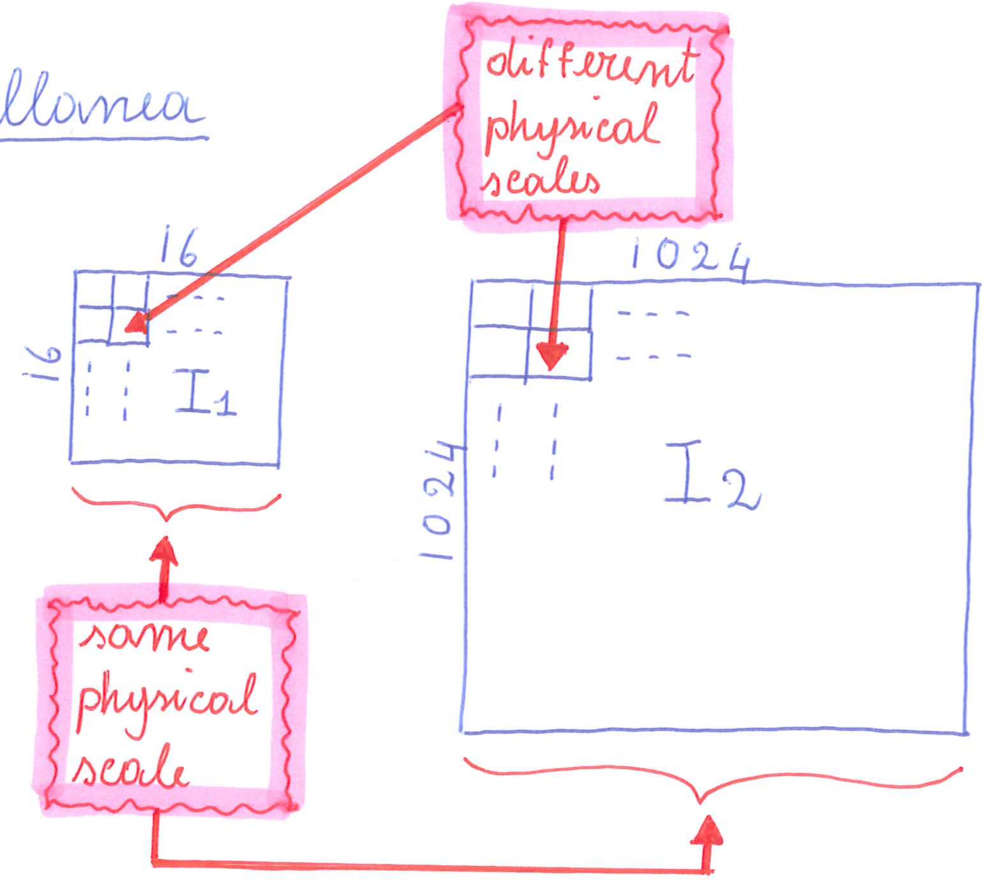
HELP TO SOME OF THE QUESTIONS

2 Miscellanea

(a)



FLOW OF ARGUMENTS

- Although the sizes of I_1 and I_2 are very different, they represent the same physical scale.
- In I_1 a pixel represents a physical scale that is 64 (=1024/16) times larger than in I_2.
- FWT the two images up to the same physical scale, which corresponds to:

* level l in I_1

  $(2^{l-1} \times \text{wavelet size} \approx 16)$,

* level $l+6$ in I_2

  $(6 = \log_2 64)$.

FLOW OF ARGUMENTS

- The compression factor is:

$$CF = \frac{N_W}{n_W} \quad \text{(same notation as in lecture notes)}$$

$$= \frac{N_{pixels}}{N_A + n_d}, \quad \text{where:}$$

* N_A = # approximation coefficients,
* n_d = # detail coefficients that are not set to zero.

- $$CF_1 = \frac{16^2}{\underbrace{N_A}_{\text{level } l} + \underbrace{n_d}_{\text{levels 1 to } l}}$$

- $$CF_2 = \frac{1024^2}{\underbrace{N_A}_{\text{level } l+6} + \underbrace{n_d}_{\text{levels 7 to } l+6} + \underbrace{\Delta n_d}_{\text{levels 1 to 6}}}$$

FLOW OF ARGUMENTS

- If the image is sufficiently regular (more low frequencies than high frequencies), then $\Delta n_d < (\ll)\ n_d$

$$\rightarrow \frac{CF_2}{CF_1} \sim \frac{1024^2}{16^2} \sim 64^2 \ !$$

$\boxed{\textbf{NOTE}}$ $CF_2 \gg CF_1$ even if $\Delta n_d \gtrsim n_d$!!

(b) To avoid significant wrap-around effects, one usually chooses the level l such that:

$$2^{l-1} \times \underbrace{\text{wavelet size}}_{\text{here 12 pixels}} \approx \underbrace{\text{image size}}_{\text{along each direction}}$$

$$\left.\begin{array}{l} \rightarrow l = 1 \text{ in } I_1 \\ \rightarrow l = 7 \text{ in } I_2 \end{array}\right\} \rightarrow N_A = 8^2$$

The theoretical maximum pre-compression can be estimated by setting $\underbrace{n_d = 0 \text{ and } \Delta n_d = 0}_{\text{this corresponds to smoothing}}$!

→ upper bound of $CF_1 = \frac{16^2}{8^2} = 2^2$

→ upper bound of $CF_2 = \frac{1024^2}{8^2} = 128^2$!?

$\boxed{\text{NOTE}}$ These are real upper limits, which can be achieved only if all detail coefficients are set to zero!

EITHER the image is very smooth, OR it gets over-smoothed !!

More advanced solution

What really matters is not the support size of the wavelet, but the range in which the wavelet is significantly different from zero, here 3 pixels

$$\left.\begin{array}{l} \rightarrow \ell = 3 \text{ in } I_1 \\ \rightarrow \ell = 9 \text{ in } I_2 \end{array}\right\} \rightarrow N_A = 2^2$$

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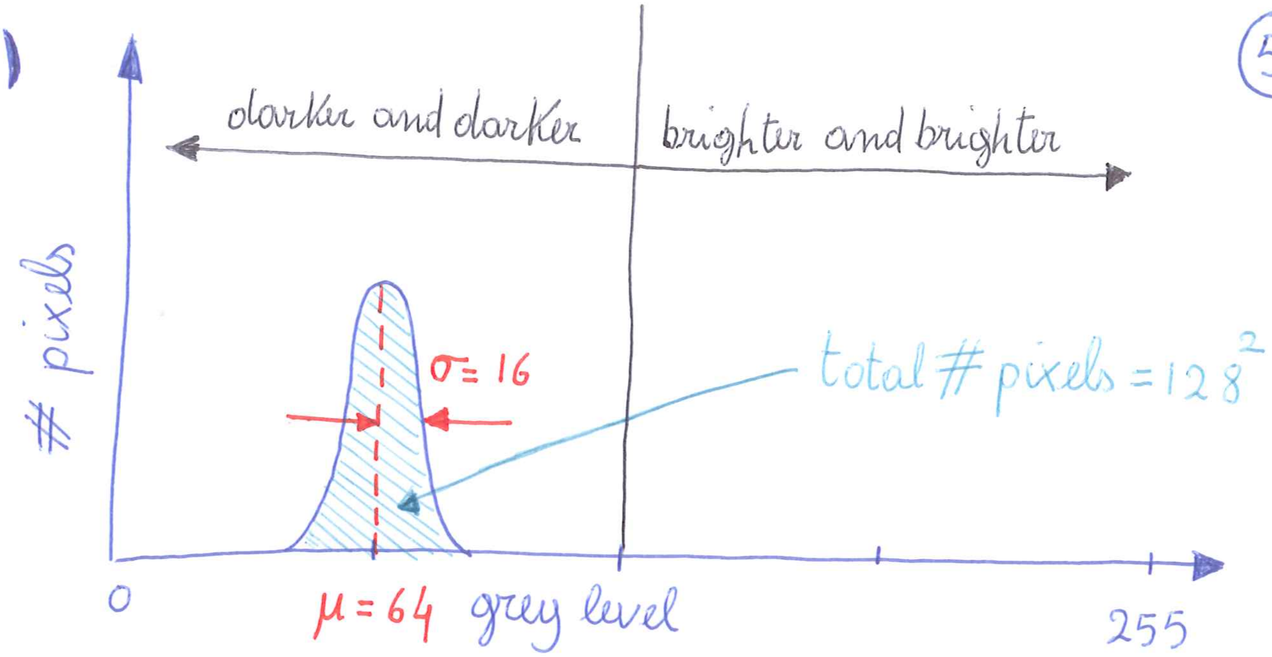
(c)



- $\mu < 127$ → the image is dark
- $\sigma \ll 255$ → the image has low contrast
- flux = total intensity
- μ = total intensity / total # pixels

→ flux = μ × total # pixels = 64×128^2

(d) Poissonian histogram → $\sigma = \sqrt{\mu}$

$\sigma = 8$ → $\mu = 64$

→ the image is as dark as the previous one, has lower contrast and the same flux!