1a) Två likadana pulser utbreder sig från olika håll. När topparna sammanfaller blir momentan y värdet två gånger större ⇒ superpositionsprincip gäller ⇒ rörelse ekv. måste vara linjär ⇒ $z = 1$.

1b)

$$\frac{d^2 y_p}{dt^2} = -s\,(y_p - y_{p-1}) - s\,(y_p - y_{p+1})$$

$$\frac{d^2 y_p}{dt^2} = +s\,(y_{p+1} - y_{p-1} - 2y_p) \qquad (1)$$

$$y(x,t) = A\cos(kx - \omega t)$$

$$x = pa \qquad (2a)$$

$$y_p(t) = A\cos(kpa - \omega t) \qquad (2b)$$

$$\frac{d^2 y_p}{dt^2} = -A\omega^2\cos(kpa - \omega t) = -\omega^2 y_p$$

(2a) + (2b) → (1)

$$-\omega^2 y_p = s\left\{A\cos[k(p+1)a - \omega t] + A\cos[k(p-1)a - \omega t] - 2A\cos(kpa - \omega t)\right\}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cdot\cos\frac{A-B}{2}$$

⇒

$$-\omega^2 y_p = sA\left[2\cos\frac{k(p+1)a - \omega t + k(p-1)a - \omega t}{2}\cdot\cos\frac{k(p+1)a - \omega t - k(p-1)a + \omega t}{2} - 2\cos(kpa - \omega t)\right]$$

$$= sA\left[2\cos(kpa - \omega t)\cdot\cos ka - 2\cos(kpa - \omega t)\right]$$

$$= -2sA\cos(kpa - \omega t)(1 - \cos ka)$$

$$-\omega^2 y_p = -2s\,y_p\cdot(1 - \cos ka)$$

$$\omega^2 = 2s\,(1 - \cos ka)$$

pga $\lambda \gg a \Rightarrow ka = \frac{2\pi a}{\lambda} \ll 1$

$\Rightarrow 1-\cos ka \approx 1-\left(1-\frac{(ka)^2}{2}\right) = \frac{1}{2}(ka)^2 = \frac{1}{2}\frac{4\pi^2 a^2}{\lambda^2} = \frac{2\pi^2 a^2}{\lambda^2}$

$\omega = 2\pi f$

$\omega^2 = 2s(1-\cos ka) \approx 2s \cdot \frac{2\pi^2 a^2}{\lambda^2}$

$4\pi^2 f^2 = \frac{4s\pi^2 a^2}{\lambda^2}$

$\lambda^2 f^2 = s a^2$

$\lambda f = v \Rightarrow$ $\boxed{v = a\sqrt{s}}$

Alternativ lösning

$\frac{d^2 y_p}{dt^2} = s\left(y_{p+1} - y_p - (y_p - y_{p-1})\right) \quad / \cdot \frac{a}{a}$

$\frac{d^2 y_p}{dt^2} = a \cdot s\left[\frac{y_{p+1}-y_p}{a} - \frac{y_p - y_{p-1}}{a}\right]$

$\frac{y_{p+1} - y_p}{a} \rightarrow \frac{\Delta y_{p+1}}{\Delta x}$

$\frac{y_p - y_{p-1}}{a} \rightarrow \frac{\Delta y_p}{\Delta x}$

$\frac{\frac{\Delta y_{p+1}}{\Delta x} - \frac{\Delta y_p}{\Delta x}}{a} \cdot a \rightarrow \frac{d^2 y_p}{dx^2} \cdot a$

$\Rightarrow \frac{d^2 y_p}{dt^2} = s a^2 \frac{d^2 y}{dx^2}$

$\Rightarrow v^2 = \frac{1}{s a^2}$

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(2) Volvo V 70 strålkastaravstånd $d \sim 1.5$ m

$\lambda \sim 5000$ Å

$D_{pupil} = 3$ mm

$\Delta\theta = 1.22\frac{\lambda}{D} = 1.22 \cdot \frac{5 \cdot 10^{-7}}{3 \cdot 10^{-3}} = 2.03 \cdot 10^{-4}$

$\Delta\theta = \frac{d}{R} \Rightarrow R = \frac{d}{\Delta\theta} = \frac{1.5}{2.03 \cdot 10^{-4}} = \underline{\underline{7{,}38 \text{ km}}}$