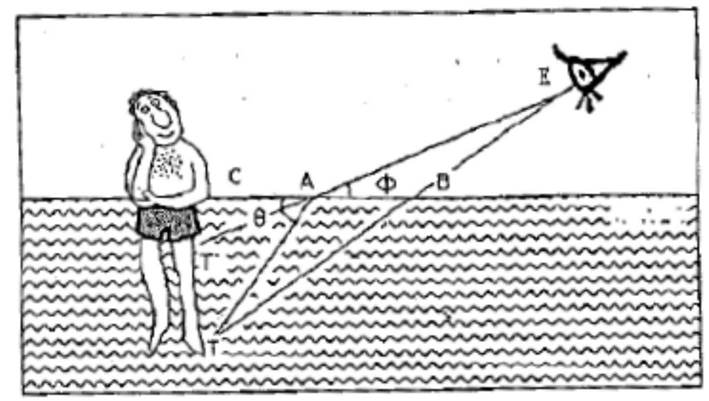
Vanligaste tillämpningen av variationskalkyl i fysiken

Minsta verkans princip

Vanligaste tillämpningen av variationskalkyl i fysiken

Minsta verkans princip

Fermats princip i optiken (1662)
en ljusstråle ”väljer” den snabbaste vägen



Vanligaste tillämpningen av variationskalkyl i fysiken

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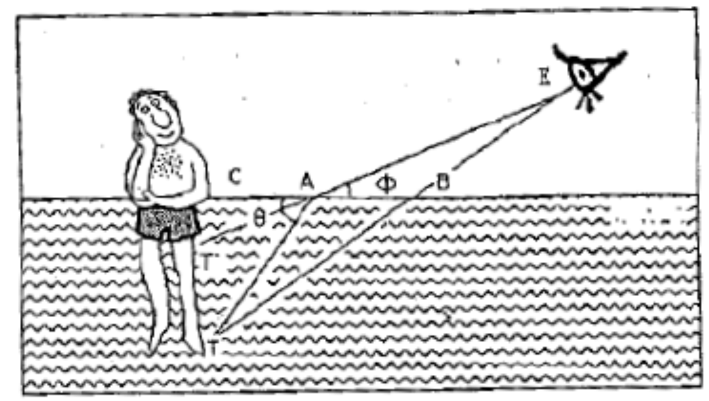
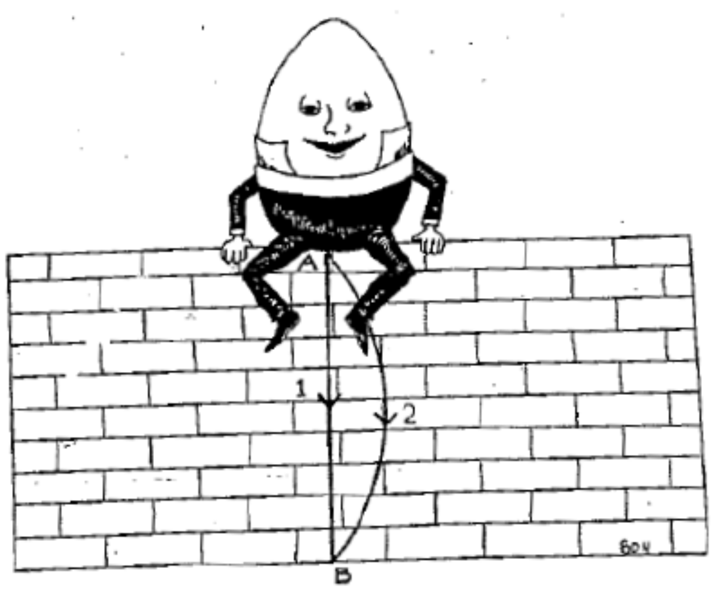


Maupertius (1748)
Lagrange (1788)
Hamilton (1834)

Minsta verkans princip *(”Hamiltons princip”)*

en partikel ”väljer” den väg som svarar mot minst verkan



$$\delta S = 0$$

$$S = \int_{t_1}^{t_2} L(q, \dot{q}, t) dt$$

$$L(q, \dot{q}, t) = K(\dot{q}) - V(q, t)$$

Vanligaste tillämpningen av variationskalkyl i fysiken

Minsta verkans princip

Fermats princip i klassisk optik (1662)
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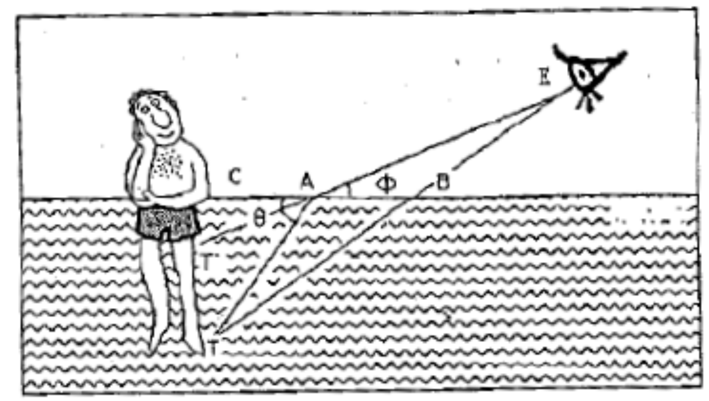

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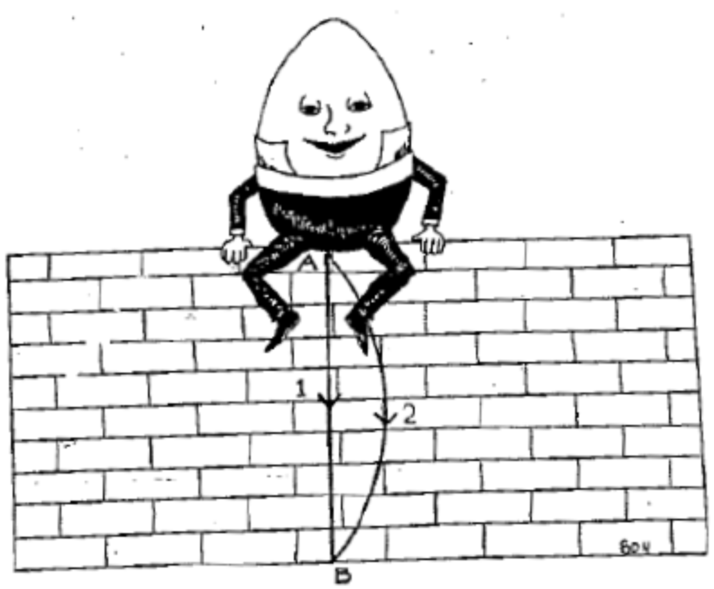


$$\delta S = 0$$

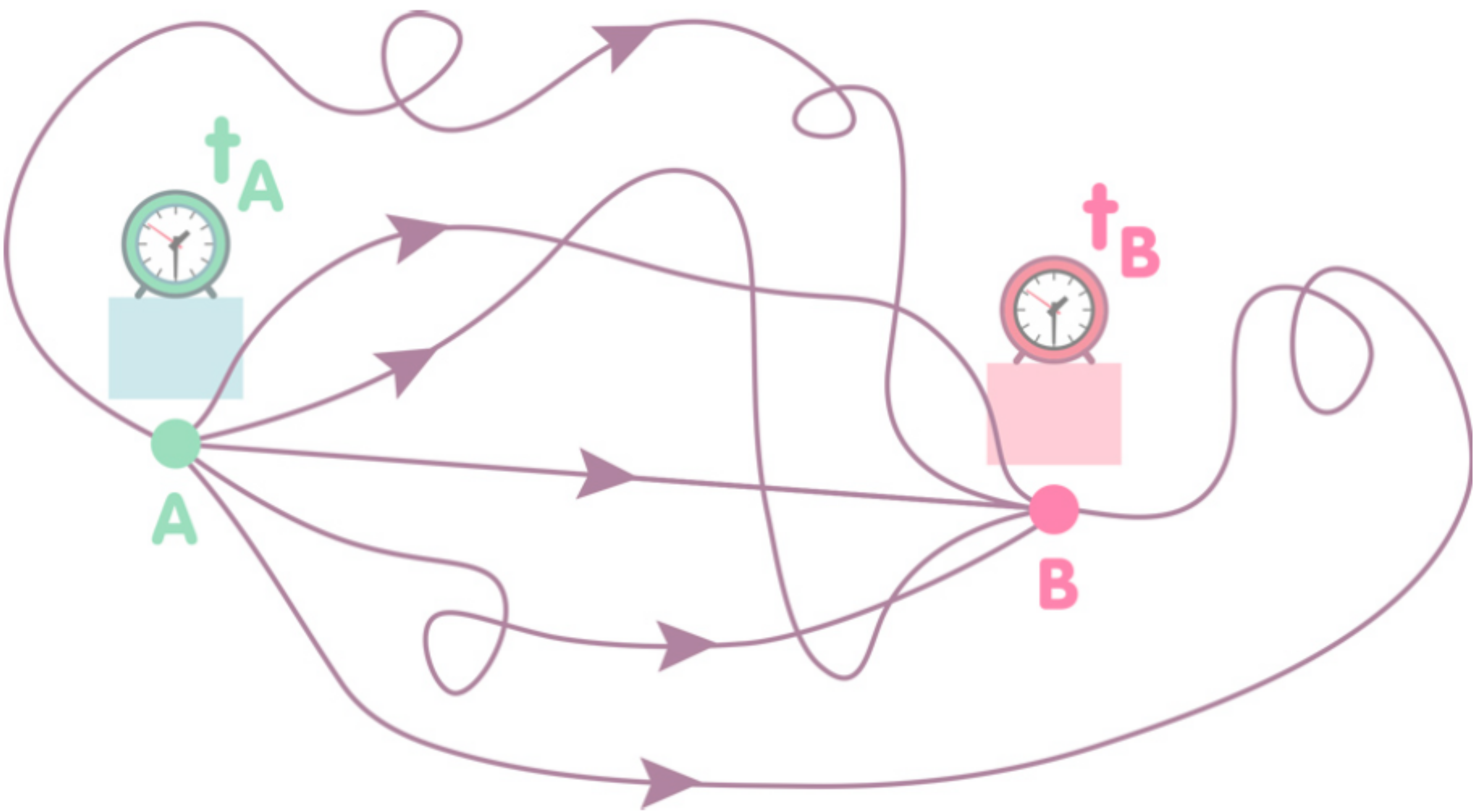
$$\frac{\partial L}{\partial q} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}} = 0$$

Vanligaste tillämpningen av variationskalkyl i fysiken

Minsta verkans princip

Hur kan vi ”förstå” **minsta verkans princip**?

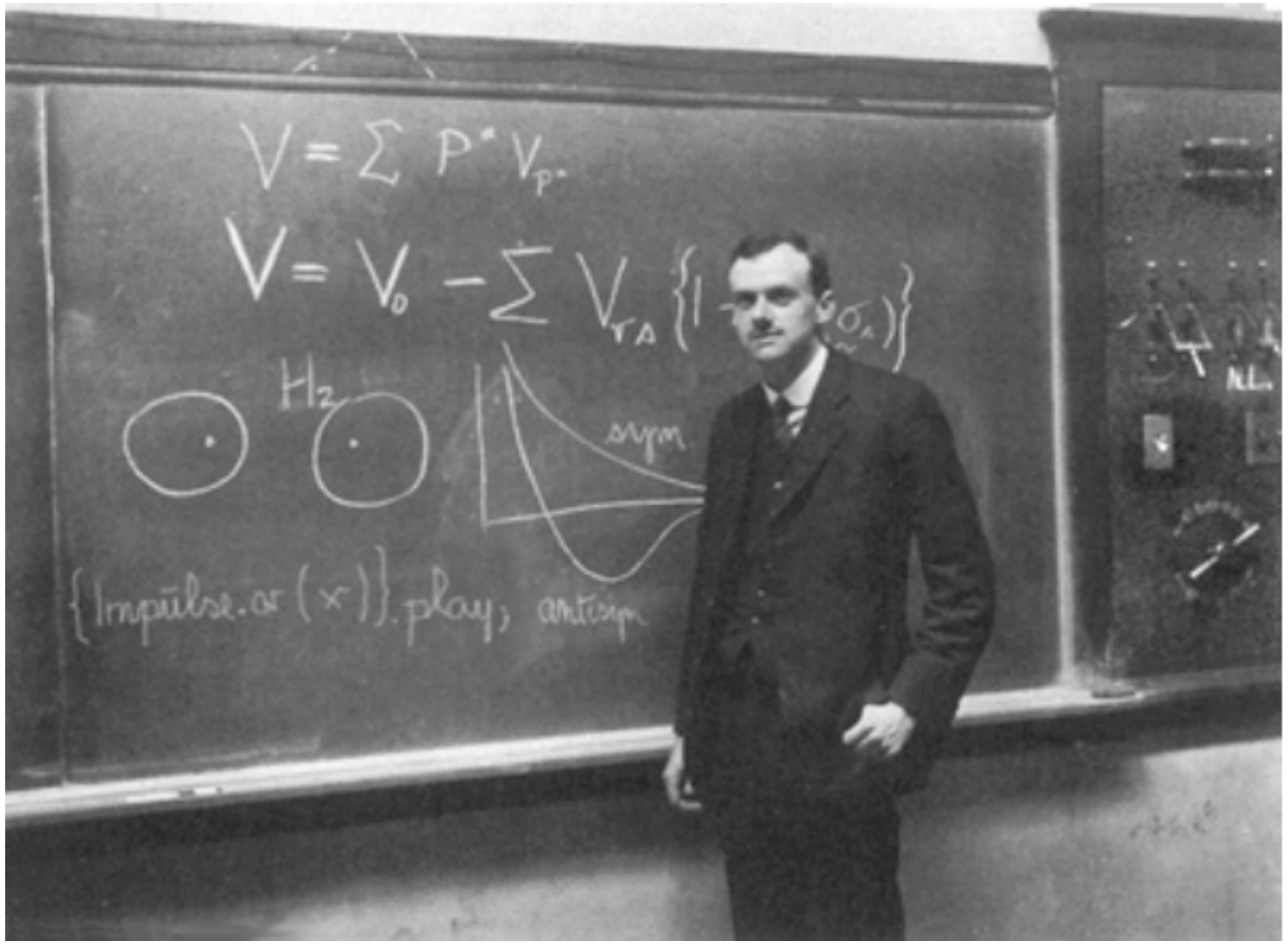
Feynmans vägintegralformulering av kvantmekaniken (intro)



Låt oss först påminna oss om ”kvantmekanikens grunder”…

4 postulat för kvantmekanik

(icke-relativistiskt)



Paul Dirac

John von Neumann

I. Ett tillstånd hos ett fysikaliskt system beskrivs av en normerad vektor $|\psi\rangle$ i ett Hilbertrum.

II. Till varje fysikalisk observabel finns en Hermitesk operator på ett Hilbertrum.

tillägg:
kanonisk kvantisering

III. Givet ett tillstånd $|\psi\rangle$ så ges de möjliga resultaten vid en mätning av en observabel svarande mot operatorn Ω av egenvärdena $\{\omega\}$ till Ω. Sannolikheten att mäta ω ges av $|\langle\omega|\psi\rangle|^2$ där $|\omega\rangle$ är motsvarande egen-vektor till Ω. Vid mätningen "kollapsar" $|\psi\rangle$ till $|\omega\rangle$.

IV. Tidsutvecklingen av tillståndet $|\psi(t)\rangle$ bestäms av systemets Hamiltonoperator:

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle$$ *(Schrödingerekvationen)*

$$|\psi(t)\rangle = e^{-iHt/\hbar}|\psi(0)\rangle$$

unitär tidsutveckling

Antalet postulat, och dess exakta formuleringar beror på hur matematiskt rigoröst man vill behandla teorin. Jfr. t.ex. Byron & Fuller (2015 års lärobok i kursen) § 5.11, som har sex postulat (varav ett är innehållet i ett av de andra).

men det är något UNDERLIGT med kvantmekaniken…

The top physicists of all time

1 Albert Einstein 1879–1955 German/Swiss/American 119 votes

2 Isaac Newton 1642–1727 British 96 votes

3 James Clerk Maxwell 1831–1879 British 67 votes

4 Niels Bohr 1885–1962 Danish 47 votes

5 Werner Heisenberg 1901–1976 German 30 votes

6 Galileo Galilei 1564–1642 Italian 27 votes

7 Richard Feynman 1918–1988 American 23 votes

8= Paul Dirac 1902–1984 British 22 votes

8= Erwin Schrödinger 1887–1961 Austrian 22 votes

10 Ernest Rutherford 1871–1937 New Zealander 20 votes

”The more succesful the quantum theory is, the sillier it looks.”
Einstein

”Anyone who thinks that he can understand quantum mechanics without getting into a state of confusion has not understood anything of the theory.”
Bohr

”No one has ever understood quantum mechanics.”
Feynman

”I regret that I had anything to do with it...”
Schrödinger

”What it all means? It is simple. You take what you need, you do what you have to do, and that’s it...”
Heisenberg

I. Ett tillstånd hos ett fysikaliskt system beskrivs av en normerad vektor $|\psi\rangle$ i ett Hilbertrum.

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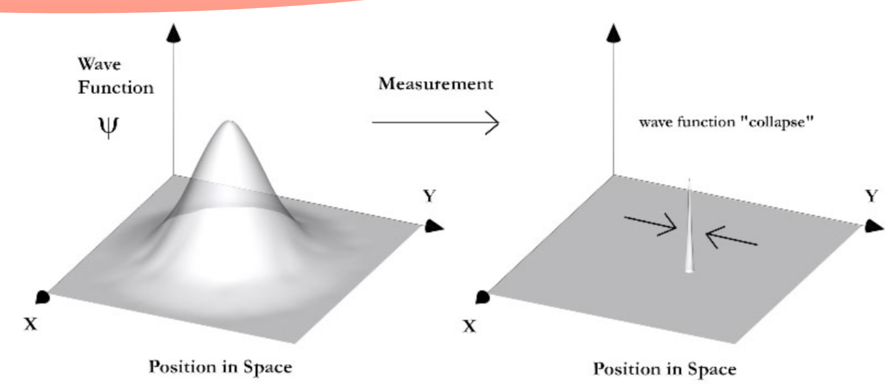
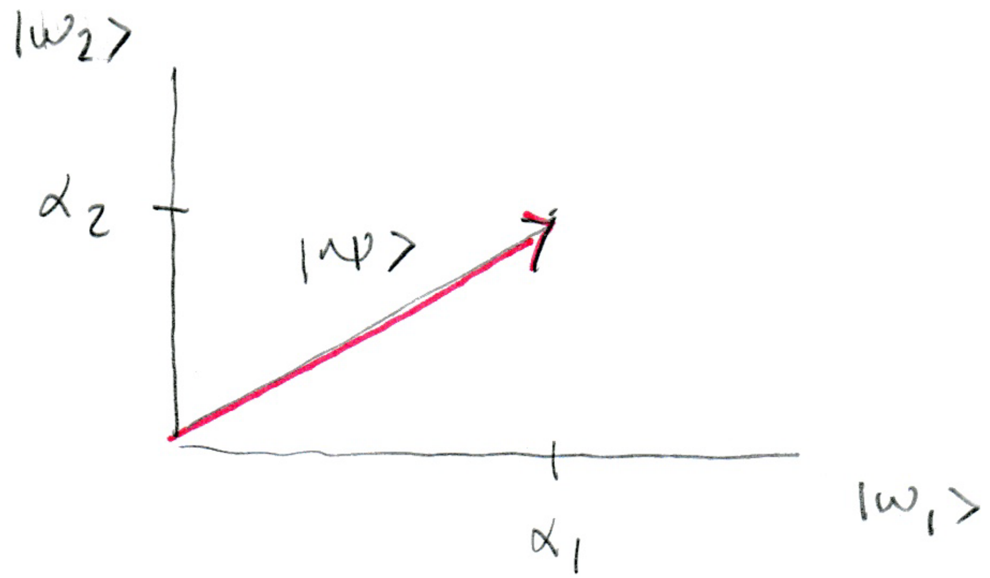


exempel: qubit $|\Psi\rangle = \alpha_1 |\omega_1\rangle + \alpha_2 |\omega_2\rangle$



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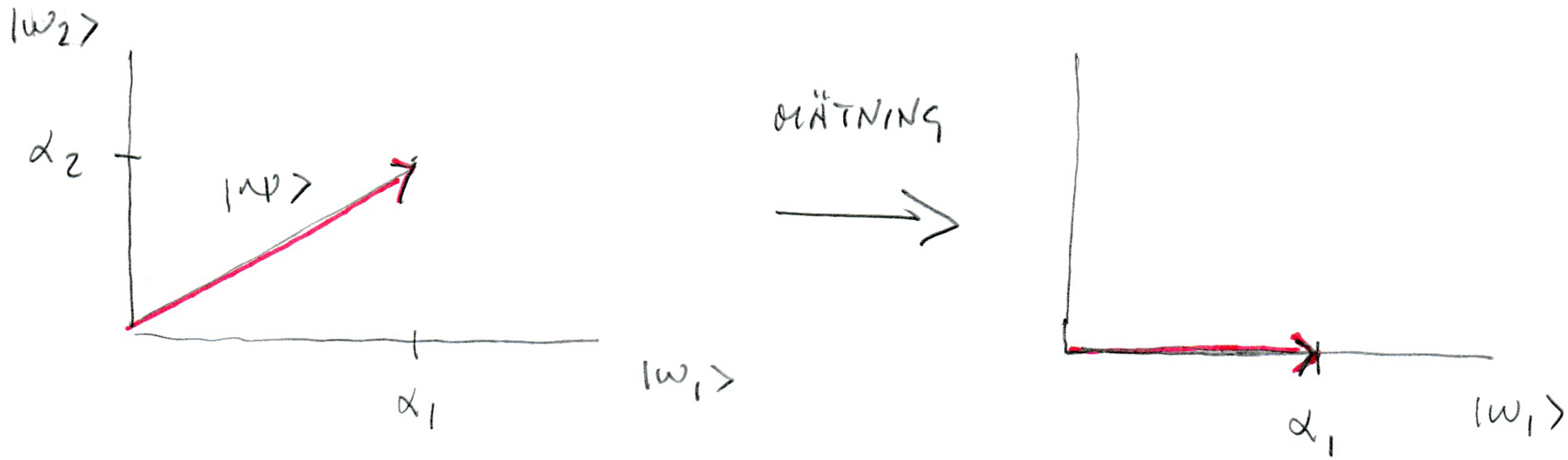

$|\psi\rangle$ "KOLLAPSAR" TILL $|\omega_1\rangle$. SANNOLIKHETEN FÖR DENNA KOLLAPS, DVS ATT VI FÅR MÄTVÄRDET ω_1 ÄR $= |\alpha_1|^2 = |\langle\omega_1|\psi\rangle|^2$

ICKE-UNITÄR PROCESS TY $|\alpha_1| < \sqrt{\alpha_1^2 + \alpha_2^2}$

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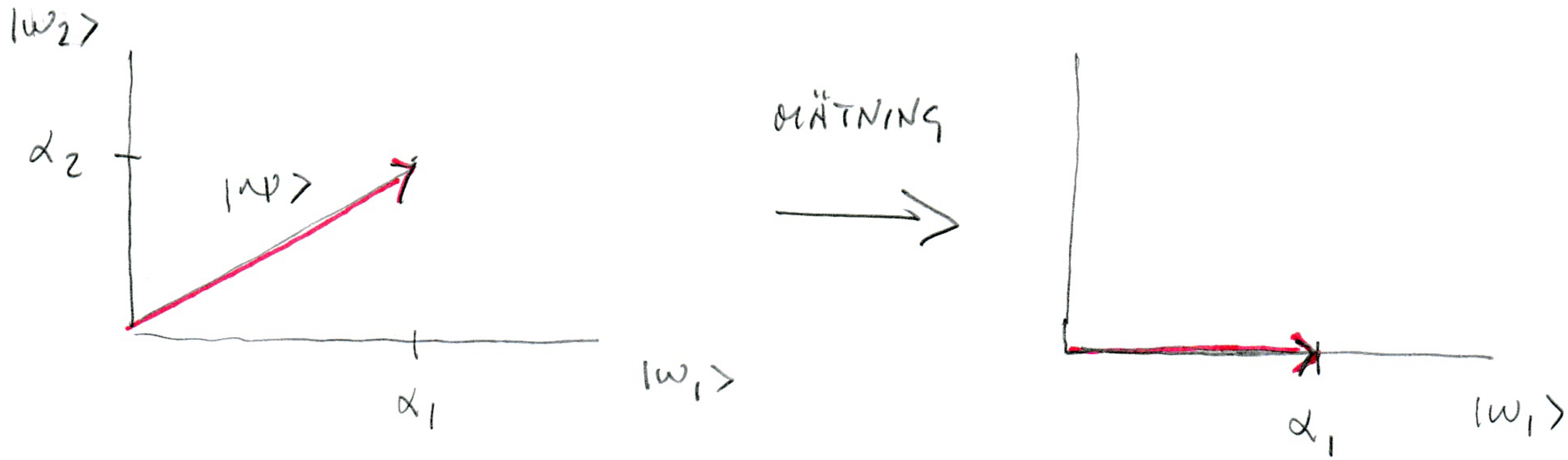

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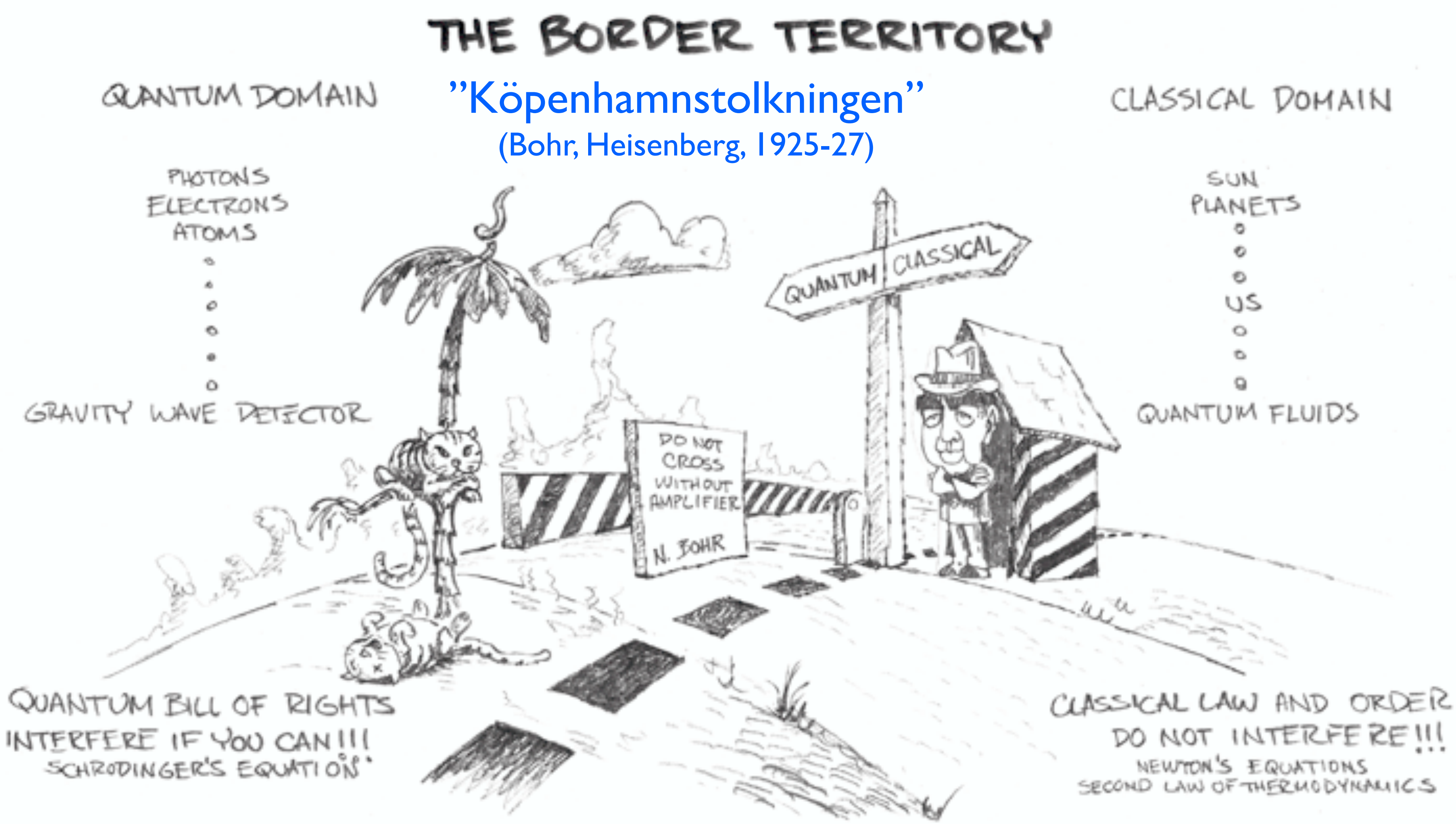
STRIDER MOT 4:e POSTULATET!



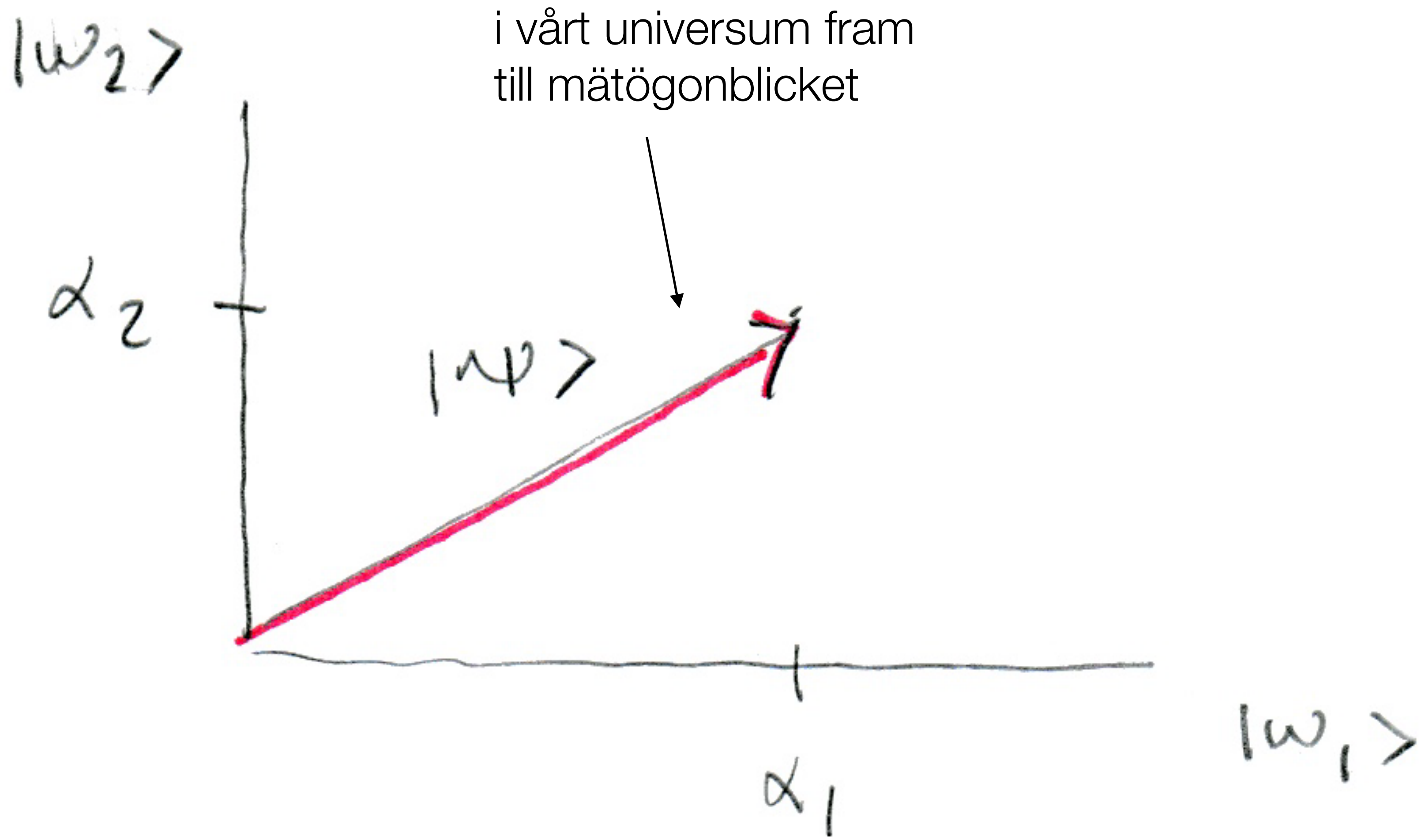
"Köpenhamnstolkningen"
(Bohr, Heisenberg, 1925-27)

"Many-worlds interpretation"
(Everett, 1957)

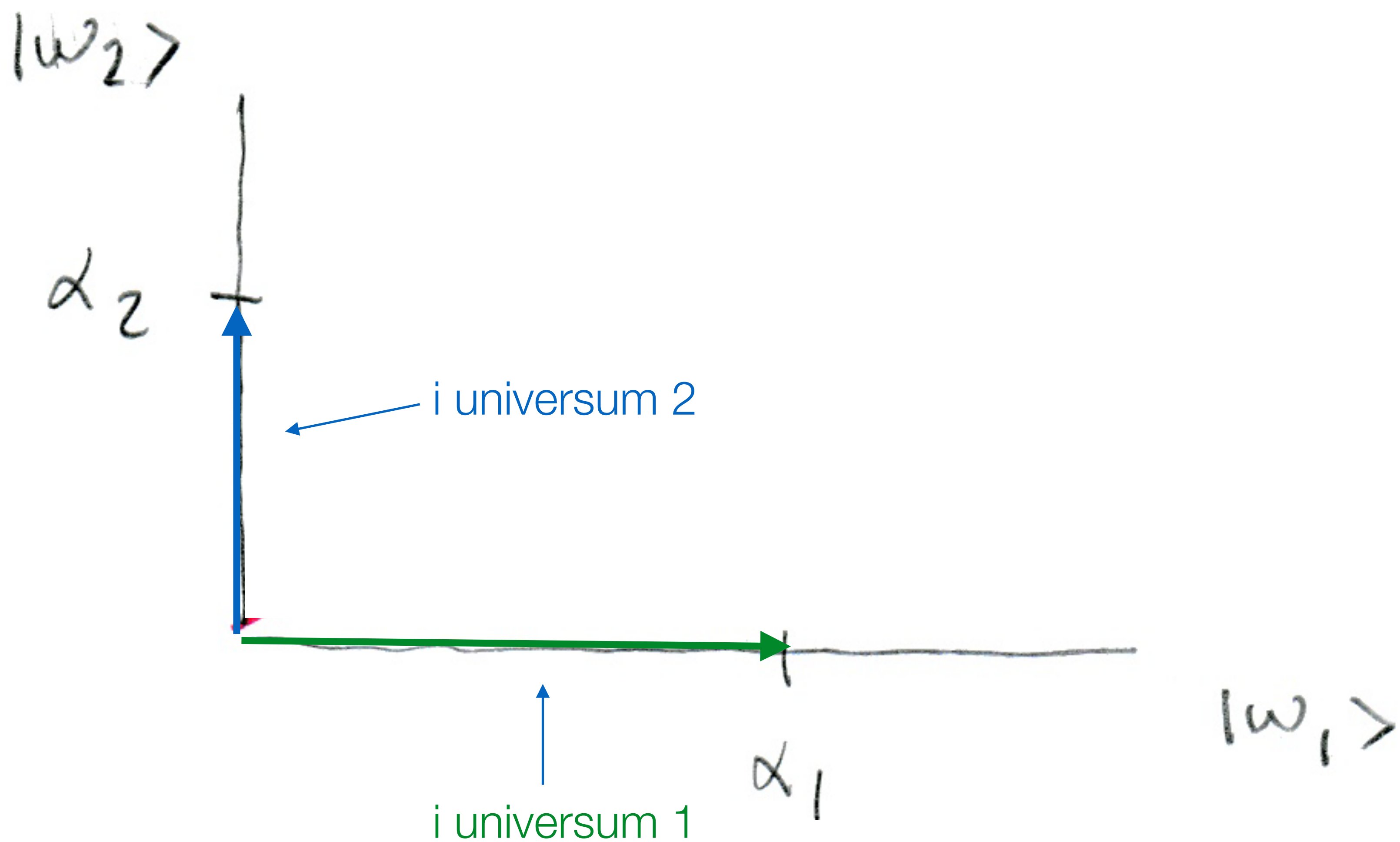
"Decoherence theory"
(Zurek 1981, Joos & Zeh, 1985)

"Consistent histories theory"
(Omnès, 1994)

"Qbism"
(Fuchs, 2010)



"Many-worlds interpretation"
(Everett, 1957)

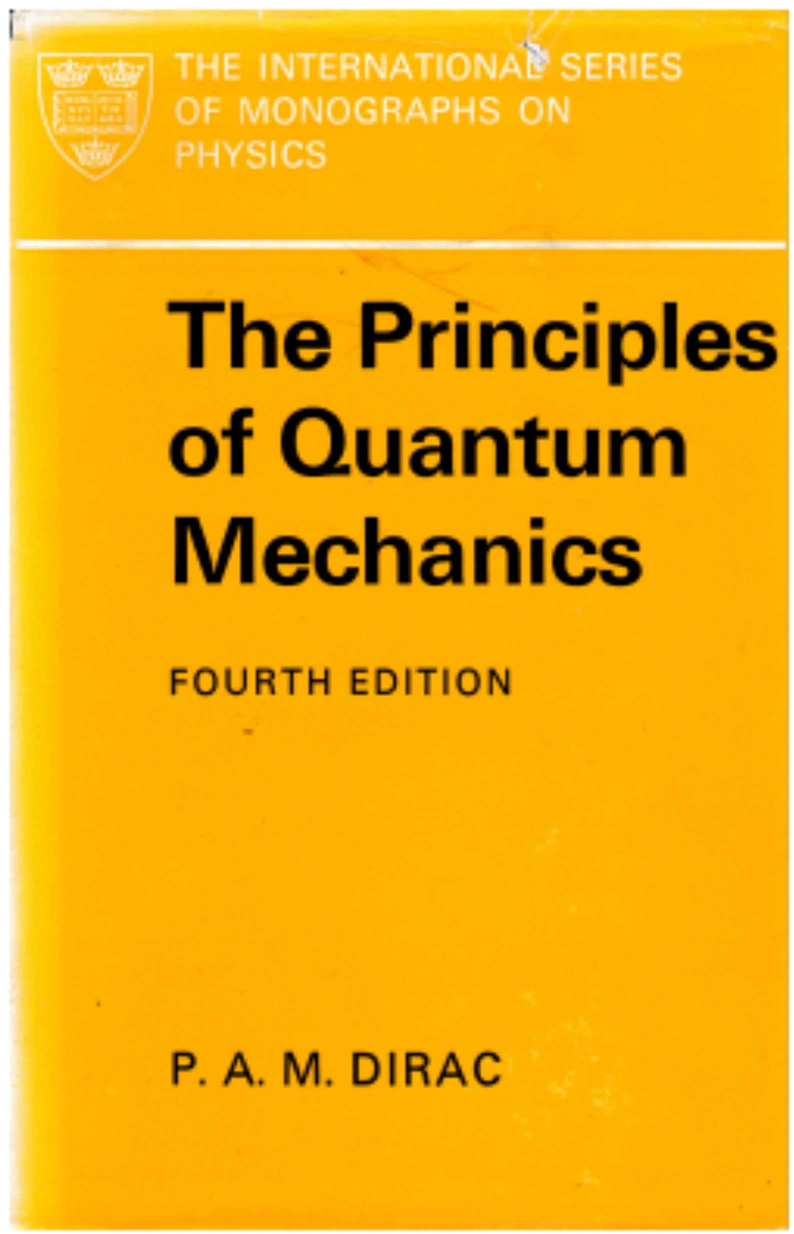


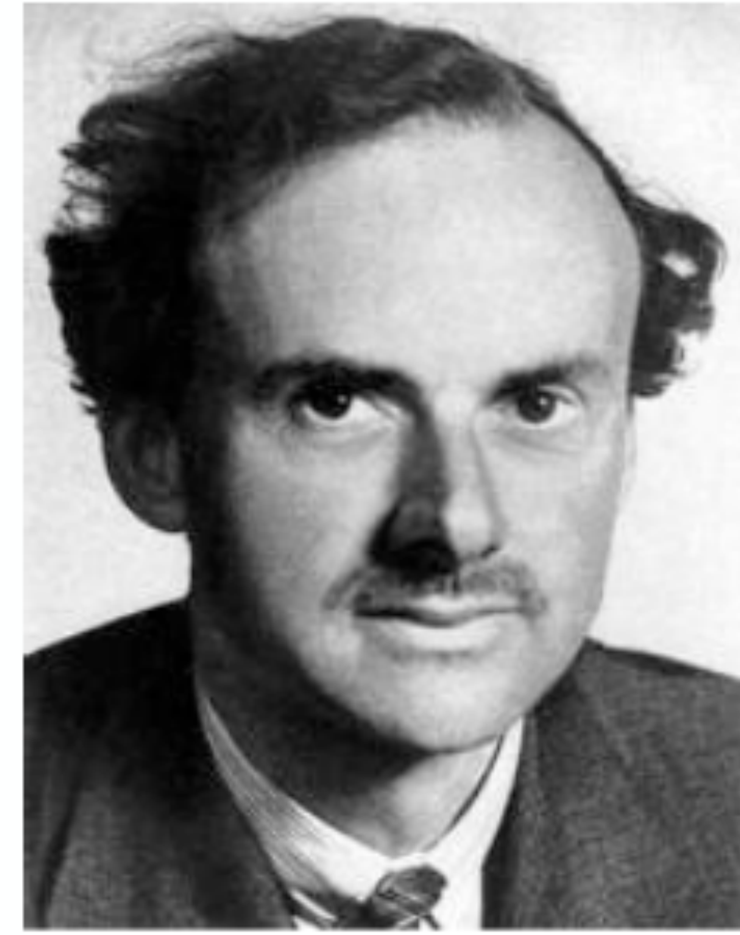
”Many-worlds interpretation”
(Everett, 1957)

Jean Dieudonné
1906 - 1992

”When one gets to the mathematical theories which are at the basis of quantum mechanics, one realizes that the attitude of many physicists in the handling of these theories borders on the delirium... One has to wonder what remains in the mind of a student who has absorbed this unbelievable accumulation of nonsense...!

I sin bok *The Principles of Quantum Mechanics* (1930) ställer Paul Dirac frågan (i en fotnot) om det går att formulera kvantmekaniken med hjälp av en Lagrangefunktion *(analytisk mekanik)* istället för att använda en Hamiltonfunktion *(Hamiltons mekanik)…*





$$(i\gamma\cdot\partial - m)\psi = 0$$

1933 utvecklar han detta i en artikel publicerad i *Physikalische Zeitschrift der Sovietunion:*

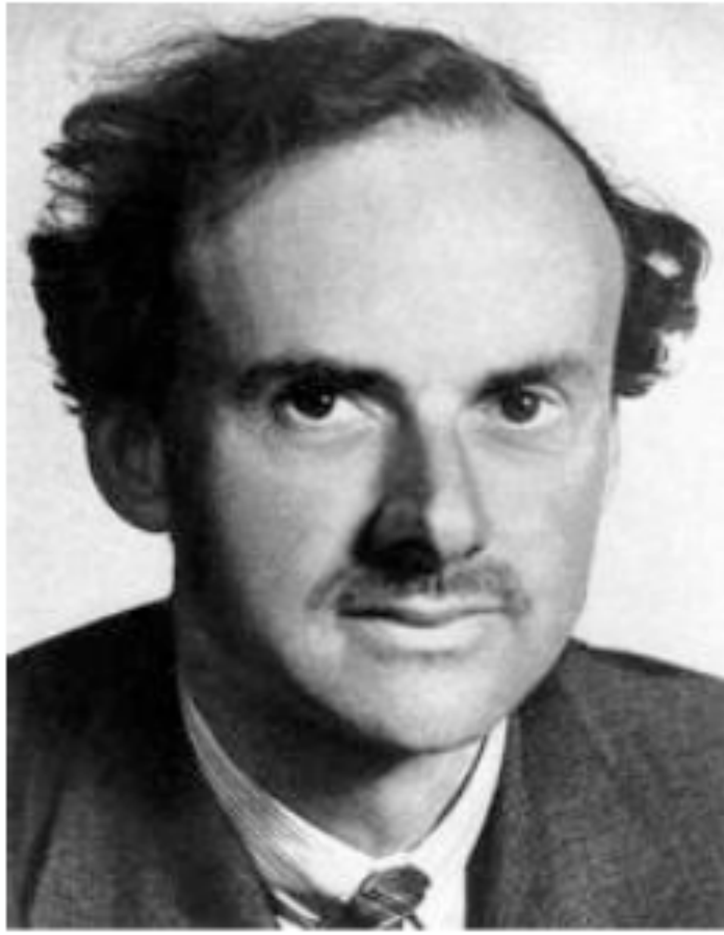
In 1933 Dirac published a paper in *Physikalische Zeitschrift der Sowjetunion* on "The Lagrangian in Quantum Mechanics." He begins by saying:

"Quantum mechanics was built up on a foundation of analogy with the Hamiltonian theory of classical mechanics. This is because the classical notion of canonical coordinates and momenta was found to be one with a very simple quantum analogue. . . .

Now there is an alternative formulation for classical dynamics, provided by the Lagrangian. This requires one to work in terms of coordinates and velocities instead of coordinates and momenta. The two formulations are, of course, closely related, but there are reasons for believing that the Lagrangian one is the more fundamental.

In the first place the Lagrangian method allows one to collect together all the equations of motion and express them as the stationary property of a certain action function. (This action function is just the time integral of the Lagrangian.) There is no corresponding action principle in terms of the coordinates and momenta of the Hamiltonian theory. [This is not true, but it doesn't matter.] Secondly the Lagrangian method can easily be expressed relativistically, on account of the action function being a relativistic invariant; while the Hamiltonian method is essentially nonrelativistic in form, since it marks out a particular time variable. . . .

For these reasons it would seem desirable to take up the question of what corresponds in the quantum theory to the Lagrangian method of the classical theory."



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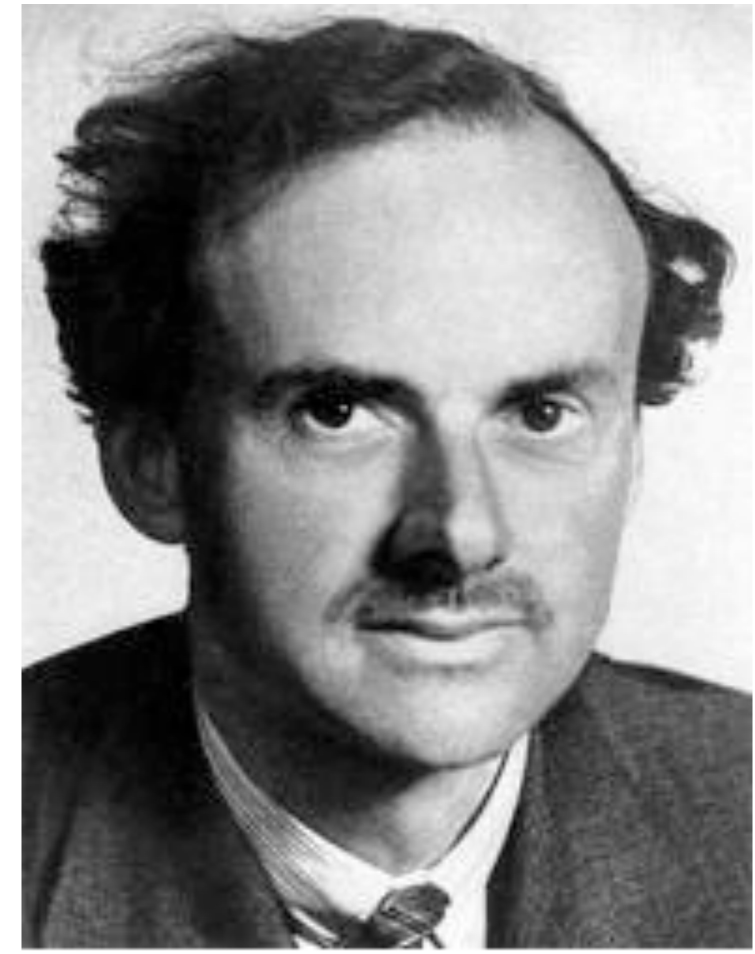

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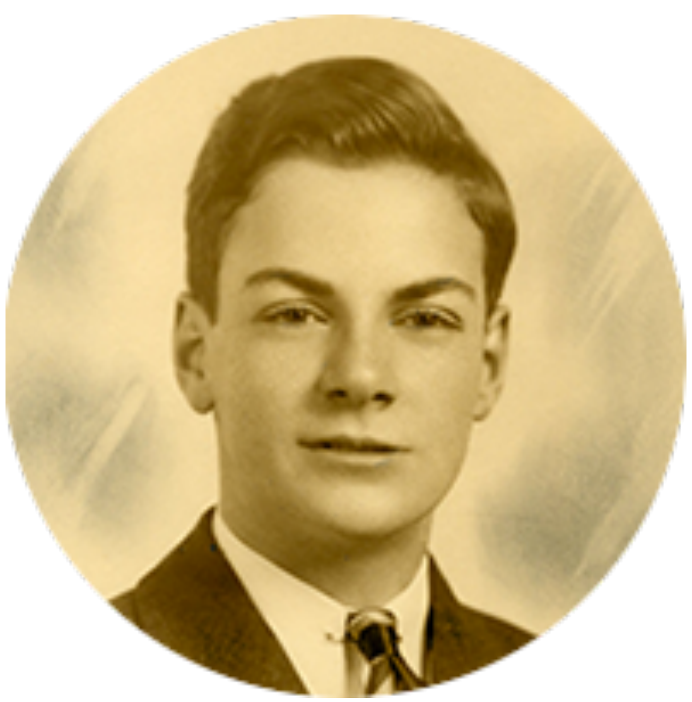


Feynman (doktorand vid Princeton University i början av 1940-talet) inspirerades av Dirac…

Ett annat sätt att göra kvantmekanik:

Feynmans vägintegralformulering

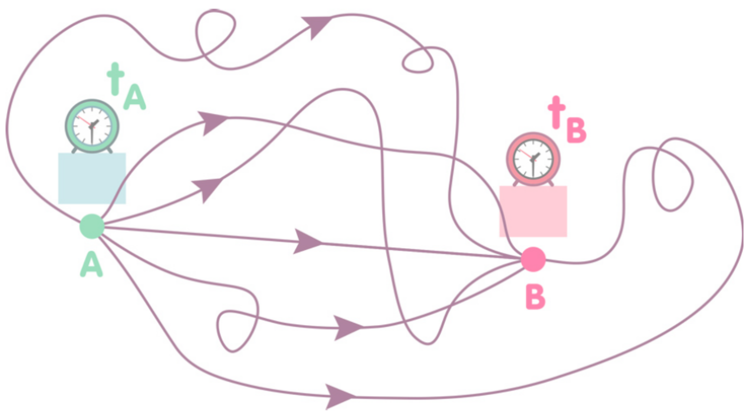
… löser inte ”tolkningsproblemet” av kvantmekaniken, men

- förenklar ofta analys och räkningar
- enklare att göra konsistent med speciell relativitetsteori ➡ kvantfältteori
- gör symmetrier och topologiska effekter transparenta
- ….
- ….
- … **och förklarar minsta verkans princip i klassisk fysik!**

Feynman vid Nobelbanketten 1965

föreläsning 1/12 (I)

Feynmans vägintegralformulering av kvantmekaniken



BAKGRUND

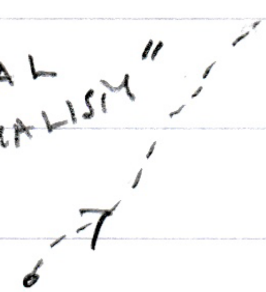
KLASSISK MEKANIK

HAMILTON FORMULERING

$$\begin{cases} \dot{x} = \dfrac{\partial H}{\partial p} \\[2ex] \dot{p} = -\dfrac{\partial H}{\partial x} \end{cases} \qquad H = \frac{p^2}{2m} + V(x), \quad F = -\frac{dV}{dx}$$

$$\Rightarrow \quad m\ddot{x} = F$$

"LOKAL FORMALISM"



$x(t=0)$

$\dot{x}(t=0)$

BAKGRUND

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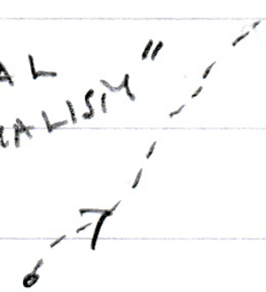

$x(t=0)$

$\dot{x}(t=0)$

ANALYTISK MEKANIK

$$S = \int_{t=0}^{t=T} dt\, L(x, \dot{x}, t)$$

$$L(x, \dot{x}, t) = K - V$$

$$\delta S = 0$$

"GLOBAL FORMALISM"

$x(t=T)$

$x(t=0)$

BAKGRUND

KLASSISK MEKANIK

HAMILTON FORMULERING

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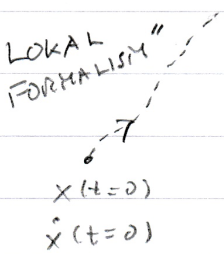

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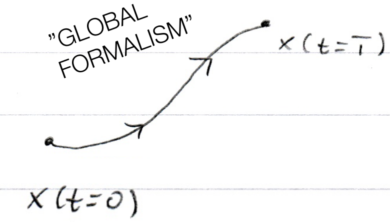

$$\{f, g\} = \frac{\partial f}{\partial x}\frac{\partial g}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial g}{\partial x}$$

$$\{x, p\} = 1$$

$$\Downarrow \text{ KANONISK KVANTISERING}$$

$$[\hat{x}, \hat{p}] = i\hbar$$

"KANONISK OPERATORFORMALISM"

Heisenberg, Born, Jordan, Schrödinger 1925–26

Dirac 1928

von Neumann 1933

BAKGRUND

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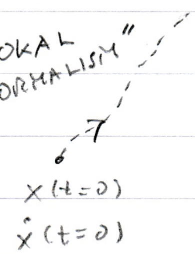

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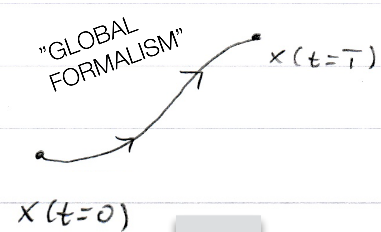

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"KANONISK OPERATORFORMALISM"

Heisenberg, Born, Jordan, Schrödinger 1925–26

Dirac 1928

von Neumann 1933

$\Downarrow$ Dirac 1933

?

BAKGRUND

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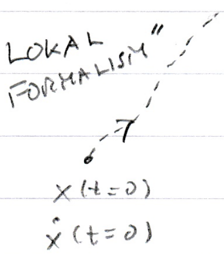

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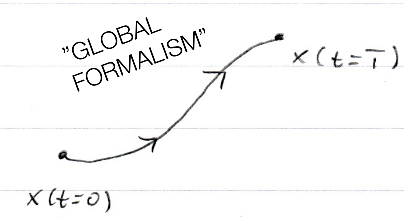

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$\Downarrow$ KANONISK KVANTISERING

$$[\hat{x}, \hat{p}] = i\hbar$$

"KANONISK OPERATORFORMALISM"

Heisenberg, Born, Jordan, Schrödinger 1925–26

Dirac 1928

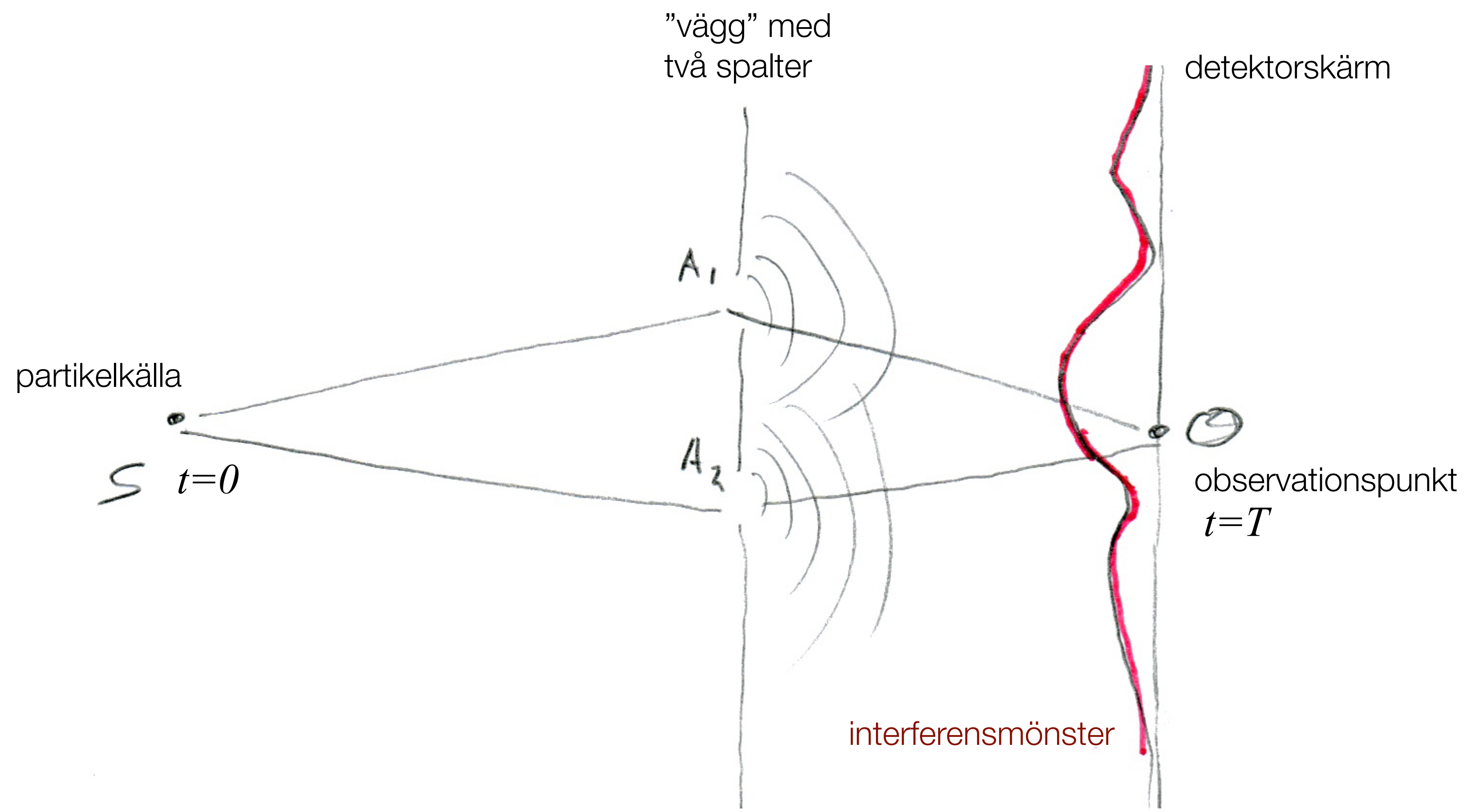
von Neumann 1933

Feynman 1948

VÄGINTEGRAL FORMULERING AV KVANTMEKANIKEN

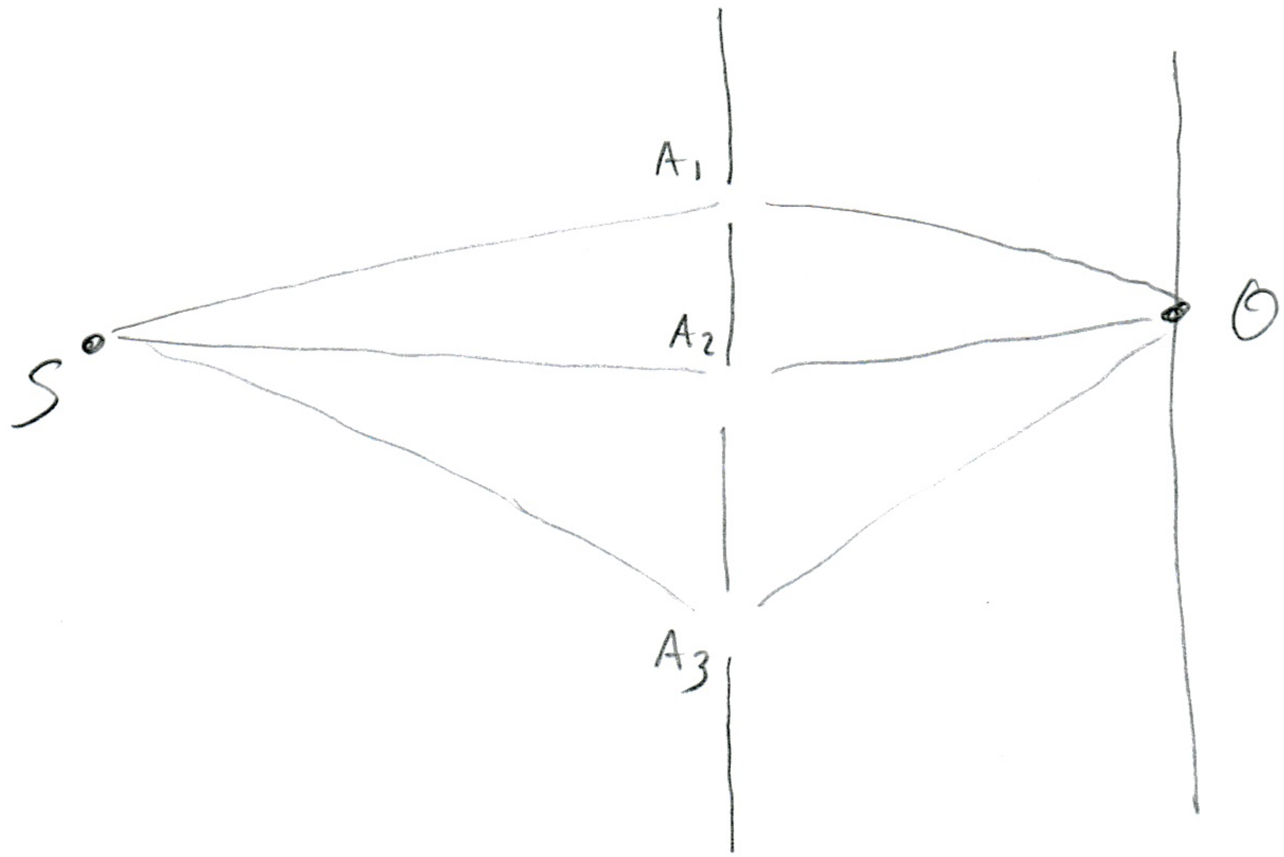
Feynmans vägintegralformulering av kvantmekaniken

motivation: dubbelspaltsexperimentet

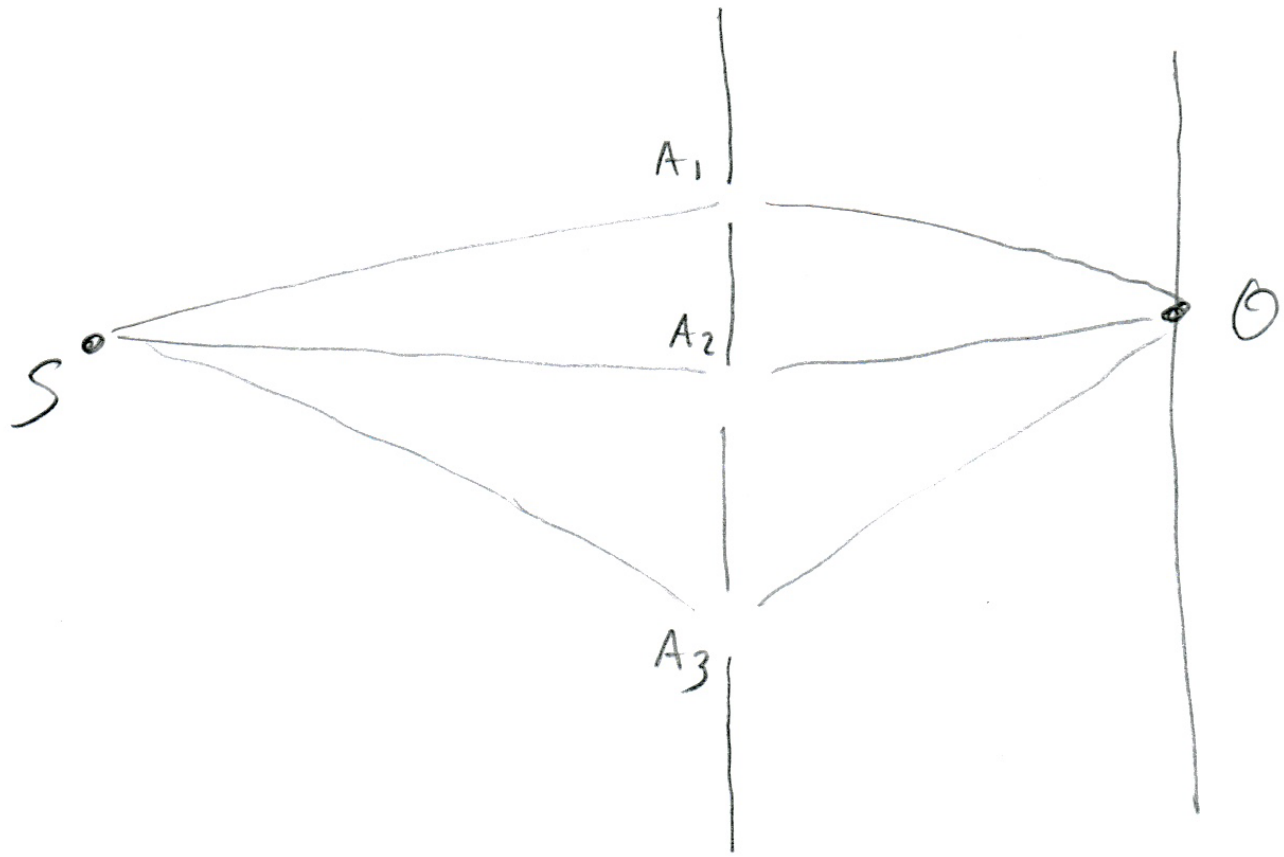


$$\mathcal{A}_{S\to O} = \mathcal{A}_{S\to A_1 \to O} + \mathcal{A}_{S\to A_2\to O}$$

$P(O) = |\mathcal{A}_{S\to O}|^2$ = sannolikheten att detektera partikeln i punkten O



$$\mathcal{A}_{S \to O} = \mathcal{A}_{S \to A_1 \to O} + \mathcal{A}_{S \to A_2 \to O} + \mathcal{A}_{S \to A_3 \to O}$$

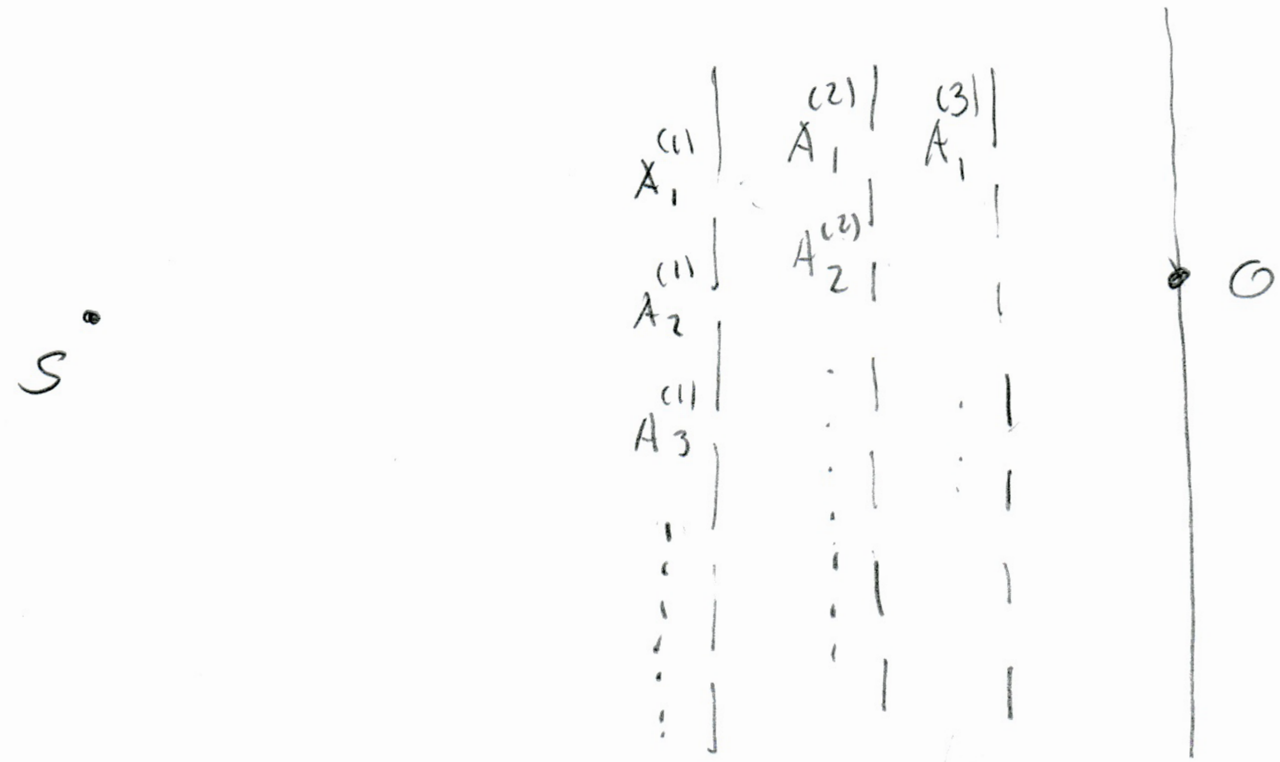


$$\mathcal{A}_{S\to O} = \mathcal{A}_{S\to A_1\to O} + \mathcal{A}_{S\to A_2\to O} + \mathcal{A}_{S\to A_3\to O}$$

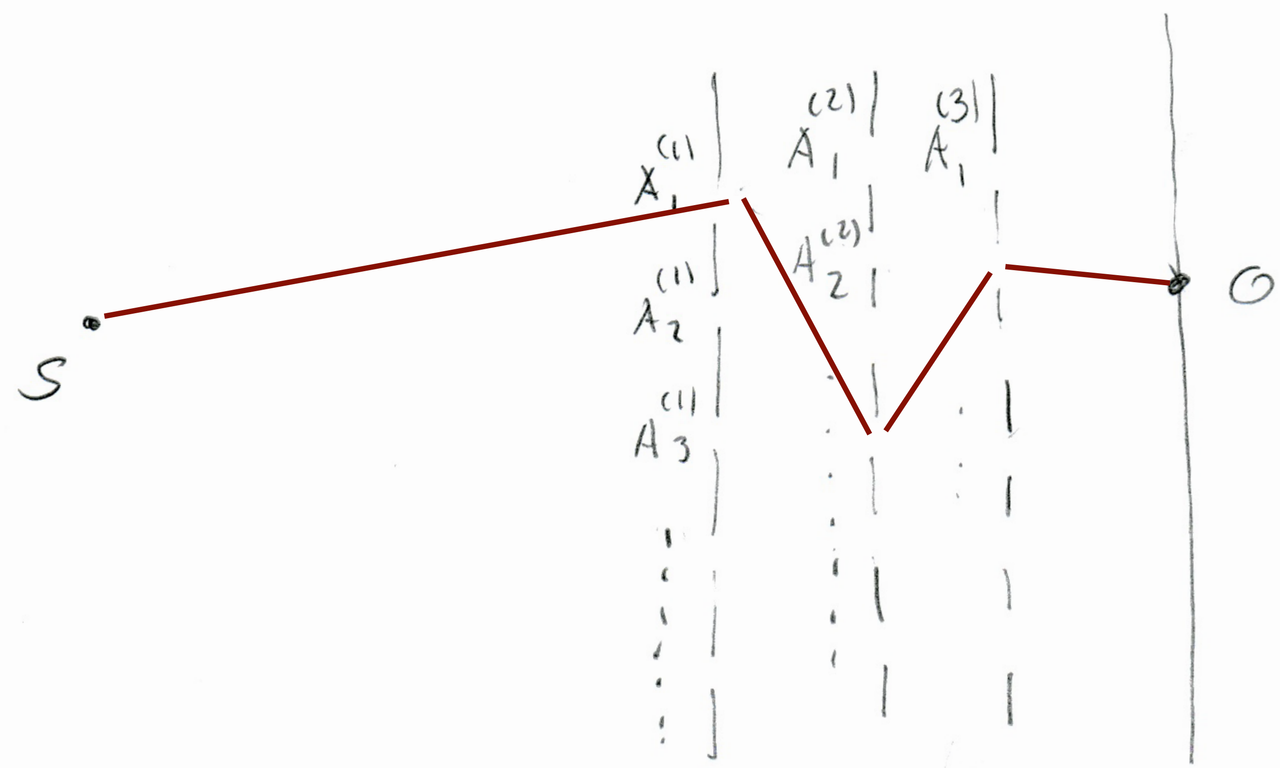
$\vdots$

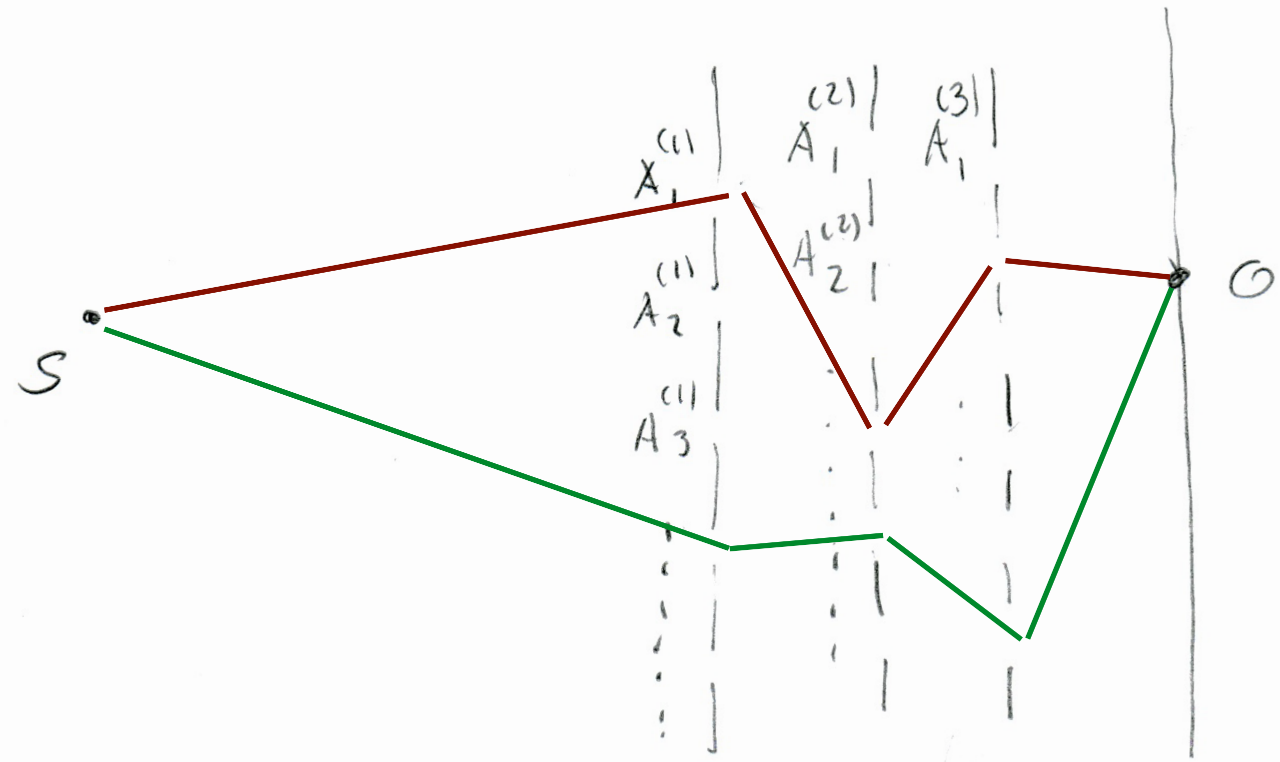
$$\mathcal{A}_{S\to O} = \sum_{i=1}^{\#\text{ spalter}} \mathcal{A}_{S\to A_i\to O}$$

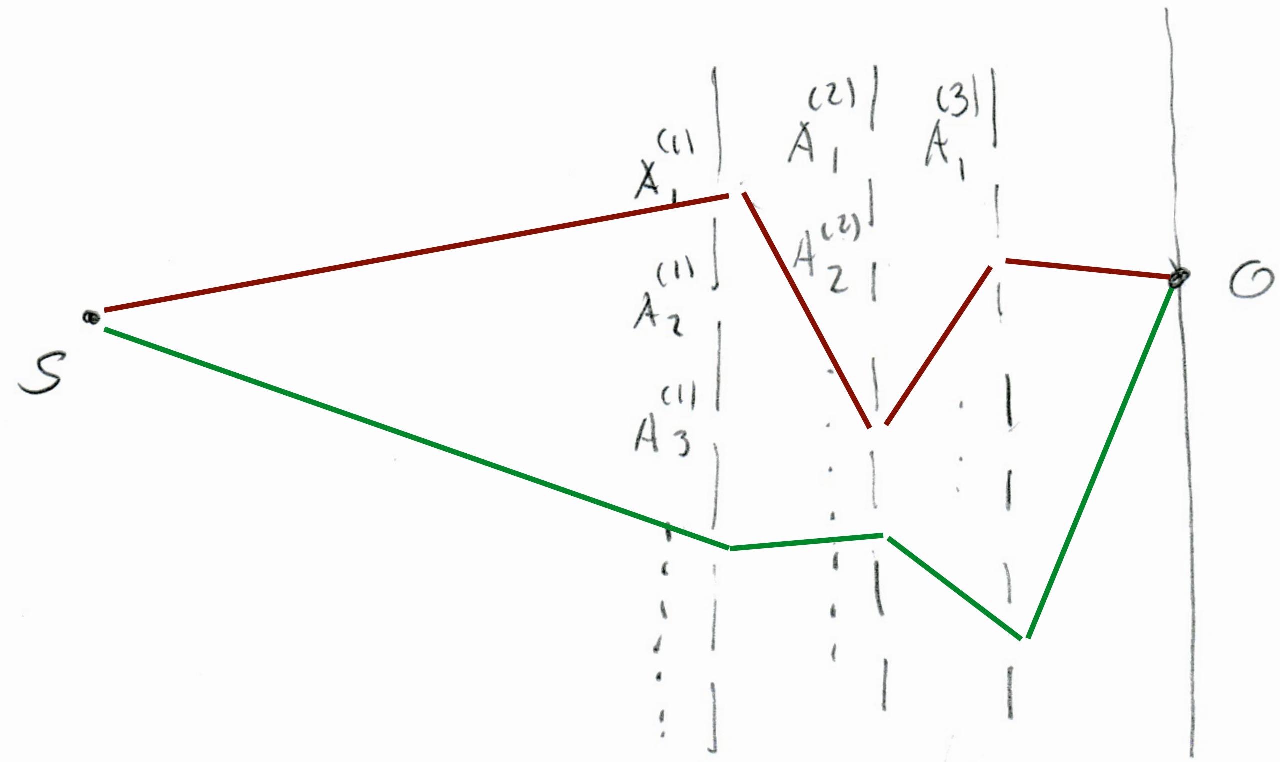
$$P(O) = |\mathcal{A}_{S\to O}|^2$$


S
$A_1^{(1)}$
$A_2^{(1)}$
$A_3^{(1)}$
$A_1^{(2)}$
$A_2^{(2)}$
$A_1^{(3)}$
O

föreläsning 1/12 (II)

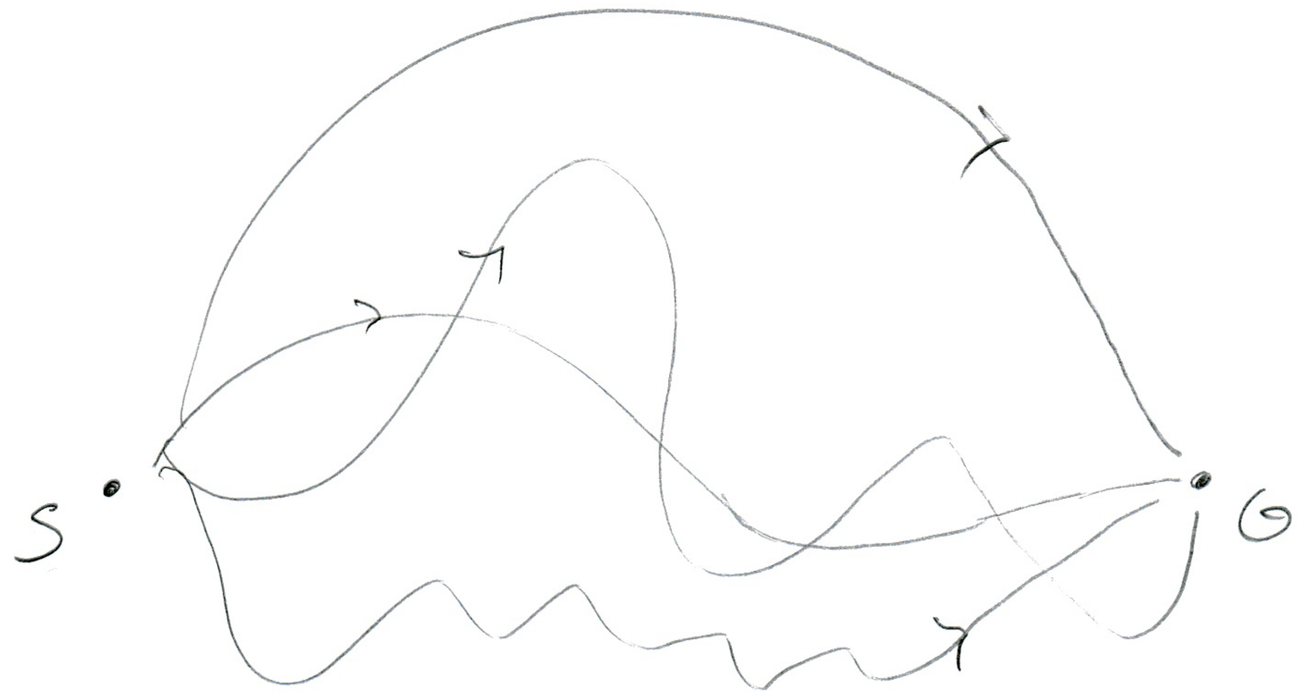



S
$A_1^{(1)}$
$A_2^{(1)}$
$A_3^{(1)}$
$A_1^{(2)}$
$A_2^{(2)}$
$A_1^{(3)}$
G



$$\mathcal{A}_{S\to O} = \sum_{i,j,k}^{\#\text{spalter}} \mathcal{A}\left(S \to A_i^{(1)} \to A_j^{(2)} \to A_k^{(3)} \to O\right)$$

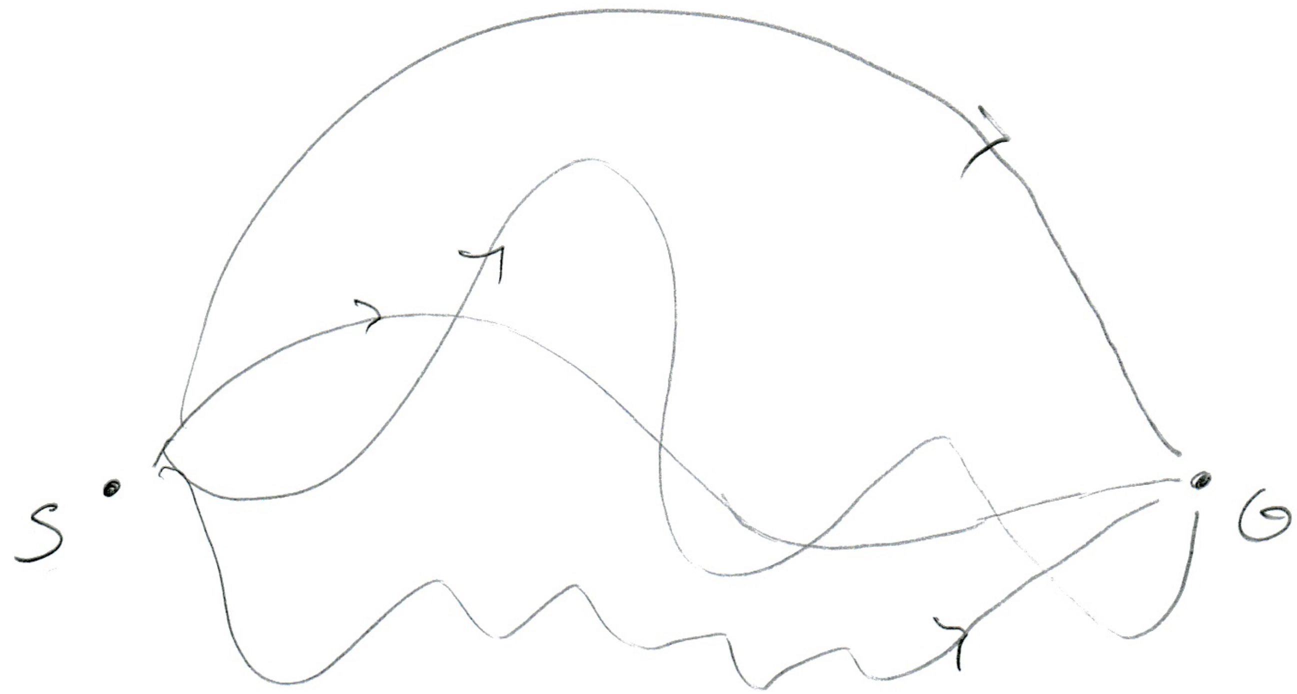
$$P(O) = |\mathcal{A}_{S\to O}|^2$$



SPALTER $\to \infty$

VÄGGAR $\to \infty$

PARTIKELN ”PRÖVAR” *ALLA*
VÄGAR FRÅN *S* *TILL* *O* !

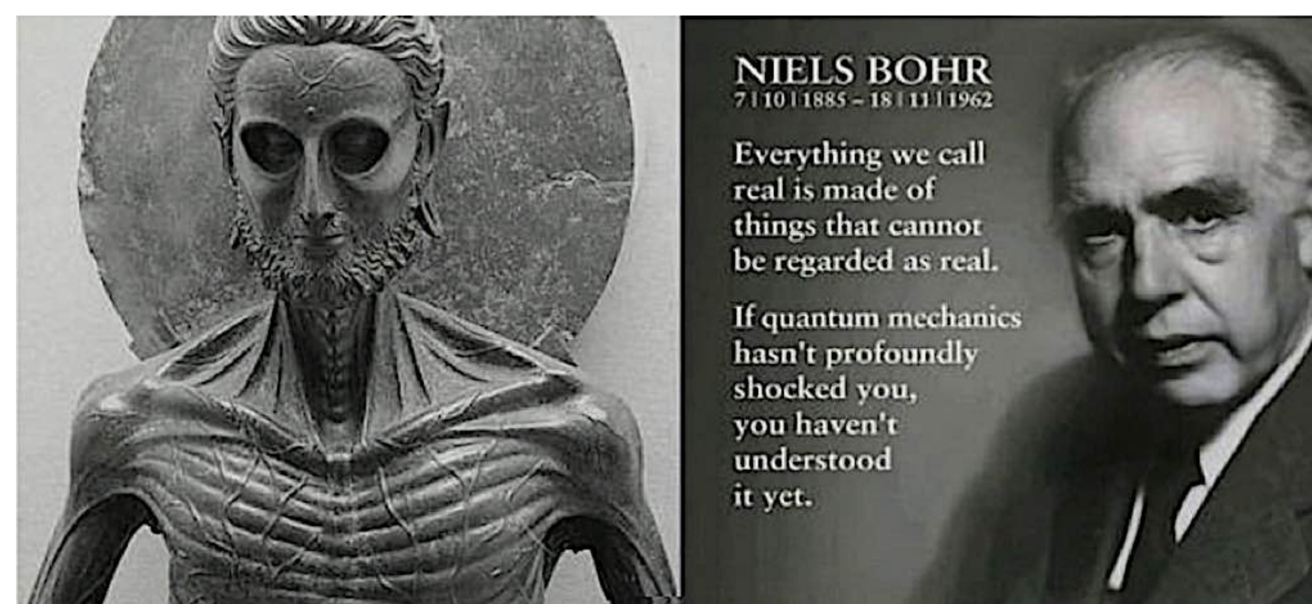


SPALTER → ∞

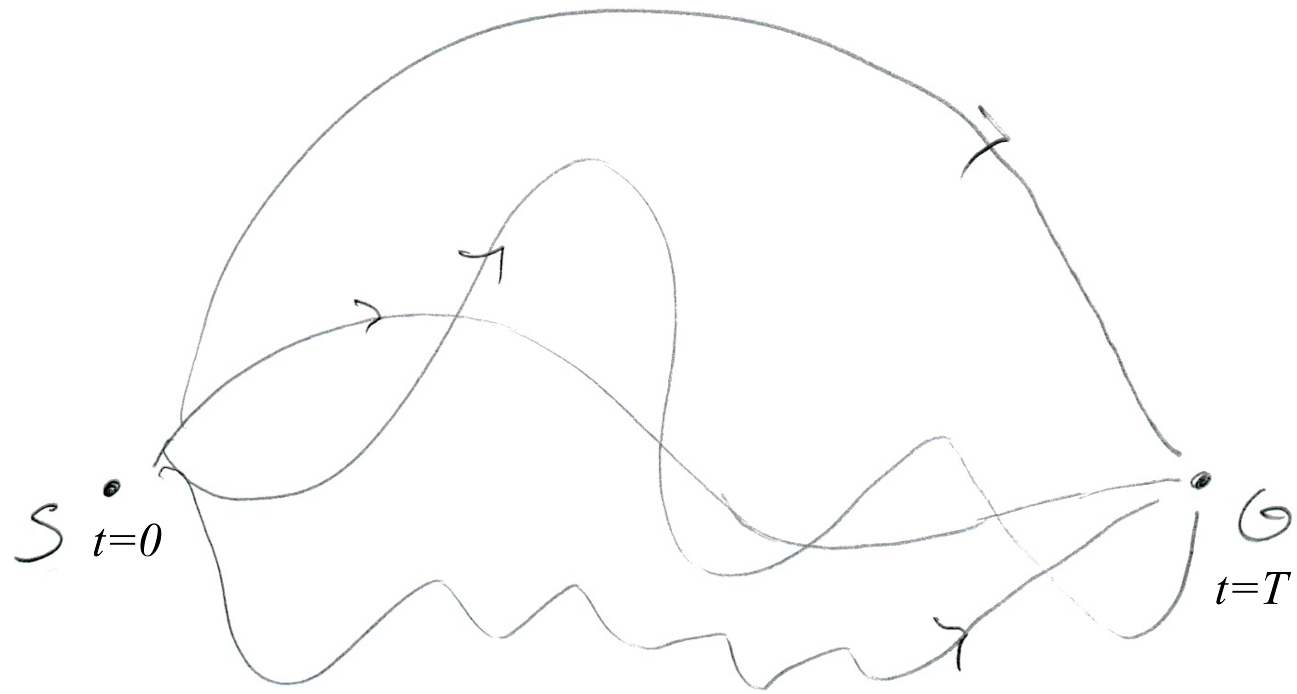
VÄGGAR → ∞

PARTIKELN ”PRÖVAR” *ALLA* VÄGAR FRÅN *S TILL O* !

The Zen of Physics...



The Lord Buddha: "The world is bound up by, and shrouded by a veil of delusion. It appears as real, and is regarded as if it were fine! The fool bound to his illusive acquisitions, blinded by darkness, assumes it as eternal, but for one who sees, and really understands, there is nothing real, stable, or ever same, neither here, nor there at all ..." [*Udāna* VII 10]



SPALTER $\to \infty$

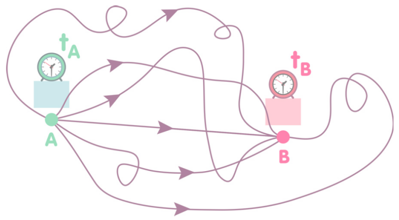
VÄGGAR $\to \infty$

*PARTIKELN "PRÖVAR" ALLA VÄGAR FRÅN **S** TILL **O** !*

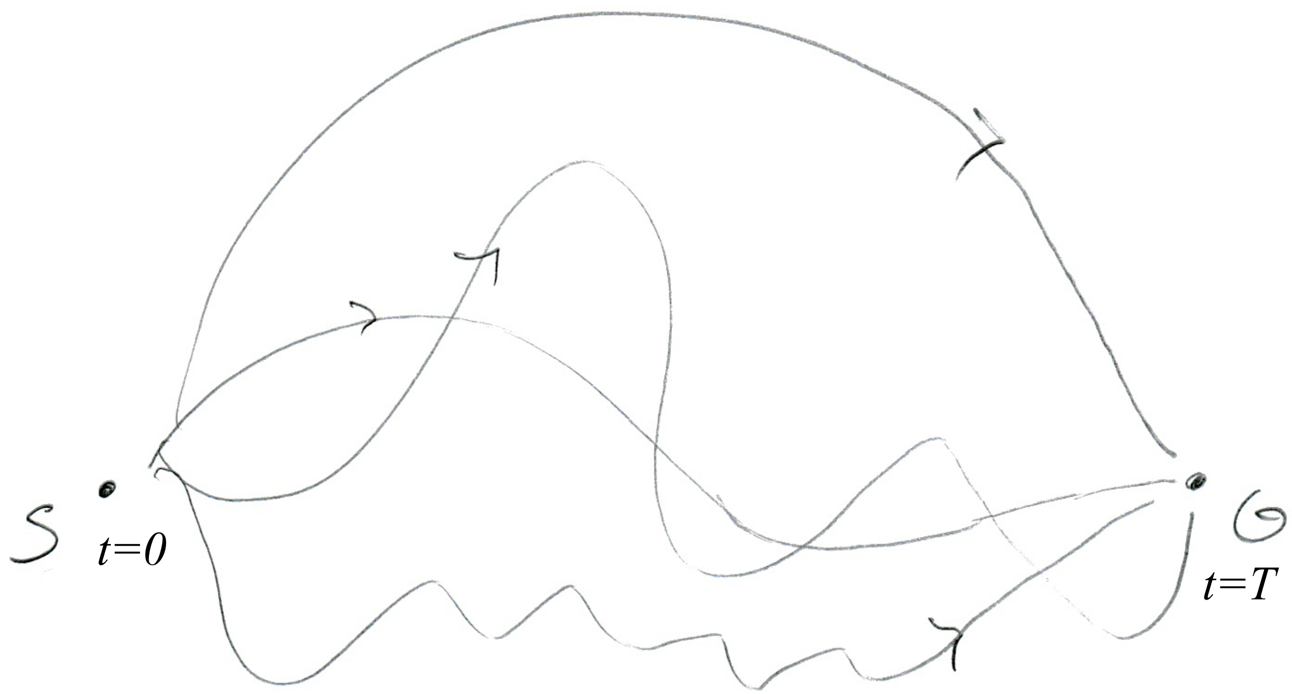
$$\mathcal{A}_{S\to O} = \lim_{n\to\infty} \sum_{v=1}^{n} \mathcal{A}(S \to O \text{ längs vägen } v \text{ under tiden } T)$$

v: vägindex

$$P(O) = |\mathcal{A}_{S\to O}|^2$$

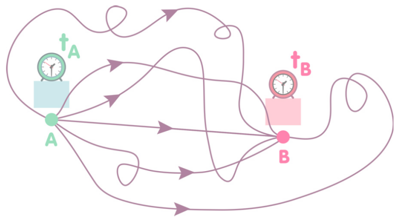


Feynmans vägintegralformulering av kvantmekaniken (forts.)



$$\mathcal{U}_{S\to O} = \lim_{n\to\infty} \sum_{\nu=1}^{n} \mathcal{A}(S \to O \text{ längs vägen } \nu \text{ under tiden } T)$$

vägindex



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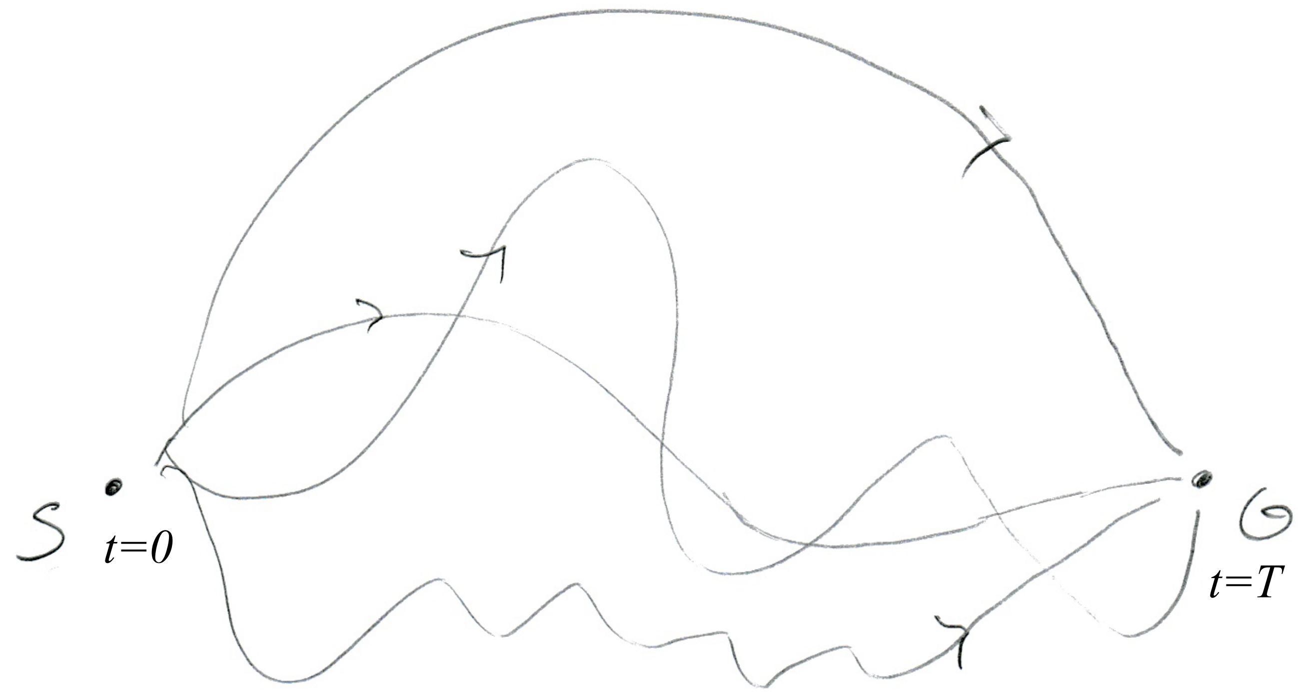
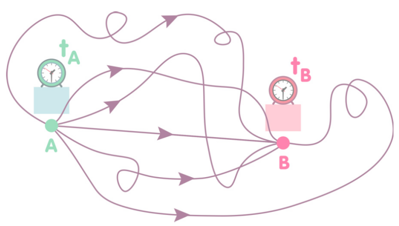


$$\mathcal{A}_{S\to G} = \lim_{n\to\infty} \sum_{v=1}^{n} \mathcal{A}\left(S \to G \text{ längs vägen } v \text{ under tiden } T\right)$$

vägindex

$$= \int_{\text{VÄGAR } v} \mathcal{A}\left(S\to G \text{ längs vägen } v \text{ under tiden } T\right) dv$$



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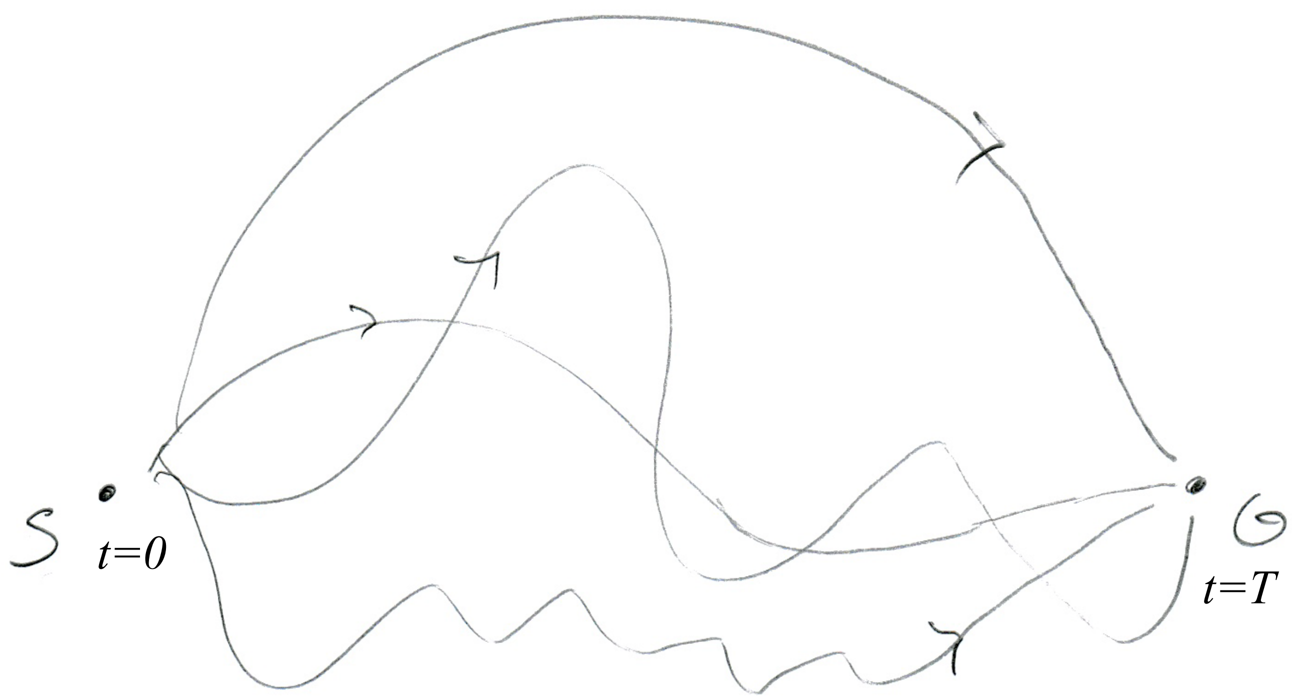


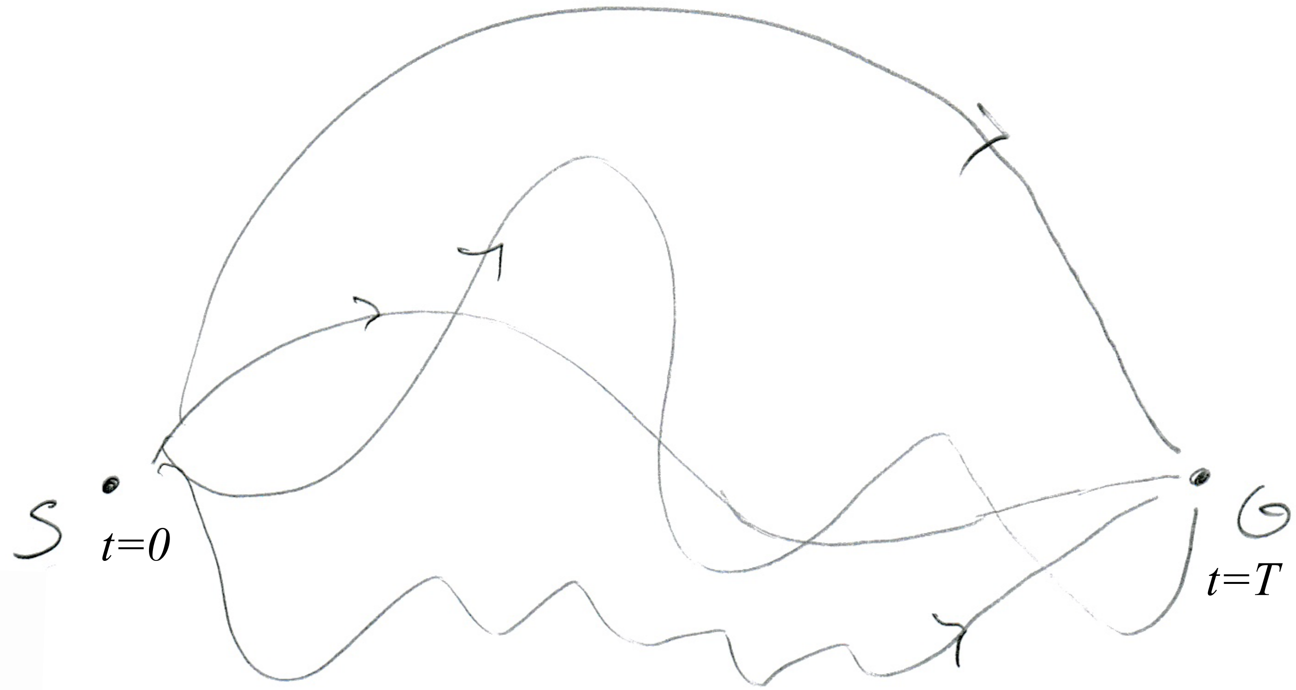
$$\mathcal{A}_{S\to O} = \lim_{n\to\infty} \sum_{v=1}^{n} \mathcal{A}(S \to O \text{ längs vägen } v \text{ under tiden } T)$$

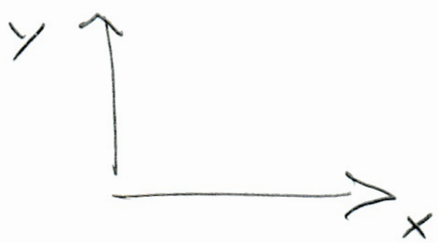
↑ vägindex

$$= \int_{\text{VÄGAR } v} \mathcal{A}\left(S \to O \text{ längs vägen } v \text{ under tiden } T\right) dv$$

Vi behöver formalisera detta!

Feynmans vägintegralformulering av kvantmekaniken (forts.)





$S = (x_I, y_I) \equiv q_I$

$O = (x_F, y_F) \equiv q_F$

$|q_I\rangle$ = TILLSTÅNDET FÖR PARTIKELN I q_I

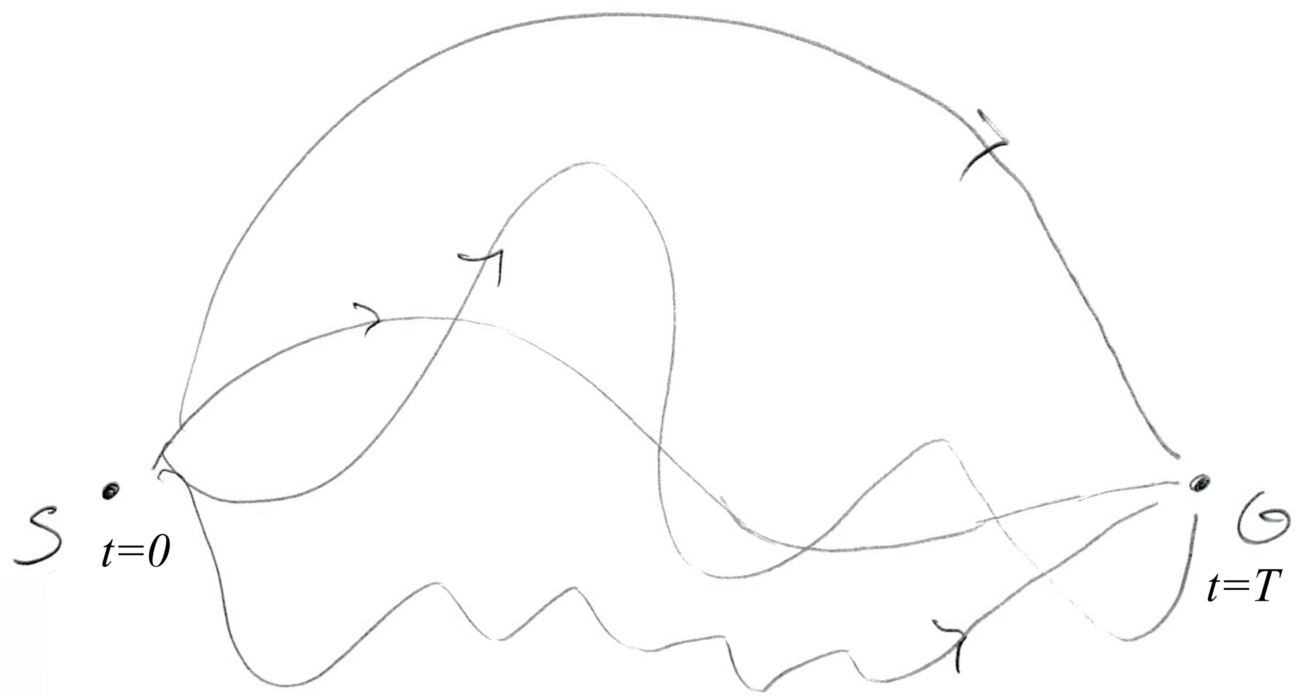
$|q_F\rangle$ = —"— q_F

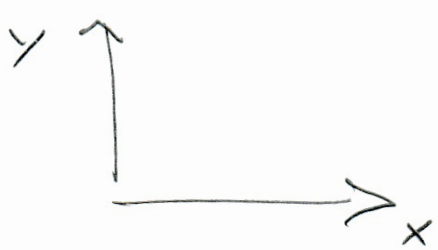
$\hbar = 1$

ON BASVEKTORER

$$\mathcal{A}_{S\to O} = \langle q_F | \underbrace{e^{-i\hat{H}T} | q_I \rangle}$$

$$|\Psi(T)\rangle = \alpha_A |q_A\rangle + \alpha_B |q_B\rangle + \ldots \alpha_F |q_F\rangle + \ldots$$





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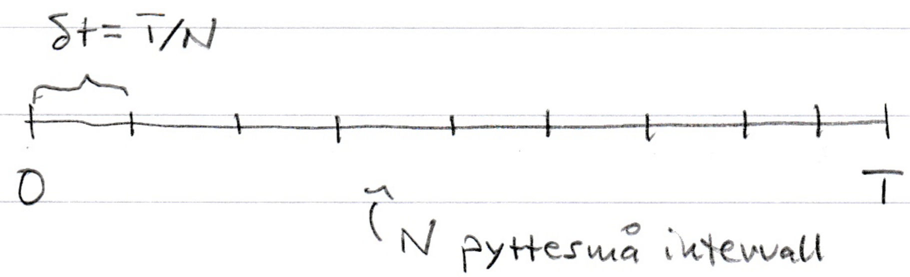
$\hbar = 1$

ON BASVEKTORER

$$\mathcal{A}_{S \to O} = \langle q_F | e^{-i\hat{H}T} | q_I \rangle = \alpha_F$$

$$|\Psi(T)\rangle = \alpha_A |q_A\rangle + \alpha_B |q_B\rangle + \dots \alpha_F |q_F\rangle + \dots$$

Låt oss nu undersöka $\mathcal{A}_{S\to 0}$ närmare!



$$\mathcal{A}_{S\to 0} = \langle q_F | e^{-i\hat{H}T} | q_I \rangle$$

$$= \langle q_F | e^{-i\hat{H}\delta t}\, \mathbb{1}\, e^{-i\hat{H}\delta t}\, \mathbb{1} \dots \mathbb{1}\, e^{-i\hat{H}\delta t} | q_I \rangle$$

"RESOLUTION OF IDENTITY"

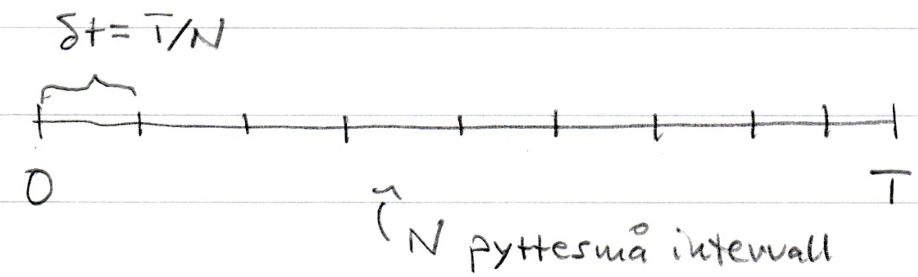
$$= \langle q_F | e^{-i\hat{H}\delta t} \underbrace{\int dq_{N-1} |q_{N-1}\rangle\langle q_{N-1}|}_{\mathbb{1}} e^{-i\hat{H}\delta t} \underbrace{\int dq_{N-2} |q_{N-2}\rangle\langle q_{N-2}|}_{\mathbb{1}} e^{-i\hat{H}\delta t} \dots$$

$$\dots \underbrace{\int dq_1 |q_1\rangle\langle q_1|}_{\mathbb{1}} e^{-i\hat{H}\delta t} |q_I\rangle$$

$$= \overbrace{\int_{q_1} \dots \int_{q_{N-1}} dq_1 \dots dq_{N-1}}^{\prod_{j=1} \int dq_j} \overbrace{\langle q_F |}^{|q_N\rangle} e^{-i\hat{H}\delta t} |q_{N-1}\rangle\langle q_{N-1}| e^{-i\hat{H}\delta t} |q_{N-2}\rangle \dots$$

$$\dots \langle q_1 | e^{-i\hat{H}\delta t} \underbrace{|q_I\rangle}_{|q_0\rangle} \qquad (1)$$

Låt oss nu undersöka $\mathcal{A}_{S\to 0}$ närmare!



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$$\dots \underbrace{\int dq_1 |q_1\rangle\langle q_1|}_{\mathbb{1}} e^{-i\hat{H}\delta t} |q_I\rangle$$

alla integrationsgränser från $-\infty$ till ∞

$$= \overbrace{\int_{q_1} \dots \int_{q_{N-1}} dq_1 \dots dq_{N-1}}^{\prod_{j=1} \int dq_j} \overbrace{\langle q_F|}^{|q_N\rangle} e^{-i\hat{H}\delta t} |q_{N-1}\rangle\langle q_{N-1}| e^{-i\hat{H}\delta t} |q_{N-2}\rangle \dots$$

$$\dots \langle q_1 | e^{-i\hat{H}\delta t} \underbrace{|q_I\rangle}_{|q_0\rangle} \qquad (1)$$

UNDERSÖK EN AV FAKTORERNA I INTEGRANDEN:

VÄLJ $\hat{H} = \frac{\hat{p}^2}{2m}$ (FRI PARTIKEL), $\hat{p} = (\hbar \hat{k})$

$$\langle q_{j+1} | e^{-i\hat{H}dt} | q_j \rangle$$

$$\hat{p}|p\rangle = p|p\rangle \qquad \frac{1}{2\pi}\int |p\rangle\langle p|\, dp = 1$$

NORMERING ”resolution of identity”

$$\langle q|p\rangle = e^{ipq}$$

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NORMERING

”resolution of identity”

$$\langle q|p\rangle = e^{ipq}$$

$$\Downarrow$$

$$\langle q_{j+1}| e^{-i\delta t \frac{\hat{p}^2}{2m}} \mathbb{1} | q_j \rangle = \langle q_{j+1}| e^{-i\delta t \frac{\hat{p}^2}{2m}} \underbrace{\frac{1}{2\pi}\int dp\, |p\rangle\langle p|}_{\mathbb{1}} q_j \rangle$$

$$= \frac{1}{2\pi}\int dp\, \underbrace{\langle q_{j+1}| e^{-i\delta t \frac{\hat{p}^2}{2m}} |p\rangle}_{e^{-i\delta t \frac{p^2}{2m}} \underbrace{\langle q_{j+1}|p\rangle}_{e^{ipq_{j+1}}}} \underbrace{\langle p|q_j\rangle}_{\langle q_j|p\rangle^* = e^{-ipq_j}}$$

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$$= \frac{1}{2\pi}\int dp\, \underbrace{\langle q_{j+1}| e^{-i\delta t \frac{\hat{p}^2}{2m}} |p\rangle}_{e^{-i\delta t \frac{p^2}{2m}} \underbrace{\langle q_{j+1}|p\rangle}_{e^{ipq_{j+1}}}} \underbrace{\langle p | q_j\rangle}_{\langle q_j|p\rangle^* = e^{-ipq_j}}$$

TY $e^{-i\delta t\,\hat{p}^2/2m}|p\rangle = e^{-i\delta t\,p^2/2m}|p\rangle$

(ALLMÄNT: $f(\hat{p})|p\rangle = f(p)|p\rangle$

BEVIS: TAYLORUTVECKLA!)

UNDERSÖK EN AV FAKTORERNA I INTEGRANDEN:

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$\Downarrow$

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$$= \frac{1}{2\pi}\int dp \, \underbrace{\langle q_{j+1} | e^{-i\delta t \frac{\hat{p}^2}{2m}} |p\rangle}_{e^{-i\delta t \frac{p^2}{2m}} \underbrace{\langle q_{j+1}|p\rangle}_{e^{ipq_{j+1}}}} \underbrace{\langle p | q_j \rangle}_{\langle q_j|p\rangle^* = e^{-ipq_j}}$$

$$= \frac{1}{2\pi}\int dp \, e^{-i\delta t \, p^2/2m} \, e^{ip(q_{j+1} - q_j)} \qquad (2)$$

$$\langle q_{j+1} | e^{-i\hat{H}dt} | q_j \rangle = \frac{1}{2\pi} \int dp \, e^{-i\delta t \, p^2/2m} \, e^{ip(q_{j+1} - q_j)} \qquad (2)$$

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"HJÄLPRESULTAT":

KOMPLEXVÄRD GAUSSISK INTEGRAL! ALLMÄNT:

$$\int dx \, e^{\frac{1}{2}iax^2 + iJx} = \sqrt{\frac{2\pi i}{a}} \, e^{-iJ^2/2a}$$

JFR

$$\int dx \, e^{-\frac{1}{2}ax^2 + Jx} = \sqrt{\frac{2\pi}{a}} \, e^{J^2/2a}$$

$$a \to -ia, \quad J \to iJ$$

$$\langle q_{j+1} | e^{-i\hat{H}\delta t} | q_j \rangle = \frac{1}{2\pi} \int dp \, e^{-i\delta t\, p^2/2m} \, e^{ip(q_{j+1}-q_j)} \qquad (2)$$

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JFR $\int dx \, e^{-\frac{1}{2}ax^2 + Jx} = \sqrt{\frac{2\pi}{a}} \, e^{J^2/2a}$

$a \to -ia \,, \quad J \to iJ$

$J = q_{j+1} - q_j$

$a = -\delta t/m \,, \quad x = p$

$$\langle q_{j+1} | e^{-i\delta t\, \hat{p}^2/2m} | q_j \rangle = \sqrt{\frac{-2\pi i m}{\delta t}} \, e^{i\delta t (m/2)\left[(q_{j+1}-q_j)/\delta t\right]^2} \qquad (3)$$

$$\mathcal{A}_{S\to O} = \langle q_F | e^{-i\hat{H}T} | q_I \rangle$$

$$= \overbrace{\int_{q_1} \cdots \int_{q_{N-1}} dq_1 \ldots dq_{N-1}}^{\prod_{j=1} \int dq_j} \underbrace{\langle q_F}_{|q_N\rangle} | e^{-i\hat{H}\delta t} | q_{N-1} \rangle \langle q_{N-1} | e^{-i\hat{H}\delta t} | q_{N-2} \rangle \ldots$$

$$\ldots \langle q_1 | e^{-i\hat{H}\delta t} \underbrace{| q_I}_{|q_0\rangle} \rangle \qquad (1)$$

$$\langle q_{j+1} | e^{-i\hat{H}\delta t} | q_j \rangle = \frac{1}{2\pi} \int dp \, e^{-i\delta t \, p^2/2m} \, e^{ip(q_{j+1}-q_j)} \qquad (2)$$

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$$\mathcal{A}_{S\to O} = \langle q_F | e^{-i\hat{H}T} | q_I \rangle$$

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$$\cdots \langle q_1 | e^{-i\hat{H}\delta t} \underbrace{| q_I \rangle}_{|q_0\rangle} \qquad (1)$$

$$\langle q_{j+1} | e^{-i\delta t\,\hat{p}^2/2m} | q_j \rangle = \sqrt{\frac{-2\pi i m}{\delta t}}\; e^{i\delta t (m/2)\left[(q_{j+1}-q_j)/\delta t\right]^2} \qquad (3)$$

$$\mathcal{A}_{s\to 0} = \langle q_F | e^{-i\hat{H}T} | q_I \rangle$$

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$$\dots \langle q_1 | e^{-i\hat{H}\delta t} \underbrace{|q_I\rangle}_{|q_0\rangle} \qquad (1)$$

$$\langle q_{j+1} | e^{-i\delta t \hat{p}^2/2m} | q_j \rangle = \sqrt{\frac{-2\pi i m}{\delta t}}\, e^{i\delta t (m/2)\left[(q_{j+1}-q_j)/\delta t\right]^2} \qquad (3)$$

sätt in (3) i (1):

$$\mathcal{A}_{s\to 0} = \langle q_F | e^{-i\hat{H}T} | q_I \rangle$$

$$= \left(-\frac{2\pi i m}{\delta t}\right)^{(N-1)/2} \prod_{j'=1}^{N-1} \int dq_{j'} \prod_{j=0}^{N-1} e^{i\delta t m/2 \left[(q_{j+1}-q_j)/\delta t\right]^2}$$

$$= \left(-\frac{2\pi i m}{\delta t}\right)^{(N-1)/2} \prod_{j'=1}^{N-1} \int dq_{j'}\, e^{i\delta t m/2 \sum_{j=0}^{N-1} \left[(q_{j+1}-q_j)/\delta t\right]^2} \qquad (4)$$

$$(q_0 = q_I,\; q_N = q_F)$$

$$\mathcal{A}_{S\to 0} = \langle q_F | e^{-i\tilde{H}T} | q_I \rangle$$

$$= \left(-\frac{2\tilde{\pi} i m}{\delta t}\right)^{(N-1)/2} \prod_{j'=1}^{N-1} \int dq_{j'} \prod_{j=0}^{N-1} e^{i\delta t m/2 \left[(q_{j+1}-q_j)/\delta t\right]^2}$$

$$= \left(-\frac{2\tilde{\pi} i m}{\delta t}\right)^{(N-1)/2} \prod_{j'=1}^{N-1} \int dq_{j'} \, e^{i\delta t m/2 \sum_{j=0}^{N-1} \left[(q_{j+1}-q_j)/\delta t\right]^2} \quad (4)$$

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$$= \left(-\frac{2\tilde{\pi} i m}{\delta t}\right)^{(N-1)/2} \prod_{j'=1}^{N-1} \int dq_{j'}\, e^{i\delta t m/2 \sum_{j=0}^{N-1} [(q_{j+1}-q_j)/\delta t]^2} \qquad (4)$$

$$(q_0 = q_I,\ q_N = q_F)$$

Vi tar nu gränsvärdena $\underline{\delta t \to 0,\ N \to \infty}$

$$[(\overbrace{q_{j+1} - q_j}^{\delta q})/\delta t]^2 \longrightarrow \dot{q}^2$$

$$\delta t \sum_{j=0}^{N-1} \ldots \longrightarrow \int_0^T dt \ldots$$

"VÄGINTEGRAL"

$$\left(-\frac{2\tilde{\pi} i m}{\delta t}\right)^{(N-1)/2} \prod_{j'=1}^{N-1} \int dq_{j'} \ldots \longrightarrow \text{“}\int \mathcal{D}[q(t)] \ldots\text{”}$$

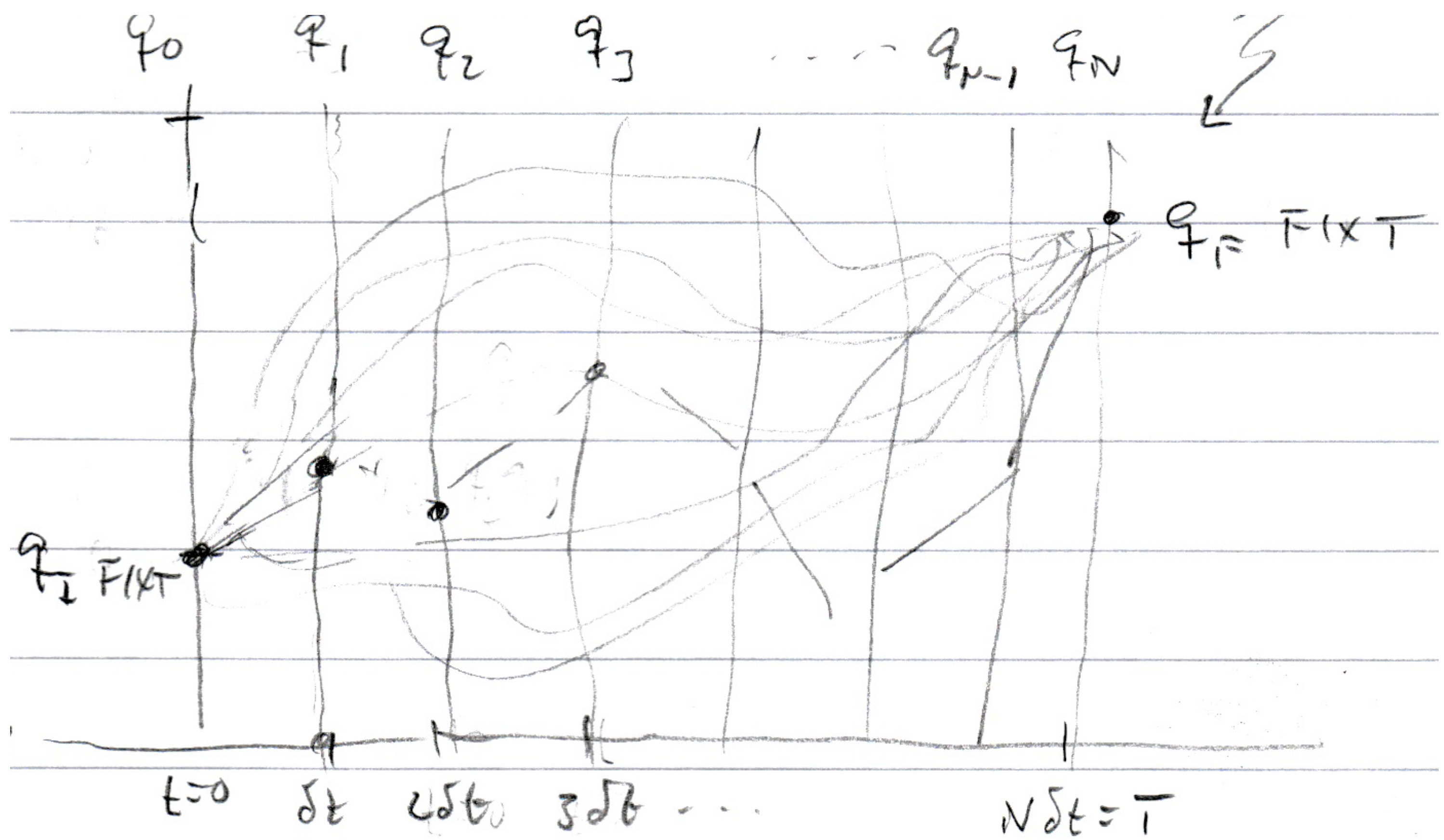
"VÄGINTEGRAL"

$$\left(-\frac{2\tilde{\pi} i m}{\delta t}\right)^{(N-1)/2} \prod_{j'=1}^{N-1} \int dq_{j'} \ldots \longrightarrow \text{"} \int \mathcal{D}[q(t)] \ldots \text{"}$$

"VÄGINTEGRAL"

$$\left(-\frac{2\pi i m}{\delta t}\right)^{(N-1)/2}\prod_{j'=1}^{N-1}\int dq_{j'}\ldots \longrightarrow \text{"}\int \mathcal{D}[q(t)]\ldots\text{"}$$

∞

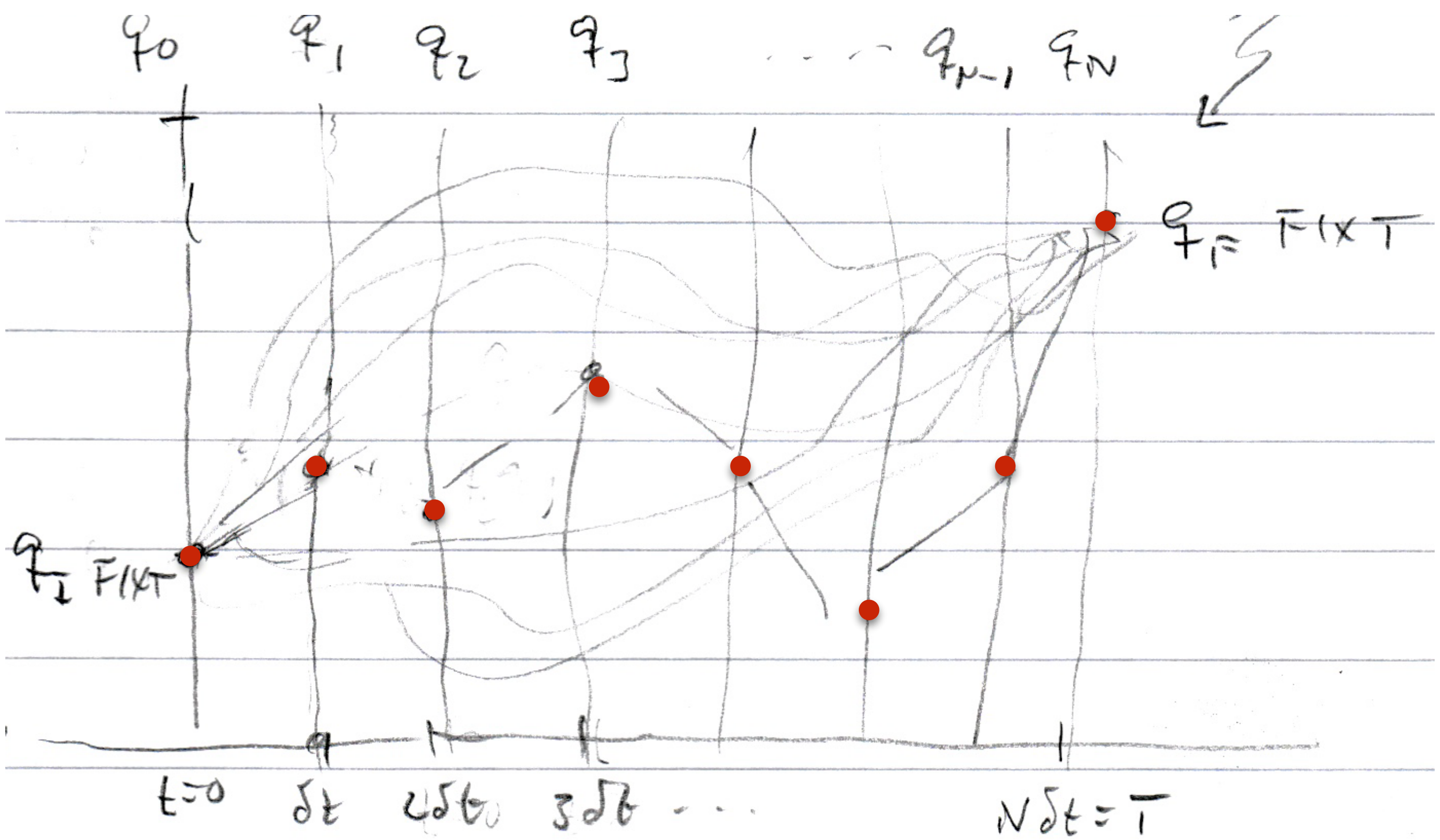


$-\infty$

"VÄGINTEGRAL"

$$\left(-\frac{2\pi i m}{\delta t}\right)^{(N-1)/2} \prod_{j'=1}^{N-1} \int dq_{j'} \ldots \longrightarrow \text{"} \int \mathcal{D}[q(t)] \ldots \text{"}$$

∞



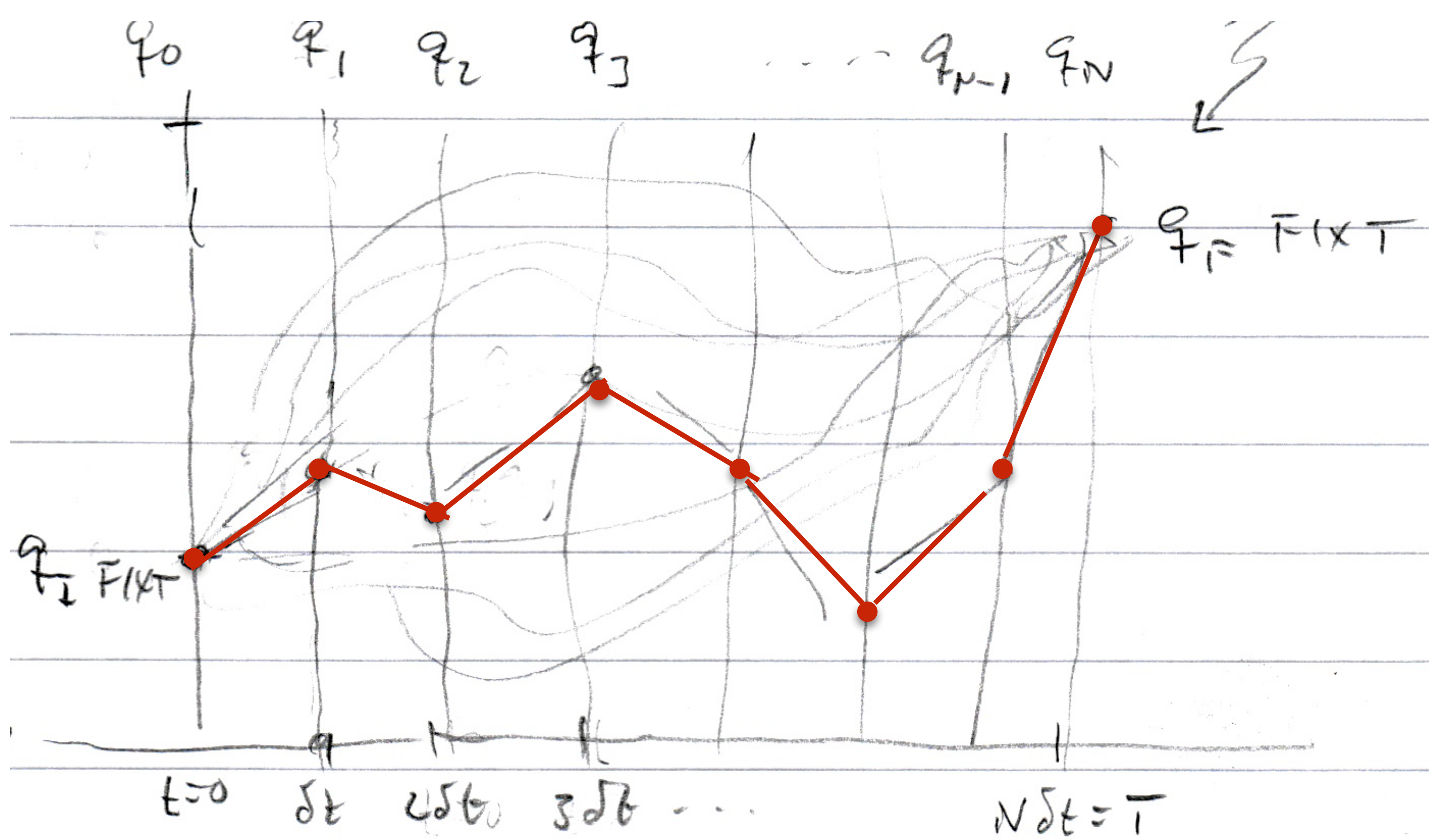
$-\infty$

föreläsning 2/12 (III)

"VÄGINTEGRAL"

$$\left(-\frac{2\pi i m}{\delta t}\right)^{(N-1)/2} \prod_{j'=1}^{N-1} \int dq_{j'} \ldots \longrightarrow \text{"} \int \mathcal{D}[q(t)] \ldots \text{"}$$

∞

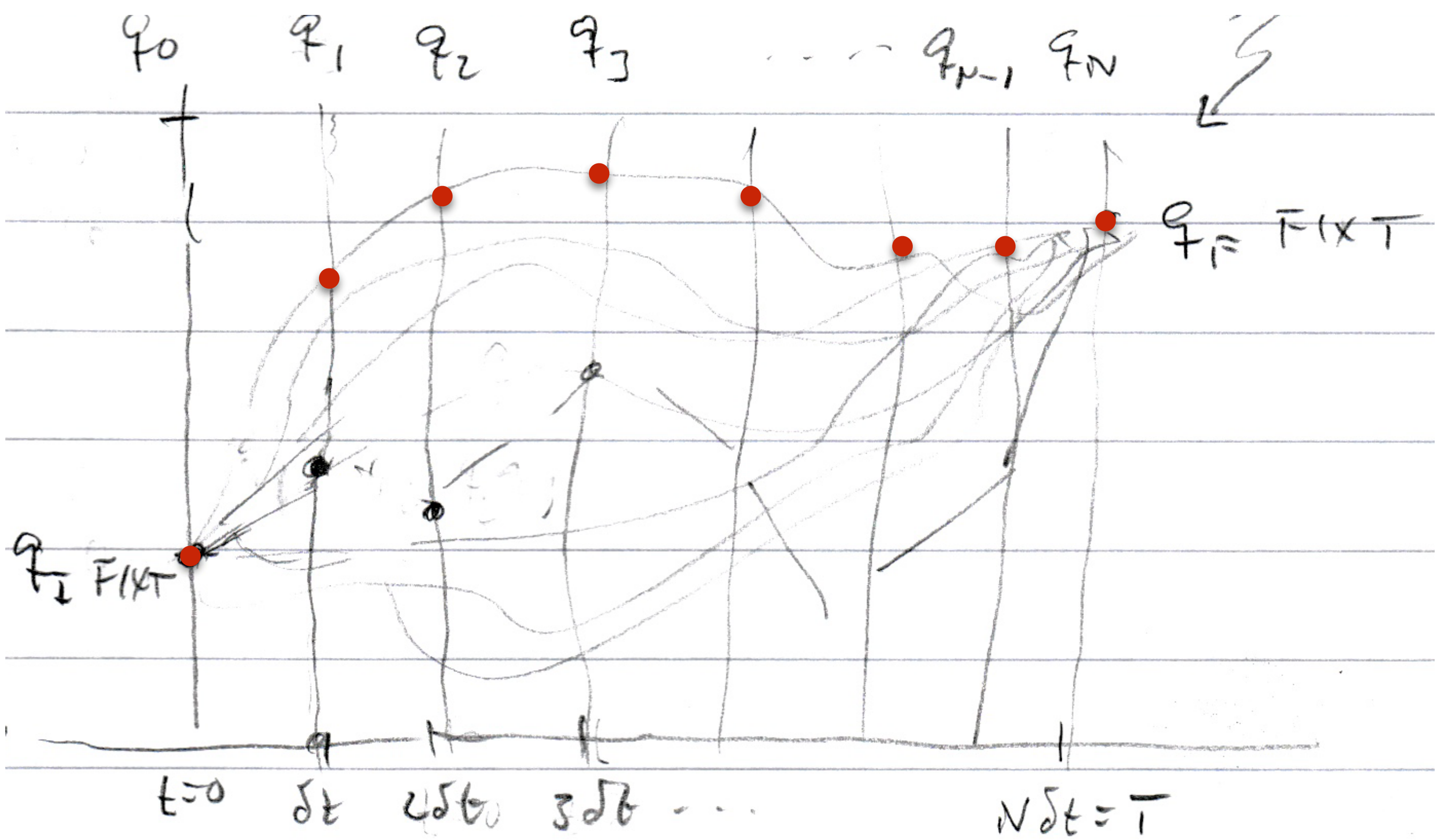


$-\infty$

"VÄGINTEGRAL"

$$\left(-\frac{2\pi i m}{\delta t}\right)^{(N-1)/2} \prod_{j'=1}^{N-1} \int dq_{j'} \ldots \longrightarrow \text{"} \int \mathcal{D}[q(t)] \ldots \text{"}$$

∞



$-\infty$

Efter att vi tagit alla gränsvärden…

VI KAN DÅ SKRIVA $\hat{H} = \hat{p}^2/2m$

$$\mathcal{A}_{s \to 0} = \langle q_F | e^{-iHT} | q_i \rangle = \int \mathcal{D}[q(t)]\, e^{i\int_0^T dt\, \frac{1}{2} m \dot{q}^2}$$

$\hat{H} = \hat{p}^2/2m$

$$\mathcal{A}_{s\to 0} = \langle q_F | e^{-iHT} | q_I \rangle = \int \mathcal{D}[q(t)]\, e^{i\int_0^T dt\, \frac{1}{2} m \dot{q}^2}$$

VI HAR GJORT HÄRLEDNINGEN FÖR EN FRI PARTIKEL…

GENERALISERA TILL FALLET MED NOLLSKILD POTENTIAL

$\hat{H} = \hat{p}^2/2m + V(q)$

$$\mathcal{A} = \langle q_F | e^{-i\hat{H}T} | q_I \rangle = \int \mathcal{D}[q(t)]\, e^{i\int_0^T dt\, \underbrace{\left(\frac{1}{2} m \dot{q}^2 - V(q)\right)}_{L(q,\dot{q})}}$$

$$\hat{H} = \hat{p}^2/2m$$

$$\mathcal{A}_{s\to 0} = \langle q_F | e^{-iHT} | q_I \rangle = \int \mathcal{D}[q(t)]\, e^{i\int_0^T dt\, \frac{1}{2} m \dot{q}^2}$$

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$$L(q, \dot{q})$$

Hamilton-operator

klassisk Lagrange-funktion

$$\hat{H} = \hat{p}^2/2m$$

$$\mathcal{A}_{s\to 0} = \langle q_F | e^{-iHT} | q_I \rangle = \int \mathcal{D}[q(t)] e^{i\int_0^T dt \, \frac{1}{2} m \dot{q}^2}$$

VI HAR GJORT HÄRLEDNINGEN FÖR EN FRI PARTIKEL…

GENERALISERA TILL FALLET MED NOLLSKILD POTENTIAL

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Hamilton-operator

klassisk Lagrange-funktion

SÄTT TILLBAKS $\hbar$! (SI ENHETER)

SÄTT IN INTEGRATIONSGRÄNSER q_F

$$\langle q_F | e^{-i\hat{H}T/\hbar} | q_I \rangle = \int_{q_I} \mathcal{D}[q(t)] e^{i \overbrace{\int_0^T dt \, L(q,\dot{q})}^{S'} /\hbar}$$

klassisk verkan S

$$\underbrace{= \int_{q_I}^{q_F} \mathcal{D}[q(t)] e^{iS'/\hbar}}_{\text{FEYNMANS VÄGINTEGRAL}}$$