

Home assignments 2, Gravitation & Cosmology, 2008

To be handed in Friday, December 12

1. In the Schwarzschild geometry, which circular orbits are lightlike geodesics?
2. Consider a spherically symmetric and static solution of Einstein's equations in three space-time dimensions, describing a point particle with mass M . Give the explicit solution. Is this space-time flat? Will there be gravitational lensing?
3. Solve one of the two following problems:
 - a. Consider a toroidal surface embedded in flat three-dimensional euclidean space. With (ρ, ϕ, z) being standard cylindrical coordinates and a/b being the radius of the torus/tube, it can be parametrised as $(\rho, \phi, z) = (a + b\cos\theta, \phi, b\sin\theta)$. Calculate the curvature scalar. Comment on the sign of R for different points on the torus. Finally, determine all Killing vectors in this space.
 - b. A space-time is described by the metric

$$ds^2 = -\left(1 - \frac{r^2}{a^2}\right)dt^2 + \frac{dr^2}{1 - \frac{r^2}{a^2}} + r^2 d\Omega_{D-2}^2 ,$$

where $d\Omega_{D-2}^2$ is the metric for a unit $(D-2)$ -sphere. How many isometries does this space-time have? Comments?