

1a) Två likadana pulser utbreder sig från olika håll. När topparna sammanfaller blir momentan  $y$  värdet två gånger större  $\Rightarrow$  superpositionsprincipen gäller  $\Rightarrow$  rörelse ekv. måste vara linjär  $\Rightarrow$   $z=1$ .

$$1b) \quad \frac{d^2 y_p}{dt^2} = -s(y_p - y_{p-1}) - s(y_p - y_{p+1})$$

$$\frac{d^2 y_p}{dt^2} = +s(y_{p+1} - y_{p-1} - 2y_p) \quad (1)$$

$$y(x, t) = A \cos(kx - \omega t)$$

$$x = pa$$

$$y_p(t) = A \cos(kpa - \omega t) \quad (2a)$$

$$\frac{d^2 y_p}{dt^2} = -A\omega^2 \cos(kpa - \omega t) = -\omega^2 y_p \quad (2b)$$

$$(2a) + (2b) \rightarrow (1)$$

$$-\omega^2 y_p = s \left\{ A \cos[k(p+1)a - \omega t] + A \cos[k(p-1)a - \omega t] - 2A \cos(kpa - \omega t) \right\}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cdot \cos \frac{A-B}{2}$$

$\Rightarrow$

$$-\omega^2 y_p = sA \left[ 2 \cos \frac{k(p+1)a - \omega t + k(p-1)a - \omega t}{2} \cdot \cos \frac{k(p+1)a - \omega t - k(p-1)a - \omega t}{2} - 2 \cos(kpa - \omega t) \right]$$

$$= sA \left[ 2 \cos(kpa - \omega t) \cdot \cos ka - 2 \cos(kpa - \omega t) \right]$$

$$= -2sA \cos(kpa - \omega t) (1 - \cos ka)$$

$$-\omega^2 y_p = -2s y_p (1 - \cos ka)$$

$$\omega^2 = 2s (1 - \cos ka)$$

2) pga  $\lambda \gg a \Rightarrow ka = \frac{2\pi a}{\lambda} \ll 1$

$$\Rightarrow 1 - \cos ka \approx 1 - \left(1 - \frac{(ka)^2}{2}\right) = \frac{1}{2}(ka)^2 = \frac{1}{2} \frac{4\pi^2 a^2}{\lambda^2} = \frac{2\pi^2 a^2}{\lambda^2}$$

$$\omega = 2\pi f$$

$$\omega^2 = c^2 (1 - \cos ka) \approx c^2 \cdot \frac{2\pi^2 a^2}{\lambda^2}$$

$$4\pi^2 f^2 = 4\pi^2 \frac{a^2}{\lambda^2} c^2$$

$$\lambda^2 f^2 = a^2 c^2$$

$$\lambda f = v \Rightarrow$$

$$v = a \sqrt{c^2}$$

Alternativ  $\ddot{y}_p$

$$\frac{d^2 y_p}{dt^2} = c (y_{p+1} - y_p - y_p + y_{p-1}) \quad | \cdot \frac{a}{a}$$

$$\frac{d^2 y_p}{dt^2} = a \cdot c \left[ \frac{y_{p+1} - y_p}{a} - \frac{y_p - y_{p-1}}{a} \right]$$

$$\frac{y_{p+1} - y_p}{a} \rightarrow \frac{\Delta y_{p+1}}{\Delta x}$$

$$\frac{y_p - y_{p-1}}{a} \rightarrow \frac{\Delta y_p}{\Delta x}$$

$$\frac{\Delta y_{p+1}}{\Delta x} - \frac{\Delta y_p}{\Delta x} \cdot a \rightarrow \frac{d^2 y_p}{dx^2} \cdot a$$

$$\Rightarrow \frac{d^2 y_p}{dt^2} = a^2 c^2 \frac{d^2 y}{dx^2}$$

$$\Rightarrow v^2 = \frac{1}{a^2} c^2$$

2)

Volvo V 70

strömkretsanslutans  $d \sim 1.5 \mu\text{m}$

$\lambda \sim 5000 \text{ \AA}$

$d_{\text{pupil}} = 3 \text{ mm}$

$$\Delta\theta = 1.22 \frac{\lambda}{D} = 1.22 \cdot \frac{5 \cdot 10^{-7}}{3 \cdot 10^{-3}} = 2.03 \cdot 10^{-4}$$

$$\Delta\theta = \frac{d}{R} \Rightarrow R = \frac{d}{\Delta\theta} = \frac{1.5}{2.03 \cdot 10^{-4}} = \underline{\underline{7.38 \text{ km}}}$$