Home assignment 1 — Symmetry TIF310/FYM310

Deadline Friday Nov. 20, 2020.

Hand in solutions produced by TEX by mail to martin.cederwall@chalmers.se or printed in a box outside room Origo 6102. Good luck!

1.1. Show that the real Lie algebra $\mathfrak{so}(4,\mathbb{R})$ of rotations in 4 euclidean dimensions is $\mathfrak{so}(4) \simeq \mathfrak{su}(2) \oplus \mathfrak{su}(2)$.

1.2. Show that if $\mathfrak{h}$ is an ideal in $\mathfrak{g}$, $\mathfrak{g}$ mod $\mathfrak{h}$ is a Lie algebra. (As a vector space, $\mathfrak{g}$ mod $\mathfrak{h}$ consists of elements $a \in \mathfrak{g}$ modulo the equivalence relations $a \approx a + b$, where b is any element in $\mathfrak{h}$.)

1.3. Show that $\mathfrak{gl}(n)$ and $\mathfrak{u}(n)$ are not semi-simple, *e.g.* by finding proper ideals.

1.4. If the ideals of problem 1.3 are modded out, which simple Lie algebras are obtained?

1.5. Consider the rank 5 simple Lie algebra $D_5 \simeq \mathfrak{so}(10)$. This algebra has a 54-dimensional representation. Describe it in a tensor language. (*I.e.*, describe elements in the representation module as tensors. This will require the use of some invariant tensor.)

1.6. Consider an "inversion" of space (or space-time), defined by

$$x^m \mapsto x'^m = \frac{x^m}{x^2} \ .$$

Show that a special conformal transformation, on the infinitesimal form given in the lecture notes, is obtained by first performing an inversion, then a translation, and finally an inversion.