Mekanik för F del B

17 augusti 2004

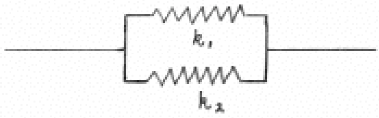
1. a. Effekt : $[P] = \frac{Nm}{s} = \frac{kg \cdot m^2}{s^3}$

Rörelsemängdsmoment : $[L] = \frac{kg \cdot m^2}{s}$

b. i.

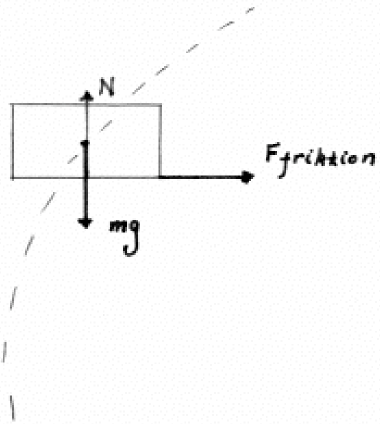
$$k = \frac{k_1 k_2}{k_1 + k_2}$$

ii.



$$k = k_1 + k_2$$

c.



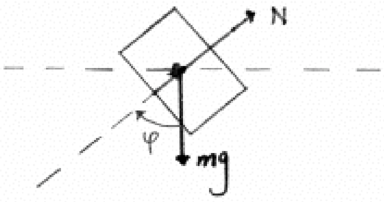
Använd cylindriska koordinater :

$$m\vec{a} = m\{(\cancel{\ddot{r}} - r\dot{\theta}^2)\hat{r} + (\cancel{r\ddot{\theta}} + \cancel{2\dot{r}\dot{\theta}})\hat{\theta} + \ddot{z}\hat{k}\}$$

$$= \vec{F}$$

$$\Rightarrow \quad mr\dot{\theta}^2\hat{r} = -\vec{F} = +F_{friktion}\hat{r}$$

Låt tåget luta så att man känner som "horisontellt" :



$$m\vec{a} = \vec{F}$$

$$\begin{cases} mg = N\cos\varphi \\ mr\dot{\theta}^2 = N\sin\varphi \end{cases}$$

$$\Rightarrow \tan\varphi = \frac{r\omega^2}{g} = \frac{v^2}{rg}$$

$$\varphi = \tan^{-1}\left(\frac{v^2}{rg}\right) = 8{,}94°$$

d. Homogen sfär

$$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3}mr^2$$

Tunn ring

$$I_{xx} = mr^2$$

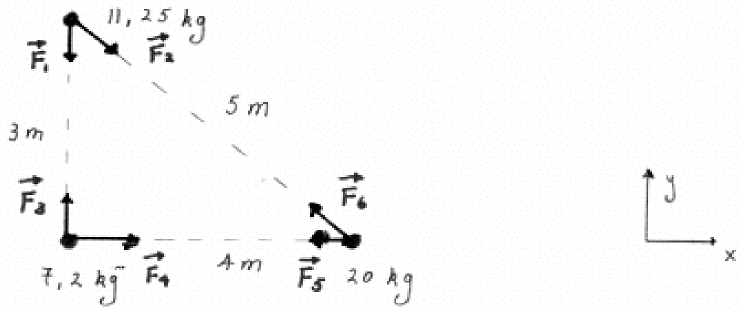
$$I_{yy} = I_{zz} = \frac{1}{2}mr^2 \qquad \left(I_{yy} = \int_0^{2\pi} (r\sin\theta)^2 \frac{m}{2\pi}\, d\theta \right)$$

Summan

Alla deviationsmoment försvinner eftersom kroppens symmetriaxlar sammanfaller med de valda koordinataxlarna.

$$I = \begin{bmatrix} \frac{7}{6}mr^2 & 0 & 0 \\ 0 & \frac{7}{6}mr^2 & 0 \\ 0 & 0 & \frac{5}{3}mr^2 \end{bmatrix}$$

e.



$$\vec{F}_1 = \frac{G \cdot 11,25 \cdot 7,2}{3^2}(-\hat{y}) = -6,00 \cdot 10^{-10}\,\hat{y} \quad N$$

$$\vec{F}_2 = \frac{G \cdot 11,25 \cdot 20}{5^2} \cdot \frac{4\hat{x} - 3\hat{y}}{\sqrt{4^2+3^2}} = 4,80 \cdot 10^{-10}\,\hat{x} - 3,60 \cdot 10^{-10}\,\hat{y} \quad N$$

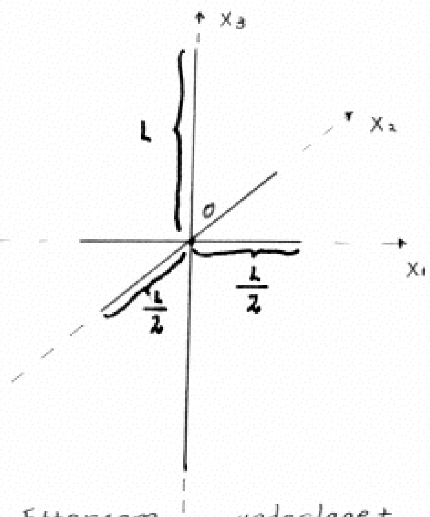
$$\vec{F}_3 = 6,00 \cdot 10^{-10}\,\hat{y} = -\vec{F}_1 \quad N$$

$$\vec{F}_4 = \frac{G \cdot 7,2 \cdot 20}{4^2}\,\hat{x} = 6,00 \cdot 10^{-10}\,\hat{x} \quad N$$

$$\vec{F}_5 = -6,00 \cdot 10^{-10}\,\hat{x} = -\vec{F}_4 \quad N$$

$$\vec{F}_6 = -\vec{F}_2 = -4,80 \cdot 10^{-10}\,\hat{x} + 3,60 \cdot 10^{-10}\,\hat{y} \quad N$$

2.



densitet ρ, $m = 4\rho L$

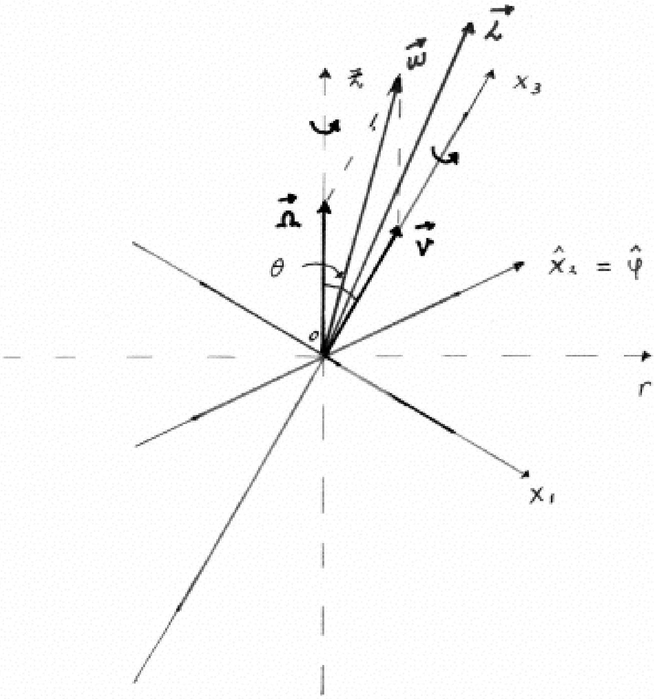
Underlaget är friktionsfritt.

Eftersom underlaget är friktionsfritt, kommer kroppens kontaktpunkten med bordet att röra sig, medan dess masscentrum befinner sig i vila.

Beräkna kroppens tröghetsmoment m.a.p. masscentrum 0.

$$\begin{cases} I_1 = 0 + \frac{1}{12}\frac{m}{4}L^2 + \frac{1}{12}\frac{m}{2}(2L)^2 = \frac{3}{16}mL^2 \\ I_2 = \frac{1}{12}\frac{m}{4}L^2 + 0 + \frac{4}{12}\frac{m}{2}L^2 = \frac{3}{16}mL^2 \\ I_3 = \frac{1}{12}\frac{m}{4}L^2 + \frac{1}{12}\frac{m}{4}L^2 + 0 = \frac{1}{24}mL^2 \end{cases}$$

$$I = \begin{bmatrix} \frac{5}{12} mL^2 & 0 & 0 \\ 0 & \frac{5}{12} mL^2 & 0 \\ 0 & 0 & \frac{1}{6} mL^2 \end{bmatrix}$$



låt $I_\perp = I_1 = I_2 = \frac{5}{12} mL^2$

$$\vec{\omega} = \omega_1 \hat{x}_1 + \omega_2 \hat{x}_2 + \omega_3 \hat{x}_3 = \Omega \hat{z} + \nu \hat{x}_3$$

$$\vec{L} = I \vec{\omega} = I_1 \omega_1 \hat{x}_1 + I_2 \omega_2 \hat{x}_2 + I_3 \omega_3 \hat{x}_3$$

$$= I_\perp \vec{\omega} + (I_3 - I_\perp) \omega_3 \hat{x}_3$$

$$= I_\perp (\Omega \hat{z} + \nu \hat{x}_3) + (I_3 - I_\perp) \omega_3 \hat{x}_3$$

$$\omega_3 = \vec{\omega} \cdot \hat{x}_3 = \Omega \hat{z} \cdot \hat{x}_3 + \nu = \Omega \cos\theta + \nu$$

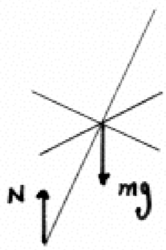
$$\Rightarrow \vec{L} = I_\perp \Omega \hat{z} + [I_\perp \nu + (I_3 - I_\perp)(\Omega \cos\theta + \nu)] \hat{x}_3$$

$$= I_\perp \Omega \hat{z} + [I_3 \nu + (I_3 - I_\perp) \Omega \cos\theta] \hat{x}_3$$

$$\Rightarrow \frac{d}{dt} \vec{L} = [I_3 \nu + (I_3 - I_\perp) \Omega \cos\theta] \frac{d\hat{x}_3}{dt},$$

där $\frac{d\hat{x}_3}{dt} = \vec{\omega} \times \hat{x}_3 = (\Omega \hat{z} + \nu \hat{x}_3) \times \hat{x}_3 = \Omega \sin\theta \, \hat{\varphi}$

$$\Rightarrow \frac{d}{dt} \vec{L} = [I_3 \nu + (I_3 - I_\perp) \Omega \cos\theta] \Omega \sin\theta \, \hat{\varphi}$$



Normalkraften : $N = mg \Rightarrow \vec{N} = mg \hat{z}$

Massan : $m = \rho \cdot (L + L + 2L) = 4\rho L$

$\Rightarrow \vec{\tau} = \vec{x} \times \vec{F} = (-L \hat{x}_3) \times mg \hat{z} = mgL \sin\theta \, \hat{\varphi}$

(m.a.p. masscentrum O)

Rörelsemängdsmomentlagen : $\frac{d}{dt}\vec{L} = \vec{\tau}$

$$[\, I_3 \nu + (I_3 - I_\perp)\,\Omega \cos\theta \,]\,\Omega \sin\theta = mgl \sin\theta$$

$$\Rightarrow \nu = \frac{1}{I_3 \Omega}[\,(I_\perp - I_3)\,\Omega^2 \cos\theta + mgl\,]$$

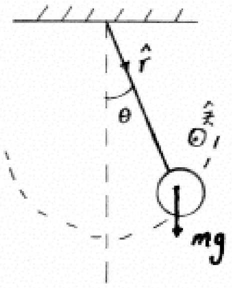
$$= \frac{6}{\Omega m l^2}\left[\,\frac{1}{4} m l^2 \,\Omega^2 \cos\theta + mgl\,\right]$$

$$= \frac{3\Omega \cos\theta}{2} + \frac{6g}{\Omega l}$$

Svar : $\nu = \frac{3\Omega\cos\theta}{2} + \frac{6g}{\Omega l}$

Kroppens masscentrum står stilla.

3.



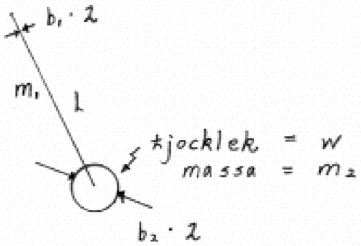
Pendelns massa : $m = \rho V = 0{,}208$ kg

Pendelns masscentrum :

$$\vec{R}_{mc} = \frac{1}{m}\left(m_1 \frac{l}{2} + m_2 l\right)\hat{r} = \frac{l}{m_1 + m_2}\left(\frac{m_1}{2} + m_2\right)\hat{r}$$

$m_1 = 0{,}0099$ kg ; $m_2 = 0{,}198$ kg

$m_1 \ll m_2$ { Kan approximera bort pinnen här ! }



$\vec{R}_{mc} \approx 0{,}98\, l\, \hat{r} = R_{mc}\,\hat{r}$

Pendels tröghetsmoment m.a.p. upphängningspunkten :

$$I = I_{pinne} + I_{vikt} = \left(\frac{1}{4} m_1 b_1^2 + \frac{1}{12} m_1 l^2 + m_1 \left(\frac{l}{2}\right)^2\right) +$$

$$+ \left(\frac{1}{2} m_2 b_2^2 + m_2 l^2\right)$$

$$\approx 0{,}033 \text{ kg m}^2$$

{ Här kan man approximera pendeln med en punktmassa : $I \approx m_2 l^2$. }

$$\left\{\begin{array}{l} \vec{L} = I\vec{\omega} = I\dot{\theta}\,\hat{z} \\ \vec{\tau} = \underbrace{-R_{mc} \cdot mg \sin\theta\,\hat{z}}_{\text{gravitation}} \; \underbrace{-\,\mu\dot{\theta}\,\hat{z}}_{\text{luftmotstånd}} \end{array}\right. \qquad \frac{d}{dt}\vec{L} = \vec{\tau}$$

$I\ddot{\theta} = -R_{mc}\, mg \sin\theta - \mu\dot{\theta}$, θ små $\Rightarrow \sin\theta \approx \theta$

$$\Rightarrow \ddot{\theta} + \frac{\mu}{I}\dot{\theta} + \frac{R_{mc}\, mg}{I}\theta = 0$$

Låt $\omega_0 = \sqrt{\frac{R_{mc}\, mg}{I}}$ Pendels frekvens i vakuum

$\gamma = \frac{\mu}{2I}$

$\ddot{\theta} + 2\gamma\dot{\theta} + \omega_0^2\theta = 0 \Rightarrow \theta = Ae^{-\gamma t}\cos(\omega t + B)$

där $\omega = \sqrt{\omega_0^2 - \gamma^2}$.

Skillnad i antal sekunder per dygn:

$$\Delta T = \left(\frac{3600 \cdot 24}{2\pi}\omega_0\right) \cdot \frac{2\pi}{\omega} - 3600 \cdot 24$$

$$= 86400\left(\frac{\omega_0}{\omega} - 1\right) = 86400\left(\frac{1}{\sqrt{1-\left(\frac{\gamma}{\omega_0}\right)^2}} - 1\right)$$

$$= 86400\left(\frac{1}{\sqrt{1 - \frac{\mu^2}{4 I R_{mc}\, mg}}} - 1\right)$$

$$\approx 0{,}000041 \text{ s}$$

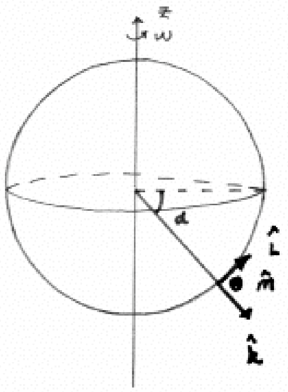
<u>Svar</u>: Om uret skulle stå i ett vakuum skulle det gå 0,000041 s för fort per dygn.

4. $m = 10^{11}$ kg, $\rho = 10^3$ kg/m^3 $\Rightarrow$ $V = 10^8$ m^3, $R = 288$ m

$v_r = 2$ m/s i rakt nordlig riktning; $\eta = 1{,}5 \cdot 10^{-3}$ kg/ms

$\Rightarrow Re = \frac{\rho \cdot 2R \cdot v_r}{\eta} \approx 7{,}7 \cdot 10^8$ $\Rightarrow$ turbulent strömnig

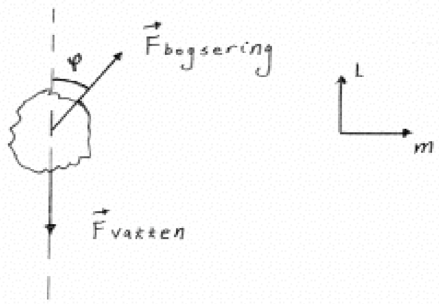
Inför ett koordinatsystem som roterar med jorden.



$\hat{k}$: ⊥ jordytan

$\hat{l}$: norrut $\qquad \hat{z} = \cos\alpha\, \hat{l} - \sin\alpha\, \hat{k}$

$\hat{m}$: österut



Dessutom påverkas isberget av coriolikraften och centrifugalkraften. Då isberget rör sig med konstant hastighet norrut, måste det råda kraftjämvikt på lm-planet. $(\ddot{l} = \ddot{m} = 0)$

$$\vec{F}_{bogsering} = F_b \left(\cos\varphi \, \hat{l} + \sin\varphi \, \hat{m} \right)$$

$$\vec{F}_{vatten} = -\frac{1}{2} \rho C_d A V_r^2 \, \hat{l} \qquad \text{(turbulent flöde)}$$

$$\vec{F}_{corioli} = -2m \left(\vec{\omega} \times \vec{V}_{rel} \right) = -2m \left(\omega \hat{z} \right) \times \left(V_r \hat{l} \right)$$

$$= -2m\omega V_r \left(\cos\alpha \, \hat{l} - \sin\alpha \, \hat{k} \right) \times \hat{l} = -2m\omega V_r \sin\alpha \, \hat{m}$$

$$\vec{F}_{centrifugal} = -2m \, \vec{\omega} \times \left(\vec{\omega} \times \vec{r} \right)$$

$$= -2m \left(\omega \hat{z} \right) \times \left[\left(\omega \hat{z} \right) \times \left(l \hat{l} \right) \right]$$

$$= -2m\omega^2 l \, \hat{z} \times \left[\left(\cos\alpha \, \hat{l} - \sin\alpha \, \hat{k} \right) \times \hat{l} \right]$$

$$= -2m\omega^2 l \left(\cos\alpha \, \hat{l} - \sin\alpha \, \hat{k} \right) \times \left(\sin\alpha \, \hat{m} \right)$$

$$= 2m\omega^2 l \sin\alpha \left(\cos\alpha \, \hat{k} + \sin\alpha \, \hat{l} \right)$$

Jämvikt i l-led: $F_b \cos\varphi - \frac{1}{2} \rho C_d A V_r^2 + 2m\omega^2 l \sin^2\alpha = 0$

Jämvikt i m-led: $F_b \sin\varphi - 2m\omega V_r \sin\alpha = 0$

$$\Rightarrow \begin{cases} F_b \sin\varphi = 2m\omega V_r \sin\alpha \\ F_b \cos\varphi = \frac{1}{2} \rho C_d A V_r^2 - 2m\omega^2 l \sin^2\alpha \end{cases}$$

$$\Rightarrow \tan\varphi = \frac{2m\omega V_r \sin\alpha}{\frac{1}{2} \rho C_d A V_r^2 - 2m\omega^2 l \sin^2\alpha}$$

Sätt in: $m = 10^{11}$ kg ; $\omega = \frac{2\pi}{3600 \cdot 24}$ s^{-1} ; $V_r = 2$ m/s ; $\alpha = 45°$

$\rho = 10^3$ kg/m^3 ; $C_d = 0{,}5$; $A = 260000$ m^2 ; $l = 0$

$\tan\varphi \approx 0{,}079 \quad \Rightarrow \quad \varphi \approx 4{,}515°$

$$\vec{F}_{bogsering} = F_b \left(0{,}997 \, \hat{l} + 0{,}079 \, \hat{m} \right)$$

<u>Svar</u>: $\varphi = 4{,}515°$

Isberget kommer att färdas väster om bogserbåten.