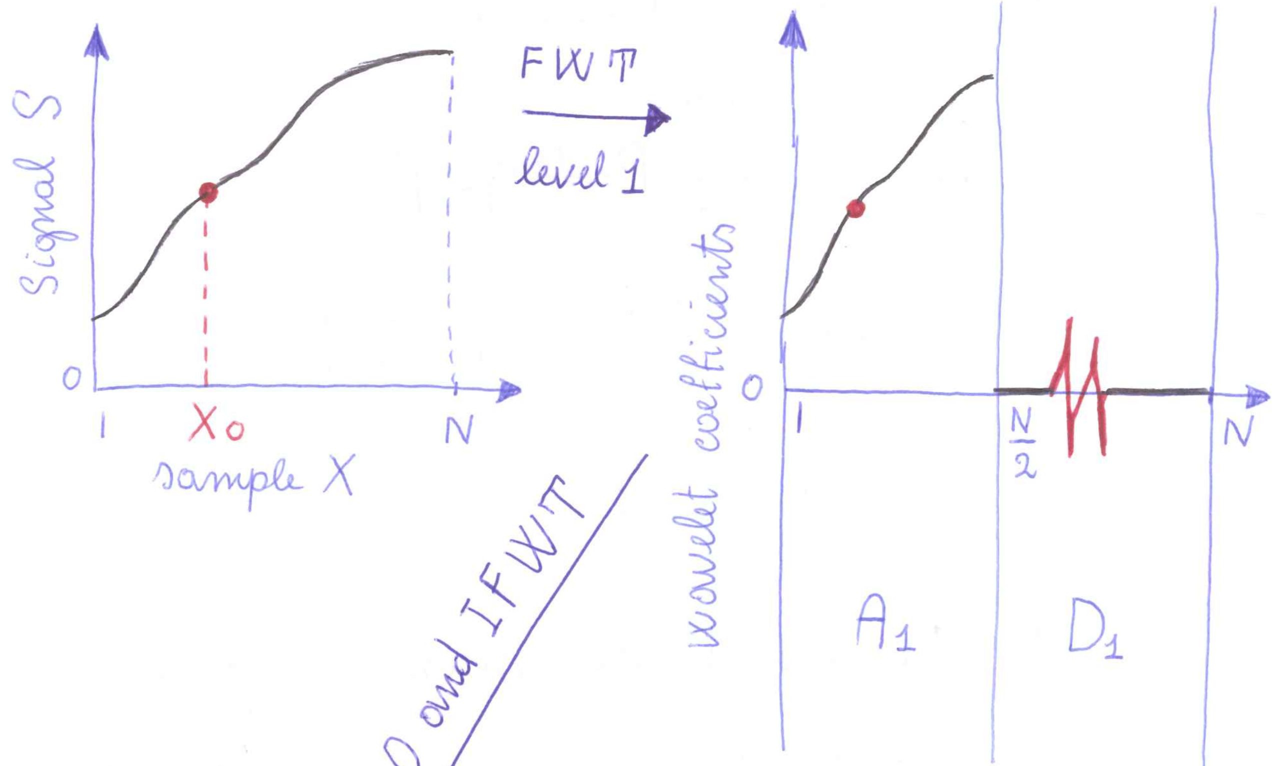
$$\frac{\text{\# bits/pixel in the image}}{\text{single-pixel entropy}} = \frac{\log_2 n}{\log_2 n} = 1$$

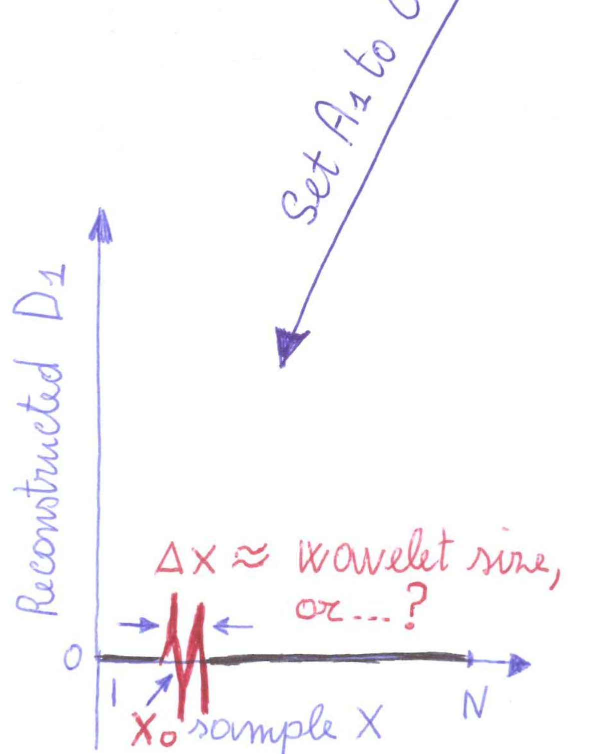
→ <u>No</u> compression!

(c) SMART solution

A wavelet with $n+1$ vanishing moments is 'blind' to polynomials of degree n.

→ Choose a wavelet with 4 vanishing moments, such as the 'FBI' wavelet bior 4.4, and then:





UNDERSTANDING MORE .....

- Why not choose a wavelet with more than 4 vanishing moments?
- Why not FWT at level 2, ...?

STANDARD solution

- The 3rd derivative of the signal shows an <u>edge</u> at $X \approx X_0$!

  (Why not a discontinuous jump at $X = X_0$?)

➡ Compute its 4th derivative and detect the edge!

<u>But how can we compute those derivatives?</u>

- $\underbrace{d_2(X)}_{\text{2nd derivative}} = S(X+1) - 2S(X) + S(X-1)$ ... we know that.
- $\underbrace{d_3(X)}_{\text{3rd derivative}} = d_2(X+1) - d_2(X-1)$ ... centred at X.

  $= S(X+2) - 2S(X+1) + 2S(X-1) - S(X-2)$
- $\underbrace{d_4(X)}_{\text{4th derivative}} = d_2(X+1) - 2d_2(X) + d_2(X-1)$ ... centred at X.

  $= S(X+2) - 4S(X+1) + 6S(X) - 4S(X-1) + S(X-2)$

<u>So what are the corresponding filters?</u>

- $d_2 = [1 \quad -2 \quad 1]$
- $d_3 = [-1 \quad 2 \quad 0 \quad -2 \quad 1]$
- $d_4 = [1 \quad -4 \quad 6 \quad -4 \quad 1]$

➡ WHAT DO WE LEARN?! ⬅