HELP TO SOME OF THE QUESTIONS

2 Noise / Noise Removal

(a) • The filter [-1 0 1] corresponds to (twice) the first derivative along the horizontal direction: x (forgive me if I am using the usual Cartesian notation :-)

• HELP →

• $$w(x) = \int_{-\infty}^{\infty} W(f)\, e^{i 2\pi f x}\, df$$

• $$\underbrace{\frac{dw}{dx}}_{v(x)} = \int_{-\infty}^{\infty} W(f)\, \frac{d}{dx} e^{i 2\pi f x}\, df$$

$$= \int_{-\infty}^{\infty} \underbrace{W(f)\, i 2\pi f}_{V(f)}\, e^{i 2\pi f x}\, df$$

• $$|V(f)|^2 = 4\pi^2 f^2\, |W(f)|^2$$

- $w(x) = \text{noise} \rightarrow \langle |V(f)|^2 \rangle = 4\pi^2 f^2 \langle |W(f)|^2 \rangle$
- $w(x) = \text{white} \rightarrow \langle |V(f)|^2 \rangle = 4\pi^2 f^2 \times \text{constant}$

→ $v(x)$ = violet noise !

(b) • The simplest lossless predictive coding:

Predict: $\bar{f}_n = f_{n-1}$

Transmit: $\underbrace{e_n}_{\text{error}} = f_n - \bar{f}_n$

$= \underbrace{f_n - f_{n-1}}_{\text{What is this?}}$

- This is the first derivative along the horizontal direction !

→ Same answer as in **(a)**:
violet noise !!

(c) • To transform white noise $w(x)$ into red noise $r(x)$, we should multiply $\langle |W(f)|^2 \rangle$ (= constant) by $1/f^2$, hence $|W(f)|^2$ by $1/f^2$ and $W(f)$ by $1/f$, apart from proportionality factors.

• Which mathematical operation does so [remembering that differentiation multiplies $W(f)$ by f] ? Integration!

• $$w(x) = \int_{-\infty}^{\infty} W(f)\, e^{i2\pi f x}\, df$$

• $$\underbrace{\int w(x)\,dx}_{r(x)} = \int_{-\infty}^{\infty} W(f) \int e^{i2\pi f x}\,dx\,df$$

$$= \int_{-\infty}^{\infty} \underbrace{W(f)\frac{1}{i2\pi f}}_{R(f)}\, e^{i2\pi f x}\,df$$

- More rigorously, $r(x) = \int w(x)\,dx$ is true apart from a constant:

  if $w(x) = \frac{dr}{dx}$, then $r(x) = \int w(x)\,dx + C$

- C can be written as:

$$C = C \int_{-\infty}^{\infty} \delta(f)\, e^{i2\pi f x}\, df$$

$$\rightarrow R(f) = \frac{W(f)}{i2\pi f} + C\,\delta(f)$$

<u>But</u> $\delta(f) = 0$ for all f except $f = 0$, and this frequency value is singular since the Fourier power spectrum of red noise ($\propto 1/f^2$) is not defined there !!

(d) • What is the most appropriate transform, given the functional form of $n(t)$?

The Fourier transform !

• $$N(f) = \frac{1}{2} a \left[\delta(f - f_0) + \delta(f + f_0) \right] +$$
$$\frac{1}{2} c \left[\delta(f - 3f_0) + \delta(f + 3f_0) \right]$$

→ $X(f)$ has four spikes: at $f = \pm f_0$ and $f = \pm 3f_0$

→ Set such coefficients to zero !!

And inverse Fourier transform ...

* Do we need to zero-pad ?

No, because such a (notch) filter has infinitesimal size !

* Anything else to worry about ?

(e) • Doesn't this problem remind you of

<u>One of the Exams in 2010/2011 : 1(e)</u>

discussed in the class?

• So what is the most appropriate transform?

The fast wavelet transform!

• Don't let me help you too much!

Try instead to understand the following:

* Choose the wavelet ...

* Choose the level ...

* FWT $x(t)$ at level l

* Compute $\sigma_l = \frac{1}{0.6745} \, MAD\{D_l\}$

* Compute $T_l = \sqrt{2 \ln N} \; \sigma_l$

* Compute $T_{n-1} = \frac{1}{2} T_n$ for $n = l, l-1, \ldots, 2$

* Set to zero the detail coefficients that are <u>ABOVE</u> the thresholds!

* IFWT