

Using Simulated Annealing in
Structure Determination and Design
of X-Ray Multilayers

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1 Introduction

This report describes the implementation of an optimization method, the so-called simulated annealing method, in

- structure determination of x-ray multilayer mirrors from their reflectivity curves
- design of multilayer mirrors with maximum reflectivity at certain energies or at certain incident angles of an x-ray beam.

The work was carried out during my stay at the European Synchrotron Radiation Facility (ESRF), Grenoble, in May and June 1992, under the supervision of and in collaboration with P. Høghøj and E. Ziegler.

2 A Short Description of the Problem

The reflectivity of x-rays on a solid is in general very small, but it can be improved by different means. X-ray multilayers (figure 1) are artificially produced stacks of materials that possess high reflectivity due to constructive interference of the reflected beams at the individual layers. The choice of materials and the characteristics of each layer decide at which angles and at which energies of the x-ray beam the reflectivity will be at maximum. Multilayers will be used at the storage ring at the ESRF.

Other means to get relatively high reflectivity for x-rays are the use of very small incident angles (grazing), and crystal based optics.

The construction of the multilayer structures depends on at which range of energies the maximum reflectivity is desired, given a fixed incident angle θ of the x-ray beam — or, given a fixed energy of the beam, at which angles of the incident beam the maximum is desired. This is a design problem; by varying the structure of the multilayers (e.g. the thickness of the layers, the roughness at the intersections) the maximum reflectivity can be obtained. Another problem consists in inferring the structure of an actual multilayer mirror given its measured reflectivity curve. This is a structure determination problem. The design problem is a special case of the structure determination problem, in that the "experimental" reflectivity curve is set to unity over the range of energies or angles where a peak in the reflectivity is desired.

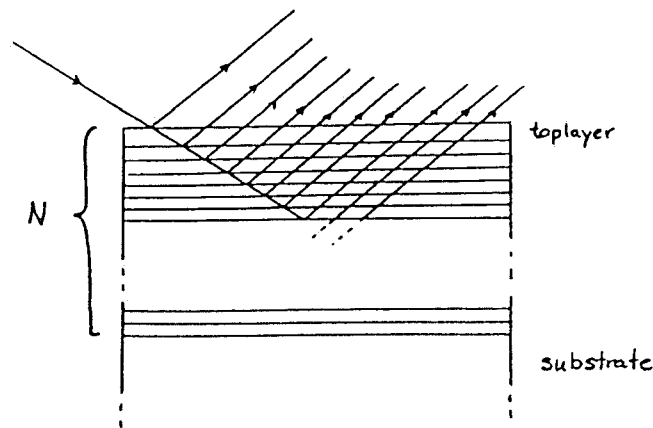


Figure 1: Multilayer structure with N layers

The reflectivity curve does not contain information about the phase of the reflected beam, and thus no direct calculation of the multilayer structure from the knowledge of its reflectivity curve is possible. One is faced with an inverse problem as in many other branches of physics, e.g. in the deconvolution of seismic reflection data. Instead, one can evaluate the theoretical reflectivity curves for different choices of structures until a structure is found whose reflectivity curve fits the experimental curve within the precision desired. This, however, is a tedious task if one needs more than a few variables to describe the multilayers.

In reality an exhaustive search among the different multilayer structures for the best fit is out of question, and one has to rely on some algorithm to evaluate a subset of all possible structures and return the best fitting structure of this subset. For this we need a measure (the cost) of how well the reflectivity curve of a suggested structure fits the experimental curve. Examples of such local optimizing algorithms are random search and pure downhill search algorithms. Usually several local (cost) minima exist among the possible structures (the structure gives a better fit than any structure obtained by a slight change of the variables). This causes the pure downhill algorithm, which only looks for better neighbouring structures, to most likely get trapped in a local minimum. A random search will not get trapped, but will, of course, only have a small chance of coming across the best structure

during the search.

Modifying the pure downhill algorithm to allow for some uphill moves reduces the risk of being trapped in the first local minimum; a further search past the barriers of the local minimum is made possible, without turning the search algorithm into a random search. The algorithm "simulated annealing" allows for such uphill moves from the start of the search, and during the search it reduces the chance of uphill moves.

3 Theoretical Computation of Reflectivity

Given the structure of a multilayer, a recursive formula involving the Fresnel reflection coefficients can be used to find the theoretical values of the reflectivity [5].

The Fresnel coefficients at the top of the $j + 1$ th layer are given by the coefficients at the top of the j th layer and by the optical indices n_j and n_{j+1} at the two interfaces:

$$r_j^p = \frac{n_{j+1} \sin \theta_{j+1} - n_j \sin \theta_j}{n_{j+1} \sin \theta_{j+1} + n_j \sin \theta_j} \quad (1)$$

$$r_j^s = \frac{n_{j+1} \sin \theta_j - n_j \sin \theta_{j+1}}{n_{j+1} \sin \theta_j + n_j \sin \theta_{j+1}} \quad (2)$$

The angle of incidence θ_j on the j th layer is found from the angle of incidence θ_m of the incoming beam using Snell-Descartes' law

$$n_j \sin \theta_j = (n_j^2 - \cos^2 \theta_m)^{1/2} \quad (3)$$

The reflected amplitude X_j at the top of a layer with reflected amplitude X_{j-1} at the bottom of the layer is

$$X_j = \frac{r_j + X_{j-1} e^{-i2\varphi}}{1 + r_j X_{j-1} e^{-i2\varphi}} \quad (4)$$

where the phase φ is

$$\varphi = (2\pi/\lambda) d_j n_j \sin \theta_j = (2\pi/\lambda) d_j (n_j^2 - \cos^2 \theta_m)^{1/2} \quad (5)$$

and d_i is the thickness of the i th layer, λ is the wavelength of the incident beam. With vanishing reflected amplitude at the substrate the reflected amplitude of the whole multilayer can be found by recursive use of equation (4)

until the uppermost layer. The reflectivity for an N -layer structure is then given by

$$R = |X_N|^2 \tag{6}$$

Roughness of the surfaces of the layers can be taken into account by multiplying the Fresnel coefficients (1) and (2) by the Debye-Waller factor

$$\exp(-(4\pi/\lambda)\sigma_j \sin \theta_j) \tag{7}$$

where σ_j is the roughness.

4 Simulated Annealing

To avoid that a downhill algorithm is trapped in the first local minimum found, the algorithm can be modified to allow for some uphill moves. The optimization algorithm "simulated annealing" is such an algorithm; it is based on an analogy of optimization problems with the annealing of physical systems with many degrees of freedom. To bring a solid to its highly ordered crystalline ground state, first the solid is melted, then the temperature is lowered until crystallization is reached. The temperature must be carefully lowered — on one hand the temperature should be slowly lowered near the crystallization temperature to avoid hardening the solid in a crystalline state with higher energy than the ground state. However, to save time, quicker cooling is acceptable right after the melting of the solid.

If the variables in the optimization problem are interpreted as (the positions of) the particles of the solid, and if the costs of the possible solutions are interpreted as the energies of the physical system, then the states of the physical system might, in this analogy, be taken to be the possible solutions to the optimization problem, the configuration space. To avoid hardening of the solid before the ground state is reached corresponds to avoiding the optimization problem being trapped in a local minimum. The "temperature" in the optimization problem is the control parameter, a parameter that controls when uphill moves are allowed.

The analogy can be summarized as follows:

Physical system (solid)		Optimization problem
annealing	\longleftrightarrow	simulated annealing
a state of the solid	\longleftrightarrow	a possible solution
particles (their positions)	\longleftrightarrow	variables
temperature	\longleftrightarrow	control parameter
energy	\longleftrightarrow	cost
sample of different states	\longleftrightarrow	ensembles

The concept of ensembles will be explained in section 4.4.

With this analogy an optimization algorithm can make use of already existing algorithms simulating the behaviour of a physical condensed matter system. The Metropolis algorithm simulates a statistical system in equilibrium. It starts from one state ω and searches the configuration space by perturbing the present state slightly to get to a new state ω' . If the energy of the new state, $E(\omega')$, is less than the energy of the old state, $E(\omega)$, the new state is always accepted, else the new state is accepted with a probability $\exp(-\Delta E/T)$ where $\Delta E = E(\omega') - E(\omega)$, and T is the temperature of the system (T is normalized such that Boltzmann's constant is unity). The higher the temperature, the greater the probability of accepting a state with a higher energy.

Simulated annealing is a series of iterations of the Metropolis algorithm for decreasing values of the control parameter. The basic elements of the algorithm are:

1. Choose a solution ω in configuration space to start from
2. Choose a start value of the control parameter T
3. Anneal
 - (a) Choose a neighbouring solution ω'
 - (b) Calculate the difference in cost $\Delta E = E(\omega') - E(\omega)$
 - (c) If $\Delta E \leq 0$ then accept the new solution ω' , if $\Delta E > 0$ then accept the new solution with the probability $\exp(-\Delta E/T)$
 - (d) Reduce the value of the control parameter T
4. Continue annealing until a stop criterion is reached

To actually use the algorithm, some parts of the algorithm must be clarified. For example, how will the algorithm find a "neighbouring" solution, and what exactly do we mean by "neighbouring"? How fast should the control parameter be lowered? And what is the stop criterion? I will use the expression "state" for a solution to the optimization problem, which in our case will be one of the possible multilayer structures.

4.1 Choice of Neighbourhood

The neighbours of a given state is called the neighbourhood. It is not always obvious how to choose the neighbourhood, and the choice depends on the nature of the optimization problem. For the same problem, different choices of neighbourhood structure can give algorithms with very different efficiencies. The neighbourhood could be chosen to be the states obtained by changing one of the variables describing the state, by a certain amount.

Let $\mathcal{N}(\omega)$ be the neighbourhood of ω . There are three criterions of the structure of the neighbourhood:

1. All states have the same number of neighbours
2. $\omega' \in \mathcal{N}(\omega) \Leftrightarrow \omega \in \mathcal{N}(\omega')$, i.e. if ω is a neighbour of ω' then ω' is a neighbour of ω
3. Given two states ω and ω' it must be possible to get from ω to ω' in a finite number of steps

In traditional simulated annealing the choice of neighbour for the next attempt is made randomly among the states of the neighbourhood. Together with the three criterions above this ensures that at equilibrium the distribution of states is a Boltzmann distribution; the probability of being at a state ω is

$$p(\omega) = \frac{e^{-E(\omega)/T}}{Z(T)} \quad (8)$$

where $Z(T)$ is the partition function for the whole set of states in the configuration space. With a Boltzmann distribution the probability of being in low energy states increases as the temperature is lowered.

4.2 Choice of Cooling Scheme

A commonly used cooling scheme is the exponential cooling scheme

$$T(t) = T_{max}e^{-t/a} \quad (9)$$

with a a constant, and t "time" passed, i.e. number of iterations executed. The exponential cooling scheme was first suggested by Kirkpatrick *et al.* in [6] and has been widely used since then. It is an *a priori* scheme; the cooling is set from start and does not depend on the performance of the system during the annealing. Special cases of *a priori* cooling schemes are random search, with a constant infinitely large control parameter (all attempted moves will be accepted), and pure downhill search, with control parameter constantly zero.

Logarithmic cooling has also been suggested:

$$T(t) = \frac{d}{\log(t+1)} \quad (10)$$

with d a constant. It is the only scheme for which it has been shown [4] that, given infinite time, the system will certainly reach the ground state, but unfortunately the scheme is much too slow for practical purposes.

Other cooling schemes make use of knowledge of the system while annealing, one such cooling scheme will be explained later (thermodynamic cooling).

In any case a start value of the control parameter (temperature) must be given before the annealing starts. The value should be large enough to allow for an almost random search from the beginning, and small enough to avoid wasting too many iterations. Some cooling schemes are more sensitive to the start value than others. A condition on the choice of start value could be that the rate of accepted moves among the attempted moves should exceed 0.8 in the first iterations of the annealing.

4.3 Other Considerations

From a simulated annealing point of view the state to start from (step 1 in the algorithm) should be chosen randomly, but from the point of view of the application a qualified guess is perhaps better as a starting point. If the start value of the control parameter is large enough it should not matter which state the algorithm starts from.

Also, a stop criterion for the annealing must be given, since infinite time is not available ... in applications to large problems the available CPU time most often sets the limit, but also the cost of the current state, or a modification thereof, can be used to decide when to stop.

4.4 Modifications of the Simulated Annealing Algorithm

Some authors (see for example [9]) define the simulated annealing algorithm to include several Metropolis steps before each cooling (steps 3.(a)–(c) in the algorithm are repeated a number of times). With more steps before cooling, the system has a chance to get closer to its equilibrium at the given temperature; in the analogy to physical systems this should improve the performance of the algorithm. A simple choice is to have a lower limit of accepted moves before each cooling.

Instead of starting the algorithm with only one state (one "walker" in configuration space), a whole ensemble of states can be chosen [12] (an ensemble of "walkers"). Each member of the ensemble "walks" the configuration space independently of the other "walkers". This allows for a statistical description of the system during the annealing, quantities like the mean energy, the variance, and even the heatcapacity of the corresponding physical system can be evaluated. These quantities can be used during the annealing — still in the analogy to the physical system — to suggest how fast to cool. A cooling scheme using the knowledge of the behaviour of the system will be explained below (thermodynamic cooling).

Even with an *a priori* cooling scheme and a fixed CPU time available an algorithm using ensembles should perform better than an algorithm using a single walker [12]. We did not find this to be the case in the problem of finding structures of the x-ray multilayer structures, see section 8.

Finally, a very different choice of neighbourhood has been suggested [10] for problems with a continuous configuration space, i.e. problems where the variables that describe a state are continuous variables. The structure determination of multilayers is a continuous problem: the thicknesses of the layers are continuous variables. The method uses the vertices of a simplex in the N -dimensional configuration space (a simple geometrical figure with $N + 1$ vertices) to search the configuration space. The difference from the ensemble approach is that with the simplex method, the moves of the $N + 1$ "walkers" are indeed very much correlated, and that the distance a "walker" covers in configuration space in one step depends on the costs of the states of the N other walkers. The simplex approach will be further explained in section 6.

5 Thermodynamic Cooling

Using ensembles during the annealing makes a statistical description of the system possible. At a given temperature each member of the ensemble is in a certain state ω which has an energy (cost) $E(\omega, T)$. Quantities like the mean energy $\langle E(T) \rangle_{\text{ensemble}}$ and the variance of energy $\sigma^2(T)_{\text{ensemble}}$ of the ensemble can be found. If the system is kept at a fixed temperature T it will eventually relax into the equilibrium distribution at that temperature, with mean energy $\langle E(T) \rangle_{\text{eq}}$ and variance $\sigma^2(T)_{\text{eq}}$. If the system is close enough to equilibrium we can use the classical theory of equilibrium thermodynamics on the ensemble. The idea in the thermodynamic cooling scheme is to lower the temperature in such a way that the mean energy of the ensembles is held at a fixed number of standard deviations from the equilibrium energy

$$\langle E(T) \rangle_{\text{ensemble}} - \langle E(T) \rangle_{\text{eq}} = v\sigma(T)_{\text{eq}} \quad (11)$$

where v is a constant (typically in the range 0.01 to 0.5). If the system is close to equilibrium (v is small) then instead of the standard deviation of the equilibrium $\sigma(T)_{\text{eq}}$ we can use the standard deviation of the ensemble $\sigma(T)_{\text{ensemble}}$.

We now want to determine the temperature T such that $\langle E(T) \rangle_{\text{eq}}$ satisfies (11). But since we do not know the properties of the equilibrium distribution of the system, the form of $\langle E(T) \rangle_{\text{eq}}$ as a function of T is unknown. One way to overcome this difficulty is the following. Let the system relax almost to equilibrium at one temperature by letting it run a large number of iterations at a constant temperature T_{max} . Then the mean energy of the ensemble can be used as $\langle E(T_{\text{max}}) \rangle_{\text{eq}}$, and values of the function $\langle E(T) \rangle_{\text{eq}}$ for lower values of T can be found by extrapolating the function. To this end we need to know the derivative $\frac{dE}{dT}$, where for simplicity I write E for $\langle E(T) \rangle_{\text{eq}}$. This is the heat capacity $C(T)$, which can also be written

$$\frac{dE}{dT} = C(T) = \frac{\sigma^2(T)_{\text{eq}}}{T^2} \quad (12)$$

and again we can replace $\sigma(T)_{\text{eq}}$ by the one of the ensemble. Simple extrapolation gives

$$\frac{dE}{dT} \simeq \frac{\langle E(T_{\text{max}}) \rangle_{\text{eq}} - \langle E(T) \rangle_{\text{eq}}}{T_{\text{max}} - T} \quad (13)$$

The cooling scheme can be summarized as the following steps:

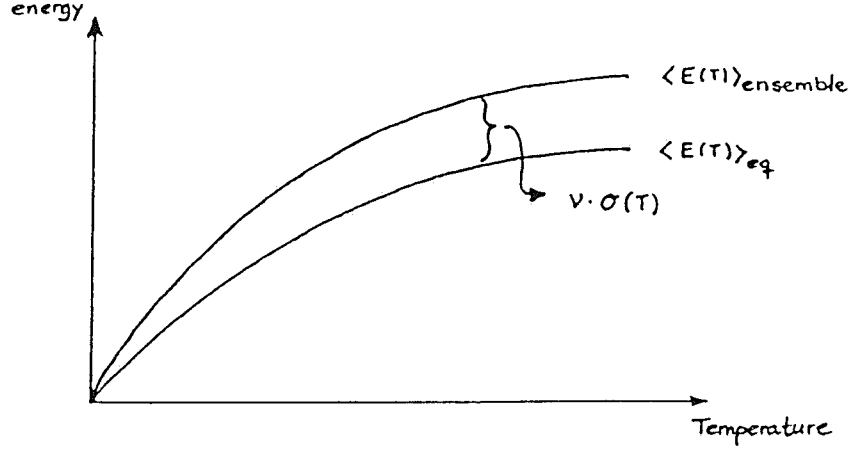


Figure 2: The ensemble mean energy $\langle E(T) \rangle_{\text{ensemble}}$ is kept at a distance $v\sigma(T)$ from the equilibrium mean energy $\langle E(T) \rangle_{\text{eq}}$

1. Run many Metropolis iterations at the temperature T_{max}
2. Evaluate mean energy and standard deviation of the ensemble and set $\langle E(T_{max}) \rangle_{\text{eq}} \simeq \langle E(T_{max}) \rangle_{\text{ensemble}}$, $\sigma(T_{max})_{\text{eq}} \simeq \sigma(T_{max})_{\text{ensemble}}$
3. Run one or more Metropolis steps
4. Evaluate $\langle E \rangle_{\text{ensemble}}$ and σ_{ensemble}
5. Let $\sigma(T_{new})_{\text{eq}} \simeq \sigma_{\text{ensemble}}$
6. Use equation (11) to find $\langle E(T_{new}) \rangle_{\text{eq}}$
7. From T_{max} , $\langle E(T_{max}) \rangle_{\text{eq}}$, $\langle E(T_{new}) \rangle_{\text{eq}}$ and $\frac{dE}{dT} = \frac{\sigma^2(T_{max})_{\text{eq}}}{T_{max}^2}$ extrapolate value of T_{new} using eq. (13)
8. Let $T_{old} = T_{new}$ and continue from step 3 with T_{old} instead of T_{max}

Another way to find $\langle E(T) \rangle_{\text{eq}}$ as a function of T is described in [7]. There the probabilities of transition between the different energy levels are estimated during the annealing, and from this the equilibrium properties of the system — like $\langle E(T) \rangle_{\text{eq}}$ — are found.



Figure 3: Simplex in two and three dimensional space

6 The Simplex Approach to Simulated Annealing

The simplex approach can be applied to optimization problems with continuous variables, and is described by W.H. Press and S.A. Teukolsky in [10].

In other neighbourhood structures the variables can only be changed with a pre-set amount, and possible minima for in-between values of the variables will never be found. Also, the next attempted neighbour is chosen with equal probability among all neighbours, this is inefficient because, even when a few local downhill moves exist, the uphill moves are almost always attempted.

The simplex approach is based on a pure downhill search algorithm by Nelder and Mead, described in [11]. A simplex is a geometrical figure in N dimensions consisting of $N + 1$ vertices, the interconnecting line segments, and the polygonal faces thus formed. In two dimensions the simplex is a triangle, in three dimensions a tetrahedron (figure 3).

When the optimization problem has N variables, then the simplex method uses a simplex in N dimensional space, where the $N + 1$ vertices of the simplex are distinct possible solutions to the optimization problem. In our case the vertices are $N + 1$ different multilayer structures. The algorithm for the pure downhill simplex method is the following (numbers according to the numbers in figure 4):

1. Evaluate the cost of all vertices, note which vertices have the highest, the second highest and the lowest cost (hereafter called the highest vertex etc.)
2. Reflect the highest vertex in the polygonal face spanned by the other N vertices; this new point x will be tried as a replacement for the currently highest vertex in the simplex.
3. Evaluate the cost of the new point x . There are three cases:

- (a) If the cost of x is higher than the second highest cost, then the direction seems to be wrong, and another direction is chosen; the point y is found at half the distance between the highest point and the polygonal face mentioned above. Evaluate the cost of y .
 - i. If the cost of y is lower than the highest cost of the simplex, then y is accepted as a vertex instead of the highest vertex.
 - ii. But if the cost of y is higher than the highest cost in the simplex, then the simplex is contracted around the lowest vertex.
- (b) If the cost of x is neither higher than the second highest cost, nor lower than the lowest cost, then x is kept as a vertex of the simplex as a replacement of the highest vertex.
- (c) If the cost of the point x is lower than the lowest cost in the simplex, then the point seems to be in the right direction, and a new point z is attempted at double the distance from the above mentioned polygonal face. Evaluate the cost of z .
 - i. If the cost of the point z is not lower than the lowest cost in the simplex, then the highest vertex is replaced by the point x , which had a low cost.
 - ii. But if the cost of z is lower than the lowest in the simplex, then z is taken to be a new vertex of the simplex, to replace the highest vertex.

4. Carry out more steps of the algorithm by starting from 1.

This is how the pure downhill simplex method works; when possible the method expands the simplex in a direction with lower costs to take larger downhill steps. The simplex converges to a local minimum. When used in connection with simulated annealing the simplex should be given a chance to escape local minima by sometimes accepting new vertices with higher costs. This is done as follows: to the cost of each vertex in the simplex a random number is added, a positive, logarithmically distributed random variable, proportional to the control parameter. Also, whenever the cost of a new point is evaluated, a similar random number is subtracted from the cost. This way a new point with a cost higher than the costs of the simplex might seem to the algorithm to have a lower cost and thus might be accepted as a new vertex.

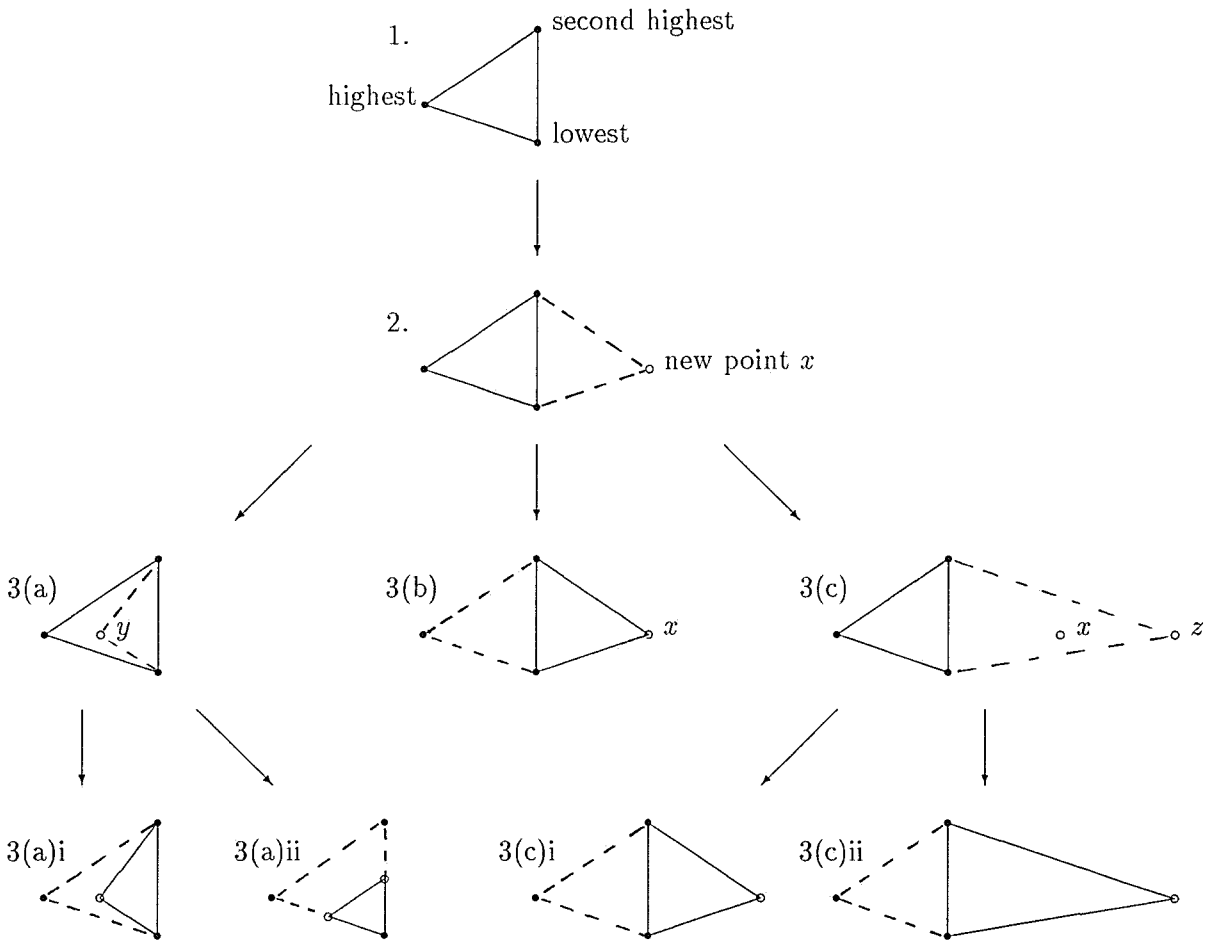


Figure 4: The pure downhill simplex algorithm in two dimensional space. Numbers correspond to the steps in the algorithm, section 6.

A ready-to-use **FORTRAN** subroutine for the simulated annealing simplex method is found in [10] and, with slight modifications, as the two last sub-routines in the print out of the **FORTRAN** program `sasi7` in appendix C.

7 Implementation

As will be clear by now, a number of decisions have to be made before implementing the algorithm simulated annealing. In the limited time available for this project I could only try out a few combinations of cooling schemes, neighbourhood structures etc., and even then only relatively short test runs were possible (time scale in the order of days). The aim was to test the simulated annealing algorithm used on reflectivity data, not so much to obtain the best possible results using all possible tricks. I have tried to keep the algorithms relatively general.

For structure determination of multilayers the inputs to the algorithm are the number of layers, the substance and/or the density of the layers and of the substrate on which the layers are grown, a qualified guess of the thickness d_i of each single layer and of the roughness σ_i of their surfaces. The thickness and roughness are taken to be the variables of the simulated annealing, but other variables could be added, as will be explained later. A maximum and minimum value of the variables (thickness and roughness) should be given to avoid wasting time evaluating structures that are not realistic — structures that can not be produced due to limitations in the actual production phase (for the design problem) or, for structure determination, structures with values that are beyond the range of values suggested by other means (e.g. if a bilayer is found to be of thickness $35\text{\AA} \pm 3\text{\AA}$ by experimental methods there is no need to test a structure with thicknesses 30\AA and 19\AA for the two layers).

The theoretical reflectivity curve of a given multilayer is calculated using the equations of section 3, with the Debye-Waller roughness factor. The cost function is a measure of how well the theoretical reflectivity curve fits the datapoints of the experimental curve. We chose to use the cost function

$$\text{cost} = \frac{1}{n} \sum_1^n \left(\frac{R_{calc} - R_{exp}}{R_{calc}} \right)^2 + \left(\frac{R_{exp} - R_{calc}}{R_{exp}} \right)^2 \quad (14)$$

where n is the number of datapoints.

Name of program	sasi7	sasi8	sasi9	sasi10
Cooling scheme	exponential	exponential	exponential	thermodyn.
Ensembles?	simplex	single walker	ensemble	ensemble

Four FORTRAN programs were written, all with the same basic structure, but with different choices for neighbourhood or cooling scheme (see table). The choice of neighbourhood structure is important, because the energy (cost) landscape mainly depends on this. A bad choice might give steep barriers that the walker can only pass at high temperatures, whereas a different choice might give a smoother landscape easy to move around in even at low temperatures. In programs `sasi8`, `sasi9` and `sasi10` a neighbour is found by changing the thickness or the roughness of one (randomly chosen) layer by a pre-set amount ($\pm\Delta$ thickness or roughness). To keep the number of neighbours the same for all states — a criterion for the neighbourhood structure — the variables are made cyclic; if a move is attempted past the upper limit of one of the variables then the lower limit is substituted before it is decided if the move will be accepted. The change in thickness is usually set to 0.5\AA , and such a change can give rise to a cost difference from almost none to the order of 10,000, which is a steep barrier compared to the best cost found (often of order 1 or 0.1). It seems that such a choice of neighbourhood structure is not very good, but except for slight modifications of it, or the totally different simplex method, no other choices have been suggested as far as I know. That is, not if all the thickness and roughness variables should be allowed to vary independently of each other. A parameter, determining the likelihood of changing the roughnesses (as opposed to the thicknesses) when choosing a neighbour, was implemented. If the value of the parameter is set to 1 only the thickness variables can be changed, and if the parameter is set to 0.5 the thickness and roughness variables are changed with equal probability.

In program `sasi7` the simplex method has been used, as described in section 6. Only variables that have a range to vary on (i.e. the maximum and minimum values are not equal) add to the number of dimensions of the space in which the simplex moves.

The cooling schemes used were exponential cooling and thermodynamic cooling. The exponential cooling scheme is easy to implement, and is used in `sasi7`, `sasi8` and `sasi9`. The scheme is sensitive to the start value of the control parameter, and should be chosen such that the accept rate is 0.8 in the beginning. A minimum value of the control parameter is also needed. This should in principle be close to zero, but to avoid spending too much

time at the final minimum (whether it is the global minimum or a local minimum) the value should be set such that the accept rate at the end of the annealing is small but not vanishing.

The thermodynamic cooling scheme was implemented in `sasi10` using the algorithm described in section 5.

Multilayer structures with a realistic ~ 100 layers will give ~ 200 variables to optimize (when both thickness and roughness is to be optimized), this is a large number of variables, even for the simulated annealing algorithm, and an optimization will take a long time on present day computers. A way to cut down the number of variables, and thereby the computation time, is to assume a periodic structure of the layers. This was not done in any of the four `FORTRAN` programs, but would be interesting to test.

It is also possible to assume that the thickness of the layers depend on the position of the layer in the stack, e.g. one could assume that the layers near the top are thicker than the layers near the substrate, the layers that only the higher energies of the beam will reach. A functional form of this dependence with some unknown parameters must be given to the simulated annealing algorithm, which then optimizes the parameters; the number of variables of the simulated annealing is the number of unknown parameters of the function.

8 Results

The aim of the project was to test simulated annealing in structure determination and design of multilayer structures, not to test the methods of calculating the reflectivity. Therefore, for most of the structure determination testruns, the "experimental" reflectivity curve is actually a curve calculated from a known structure (unknown to the program, of course), thereby eliminating possible effects arising from the reflectivity calculations.

Two design problems were tested. They both consist in maximizing the reflectivity of a Platinum/Carbon multilayer structure for energies of the beam in the range 5.7keV to 7.0keV. The number of periods is given to be 12 (25 layers including the substrate), and the incident angle of the beam is $\theta = 1.53^\circ$ for the first problem and $\theta = 1.00^\circ$ for the second problem. For design no roughness of the interfaces of the layers is assumed.

The $\theta = 1.00^\circ$ problem was run with a single walker (`sasi8`), with a simplex (`sasi7`) and with thermodynamic cooling (`sasi10`). The three programs found different optimal structures, but all with almost the same

reflectivity in the energy range considered. The best costs found (using equation (14)) by the three programs were 0.849, 0.906 and 0.941 respectively; the reflectivity curve found by `sasi8` is in appendix B. The numbers on the y-axis are the \log_{10} values of the reflectivity, and in the energy range considered the reflectivity of the found structure is between $0.44(= 10^{-0.36})$ and $0.66(= 10^{-0.18})$. These results are better than what a manual fit gives, and they are certainly less tiresome to obtain.

Most of the design testruns were run with 500,000 iterations, which on the machines used took one night. For the programs using exponential cooling a maximum and a minimum value of the control parameter should be given. The testruns where the maximum value was set to 10^{-1} had an acceptance rate around 0.8 to start with. The minimum value of the control parameter was set to 10^{-4} , and the acceptance rate at the end of the testruns was around 0.3. At the testrun with the simplex program the simplex "collapsed" after few iterations, i.e. the vertices in the simplex all got the same values of the coordinates except for one coordinate. For the multilayer structures this means that all the supposedly different structures making up the vertices of the simplex after a few iterations all got the same thicknesses except for *one* layer, in this case layer number five. This problem of a collapsing simplex is not specific for the design problem, also in structure determination the simplex tends to collapse. The problem will be discussed below.

For the program using thermodynamic cooling the maximum value of the control parameter should not matter too much, as long as the value is not set too low. In the light of the acceptance rates found for a single walker, using 10^{-1} as the maximum value, it was decided to use the values 2 and 20 for two testruns of the thermodynamic cooling program. In the first testrun the constant v (see section 5 for explanation) was set to 0.05, but after almost 500,000 iterations the algorithm still seemed able to find better structures given more time. $v = 0.05$ was perhaps too strict a condition, and $v = 0.25$ was tried in the next testrun. This, on the other hand, seemed to be too weak a condition: the temperature does not fall off monotonously. Therefore, at later testruns (for structure determination), the constant v is kept at the value 0.10.

For the incident angle $\theta = 1.53^\circ$ a best cost of 12 was found, the reflectivity is approximately 0.25. The lower values of the reflectivity found at $\theta = 1.53^\circ$ do not surprise, since the reflectivity of solids at larger angles is small.

The structure determination was tested on three different Tungsten/

Silicon structures: a periodic structure with 11 layers (including the substrate), an a-periodic structure with 11 layers, and a periodic structure with 51 layers. All three "experimental" reflectivity curves, from which the programs should find the multilayer structures, are given by 301 datapoints in the range of angles $\theta = 0^\circ$ to 3° . The energy of the beam is 8keV. The "experimental" reflectivity curves to be used at the tests were found from theoretical multilayer structures without roughness of the interfaces, and thus in the testruns roughness was not considered. Adding roughness to the variables in the test is computationally approximately the same as doubling the number of layers.

The 11 layers periodic structure was tested with the programs `sasi8`, `sasi7` and `sasi10`. The single walker, `sasi8`, finds the original structure after 50,000 iterations in one of the testruns, but gets stuck at a very high cost after only 34 iterations (!) in another testrun. Also, in the first testrun, the original structure is *almost* found after approx. 20,000 iterations, with only the thickness of one layers off by 0.5Å, and the algorithm wanders off in a worse direction from there. This suggests that there is a great deal of randomness involved in finding the right structure using the single walker, and the idea in using ensembles or simplexes is to be more certain to get to the best structure, though no guarantee is given.

With the time available for tests (approx. 250,000 iterations per test) this did not seem to be the case. Neither the ensemble program with exponential cooling (`sasi9`), the ensemble program with thermodynamic cooling (`sasi10`), nor the simplex program (`sasi7`) find the original structure of the 11 layers periodic structure. `sasi10` finds a structure with reflectivity curve and cost (2.17) not far from reflectivity of the original, whereas a random search after 328,000 iterations still has not found anything reasonable, the best cost is 580. A pure downhill search was conducted twice; the program got stuck at structures with costs 26 and 400 after 400 iterations. No matter what the hopes for the simulated annealing programs were, at least the programs prove to be — by far — better than random searches or pure downhill searches!

One test was run on the 51 layers periodic structure. The original structure was not found, and it became obvious that to get any reasonable test results the program would have to run for a longer time than was available. Whenever the number of layers is increased by a factor n , the time needed for the reflectivity calculations is increased by at least a factor n , and the number of iterations necessary to change each layer a given number of times (in average) is also increased by a factor n , so that the necessary compu-

tational time is increased by at least a factor n^2 . A typical test of the 11 layers structure took one night, and a test of the 51 layers structure would then take a week or two.

Instead the tests were concentrated on the 11 layers a-periodic structure. None of the testruns found the original structure. The best fit to the reflectivity curve of the original structure was found by the single walker. The cost is low (0.124), and the two reflectivity curves look very similar. The curves are shown in the second diagram in appendix B, the dark grey curve is from the original structure, the light grey curve is the best fit. Although the two reflectivity curves look almost the same, the multilayer structures are different:

	Original structure (thickness in Å)	Structure for best fit (thickness in Å)
substrate	10^8	10^8
layer 1	17.5	17.0
layer 2	43.5	44.5
layer 3	19.0	18.0
layer 4	45.5	46.5
layer 5	15.0	14.5
layer 6	45.5	45.0
layer 7	17.0	18.0
layer 8	44.5	43.0
layer 9	17.0	18.0
toplayer	45.5	48.0

This is a problem not only for the simulated annealing algorithm. That more multilayer structures give almost the same reflectivity curves makes it difficult to determine the structure from which an experimental reflectivity curve originates, no matter which optimization algorithm is used. For the design of multilayers this is not a problem, all we are interested in is to find *one* structure that gives the desired reflectivity.

The testruns using thermodynamic cooling give best fits with costs 0.617 and 0.738. The reflectivity curve for the cost 0.738 is shown in the third diagram in appendix B. Again the dark grey curve is the original, and the light grey curve is the best fit.

In most of the testruns the program using thermodynamic cooling gives results not quite as good as the single walker. The temperature is lowered

according to the behaviour of the members of the ensemble. In all tests an ensemble of 50 walkers was used, this might not be enough to find reliable statistical quantities. In some of the testruns the temperature falls off very slowly and it even *rises* once in a while. In program `sasi10` the temperature is lowered after each Metropolis step of the ensemble. As mentioned before more steps might be necessary to let the system get closer to its equilibrium distribution before each lowering of the temperature. One test was run on a modification of `sasi10` with 10 Metropolis steps of the ensemble before each cooling step. Still the temperature rose once in a while, but not quite as often. The best fit was not better than obtained by one Metropolis step per cooling step. It would be interesting to test a thermodynamic cooling with more Metropolis steps.

In general the simplex program did not perform very well. In almost all testruns the simplex collapsed, with only one or two variables not stuck. Once, in a very long testrun, the simplex collapsed and stayed collapsed for many iterations but then very slowly began to move, though not enough to change the best fit. The cause might be rounding errors after many iterations, a change to higher precision variables in the `FORTRAN` program will show if this is the case. But higher precision variables (probably) still will not prevent the simplex from collapsing. One solution will be to set a minimum difference of the variables of the simplex, e.g. for each layer there should be a minimum difference between the thickness of the thickest and the thinnest of the structures in the simplex. The minimum difference should then decrease with the temperature. I am not sure that this will prevent the simplex from collapsing, but it is the only solution I can imagine.

The testruns using simplexes did not perform very well because the simplexes collapsed, but even if they had not collapsed, they would probably have needed more CPU than the other programs to find good fits. This is because the simplex searches a *continuous* space, whereas the other programs only search for (in our tests) half integer values of the thicknesses. The original structures have half integer values, and thus the tests favourize the other programs. In a test with real experimental data this would not be the case, of course.

9 The FORTRAN Programs

For this project five `FORTRAN` programs were written, simulated annealing in `sasi7-10`, and `graphics`, a small program to show diagrams after the

annealing. The programs `sasi7`, `sasi8` and `sasi10` are in appendix C.

Name of program	sasi7	sasi8	sasi9	sasi10
Cooling scheme	exponential	exponential	exponential	thermodyn.
Ensembles?	simplex	single walker	ensemble	ensemble

In this section I will use the term "S.A. variable" about a variable in the simulated annealing (thickness, roughness), and "FORTRAN variable" or simply "variable" about a variable in the FORTRAN programs.

A FORTRAN program `mlp`, written by P. Høghøj, calculates the reflectivity curve for a given multilayer using the method described in section 3. `mlp` was taken as a starting point for the implementation of simulated annealing. `sasi8` is the simplest of the four simulated annealing programs, and it was also the first program to be written. The main structure of `sasi8` is:

1. read input (everything except experimental reflectivity curve) (`rdfile`)
2. anneal: (`anneal`)
 - (a) read experimental curve (`rddat`)
 - (b) prepare optical constants (`optconan` or `optconen`)
 - (c) for a number of steps do:
 - choose a neighbour (a multilayer structure) (`neighbour`)
 - calculate reflectivity for neighbour (`reflecan` or `reflecan`)
 - calculate the cost of the neighbour (`cost1`)
 - decide if neighbour should be accepted (`accept`)
 - move to neighbour if it was accepted
 - lower the control parameter (`cool`)
 - (d) write output (`wrtfile`)

The S.A. variables are kept in a one-dimensional array `thick`. The variables used for the optical calculations are all kept by pointers, that way the size of the variables does not need to be declared before the program is compiled and run, and the size can be decided after reading the input file (variables `delta`, `beta`, `theta`, `energy`, `cd_index`). The reflectivity data are also kept by pointers, the variables are `calcrefl`, `exprefl`, `bestrefl`.

More S.A. variables can be added by changing the size of the array `thick` and the arrays connected to it (`bestthick`, `maxthick` etc.).

During the annealing the program will show a diagram with the experimental reflectivity curve, a curve for the best structure found so far, and

a curve for the structure the program attempts now, all versus the angle or energy. The diagram is updated according to the value read from the input data. The graphs are drawn using a set of subroutines `EXPG` written by A. Hammersley, ESRF.

The subroutine `cost1` calculates the cost of a given reflectivity curve and uses the cost function (14). Two other cost function subroutines are included (`cost2` and `cost4`), they are not used by the program, but can be used if `cost1` is changed to `cost2` or `cost4` everywhere except at the subroutine itself.

`sasi9` is similar to `sasi8` except that ensembles have been implemented. `thick` is now a two dimensional array where each row represents a member of the ensemble, i.e. one row of `thick` in `sasi9` is the same as the whole one dimensional array `thick` in `sasi8`. The steps 2.(c) in the algorithm are carried out for each member of the ensemble.

The thermodynamic cooling scheme is implemented in `sasi10`. A number of Metropolis steps are carried out at the maximum value of the control parameter before the annealing takes place. This is done to let the ensemble approach its equilibrium distribution, as explained in section 5. The maximum number of steps is set to 1000 per member of the ensemble, but the ensemble is considered close enough to equilibrium if in 10 steps the change in mean energy of the ensemble is less than its standard deviation times the constant v divided by 10:

$$\Delta\langle E(T)\rangle_{\text{ensemble}} < v\sigma(T)_{\text{eq}}/10 \quad (15)$$

In practice the mean energy remains reasonably stable at the last iterations before (15) stops the search for the equilibrium distribution.

The structure of `sasi10` is similar to `sasi8` except for the change to ensembles and for the extra steps needed to get close to the equilibrium before annealing:

1. read input (`rdfile`)
2. anneal: (`anneal`)
 - (a) read experimental curve (`rddat`)
 - (b) prepare optical constants (`optconan` or `optconen`)
 - (c) find equilibrium properties at max. control parameter
 - (d) for a number of steps do:
 - for each member of the ensemble do: (`step_ensemble`)
 - choose a neighbour (`neighbour`)
 - calculate reflectivity for neighbour (`reflecan` / `refleceen`)
 - calculate the cost of the neighbour (`cost1`)
 - decide if neighbour should be accepted (`accept`)
 - move to neighbour if it was accepted
 - lower the control parameter (`cool`)
 - (e) write output (`wrtfile`)

In `sasi10` the control parameter is changed when all the members of the ensemble have gone through one iteration. The program can easily be changed to take more iterations per member before next change of control parameter. The subroutine `cool` uses the equations and the algorithm of section 5 to lower the control parameter.

`sasi7` uses the simplex method to choose neighbours. Of the S.A. variables describing the multilayer structure only actual variables are used in the simplex, as opposed to variables that are fixed by the input data, e.g. with Δ thickness zero, or with the maximum value equal to the minimum value. Let N be the number of such actual variables. The data is first read to the FORTRAN variable `thick`, including any fixed variables. The N actual variables are then transferred to the first row of an $(N + 1) \times N$ -sized array `sim_p` as the first vertex of the simplex, and the array `thickconst` keeps track of the correspondance between the multilayer structure and the dimensions of the simplex space. The $N + 1$ rows of `sim_p` are the $N + 1$ vertices of of the simplex, and the start values of the vertices are found by adding a multiplum of Δ thickness or Δ roughness to either the thickness or the roughness of one of the layers, provided it is not a fixed variable.

The subroutines for the simplex movements are kept as close as possible to the original subroutines suggested in [10]. To do this, the calculation of the reflectivity was moved to a subroutine of the cost evaluation. It does not alter the way the program calculates the cost, but does make the program structure look a bit different:

1. read input (**rdfile**)
2. get rid of fixed variables
3. anneal: (**anneal**)
 - (a) read experimental curve (**rddat**)
 - (b) prepare optical constants (**optconan** or **optconen**)
 - (c) prepare a start simplex
 - (d) for a number of steps do:
 - call the simplex subroutines (**amebsa**)
 - lower the control parameter (**cool**)
 - (e) write output (**wrtfile**)

10 Format of In- and Outdata

An example of an input file is shown in appendix A. It is essential to keep the exact format of the input file. The **Input data filename** is the name of the file containing the datapoints of the experimental reflectivity curve. It has four text lines of no importance to the program, and two columns of numbers: the first column contains the x-axis values (energy in eV or angle in degrees), the second column contains the corresponding values of the experimental reflectivity.

The **Output data filename** is the name of the file to where the best structure found will be written at the end of the annealing, along with the calculated and the experimental reflectivity, and the **Name of .log file** is the file where the results during the annealing will be written (i.e. the number of iterations, the control parameter, the best cost so far, the rate of accepted moves ...).

The **Number of loops before output to screen** at the end of the input file is the number of iterations run before the graph on the screen is updated and the values are written in the **.log** file and on the screen. When using ensembles the "loop" is one step for each member of the ensemble, and thus the value should be set accordingly lower than for the program that uses only a single walker. Likewise the simplex program counts one "loop" as one call of the **amebsa** subroutine.

After termination of the simulated annealing program the **.log** file can be inspected using the program **graphs**, a simple program that gives choices for the x- and y-axis and draws a graph. A **.cgm** file with the graphs can

be made for later print out (use `ralcgm`). `graphs` also uses the graphic subroutines `EXPG`.

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Appendix A

INPUT FILE

This is an inputfile for the multilayer simulation program mlp ver.1.00.
Please edit the file to describe your multilayer and simulation, while
preserving the exact format of the file. Have fun. Nov-91 PH sisi

Input data filename: testdata/s0102.indat
Output data filename: testdata/s0102b.outdat
Name of .log file: testdata/s0102b.log

The simulation:

Do you want to calculate the reflectivity as a function of angle or
energy [a/e]? a

Initial value of the control parameter=100.

Final value of the control parameter=0.05

Number of iterations=250000

Number of datapoints=301

If angle:

Energy[ev]=8000.

Minimum angle[degrees]=0.

Maximum angle[degrees]=3.0

Angle increment[degrees]=0.01

Do you wish to include the angular dependence of the optical constants?
(or just use the value for the mean angle) [y/n]? y

If energy:

Angle[degrees]=0.42

Minimum energy[eV]=6000

Maximum energy[eV]=8000

Energy increment[eV]=100

The multilayer[optional name and description]:

"This is a test of the Simulated Annealing Multilayer program sisi,
development version. "

Number of layers (incl. substrate)=11

Do you wish to specify all layers or only the substrate and the next two
layers (mlp will assume the rest of the multilayer to be a sequence of
these two layers) [all/2]? all

Formula	rho [g/ccm]	rough [A]	min [A]	max [A]	delta [A]	thick [A]	min [A]	max [A]	delta [A]
Si	0.0	0.0	0.0	0.0	0.0	1.e8	1.e8	1.e8	0.0
W	0.0	0.0	0.0	0.0	0.0	20.0	10.0	30.0	0.5
Si	0.0	0.0	0.0	0.0	0.0	40.0	30.0	50.0	0.5
W	0.0	0.0	0.0	0.0	0.0	20.0	10.0	30.0	0.5
Si	0.0	0.0	0.0	0.0	0.0	40.0	30.0	50.0	0.5
W	0.0	0.0	0.0	0.0	0.0	20.0	10.0	30.0	0.5
Si	0.0	0.0	0.0	0.0	0.0	40.0	30.0	50.0	0.5
W	0.0	0.0	0.0	0.0	0.0	20.0	10.0	30.0	0.5
Si	0.0	0.0	0.0	0.0	0.0	40.0	30.0	50.0	0.5
W	0.0	0.0	0.0	0.0	0.0	20.0	10.0	30.0	0.5
Si	0.0	0.0	0.0	0.0	0.0	40.0	30.0	50.0	0.5

Parameters:

Number of loops before output to screen=40

If ensemble program: ensemble size=50

If thermodyn. cooling: v=0.10

If simplex program: ftol=1e-5

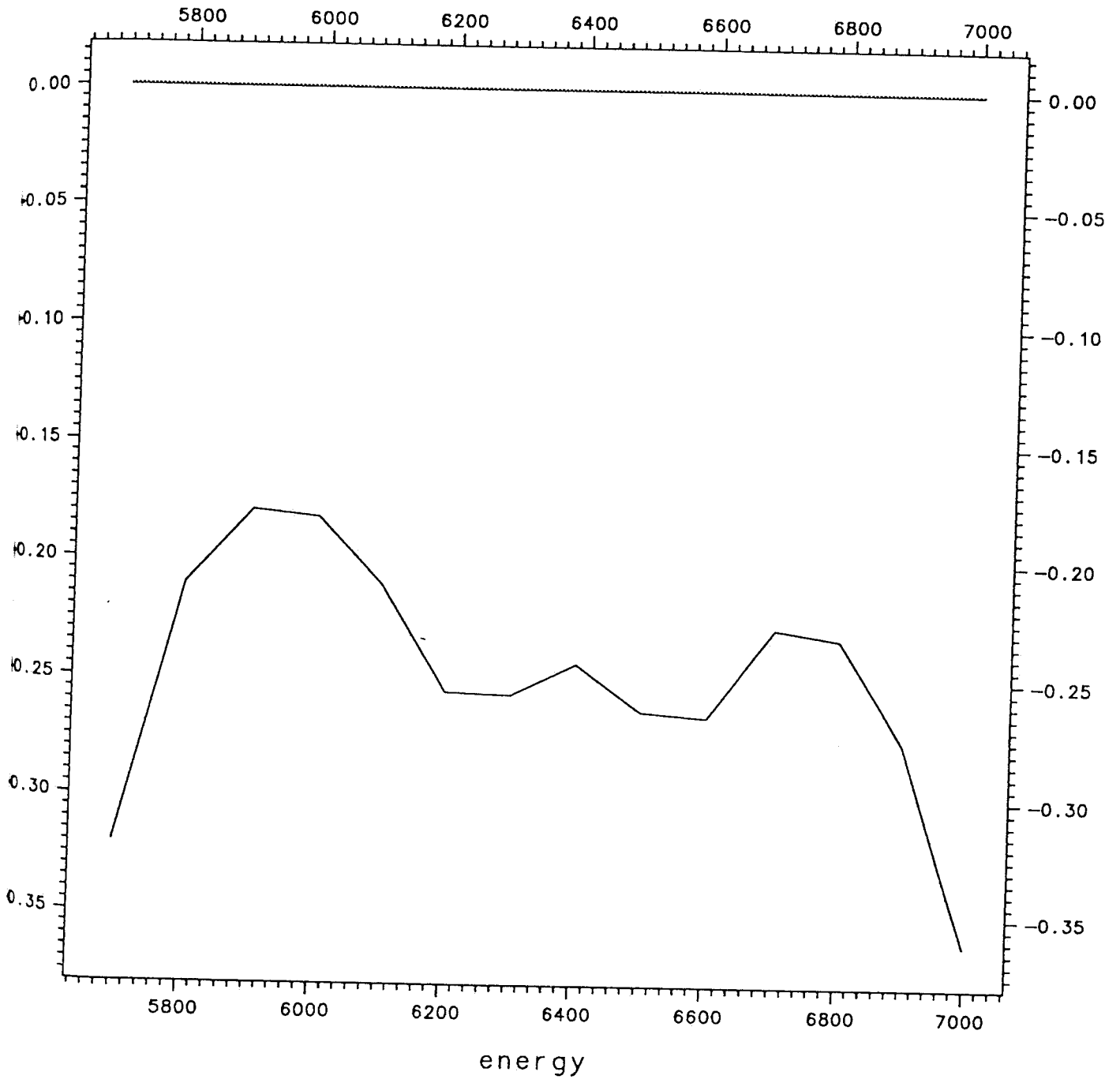
Number of itrs between change of controlparameter=100

!23456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*1234

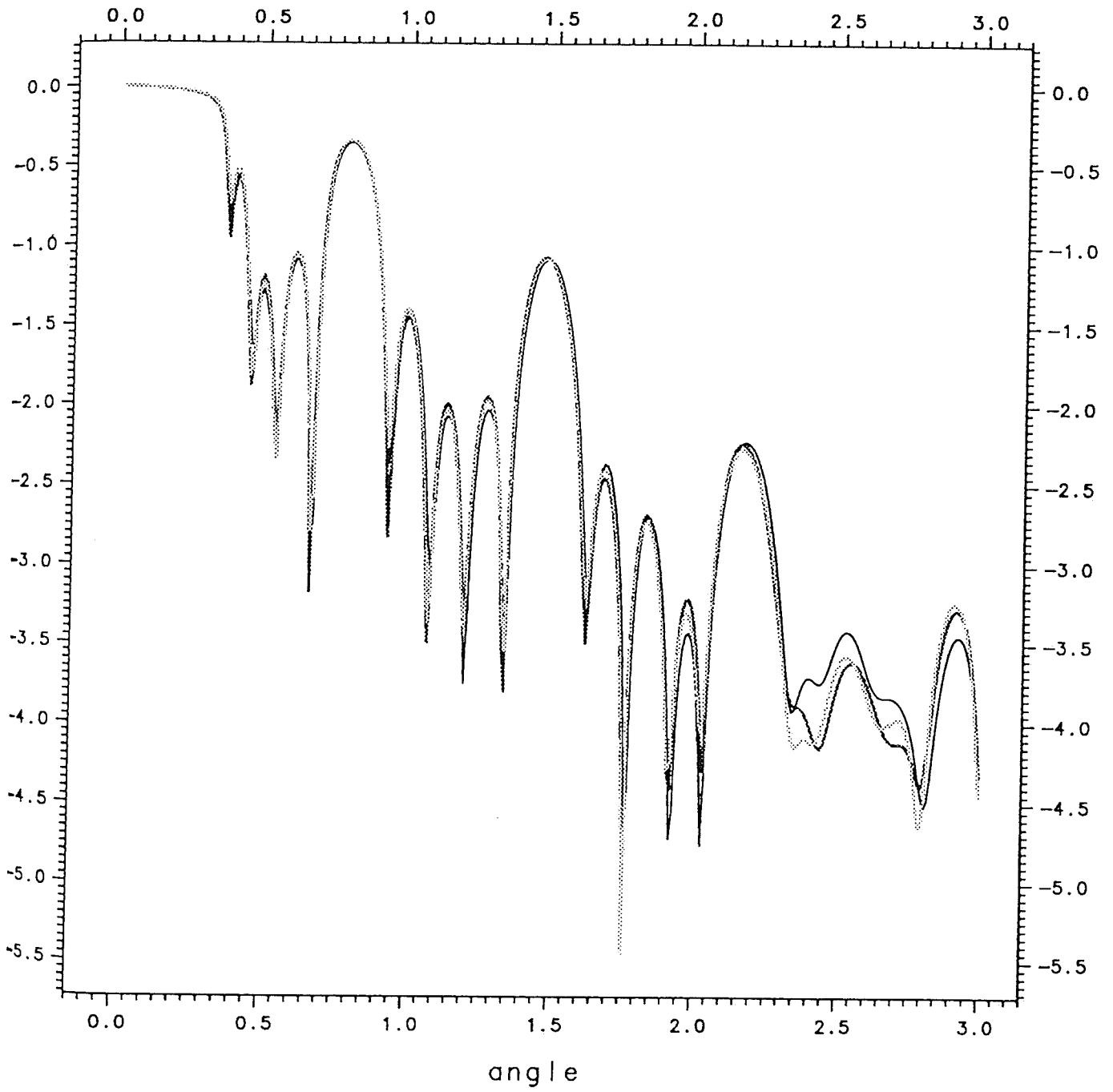
Appendix B

DIAGRAMS

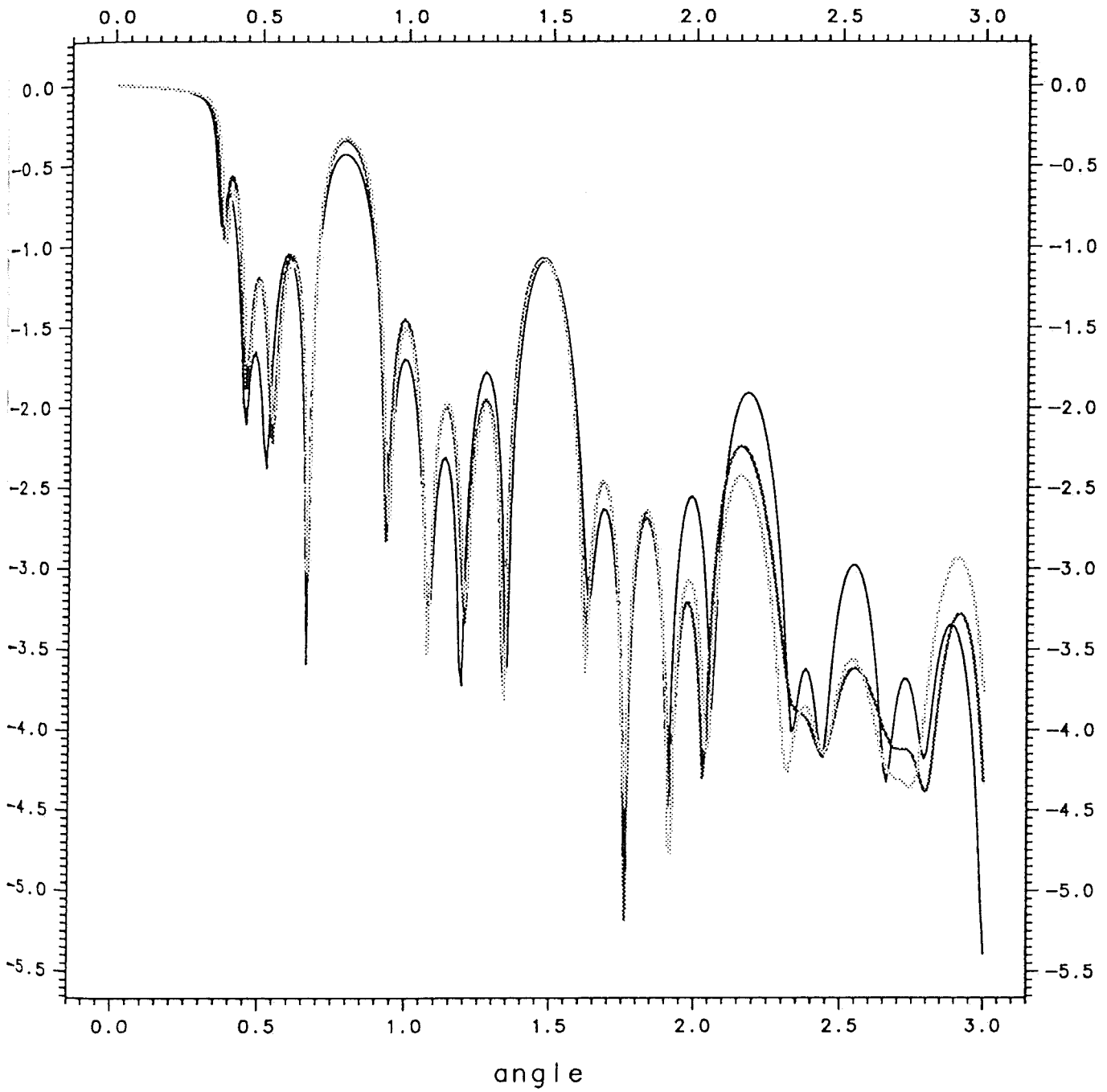
test graph, sasi8



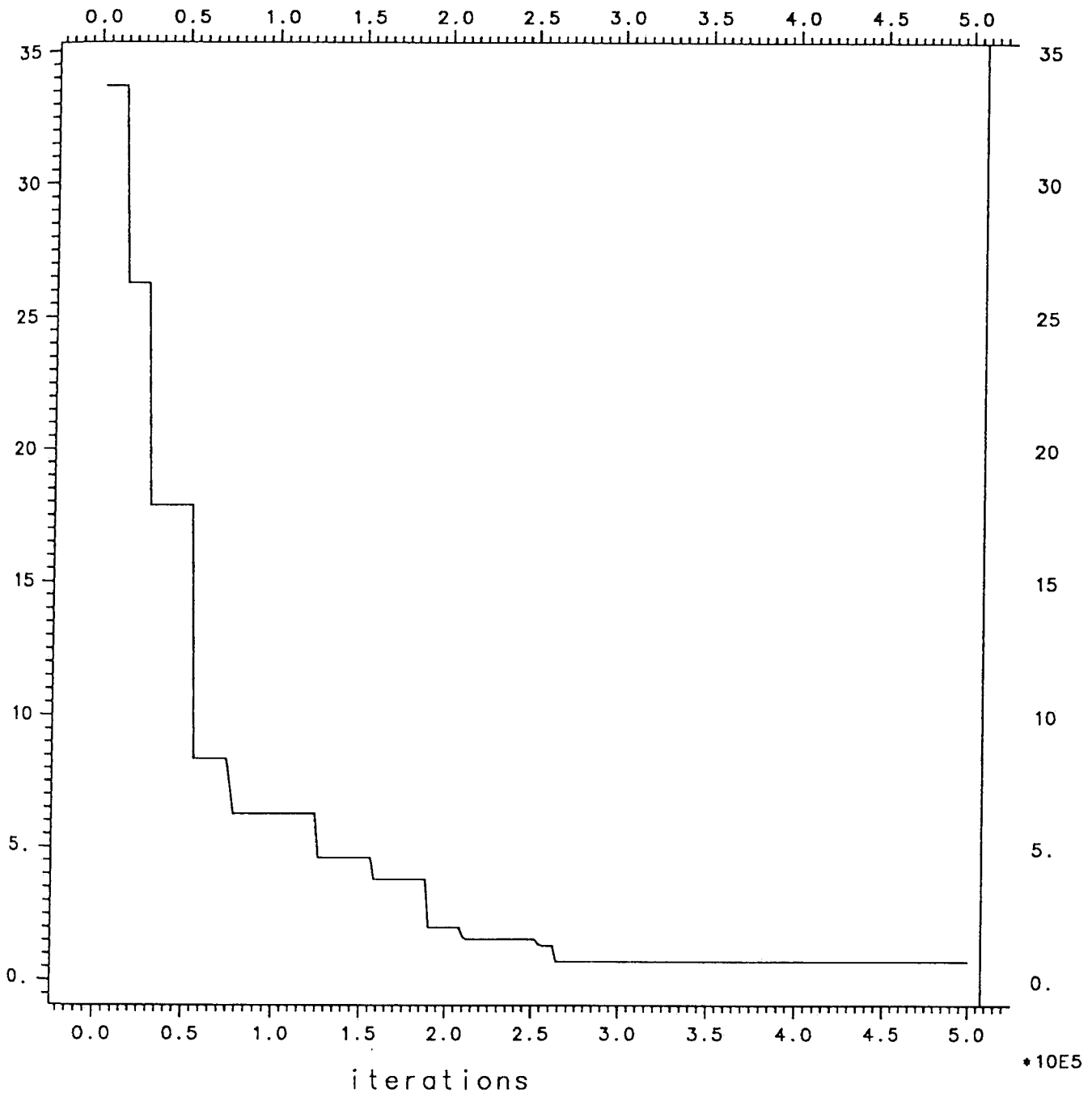
single, expon. cooling



ensemble, thermodyn. cooling



testdata/s0202g.log



Appendix C

FORTRAN PROGRAMS


```

write(3,'(a)') title
do i=1,nlayers
write(3,*) i,thick(i),thick(i+nlayers)
end do
write(3,'(a)') xleg
write(3,'(a)') yleg
write(3,'(i6)') npts
do i=1,npts
write(3,'(3(lpgl4.7))',xval(i),yval(i),y2val(i))
end do
close(3)
end
C*****The simplex subroutines
subroutine ambsa(p,y,pb,yb,ftol,funk,iter,temptr,exprefl,
cd_index,energy,theta,init,minp,maxp,bestthick,
thickconst,bestj)
integer iter,ndim,npts,nlayers,thickconst(2*nlayers),aoe,bestj,
ngood,n_attempts
real*4 ftol,temptr,yb,p(ndim+1,ndim),pb(ndim),y(ndim+1),
funk,ran,amotsa,exprefl(npts)
1 energy(((aoe-1)*npts+(aoe-1))/2),!(1)=1 if ang,=npts if endep
1 theta(((aoe-1)*npts+(aoe-1))/2),!(1)=1 if endep, =npts if ang
1 minp(ndim),maxp(ndim),bestthick(2*nlayers)
C uses amotsa,funk,ran
integer i,init,ih,iio,inh,i,j,m,n
real rtoi,sum,swap,tt,yhi,ylo,ynhi,ysave,yt,ytry,psum(3001*2),
complex*16 cd_index(nlayers+1,npts)
common /points/npts,nlayers,thickconst(2*nlayers),aoe,bestj,
common /quality/ngood,n_attempts
external funk
implicit none
Executable code:
C
tt=temptr
do n=1,ndim
sum=0.
do m=1,ndim+1
sum=sum+p(m,n)
end do
psum(n)=sum
end do
iio=1
inh=1
ih=2
ylo=y(1)+tt*log(ran(init))
ynhi=ylo
yhi=y(2)+tt*log(ran(init))
if (ylo.gt.yhi) then
ihi=1
ih=2
iio=2
ynhi=yhi
yhi=ylo
ylo=ynhi
endif
do i=3,ndim+1
yt=y(1)+tt*log(ran(init))
if (yt.le.ylo) then
iio=i
ylo=yt
end if
if (yt.gt.yhi) then
inh=ihi
ynhi=yhi
ih=i
yhi=yt
else if (yt.gt.ynhi) then
inh=i
ynhi=yt
end if
end if

```

```

read(2,'(a72)') text
n=3
call blank(allortwo,n)
if (allortwo.eq.'2') then
do i=1,3
read(2,'(al5,f8.1,f8.1,f8.1,f8.1,f8.1,f8.1,f8.1,
f8.1,f8.1,f8.1) sym(i),rho(i),bestthick(i+nlayers),
minthick(i+nlayers),maxthick(i+nlayers),
deltathick(i+nlayers),bestthick(i),minthick(i),
maxthick(i),deltathick(i))
end do
do i=4,nlayers,2
sym(i)=sym(2)
rho(i)=rho(2)
bestthick(i+nlayers)=bestthick(2+nlayers)
minthick(i+nlayers)=minthick(2+nlayers)
maxthick(i+nlayers)=maxthick(2+nlayers)
deltathick(i+nlayers)=deltathick(2+nlayers)
bestthick(i)=bestthick(2)
minthick(i)=minthick(2)
maxthick(i)=maxthick(2)
deltathick(i)=deltathick(2)
sym(i+1)=sym(3)
rho(i+1)=rho(3)
bestthick(i+1+nlayers)=bestthick(3+nlayers)
minthick(i+1+nlayers)=minthick(3+nlayers)
maxthick(i+1+nlayers)=maxthick(3+nlayers)
deltathick(i+1)=deltathick(3)
bestthick(i+1)=bestthick(3)
minthick(i+1)=minthick(3)
maxthick(i+1)=maxthick(3)
deltathick(i+1)=deltathick(3)
end do
end if
if (allortwo.eq.'al') then
do i=1,nlayers
read(2,'(al5,f8.1,f8.1,f8.1,f8.1,f8.1,f8.1,f8.1,
f8.1,f8.1,f8.1) sym(i),rho(i),bestthick(i+nlayers),
minthick(i+nlayers),maxthick(i+nlayers),
deltathick(i+nlayers),bestthick(i),minthick(i),
maxthick(i),deltathick(i))
n=i5
call blank(sym(i),n)
type *,bestthick(i),bestthick(i+nlayers)
end do
end if
read(2,'(a72)') text
read(2,'(a72)') text
read(2,'(a72)') text
read(2,'(40x,i8)') conskip
read(2,'(a72)') text
read(2,'(a72)') text
read(2,'(25x,f7.3)') ftol
read(2,'(50x,i8)') itrtemp
close(2)
end
I*****
I23456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*
subroutine wrfile(outname,npts,xval,yval,y2val,title,xleg,
ylleg,nlayers,thick)
real*4 xval(2001),yval(2001),y2val(2001),thick(3001)
integer*4 npts,nlayers,n,i
character*80 title,xleg,yleg
character*80 outname
implicit none
n=80
call blank(outname,n)
call blank(title,n)
call blank(xleg,n)
call blank(ylleg,n)
open(unit=3,file=outname,status='unknown',access='sequential',
form='formatted')

```

```

enddo
rtol=abs(yhi-ylo)/(abs(yhi)+abs(ylo)) ! factor 2 diff from orig.
if (rtol.lt.ftol.or.iter.lt.0) then
  type*, rtol, rtol
  swap=y(1)
  y(1)=y(iilo)
  y(iilo)=swap
  do n=1, ndim
    swap=p(1,n)
    p(1,n)=p(iilo,n)
    p(iilo,n)=swap
  enddo
  return
end if
iter=iter-2
ytry=amotsa(p,y,psum,pb,yb,funk,ih,i,yhi,-1.0,
  exprefl,cd_index,energy,theta,init,tt,
  minp,maxp,bestthick,thickconst,bestj,iter)
if (ytry.le.ylo) then
  ytry=amotsa(p,y,psum,pb,yb,funk,ih,i,yhi,2.0,
    exprefl,cd_index,energy,theta,init,tt,
    minp,maxp,bestthick,thickconst,bestj,iter)
else if (ytry.ge.yshi) then
  ysave=yhi
  ytry=amotsa(p,y,psum,pb,yb,funk,ih,i,yhi,.5,
    exprefl,cd_index,energy,theta,init,tt,
    minp,maxp,bestthick,thickconst,bestj,iter)
if (ytry.ge.ysave) then
  do i=1,ndim+1
    if (i.ne.iilo) then
      do j=1,ndim
        psum(j)=0.5*(p(i,j)+p(iilo,j))
      enddo
      y(i)=funk(psum,exprefl,cd_index,energy,theta,
        exprefl,thickconst,bestthick,thickconst)
    end if
  enddo
  iter=iter-ndim
  goto 1
end if
else
  iter=iter+1
end if
goto 2
end

function amotsa(p,y,psum,pb,yb,funk,ih,i,yhi,fac,
  exprefl,cd_index,energy,theta,init,tt,minp,
  maxp,bestthick,thickconst,bestj,iter)
  integer ih,i,ndim,thickconst(2*nlayers),aoe,npts,nlayers,
  ngood,n_attempts
  real*4 amotsa,fac,yb,yhi,p(ndim+1,ndim),pb(ndim),psum(ndim),
  y(ndim+1),funk,ran,exprefl(npts),
  energy(((aoe-1)*npts+(aoe+1))/2),!(1)=1 if ang,=npts if endep
  theta(((aoe-1)*npts+(aoe+1))/2),!(1)=1 if endep,=npts if ang
  minp(ndim),maxp(ndim),bestthick(2*nlayers)
  uses funk,ran
  integer init,j,bestj,iter
  real fac1,fac2,tt,yflu,ytry,ptry(3001*2)
  complex*16 cd_index(nlayers+1,npts)
  common /points/ npts,nlayers,ndim,aoe
  common /quality/ ngood,n_attempts
  implicit none
  Executable code :
  fac1=(1.-fac)/ndim
  fac2=fac1-fac
  do j=1,ndim
    ptry(j)=psum(j)*fac1-p(ihi,j)*fac2
    if (ptry(j).lt.minp(j)) then
      ptry(j)=ptry(j)+maxp(j)-minp(j)
    else if (ptry(j).gt.maxp(j)) then
      ptry(j)=ptry(j)-maxp(j)+minp(j)
    end if
  enddo
  end if
enddo
ytry=funk(pty,exprefl,cd_index,energy,theta,bestthick,
  thickconst)
do j=1,ndim
  pb(j)=pty(j)
enddo
yb=ytry
bestj=iter
end if
yflu=ytry-tt*log(ran(init))
n_attempts=n_attempts+1
if (yflu.lt.yhi) then
  ngood=ngood+1
  y(ihi)=ytry
  do j=1,ndim
    psum(j)=psum(j)-p(ihi,j)+pty(j)
  enddo
  end if
  amotsa=yflu
  return
end

```

```

enddo
rtol=abs(yhi-ylo)/(abs(yhi)+abs(ylo)) ! factor 2 diff from orig.
if (rtol.lt.ftol.or.iter.lt.0) then
  type*, rtol, rtol
  swap=y(1)
  y(1)=y(iilo)
  y(iilo)=swap
  do n=1, ndim
    swap=p(1,n)
    p(1,n)=p(iilo,n)
    p(iilo,n)=swap
  enddo
  return
end if
iter=iter-2
ytry=amotsa(p,y,psum,pb,yb,funk,ih,i,yhi,-1.0,
  exprefl,cd_index,energy,theta,init,tt,
  minp,maxp,bestthick,thickconst,bestj,iter)
if (ytry.le.ylo) then
  ytry=amotsa(p,y,psum,pb,yb,funk,ih,i,yhi,2.0,
    exprefl,cd_index,energy,theta,init,tt,
    minp,maxp,bestthick,thickconst,bestj,iter)
else if (ytry.ge.yshi) then
  ysave=yhi
  ytry=amotsa(p,y,psum,pb,yb,funk,ih,i,yhi,.5,
    exprefl,cd_index,energy,theta,init,tt,
    minp,maxp,bestthick,thickconst,bestj,iter)
if (ytry.ge.ysave) then
  do i=1,ndim+1
    if (i.ne.iilo) then
      do j=1,ndim
        psum(j)=0.5*(p(i,j)+p(iilo,j))
      enddo
      y(i)=funk(psum,exprefl,cd_index,energy,theta,
        exprefl,thickconst,bestthick,thickconst)
    end if
  enddo
  iter=iter-ndim
  goto 1
end if
else
  iter=iter+1
end if
goto 2
end

function amotsa(p,y,psum,pb,yb,funk,ih,i,yhi,fac,
  exprefl,cd_index,energy,theta,init,tt,minp,
  maxp,bestthick,thickconst,bestj,iter)
  integer ih,i,ndim,thickconst(2*nlayers),aoe,npts,nlayers,
  ngood,n_attempts
  real*4 amotsa,fac,yb,yhi,p(ndim+1,ndim),pb(ndim),psum(ndim),
  y(ndim+1),funk,ran,exprefl(npts),
  energy(((aoe-1)*npts+(aoe+1))/2),!(1)=1 if ang,=npts if endep
  theta(((aoe-1)*npts+(aoe+1))/2),!(1)=1 if endep,=npts if ang
  minp(ndim),maxp(ndim),bestthick(2*nlayers)
  uses funk,ran
  integer init,j,bestj,iter
  real fac1,fac2,tt,yflu,ytry,ptry(3001*2)
  complex*16 cd_index(nlayers+1,npts)
  common /points/ npts,nlayers,ndim,aoe
  common /quality/ ngood,n_attempts
  implicit none
  Executable code :
  fac1=(1.-fac)/ndim
  fac2=fac1-fac
  do j=1,ndim
    ptry(j)=psum(j)*fac1-p(ihi,j)*fac2
    if (ptry(j).lt.minp(j)) then
      ptry(j)=ptry(j)+maxp(j)-minp(j)
    else if (ptry(j).gt.maxp(j)) then
      ptry(j)=ptry(j)-maxp(j)+minp(j)
    end if
  enddo
  end if
enddo
ytry=funk(pty,exprefl,cd_index,energy,theta,bestthick,
  thickconst)
do j=1,ndim
  pb(j)=pty(j)
enddo
yb=ytry
bestj=iter
end if
yflu=ytry-tt*log(ran(init))
n_attempts=n_attempts+1
if (yflu.lt.yhi) then
  ngood=ngood+1
  y(ihi)=ytry
  do j=1,ndim
    psum(j)=psum(j)-p(ihi,j)+pty(j)
  enddo
  end if
  amotsa=yflu
  return
end

```



```

1 integer nsteps,nlayers,npts,good,i,j,bestj,consskip,
1 init,aoe,n,upgood,
1 status,start_coordinates(5),end_coordinates(5)
1 character*80 inname,indat,outdat,outlo1,sym(nlayers),xleg,yleg
1 character*20 angdep
1 character*6 x_label
1 complex*16 cd_index(nlayers+1,npts)
1 real*4 control,mincontrol,maxcontrol,thetamean,
1 deltathick(2*nlayers),newthick(2*3001),
1 bestthick(2*nlayers),
1 energy(((aoe-1)*npts+(aoe+1))/2),!(i)=1 if ang,=npts if endep
1 theta((aoe-1)*npts+(aoe+1))/2,!(i)=1 if endep,=npts if ang
1 delta((aoe+1)*npts+(aoe-1))/2,((aoe-1)*npts+(aoe+1))/2),
1 beta((aoe+1)*npts+(aoe-1))/2,((aoe-1)*npts+(aoe+1))/2),
1 rho(nlayers),
1 oldcost,newcost,bestcost,exprefl(npts),calcrefl(npts),
1 bestrefl(npts),
1 rate,uprate,time,time1,time2,seconds,
1 x_coordinates(npts,5),y_coordinates(npts,5)
1 parameter (degrad=0.01745329)
1 implicit none
1 common/points/npts,nlayers,aoe

C Executable code:
1 time=seconds(0.)
1 init=nint(time/2)*2+1
1 type *,'initvalue ',init
1 time1=seconds(0.)
1 if (aoe.eq.1) then
1 call rddat(indat,npts,theta,exprefl)
1 call optconan(npts,energy,theta,nlayers,sym,rho,cd_index,
1 delta,beta,angdep,thetamean)
1 call reflecan(bestrefl,energy,theta,thick,cd_index)
1 else
1 call rddat(indat,npts,energy,exprefl)
1 call optconen(npts,energy,theta,nlayers,sym,rho,cd_index,
1 delta,beta)
1 call reflecan(bestrefl,energy,theta,thick,cd_index)
1 end if
1 time2=seconds(time1)
1 type *,'sec.s:',time2
1 call costl(exprefl,bestrefl,npts,oldcost)
1 bestcost=oldcost
1 do i=1,nlayers*2
1 bestthick(i)=thick(i)
1 newthick(i)=thick(i)
1 end do
1 control=maxcontrol
1 uprate=0
1 rate=0

C
1 Graphics before annealing
1 if (aoe.eq.1) then
1 do i=1,npts
1 x_coordinates(i,2)=theta(i)/degrad
1 x_coordinates(i,1)=theta(i)/degrad
1 end do
1 x_label='angle '
1 do i=1,npts
1 x_coordinates(i,2)=energy(i)
1 x_coordinates(i,1)=energy(i)
1 end do
1 x_label='energy'
1 end if
1 do i=1,2
1 start_coordinates(i)=1
1 end_coordinates(i)=npts
1 end do
1 do i=1,npts
1 y_coordinates(i,2)=Alog10(exprefl(i))
1 y_coordinates(i,1)=Alog10(bestrefl(i))
1 end do
1 call EXPG GR XYGRAPH(npts,5,2,
1 start_coordinates,end_coordinates,
1 x_coordinates,y_coordinates,
1 x_label,'reflectivity',status)

1 integer nsteps,nlayers,npts,good,i,j,bestj,consskip,
1 init,aoe,n,upgood,
1 status,start_coordinates(5),end_coordinates(5)
1 character*80 inname,indat,outdat,outlo1,sym(nlayers),xleg,yleg
1 character*20 angdep
1 character*6 x_label
1 complex*16 cd_index(nlayers+1,npts)
1 real*4 control,mincontrol,maxcontrol,thetamean,
1 deltathick(2*nlayers),newthick(2*3001),
1 bestthick(2*nlayers),
1 energy(((aoe-1)*npts+(aoe+1))/2),!(i)=1 if ang,=npts if endep
1 theta((aoe-1)*npts+(aoe+1))/2,!(i)=1 if endep,=npts if ang
1 delta((aoe+1)*npts+(aoe-1))/2,((aoe-1)*npts+(aoe+1))/2),
1 beta((aoe+1)*npts+(aoe-1))/2,((aoe-1)*npts+(aoe+1))/2),
1 rho(nlayers),
1 oldcost,newcost,bestcost,exprefl(npts),calcrefl(npts),
1 bestrefl(npts),
1 rate,uprate,time,time1,time2,seconds,
1 x_coordinates(npts,5),y_coordinates(npts,5)
1 parameter (degrad=0.01745329)
1 implicit none
1 common/points/npts,nlayers,aoe

C Executable code:
1 time=seconds(0.)
1 init=nint(time/2)*2+1
1 type *,'initvalue ',init
1 time1=seconds(0.)
1 if (aoe.eq.1) then
1 call rddat(indat,npts,theta,exprefl)
1 call optconan(npts,energy,theta,nlayers,sym,rho,cd_index,
1 delta,beta,angdep,thetamean)
1 call reflecan(bestrefl,energy,theta,thick,cd_index)
1 else
1 call rddat(indat,npts,energy,exprefl)
1 call optconen(npts,energy,theta,nlayers,sym,rho,cd_index,
1 delta,beta)
1 call reflecan(bestrefl,energy,theta,thick,cd_index)
1 end if
1 time2=seconds(time1)
1 type *,'sec.s:',time2
1 call costl(exprefl,bestrefl,npts,oldcost)
1 bestcost=oldcost
1 do i=1,nlayers*2
1 bestthick(i)=thick(i)
1 newthick(i)=thick(i)
1 end do
1 control=maxcontrol
1 uprate=0
1 rate=0

C
1 Graphics before annealing
1 if (aoe.eq.1) then
1 do i=1,npts
1 x_coordinates(i,2)=theta(i)/degrad
1 x_coordinates(i,1)=theta(i)/degrad
1 end do
1 x_label='angle '
1 do i=1,npts
1 x_coordinates(i,2)=energy(i)
1 x_coordinates(i,1)=energy(i)
1 end do
1 x_label='energy'
1 end if
1 do i=1,2
1 start_coordinates(i)=1
1 end_coordinates(i)=npts
1 end do
1 do i=1,npts
1 y_coordinates(i,2)=Alog10(exprefl(i))
1 y_coordinates(i,1)=Alog10(bestrefl(i))
1 end do
1 call EXPG GR XYGRAPH(npts,5,2,
1 start_coordinates,end_coordinates,
1 x_coordinates,y_coordinates,
1 x_label,'reflectivity',status)

```

```

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
integer good,init,upgood
real*4 oldcost, newcost,control,deltacost,probb,random,ran
implicit none
C Executable code:
deltacost=newcost-oldcost
if (deltacost .le. 0) then
  good=1
  upgood=0
else
  random=ran(init)
  probb=exp(-deltacost/control)-random
  if (probb.ge. 0) then
    good=1
    upgood=1
  else
    good=0
    upgood=0
  endif
endif
return
end
C*****123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*12345
C Jan-92
1 subroutine neighbour(thick,newthick,minthick,maxthick,
  deltathick,nlayers,init)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C Randomly chooses a neighbour to the current solution
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
integer nlayers,changel,init
real*4 thick(2*nlayers),minthick(2*nlayers),maxthick(2*nlayers),
deltathick(2*nlayers),addthick,random,userough,
parameter (userough=0.0)
implicit none
C Executable code
random=ran(init)
changel=int(random*nlayers-le-15)+1
random=ran(init)
if (random.ge. userough) then
  random=ran(init)
  addthick=(int(random*2-le-15)*2-1)*deltathick(changel)
  newthick(changel)=thick(changel)+addthick
  if (newthick(changel).lt. minthick(changel)) then
    newthick(changel)=maxthick(changel)
  else if (newthick(changel).gt. maxthick(changel)) then
    newthick(changel)=minthick(changel)
  endif
endif
random=ran(init)
addthick=(int(random*2-le-15)*2-1)*
deltathick(changel+nlayers)
newthick(changel+nlayers)=thick(changel+nlayers)+addthick
if (newthick(changel+nlayers).lt.
minthick(changel+nlayers)) then
  newthick(changel+nlayers)=maxthick(changel+nlayers)
else if (newthick(changel+nlayers).gt.
maxthick(changel+nlayers)) then
  newthick(changel+nlayers)=minthick(changel+nlayers)
endif
return
end
C*****123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*12345
C Jan-92
1 subroutine accept(oldcost,newcost,control,good,upgood,init)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C Decides whether to accept neighbour to the current solution
C

```

```

write(*,*)
write(*, '(x,a9,4x,a9,4x,a4,6x,a7,4x,a6)') 'iter. no.',
'best iter.',rate,'control',uprate'
write(*, '(x,i8,5x,i8,3x,g10.5,2x,g10.5)') j,
bestj,rate,control,uprate
write(*, '(x,a15,x,a19)', bestcost
cost now
write(*, '(x,g15.7,3x,g15.7)') bestcost,oldcost
write(2, '(a9,2x,a9,3x,a4,3x,a7,3x,a6)') 'iter. no.',
'best iter.',rate,'control',uprate'
write(2, '(x,i8,3x,i8,x,g10.5,g10.5)') j,
bestj,rate,control,uprate
write(2, '(x,a15,x,a19)') bestcost
cost now
write(2, '(x,g15.7,3x,g15.7)') bestcost,oldcost
write(*,*)
write(*, '(i3x,a13,6x,a15)') 'best till now',
layers now
write(*, '(a9,3x,4(a5,5x))') 'layer no.', 'thick', 'rough',
'thick', 'rough'
do i=1,nlayers
  write(*, '(i8,2x,4(g10.5,g10.5)') i,bestthick(i),
  bestthick(i+nlayers),thick(i),thick(i+nlayers)
end do
rate=0
uprate=0
endif
call cool(control,maxcontrol,mincontrol,nsteps)
close(2)
if (aoe.eq.1) then
  xleg='angle (degrees)'
  yleg='reflectivity exp calc'
  call wrfile(outdat,npts,theta,exprefl,bestrefl,inname,
  xleg,yleg,nlayers,bestthick)
else
  xleg='energy (ev)'
  yleg='reflectivity exp calc'
  call wrfile(outdat,npts,energy,exprefl,bestrefl,inname,
  xleg,yleg,nlayers,bestthick)
endif
return
end
C*****123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*12345
C Jan-92
C
subroutine cool(control,maxcontrol,mincontrol,nsteps)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C Lowers the control parameter (temperature) exponentially
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
integer nsteps
real*4 control,maxcontrol,mincontrol
implicit none
C Executable code:
control=control/exp(log(maxcontrol/mincontrol)/nsteps)
return
end
C*****123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*12345
C Jan-92
1 subroutine accept(oldcost,newcost,control,good,upgood,init)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C Decides whether to accept neighbour to the current solution
C

```

```

C23456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*12345
C Jan-92

subroutine cost1(expdat,calcdat,npts,kost)
=====
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C Calculates the cost function of a calculated reflectivity curve C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

integer npts,i
real*4 expdat(2001),calcdat(2001), kost
implicit none

C Executable code:
kost=0
do i=1,npts
  kost=kost+(1-(calcdat(i)/expdat(i)))**2
  + (1-(expdat(i)/calcdat(i)))**2
end do
kost=kost/npts
return
end

C*****
C23456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*12345
C Jan-92

subroutine cost2(expdat,calcdat,npts,kost)
=====
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C Calculates the cost function of a calculated reflectivity curve C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

integer npts,i
real*4 expdat(2001),calcdat(2001), kost
implicit none

C Executable code:
kost=0
do i=1,npts
  kost=kost+(1-(calcdat(i)/expdat(i)))**2
end do
kost=kost/npts
return
end

C*****
C23456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*12345
C Jan-92

subroutine cost4(expdat,calcdat,npts,kost)
=====
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C Calculates the cost function of a calculated reflectivity curve C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

integer npts,i
real*4 expdat(2001),calcdat(2001), kost
implicit none

C Executable code:
kost=0
do i=1,npts
  kost=kost*log(1+(1-(calcdat(i)/expdat(i))))**2
  + (1-(expdat(i)/calcdat(i)))**2
end do
kost=kost/npts
return
end

C*****

```

```

C23456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*12345
C Jan-92

subroutine optconan(npts,energy,theta,nlayers,sym,rho,cd_index,
  delta,beta,angdep,thetamean)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C Finds optical constants for each layer and for each angle C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

character*20 unit,angdep
integer nlayers,npts,i,n,k,j
character*80 sym(3001)
complex*16 cd_index(nlayers+1,npts)
real*4 delta(npts,1),beta(npts,1),
  energy(1),theta(npts),rho(3001),thetamean
parameter (degrad=0.01745329)
implicit none

C Executable code
type *,in optconan'
unit='ev'
do i=1,npts
  theta(i)=theta(i)*degrad
end do
n=3
call blank(angdep,n)
if (angdep .eq. 'y ') then
  type *,sym(1)
call fi2(npts,1,sym(1),energy,unit,theta,delta,beta,rho(1))
type *, called fi2 (1),
do i=1,npts
  cd_index(1,i)= dcmplx(1-delta(i,1),-beta(i,1))
end do
do j=2,nlayers
  do k=1,j-1
    if (sym(j) .eq. sym(k)) then
      do i=1,npts
        cd_index(j,i)=cd_index(k,i)
        type *,cd_index(j,i)
      end do
      go to 111
    end if
  end do
call fi2(npts,1,sym(j),energy,unit,theta,delta,beta,rho(j))
type *, called fi2,
do i=1,npts
  cd_index(j,i)= dcmplx(1-delta(i,1),-beta(i,1))
  type *,cd_index(j,i)
end do
go to 111
end do
type *,'Found optical constants'
end if

if (angdep .eq. 'n ') then
  type *,'thetamean=',thetamean
call fi2(1,1,sym(1),thetamean,energy,unit,thetamean,delta,beta,rho(1))
type *, called fi2 (1),
do i=1,npts
  cd_index(1,i)= dcmplx(1-delta(1,1),-beta(1,1))
  cd_index(nlayers+1,i) =(1.,0.)
end do
do j=2,nlayers
  do k=1,j-1
    if (sym(j) .eq. sym(k)) then
      do i=1,npts
        cd_index(j,i)=cd_index(k,i)
        type *,cd_index(j,i)
      end do
      go to 112
    end if
  end do
call fi2(1,1,sym(j),energy,unit,thetamean,delta,beta,rho(j))
type *, called fi2,
do i=1,npts

```



```

C executable code
type *,in optconen'
theta(1)=theta(1)*degrad
unit='ev'
call fl2(1,npts,sym(1),energy,unit,theta,delta,beta,rho(1))
!
type *, called fl2 (1),
do i=1,npts
  cd_index(1,i)= dcmplx(1-delta(1,i),-beta(1,i))
  cd_index(nlayers+1,i) =(1.,0.)
end do
do j=2,nlayers
  do k=i,j-1
    if (sym(j) .eq. sym(k)) then
      do i=1,npts
        cd_index(j,i)=cd_index(k,i)
        type *,cd_index(j,i)
      end do
    go to 111
  end if
end do
call fl2(1,npts,sym(j),energy,unit,theta,delta,beta,rho(j))
!
type *, called fl2',
do i=1,npts
  cd_index(j,i)= dcmplx(1-delta(1,i),-beta(1,i))
  type *,cd_index(j,i)
end do
go to 111
end do
!
type *,'Found optical constants'
return
end

C*****123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*
C Jan-92

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C Calculates reflectivity of a multilayer structure as a function of angle C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

integer nlayers,npts,aoe,i,j
complex*16 cd_index(nlayers+1,npts), cd_r, cd_x_zero, cd_x_top
real*8 four_pi_on_lam,cos_sqr_th,sinthe
parameter (four_pi_hc= 1.013536e-3) ! =4*pi/hc
common/points/npts,nlayers,aoe
implicit none

C Executable code:
four_pi_on_lam= four_pi_hc* energy(1)
do i=1,npts
  cos_sqr_th= cos( theta(i))**2
  sinthe= sin( theta(1))**2
  cd_x_zero= (0., 0.) ! reflected amplitude=0 at substrate
  cd_x_top= (0., 0.)
  do j=1,nlayers
    ! njsinthej= sqrt(nj**2-cos**2(th inc))
    cd_thet2= cdsqrt( cd_index(j,i)**2- cos_sqr_th)
    if(dimag(cd_thet2) .gt. 0.) cd_thet2= dconjg(cd_thet2)
    cd_thet1= cdsqrt( cd_index(j+1,i)**2-cos_sqr_th)
    ! x = (cd_thet1- cd_thet2)/( cd_thet1+ cd_thet2)
    ! include debye-waller roughness term
    cd_r=cd_r*exp(-(four_pi_on_lam*thick(j)+nlayers)*sinthe)**2
    cd_exp_phi=cexp( (0.,-1.d0)*
      four_pi_on_lam* thick(j) * cd_thet2)
    cd_x_top= ( cd_r+ cd_x_zero*cd_exp_phi)
    / ( 1.+ cd_r* cd_x_zero*cd_exp_phi)
    cd_x_zero= cd_x_top
  end do
  refl(i) = cdabs(cd_x_top)**2
end do
return
end

!*****123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*
! sept-91

subroutine optconen(npts,energy,theta,nlayers,sym,rho,cd_index,
  delta,beta)
  integer nlayers,npts,i,k,j
  character*80 sym(3001)
  complex*16 cd_index(nlayers+1,npts)
  real*4 delta(1,npts),beta(1,npts),energy(npts)
  parameter (delta(1,3001),beta(1,3001),degrad
    parameter (degrad=0.01745329)
  implicit none

```

```

C executable code
type *,in optconen'
theta(1)=theta(1)*degrad
unit='ev'
call fl2(1,npts,sym(1),energy,unit,theta,delta,beta,rho(1))
!
type *, called fl2 (1),
do i=1,npts
  cd_index(1,i)= dcmplx(1-delta(1,i),-beta(1,i))
  cd_index(nlayers+1,i) =(1.,0.)
end do
do j=2,nlayers
  do k=i,j-1
    if (sym(j) .eq. sym(k)) then
      do i=1,npts
        cd_index(j,i)=cd_index(k,i)
        type *,cd_index(j,i)
      end do
    go to 111
  end if
end do
call fl2(1,npts,sym(j),energy,unit,theta,delta,beta,rho(j))
!
type *, called fl2',
do i=1,npts
  cd_index(j,i)= dcmplx(1-delta(1,i),-beta(1,i))
  type *,cd_index(j,i)
end do
go to 111
end do
!
type *,'Found optical constants'
return
end

C*****123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*
C Jan-92

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C Calculates reflectivity of a multilayer structure as a function of energy C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

integer nlayers,npts,aoe,i,j
complex*16 cd_index(nlayers+1,npts), cd_r, cd_x_zero, cd_x_top
real*8 four_pi_on_lam,cos_sqr_th,sinthe
parameter (four_pi_hc= 1.013536e-3) ! =4*pi/hc
common/points/npts,nlayers,aoe
implicit none

C executable code:
cos_sqr_th= cos( theta(1))**2
sinthe= sin( theta(1))**2
do i=1, npts
  four_pi_on_lam= four_pi_hc* energy(i)
  cd_x_zero= (0., 0.) ! rreflected amplitude=0 at substrate
  cd_x_top= (0., 0.)
  do j=1,nlayers
    ! njsinthej= sqrt(nj**2-cos**2(th inc))
    cd_thet2= cdsqrt( cd_index(j,i)**2- cos_sqr_th)
    if(dimag(cd_thet2) .gt. 0.) cd_thet2= dconjg(cd_thet2)
    cd_thet1= cdsqrt( cd_index(j+1,i)**2-cos_sqr_th)
    cd_x = (cd_thet1- cd_thet2)/( cd_thet1+ cd_thet2)
    ! include debye-waller roughness term
    cd_r=cd_r*exp(-(four_pi_on_lam*thick(j)+nlayers)*sinthe)**2
    cd_exp_phi=cexp( (0.,-1.d0)*
      four_pi_on_lam* thick(j) * cd_thet2)
    cd_x_top= ( cd_r+ cd_x_zero*cd_exp_phi)
    / ( 1.+ cd_r* cd_x_zero*cd_exp_phi)
    cd_x_zero= cd_x_top
  end do
  cd_exp_phi=cexp( (0.,-1.d0)*
    thick(j+nlayers)*sinthe)**2
  cd_x_top= ( cd_r+ cd_x_zero*cd_exp_phi)
  / ( 1.+ cd_r* cd_x_zero*cd_exp_phi)
  cd_x_zero= cd_x_top
end do

```

```

    refl(i) = cdabs(cd_x_top)**2
end do
return
end

C*****
C23456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*12345
C Jan-92
    subroutine rddat(datname,npts,xval,yval)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C Reads the data file
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
    real*4 xval(npts),yval(npts)
    integer*4 npts,nskip,erval,i,n,fgetc
    character*80 datname
    character char
    parameter (nskip=4)
    implicit none

! Executable code:
    type *,npts in rddat',npts
n=80
    call blank(datname,n)
    open(unit=3,file=datname,status='old',access='sequential',
         form='formatted')
    1 i=0
    do while (.i.lt.nskip)
        erval=fgetc(3,char)
        if (char.eq.'\n') then ! If char = return-character
            i=i+1
        end if
    end do
    type *,npts in rddat(i)',npts
do i=1,npts
    read(3,*) xval(i),yval(i)
    type *,xval(i),yval(i),npts
end do
close(3)
return
end
!*****
!23456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*12345
    subroutine rdfile(inname,indat,outdat,outl0l,anoren,mincontrol,
1 maxcontrol,nsteps,eng,thetmin,thetmax,dthet,angdep,the,
1 engmin,engmax,deng,nlayers,allortwo,sym, rho,
1 thick,minthick,maxthick,deltathick,npts,conseqkip)
    character*80 inname,outdat,indat,outl0l,sym(3001)
    character*72 text
    character*5 anoren,angdep,allortwo
    real*4 eng,thetmin,thetmax,dthet,the,engmin,engmax,deng,
1 thick(2*3001),
1 rho(3001),mincontrol,maxcontrol,
1 minthick(2*3001),maxthick(2*3001),deltathick(2*3001)
integer*4 nlayers,nsteps,npts,n,i,conseqkip
implicit none

! Executable code:
1 open(unit=2,file=inname,status='old',
access='sequential',form='formatted')
    read(2,(a72)) text
    read(2,(a72)) text
    read(2,(a72)) text
    read(2,(a72)) text
    read(2,(20x,a80)) indat
    read(2,(21x,a80)) outdat
    read(2,(18x,a80)) outl0l
    read(2,(a72)) text
    read(2,(a72)) text

```

```

    read(2,(13x,a3)) anoren
    read(2,(a72)) text
    read(2,(39x,f5.1)) maxcontrol
    read(2,(37x,f5.1)) mincontrol
    read(2,(21x,18)) nsteps
    read(2,(21x,18)) npts
    read(2,(a72)) text
    read(2,(11x,f8.1)) eng
    read(2,(23x,f5.1)) thetmin
    read(2,(23x,f5.1)) thetmax
    read(2,(25x,f8.1)) dthet
    read(2,(a72)) text
    read(2,(48x,a3)) angdep
    read(2,(a72)) text
    read(2,(a72)) text
    read(2,(15x,f5.1)) the
    read(2,(19x,f8.1)) engmin
    read(2,(19x,f8.1)) engmax
    read(2,(21x,f5.1)) deng
    read(2,(a72)) text
    read(2,(a72)) text
    read(2,(35x,i5)) nlayers
    read(2,(a72)) text
    read(2,(a72)) text
    read(2,(a72)) text
    read(2,(25x,a5)) allortwo
    read(2,(a72)) text
    read(2,(a72)) text
    read(2,(a72)) text
    call blank(allortwo,n)
    if (allortwo.eq.'2') then
do i=1,3
    read(2,(a15,f8.1,f8.1,f8.1,f8.1,f8.1,f8.1,f8.1,f8.1,f8.1,
1 ,f8.1,f8.1,f8.1)) sym(i),rho(i),thick(i+nlayers),
1 minthick(i+nlayers),maxthick(i+nlayers),
1 deltathick(i+nlayers),thick(i),minthick(i),
1 maxthick(i),deltathick(i)
end do
do i=4,nlayers,2
    sym(i)=sym(2)
    rho(i)=rho(2)
    thick(i+nlayers)=thick(2+nlayers)
    minthick(i+nlayers)=minthick(2+nlayers)
    maxthick(i+nlayers)=maxthick(2+nlayers)
    deltathick(i+nlayers)=deltathick(2+nlayers)
    thick(i)=thick(2)
    minthick(i)=minthick(2)
    maxthick(i)=maxthick(2)
    deltathick(i)=deltathick(2)
    sym(i+1)=sym(3)
    rho(i+1)=rho(3)
    thick(i+1+nlayers)=thick(3+nlayers)
    minthick(i+1+nlayers)=minthick(3+nlayers)
    maxthick(i+1+nlayers)=maxthick(3+nlayers)
    deltathick(i+1)=deltathick(3)
    thick(i+1)=thick(3)
    minthick(i+1)=minthick(3)
    maxthick(i+1)=maxthick(3)
    deltathick(i+1)=deltathick(3)
end do
end if
if (allortwo.eq.'al') then
do i=1,nlayers
    read(2,(a15,f8.1,f8.1,f8.1,f8.1,f8.1,f8.1,f8.1,f8.1,f8.1,
1 ,f8.1,f8.1,f8.1)) sym(i),rho(i),thick(i+nlayers),
1 minthick(i+nlayers),maxthick(i+nlayers),
1 deltathick(i+nlayers),thick(i),minthick(i),
1 maxthick(i),deltathick(i)
n=15
    call blank(sym(i),n)
    type *,thick(i),thick(i+nlayers)
end do
end if
    read(2,(a72)) text
end(2,(a72)) text

```



```

1  deltathick,cd_index,delta,beta,theta,ensemble,
1  calcrefl,bestrefl,bestthick,v,status,
1  conskip,x_coordinates
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C  Performs the annealing using the subroutines
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
integer nsteps,nlayers,npts,i,j,bestj,conskip,k,
1  init,aoe,n,enssize,ens_index,best_index,
1  status,start_coordinates(5),end_coordinates(5)
character*80 inname,indat,oumdat,outlol,SYM(nlayers),xleg,yleg
character*20 angdep
complex*16 cd_index(nlayers+1,npts)
real*4 control,mincontrol,maxcontrol,thetamean,
1  thick(2*nlayers),minthick(2*nlayers),maxthick(2*nlayers),
1  deltathick(2*nlayers),
1  bestthick(2*nlayers),
1  energy(((aoe-1)*npts+(aoe+1))/2),!(0=1 if ang,=npts if endep
1  theta(((aoe+1)*npts+(aoe-1))/2),!(0=1 if endep,=npts if ang
1  delta(((aoe+1)*npts+(aoe-1))/2),((aoe-1)*npts+(aoe+1))/2),
1  beta(((aoe+1)*npts+(aoe-1))/2),((aoe-1)*npts+(aoe+1))/2),
1  rho(nlayers),ensemble(enssize,2*nlayers),
1  cost(1000),bestcost,exprefl(npts),calcrefl(npts),
1  bestrefl(npts),mint,maxt,delt,v,eqcost,meancost,variance,
1  oldmeancost,
1  rate,urate,time,seconds,ran,random,moment_2,sum1,sum2,
1  x_coordinates(npts,5),y_coordinates(npts,5)
1  parameter (degrad=0.01745329)
implicit none
common/points/npts,nlayers,aoe,enssize
C Executable code:
time=seconds(0.)
init=nint(time/2)*2+1
type *,initvalue',init
do i=1,nlayers*2
  bestthick(i)=thick(i)
end do
if (aoe.eq.1) then
  call rddat(indat,npts,theta,exprefl)
  call optconan(npts,energy,theta,nlayers,SYM,rho,cd_index,
1  delta,beta,angdep,thetamean)
  call reflcan(bestrefl,energy,theta,bestthick,cd_index)
else
  call rddat(indat,npts,energy,exprefl)
  call optconen(npts,energy,theta,nlayers,SYM,rho,cd_index,
1  delta,beta)
  call reflcen(bestrefl,energy,theta,bestthick,cd_index)
end if
call costl(exprefl,bestrefl,npts,bestcost)
C*****Prepare ensemble and costs
do i=1,2*nlayers
  ensemble(i,i)=thick(i)
end do
cost(1)=bestcost
do ens_index=2,enssize
  do i=1,2*nlayers
    random=ran(init)
    maxt=maxthick(i)
    mint=minthick(i)
    delt=deltathick(i)
    thick(1)=mint+int(random*(1+(maxt-mint)/delt))*delt
    ensemble(ens_index,i)=thick(i)
  end do
  if (aoe.eq.1) then
    call reflcan(calcrefl,energy,theta,thick,cd_index)
  else
    call reflcen(calcrefl,energy,theta,thick,cd_index)
  end if
  call costl(exprefl,calcrefl,npts,cost(ens_index))
end do
C*****Find equilibrium cost value at "temperature" maxcontrol
sum1=0
do ens_index=1,enssize
  sum1=sum1+cost(ens_index)
end do
oldmeancost=sum1/enssize
do i=1,10
  call step_ensemble(ensemble,minthick,maxthick,deltathick,
  bestthick,energy,theta,cd_index,exprefl,bestrefl,calcrefl,
  cost,bestcost,maxcontrol,0,bestj,rate,urate,init)
end do
sum1=0
do ens_index=1,enssize
  sum1=sum1+cost(ens_index)
  sum2=sum2+cost(ens_index)**2
end do
meancost=sum1/enssize
moment_2=sum2/enssize
variance=moment_2-meancost**2
type *,mean',oldmeancost
type *,mean',meancost', variance', variance
type *,',start finding eqcost'
do k=1,100
  if ((oldmeancost-meancost)**2 .gt. v*v*variance/100.) then
    oldmeancost=meancost
    do i=1,10
      call step_ensemble(ensemble,minthick,maxthick,
      deltathick,bestthick,energy,theta,cd_index,
      exprefl,bestrefl,calcrefl,
      cost,bestcost,maxcontrol,0,bestj,rate,urate,init)
    end do
    sum1=0
    sum2=0
    do ens_index=1,enssize
      sum1=sum1+cost(ens_index)
      sum2=sum2+cost(ens_index)**2
    end do
    meancost=sum1/enssize
    moment_2=sum2/enssize
    variance=moment_2-meancost**2
  type *,',mean',oldmeancost
  type *,',mean',meancost', variance', variance
  do k=1,100
    if ((oldmeancost-meancost)**2 .gt. v*v*variance/100.) then
      oldmeancost=meancost
    else
      type *,',eqcost found after',k*10,' steps per member'
      goto 7
    end if
  end do
type *,',eqcost found after 1000 (max) steps per member'
eqcost=meancost
control=maxcontrol
urate=0
rate=0
C Graphics before annealing
if (aoe.eq.1) then
  do i=1,npts
    x_coordinates(i,2)=theta(i)/degrad
    x_coordinates(i,1)=theta(i)/degrad
  end do
  else
    x_label='angle'
  end do
  do i=1,npts
    x_coordinates(i,2)=energy(i)
    x_coordinates(i,1)=energy(i)
  end do
  x_label='energy'
end if
do i=1,2
  start_coordinates(i)=1
end_coordinates(i)=npts
end do
do i=1,npts
  y_coordinates(i,2)=Alog10(exprefl(i))
  y_coordinates(i,1)=Alog10(bestrefl(i))
end do
call EXPG_GR_XYGRAPH(npts,5,2,
1  start_coordinates,end_coordinates,
1  x_coordinates,y_coordinates,
1  'ensemble, thermodyn. cooling',x_label,'reflectivity',
1  status)

```

```

do i=1,nlayers
  type *,bestthick(i),bestthick(i+nlayers)
end do
n=80
call blank(outlol,n)
open(unit=2,file=outlol,status='unknown',
  access='sequential',form='formatted')
write(2,'(i8)') int(nsteps/consnskip/enssize)
C start annealing
do j=1,int(nsteps/enssize)
  call step_ensemble(ensemble,minthick,maxthick,deltathick,
  bestthick,energy,theta,cd_index,exprefl,bestrefl,calcrefl,
  cost,bestcost,control,j,bestj,rate,update,init)
  if (mod(j,consnskip).eq.0) then
    C***** Graphics
    best_index=1
    do ens_index=2,enssize
      if (cost(best_index).gt.cost(ens_index)) then
        best_index=ens_index
      end if
    end do
    do i=1,2*nlayers
      newthick(i)=ensemble(best_index,i)
    end do
    do i=1,3
      start_coordinates(i)=1
      end_coordinates(i)=npts
    end do
    if (aoe.eq.1) then
      do i=1,npts
        x_coordinates(i,1)=theta(i)/degrad
        x_coordinates(i,2)=theta(i)/degrad
        x_coordinates(i,3)=theta(i)/degrad
      end do
      call reflecan(calcrefl,energy,theta,newthick,
      cd_index)
    else
      do i=1,npts
        x_coordinates(i,1)=energy(i)
        x_coordinates(i,2)=energy(i)
        x_coordinates(i,3)=energy(i)
      end do
      call reflecan(calcrefl,energy,theta,newthick,
      cd_index)
    end if
  do i=1,npts
    Y_coordinates(i,1)=Alog10(calcrefl(i))
    Y_coordinates(i,2)=Alog10(exprefl(i))
    Y_coordinates(i,3)=Alog10(bestrefl(i))
  end do
  call EXPG GR_XYGRAPH(npts,5,3,
  start_coordinates,end_coordinates,
  x_coordinates,y_coordinates,
  'ensemble,thermodyn.cooling',
  x_label,'reflectivity',status)
  rate=rate/consnskip/enssize
  uprate=update/consnskip/enssize
  write(*,*)
  write(*, '(x,a9,4x,a9,4x,a4,6x,a7,4x,a6)') 'iter. no.',
  'best iter.','rate','control','uprate'
  write(*, '(x,i8,5x,i8,3x,g10.5,2x,g10.5,2x,g10.5)')
  j*enssize,bestj*enssize,rate,control,uprate
  write(*, '(x,a15,x,a19,x,a8,x,a8)') bestcost
  'bestcost in ensemble','meancost','variance'
  write(*, '(x,g15.7,3x,g15.7,4x,g15.7,5x,g15.7)') bestcost,
  cost(best_index),meancost,variance
  write(2,'(a9,2x,a9,3x,a4,3x,a7,3x,a6)') 'iter. no.',
  'best iter.','rate','control','uprate'
  write(2,'(x,i8,3x,i8,x,g10.5,x,g10.5)') j*enssize,
  bestj*enssize,rate,control,uprate
  write(2,'(x,a15,x,a19)') bestcost
  'bestcost in ensemble'
  write(2,'(x,g15.7,3x,g15.7)') bestcost,cost(best_index)
  write(*,*)

```

```

write(*, '(l3x,a13,6x,a15)') 'best till now',
'best in ensemble'
write(*, '(a9,3x,4(a5,5x))') 'layer no.','thick','rough',
'thick','rough'
do i=1,nlayers
  write(*, '(i8,2x,4(g10.5,g10.5))') i,bestthick(i),
  bestthick(i+nlayers),newthick(1),newthick(i+nlayers)
end do
rate=0
uprate=0
endif
call cool(control,cost,eqcost,v,meancost,variance)
end do
close(2)
if (aoe.eq.1) then
  xleg='angle (degrees)'
  yleg='reflectivity exp calc'
  call wrtfile(outdat,npts,theta,exprefl,bestrefl,inname,
  xleg,yleg,nlayers,bestthick)
else
  xleg='energy (eV)'
  yleg='reflectivity exp calc'
  call wrtfile(outdat,npts,energy,exprefl,bestrefl,inname,
  xleg,yleg,nlayers,bestthick)
end if
return
end
C*****
C June 92
subroutine step_ensemble(ensemble,minthick,maxthick,deltathick,
  bestthick,energy,theta,cd_index,exprefl,bestrefl,calcrefl,
  cost,bestcost,control,j,bestj,rate,update,init)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C Takes one step for each member of the ensemble
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
integer npts,nlayers,aoe,enssize,ens_index,init,i,good,upgood
real*4 ensemble(2*nlayers),deltathick(2*nlayers),
  maxthick(2*nlayers),minthick(2*nlayers),
  bestthick(2*nlayers),newthick(2*3001),
  energy(((aoe-1)*npts+(aoe+1))/2),!(i)=1 if ang,=npts if endep
  theta(((aoe+1)*npts+(aoe-1))/2),!(i)=1 if endep,=npts if ang
  exprefl(npts),calcrefl(npts),bestrefl(npts),control,
  cost(enssize),bestcost,newcost,j,bestj,rate,update,
  complex*16 cd_index(nlayers+1,npts)
common/points/npts,nlayers,aoe,enssize
implicit none
do ens_index=1,enssize
  do i=1,2*nlayers
    newthick(i)=ensemble(ens_index,i)
  end do
  call neighbour(newthick,minthick,maxthick,deltathick,
  layers,init)
  if (aoe.eq.1) then
    call reflecan(calcrefl,energy,theta,newthick,
    cd_index)
  else
    call reflecan(calcrefl,energy,theta,newthick,
    cd_index)
  end if
  call costl(exprefl,calcrefl,npts,newcost)
  call accept(cost(ens_index),newcost,control,good,
  if (good .eq. 1) then
    upgood=init
  do i=1,2*nlayers
    ensemble(ens_index,i)=newthick(i)
  end do
  cost(ens_index)=newcost
  if (newcost .lt. bestcost) then

```

```

if (prob .ge. 0) then
  good=1
  upgood=1
else
  good=0
  upgood=0
endif
end if
return
end

C*****
C23456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*12345
C Jan-92
1 subroutine neighbour(newthick,minthick,maxthick,
  deltathick,nlayers,init)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
Randomly chooses a neighbour to the current solution C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
integer nlayers,changel,init
real*4 minthick(2*nlayers),maxthick(2*nlayers),
1 deltathick(2*nlayers),addthick,random,userough,
1 parameter (userough=0.0)
implicit none

C Executable code
random=ran(init)
changel=int(random*nlayers-le-15)+1
random=ran(init)
if (random .ge. userough) then
  random=ran(init)
  addthick=(int(random*2-le-15)*2-1)*deltathick(changel)
  newthick(changel)=newthick(changel)+addthick
  if (newthick(changel) .lt. minthick(changel)) then
    newthick(changel)=maxthick(changel)
  else if (newthick(changel) .gt. maxthick(changel)) then
    newthick(changel)=minthick(changel)
  endif
else
  random=ran(init)
  addthick=(int(random*2-le-15)*2-1)*
  deltathick(changel+nlayers)
  newthick(changel+nlayers)=newthick(changel+nlayers)+addthick
  if (newthick(changel+nlayers) .lt.
1 minthick(changel+nlayers)) then
1 newthick(changel+nlayers)=maxthick(changel+nlayers)
  else if (newthick(changel+nlayers) .gt.
1 maxthick(changel+nlayers)) then
1 newthick(changel+nlayers)=minthick(changel+nlayers)
endif
end if
return
end

C*****
C23456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*12345
C Jan-92
1 subroutine costl(expdat,calcdat,npts,kost)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
Calculates the cost function of a calculated reflectivity curve C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
integer npts,i
real*4 expdat(2001),calcdat(2001), kost
implicit none

C Executable code:

```

```

bestcost=newcost
bestj=j
do i=1,npts
  bestrefl(i)=calcrefl(i)
end do
do i=1,nlayers*2
  bestthick(i)=newthick(i)
end do
endif
endif
rate=rate+good
uprate=uprate+upgood
end do

return
end

C*****
C23456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*12345
C Jan-92
subroutine cool(control,cost,eqcost,v,meancost,variance)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
Lowers the control parameter (temperature) according to costs C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
integer npts,nlayers,aoe,enssize,ens_index
real*4 cost(enssize),v,meancost,moment_2,variance,eqcost,
1 neweqcost,control,sum1,sum2,oldvariance
1 common/points/npts,nlayers,aoe,enssize
implicit none

C Executable code:
sum1=0.
sum2=0.
oldvariance=variance
do ens_index=1,enssize
  sum1=sum1+cost(ens_index)
  sum2=sum2+cost(ens_index)**2
end do
meancost=sum1/enssize
moment_2=sum2/enssize
variance=moment_2-meancost**2
neweqcost=meancost-v*sqrt(variance)
control=control*(1+(neweqcost-eqcost)/control/oldvariance)
eqcost=neweqcost
if (control .lt. le-10) then
  control=le-10
end if
return
end

C*****
C23456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*12345
C Jan-92
subroutine accept(oldcost,newcost,control,good,upgood,init)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
Decides whether to accept neighbour to the current solution C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
integer good,init,upgood
real*4 oldcost, newcost,control,deltacost,prob,random,ran
implicit none

C Executable code:
deltacost=newcost-oldcost
if (deltacost .le. 0) then
  good=1
  upgood=0
else
  random=ran(init)
  prob=exp(-deltacost/control)-random

```

```
1 energy(l),theta(npts),rho(3001),thetamean
parameter (degrad=0.01745329)
implicit none

C Executable code
unit='ev'
do i=1,npts
theta(i)=theta(i)*degrad
end do
n=3
call blank(angdep,n)
if (angdep .eq. 'y ') then
type *,sym(l)
call fl2(npts,l,sym(l),energy,unit,theta,delta,beta,rho(l))
type *, called fl2 (l),
do i=1,npts
cd_index(l,i)= dcmplx(1-delta(i,l),-beta(i,l))
cd_index(nlayers+1,i) =(1.,0.)
end do
do j=2,nlayers
do k=1,j-1
if (sym(j) .eq. sym(k)) then
do i=1,npts
cd_index(j,i)=cd_index(k,i)
type *,cd_index(j,i)
end do
go to 111
end if
end do
call fl2(npts,l,sym(j),energy,unit,theta,delta,beta,rho(j))
type *, called fl2,
do i=1,npts
cd_index(j,i)= dcmplx(1-delta(i,l),-beta(i,l))
type *,cd_index(j,i)
end do
go to 111
end do
type *,'Found optical constants'
end if

if (angdep .eq. 'n ') then
type *,thetamean=,thetamean
call fl2(l,1,sym(l),energy,unit,thetamean,delta,beta,rho(l))
type *, called fl2 (l),
do i=1,npts
cd_index(l,i)= dcmplx(1-delta(1,l),-beta(1,l))
cd_index(nlayers+1,i) =(1.,0.)
end do
do j=2,nlayers
do k=1,j-1
if (sym(j) .eq. sym(k)) then
do i=1,npts
cd_index(j,i)=cd_index(k,i)
type *,cd_index(j,i)
end do
go to 112
end if
end do
call fl2(l,1,sym(j),energy,unit,thetamean,delta,beta,rho(j))
type *, called fl2,
do i=1,npts
cd_index(j,i)= dcmplx(1-delta(1,l),-beta(1,l))
type *,cd_index(j,i)
end do
go to 112
end do
type *,'Found optical constants'
return
end

112
do
type *,'Found optical constants'
end if

return
end

C*****
C23456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*
C Jan-92
subroutine reflcan(refl,energy,theta,thick,cd_index)
```

```
kost=0
do i=1,npts
kost=kost+(1-calcdat(i)/expdat(i))**2
+(1-(expdat(i)/calcdat(i))**2)
end do
kost=kost/npts

return
end

C*****
C23456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*
C Jan-92
subroutine cost2(expdat,calcdat,npts,kost)
=====
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C Calculates the cost function of a calculated reflectivity curve C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

integer npts,i
real*4 expdat(2001),calcdat(2001), kost
implicit none

C Executable code:
kost=0
do i=1,npts
kost=kost+(1-calcdat(i)/expdat(i))**2
end do
kost=kost/npts

return
end

C*****
C23456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*
C Jan-92
subroutine cost4(expdat,calcdat,npts,kost)
=====
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C Calculates the cost function of a calculated reflectivity curve C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

integer npts,i
real*4 expdat(2001),calcdat(2001), kost
implicit none

C Executable code:
kost=0
do i=1,npts
kost=kost*log(1+(1-calcdat(i)/expdat(i))**2)
+(1-(expdat(i)/calcdat(i))**2)
end do
kost=kost/npts

return
end

C*****
C23456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*
C Jan-92
subroutine optconan(npts,energy,theta,nlayers,sym,rho,cd_index,
delta,beta,angdep,thetamean)
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C Finds optical constants for each layer and for each angle C
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

character*20 unit,angdep
integer nlayers,npts,i,n,k,j
character*80 sym(3001)
complex*16 cd_index(nlayers+1,npts)
real*4 delta(npts,l),beta(npts,l),
```

C*****
C23456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*123456789*
C Jan-92
subroutine reflcan(refl,energy,theta,thick,cd_index)


```
character*80 outname
n=80
call blank(outname,n)
call blank(title,n)
call blank(xleg,n)
call blank(xleg,n)

open(unit=3,file=outname,status='unknown',access='sequential',
      form='formatted')

write(3,'(a)') title
do i=1,nlayers
  write(3,'(i16)') i,thick(i)
end do
write(3,'(a)') xleg
write(3,'(a)') yleg
write(3,'(i16)') npts

do i=1,npts
  write(3,'(3(lp14.7))',xval(i),yval(i),y2val(i))
end do
close(3)
end
```