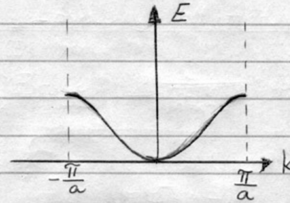
(4.)



$$E(k) = \frac{\hbar^2}{ma^2}\left[\frac{7}{8} - \cos(ka) + \frac{1}{8}\cos(2ka)\right]$$

$$\frac{dE(k)}{dk} = \frac{\hbar^2}{ma^2}\left[a\sin(ka) - \frac{1}{4}a\sin(2ka)\right]$$

a)

$$m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$$

där $\frac{d^2E}{dk^2} = \frac{\hbar^2}{ma^2}\left[a^2\cos(ka) - \frac{1}{2}a^2\cos(2ka)\right]$

$$\therefore m^* = \frac{m}{\cos(ka) - \frac{1}{2}\cos(2ka)} = \begin{cases} \underline{2m} & \text{vid } k = 0 \\ \underline{-\frac{2}{3}m} & \text{vid } k = \frac{\pi}{a} \end{cases}$$

b) $v_g = \frac{1}{\hbar}\frac{dE}{dk} = \frac{\hbar}{ma}\left[\sin(ka) - \frac{1}{4}\sin(2ka)\right]$

$\underline{v_g = 0}$ vid $k = \pm\frac{\pi}{a}$ $\therefore$ bandet ⊥ zon-gränsen

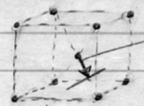
$\underline{v_g \text{ också} = 0}$ då $k = 0$ (dvs vid bandbotten)

(5.) Vanliga rummet

fcc

4 atomer/cell.

Rec. gittret (bcc med gitterkonst = $= \frac{4\pi}{a}$)



$k_{BZ} = \frac{1}{4}\cdot\sqrt{3}\cdot\frac{4\pi}{a}$ ty kortaste rec. gittervektor $= \frac{1}{2}\cdot\sqrt{3}\cdot\frac{4\pi}{a}$

antag:
x el per atom

$\therefore \frac{N_e}{V} = \text{el.tätheten} = \frac{4\cdot x}{a^3}$

enl. uppgift: $k_F = k_{BZ} = \frac{\sqrt{3}}{4}\cdot\frac{4\pi}{a} = 1{,}3\cdot10^{10}\,m^{-1}$

Friel. modellen: $k_F = \left(3\pi^2\frac{N_e}{V}\right)^{1/3}$

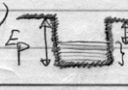
a) $k_F = k_{BZ} \Rightarrow \left(3\pi^2\frac{4x}{a^3}\right)^{1/3} = \frac{\sqrt{3}}{4}\cdot\frac{4\pi}{a} \Rightarrow \underline{x} = \frac{\sqrt{3}\,\pi}{4} \approx \underline{\underline{1{,}36}}$

b) $k_{BZ} = k_F = 1{,}3\cdot10^{10}\,m^{-1} \Rightarrow \sqrt{3}\frac{\pi}{a} = 1{,}3\cdot10^{10}\,m^{-1} \Rightarrow \underline{\underline{a = 4{,}18\,Å}}$

c) 2 el per atom (ty här en atom per gitterpunkt) i en B-zon.

Vi har här 1,36 el per atom $\therefore \frac{\text{Fermisfärens volym}}{\text{Volymen av 1:a B-zonen}} = \frac{1{,}36}{2} = \underline{\underline{0{,}68}}$

d)



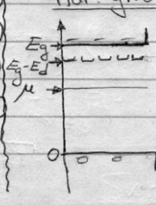
$\phi = h\frac{c}{\lambda} - E_{k,max} = 4{,}0 - 0{,}5 = 3{,}5\,eV$

$E_F = \frac{\hbar^2}{2m}\cdot k_F^2 = 6{,}4\,eV$

$\therefore E_p = E_F + \phi = \underline{\underline{9{,}9\,eV}}$

(6.)

$$\begin{cases} n = n_0 e^{\frac{\mu - E_g}{kT}} & (1) \\ p = p_0 e^{-\mu/kT} & (2) \\ n\cdot p = n_i^2 & (3) \\ \sigma = ne\mu_e + pe\mu_h & (4) \\ n = p + N_d^+ & (5) \\ \text{där } N_d^+ \approx N_d \text{ vid RT} & \end{cases}$$



Här: givet: $f(E_g) = \frac{1}{e^{\frac{E_g-\mu}{kT}}+1} = 2\cdot10^{-4} \Rightarrow e^{\frac{\mu - E_g}{kT}} = 2\cdot10^{-4}$

ekv (1) $\Rightarrow n = 2{,}5\cdot10^{25}\cdot(0{,}26)^{3/2}\cdot2\cdot10^{-4}\,m^{-3} = 6{,}6\cdot10^{20}\,m^{-3}$

ekv (3) $\Rightarrow p = \frac{n_i^2}{n} = \{n_i^2 \approx 10^{32}\} \lesssim 1{,}5\cdot10^{11}\,m^{-3} \lll n$

ekv (4) $\Rightarrow \sigma = ne\mu_e + \cancel{pe\mu_h} \approx 6{,}6\cdot10^{20}\cdot1{,}6\cdot10^{-19}\cdot0{,}16 = 17\,(\Omega m)^{-1}$ Svar i

ekv (5) $\Rightarrow N_d = n - p = n = \underline{6{,}6\cdot10^{20}\,m^{-3}}$ = Svar i (ii)

Svar i (iii): $f(E_g - E_d) \approx \frac{1}{e^{\frac{E_g - E_d - \mu}{kT}}} \approx 2\cdot10^{-4}\cdot e^{+\frac{E_d}{kT}} \approx \underline{\underline{1{,}4\cdot10^{-3}}}$