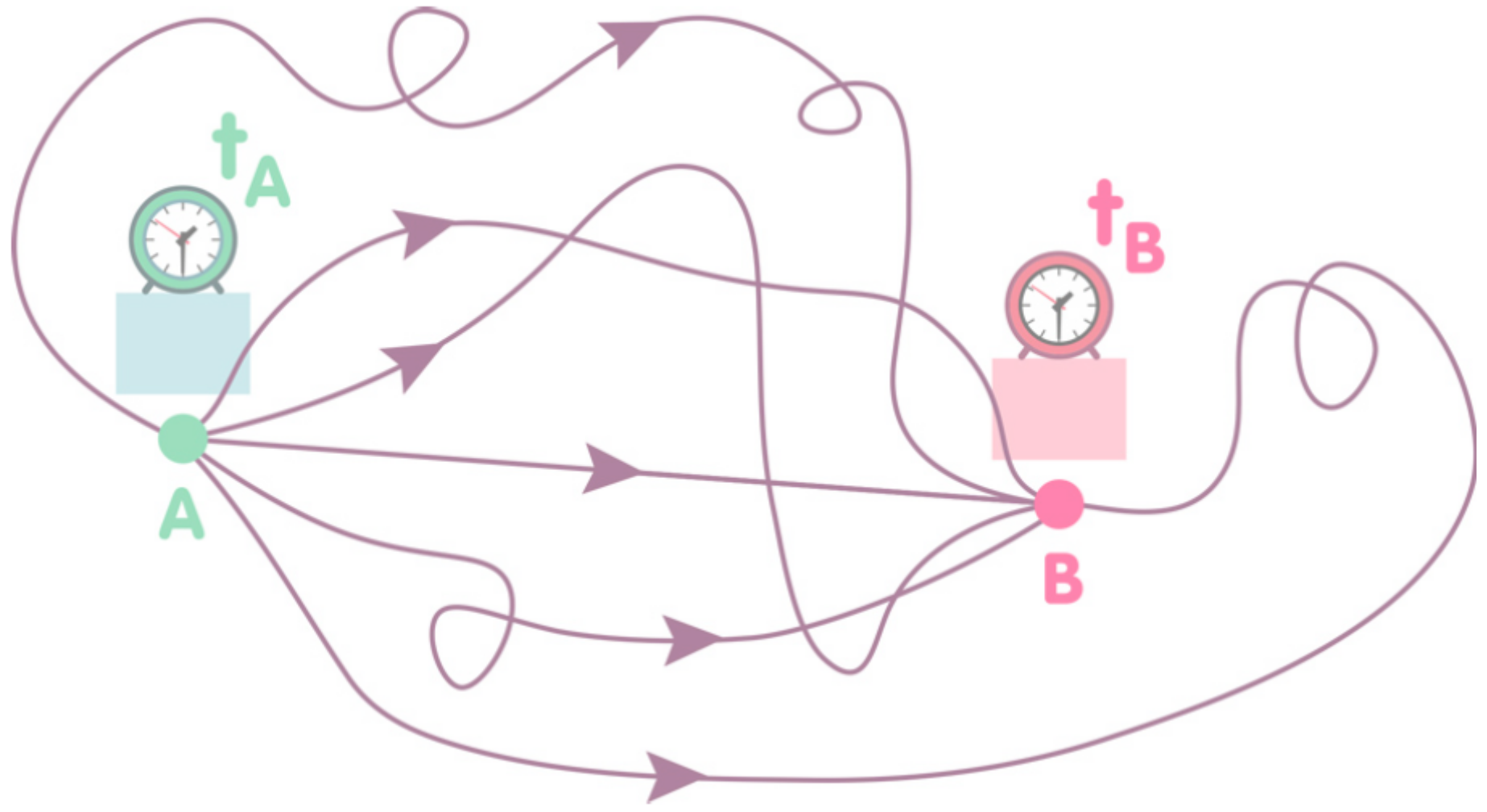
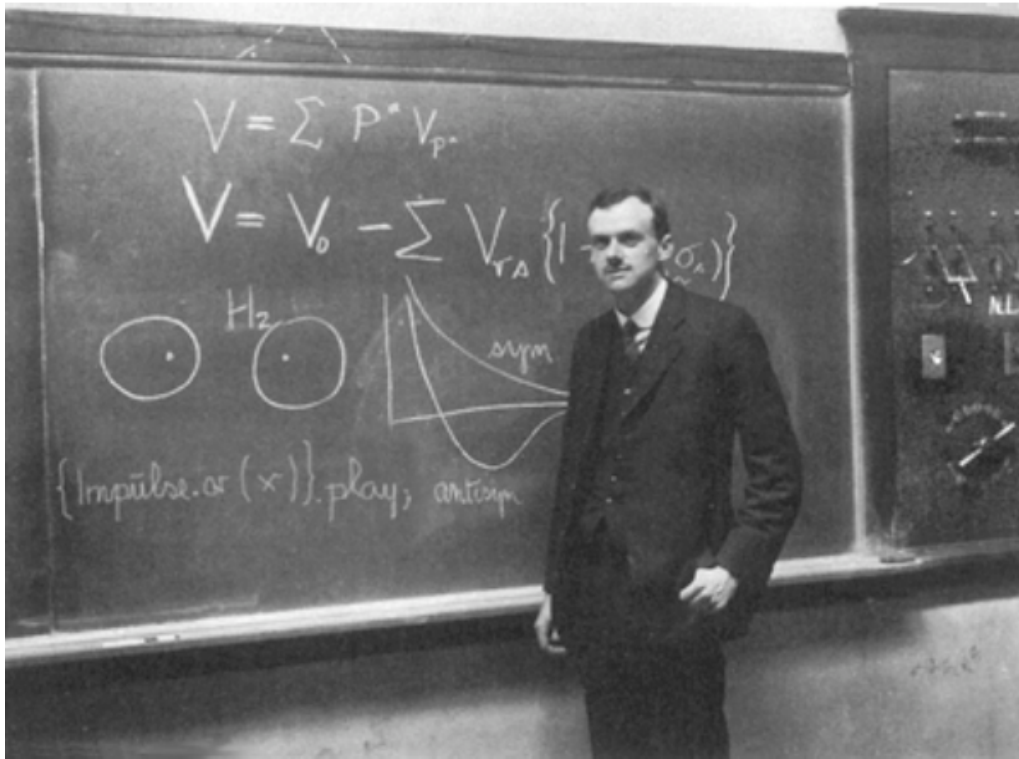


Feynmans vägintegralformulering av kvantmekaniken (intro)



Låt oss först påminna oss om "kvantmekanikens grunder"...

4 postulat för kvantmekanik (icke-relativistiskt)



Paul Dirac

John von Neumann



I. Ett tillstånd hos ett fysikaliskt system beskrivs av en normerad vektor $|\psi\rangle$ i ett Hilbertrum.

II. Till varje fysikalisk observabel finns en Hermitesk operator på ett Hilbertrum.

tillägg:
kanonisk kvantisering

III. Givet ett tillstånd $|\psi\rangle$ så ges de möjliga resultaten vid en mätning av en observabel svarande mot operatören Ω av egenvärdena $\{\omega\}$ till Ω . Sannolikheten att mäta ω ges av $|\langle\omega|\psi\rangle|^2$ där $|\omega\rangle$ är motsvarande egenvektor till Ω . Vid mätningen “kollapsar” $|\psi\rangle$ till $|\omega\rangle$.

IV. Tidsutvecklingen av tillståndet $|\psi(t)\rangle$ bestäms av systemets Hamiltonoperator:

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle \quad (\text{Schrödingerekvationen})$$



$$|\psi(t)\rangle = e^{-iHt/\hbar} |\psi(0)\rangle$$

unitär tidsutveckling

Antalet postulat, och dess exakta formuleringar beror på hur matematiskt rigoröst man vill behandla teorin. Jfr. t.ex. Byron & Fuller (2015 års lärobok i kursen) § 5.11, som har sex postulat (varav ett är innehållet i ett av de andra).

men det är något UNDERLIGT med kvantmekaniken...

The top physicists of all time



1 Albert Einstein 1879-1955
German/Swiss/American
119 votes



2 Isaac Newton
1642-1727 British
96 votes



3 James Clerk Maxwell
1831-1879 British
67 votes



4 Niels Bohr
1885-1962 Danish
47 votes



5 Werner Heisenberg
1901-1976 German
30 votes



6 Galileo Galilei
1564-1642 Italian
27 votes



7 Richard Feynman
1918-1988 American
23 votes



8= Paul Dirac
1902-1984 British
22 votes



8= Erwin Schrödinger
1887-1961 Austrian
22 votes



10 Ernest Rutherford
1871-1937 New Zealander
20 votes



**"The more successful the quantum theory is, the sillier it looks."
Einstein**

**"Anyone who thinks that he can understand quantum mechanics
without getting into a state of confusion has not understood anything
of the theory."
Bohr**

**"No one has ever understood quantum mechanics."
Feynman**

**"I regret that I had anything to do with it..."
Schrödinger**

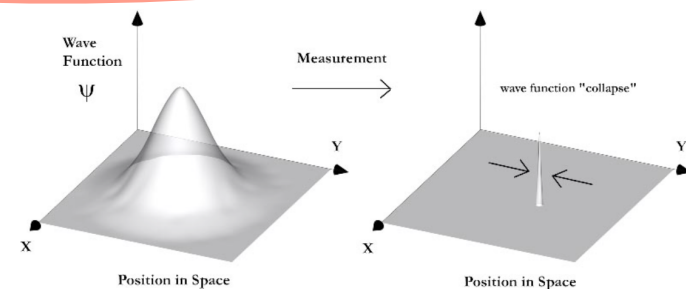
**"What it all means? It is simple. You take what you need,
you do what you have to do, and that's it..."
Heisenberg**

I. Ett tillstånd hos ett fysikaliskt system beskrivs av en normerad vektor $|\psi\rangle$ i ett Hilbertrum.

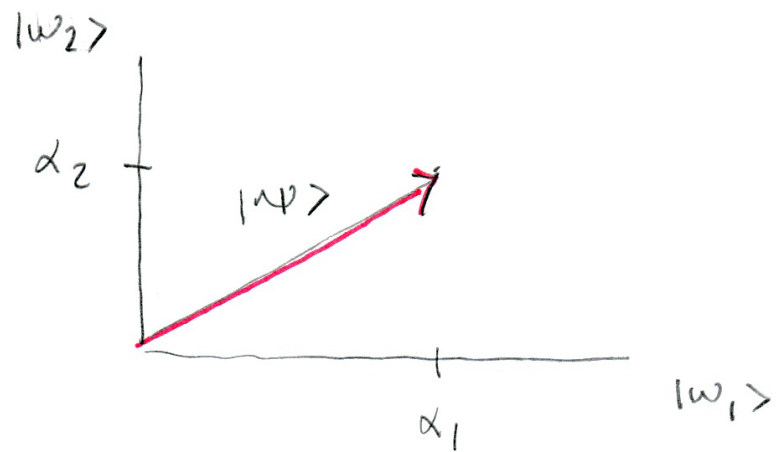
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tillägg:
kanonisk kvantisering

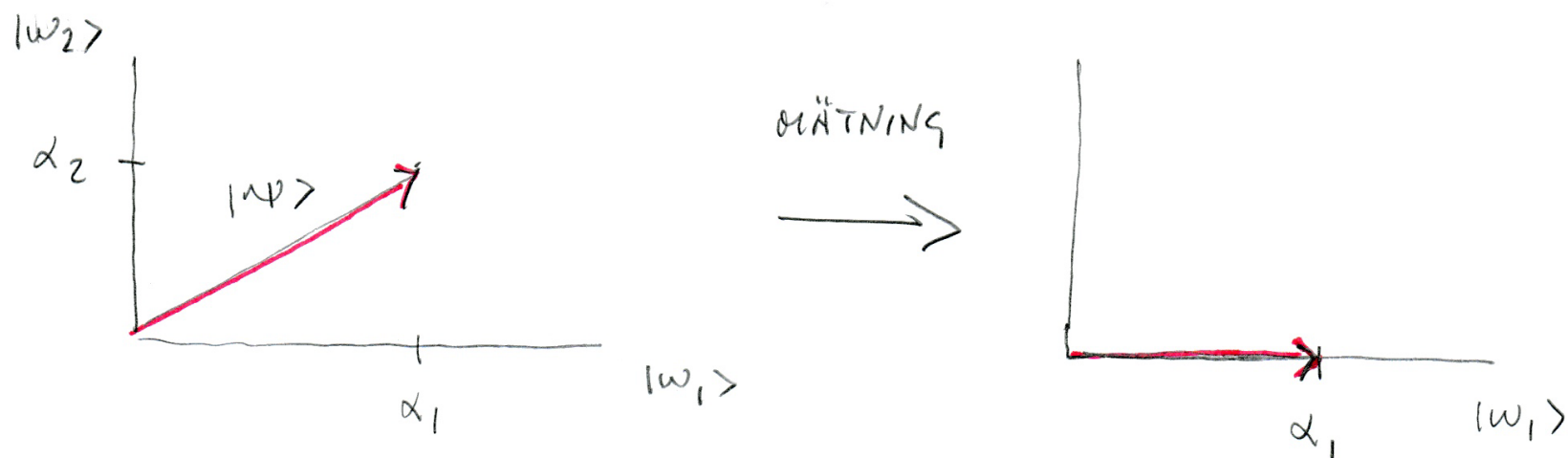
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exempel: qubit $|\psi\rangle = \alpha_1|\omega_1\rangle + \alpha_2|\omega_2\rangle$



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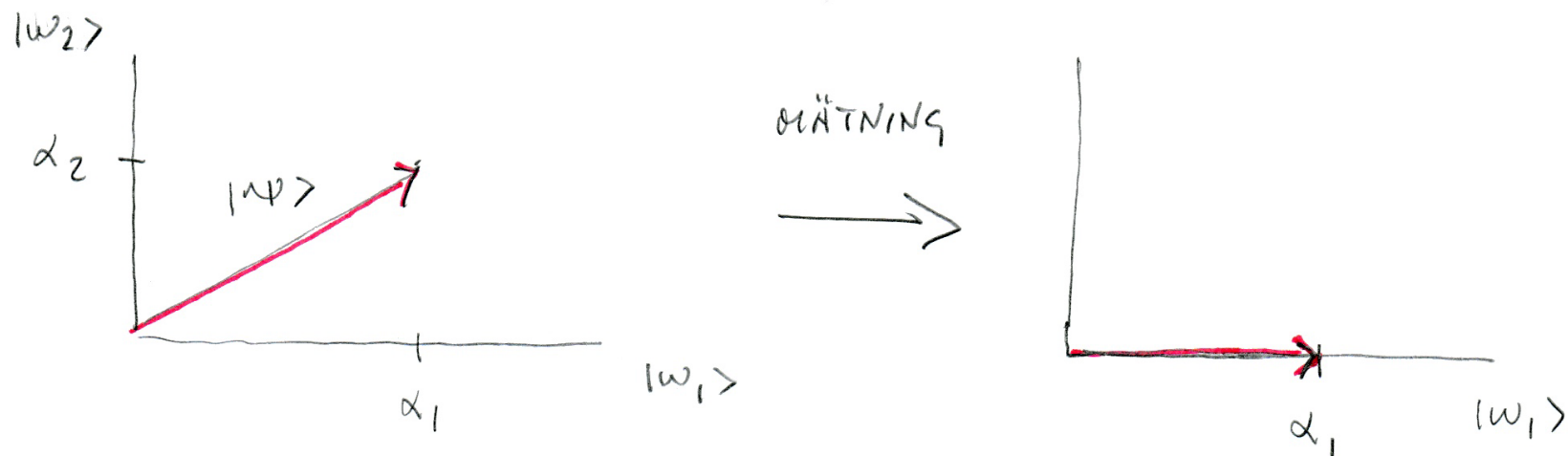


$|\psi\rangle$ "KOLLAPSAR" TILL $|\omega_1\rangle$. SANNOLIKHETEN FÖR DENNA KOLLAPS, DVS ATT VI FÅR MÄTVÄRDENET ω_1

$$\text{ÄR} = |\alpha_1|^2 = |\langle \omega_1 | \psi \rangle|^2$$

ICKÉ-UNITÄR PROCESS TY $|\alpha_1| < \sqrt{\alpha_1^2 + \alpha_2^2}$

exempel: qubit $|\psi\rangle = \alpha_1|\omega_1\rangle + \alpha_2|\omega_2\rangle$



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ICKĒ-UNITÄR PROCESS TY $|\alpha_1| < \sqrt{\alpha_1^2 + \alpha_2^2}$

STRIDER MOT 4:e POSTULATET!

THE BORDER TERRITORY

”Köpenhamnstolkningen”
(Bohr, Heisenberg, 1925-27)

QUANTUM DOMAIN

CLASSICAL DOMAIN



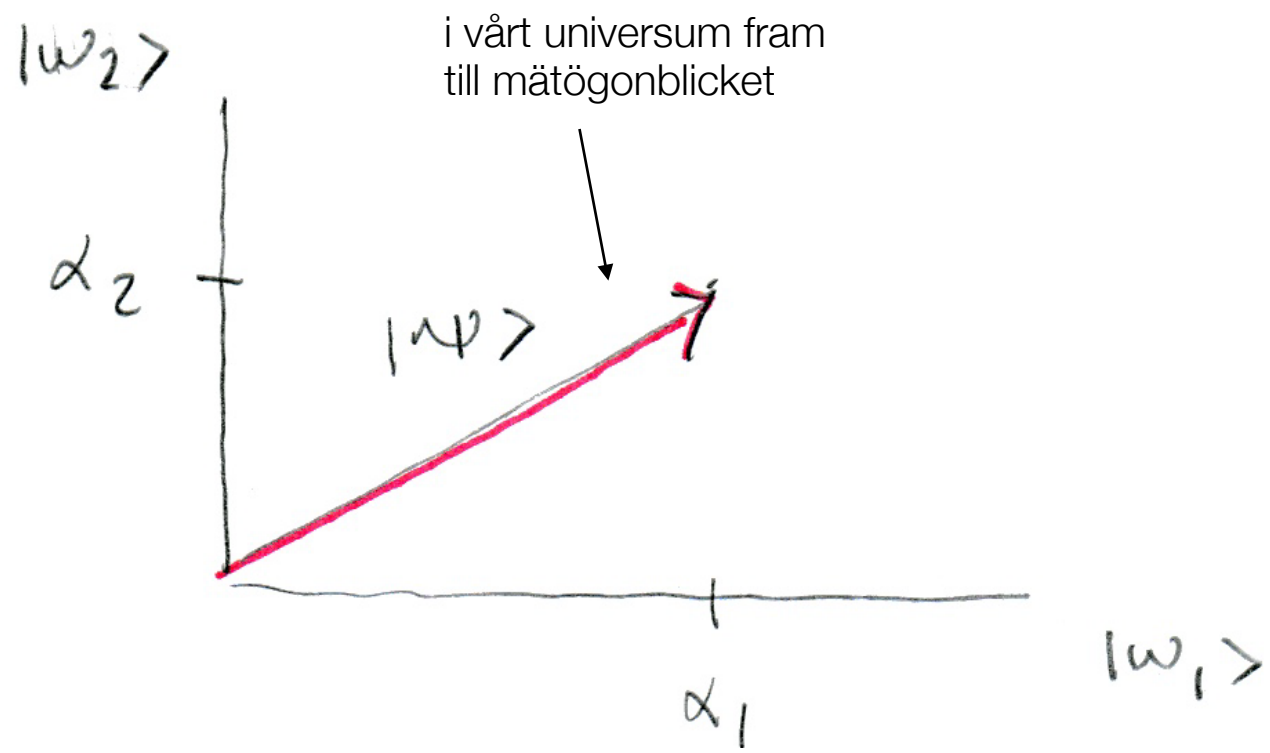
”Many-worlds interpretation”
(Everett, 1957)

”Decoherence theory”
(Zurek 1981, Joos & Zeh, 1985)

”Consistent histories theory”
(Omnès, 1994)

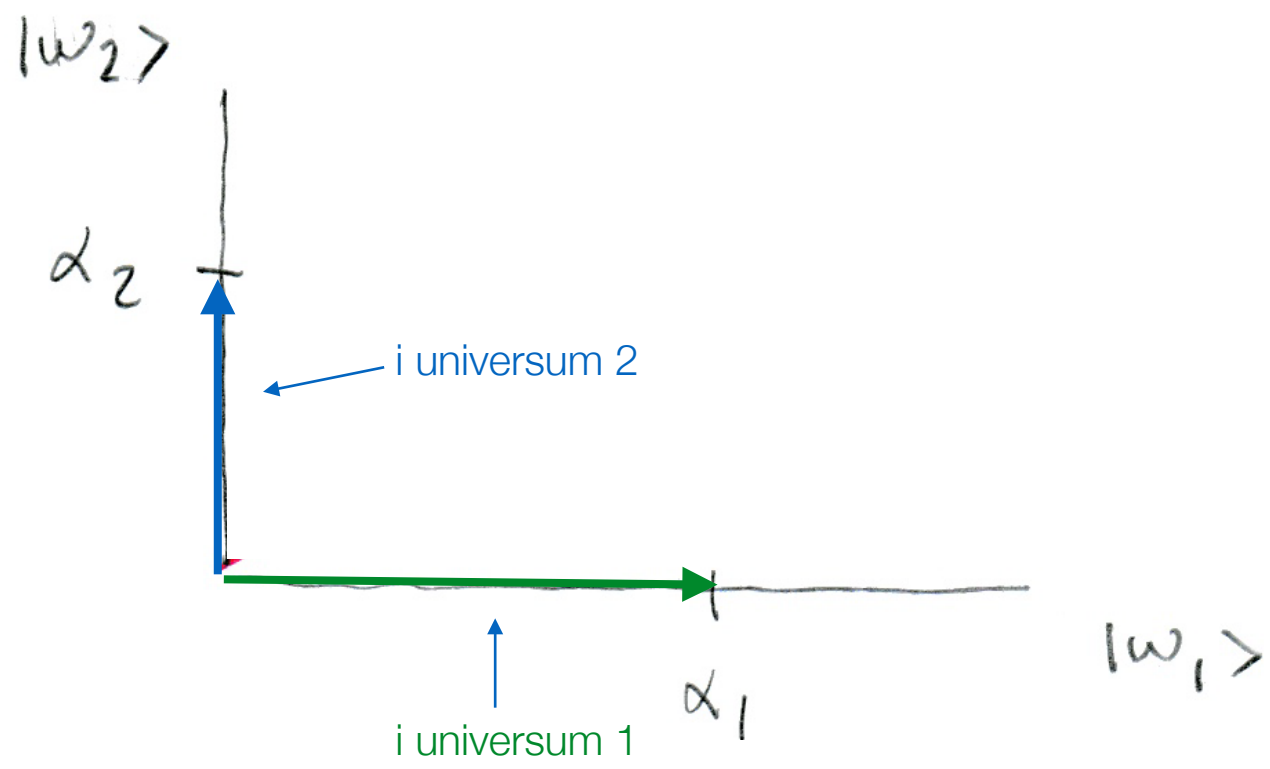
”Qbism”
(Fuchs, 2010)





”Many-worlds interpretation”
(Everett, 1957)





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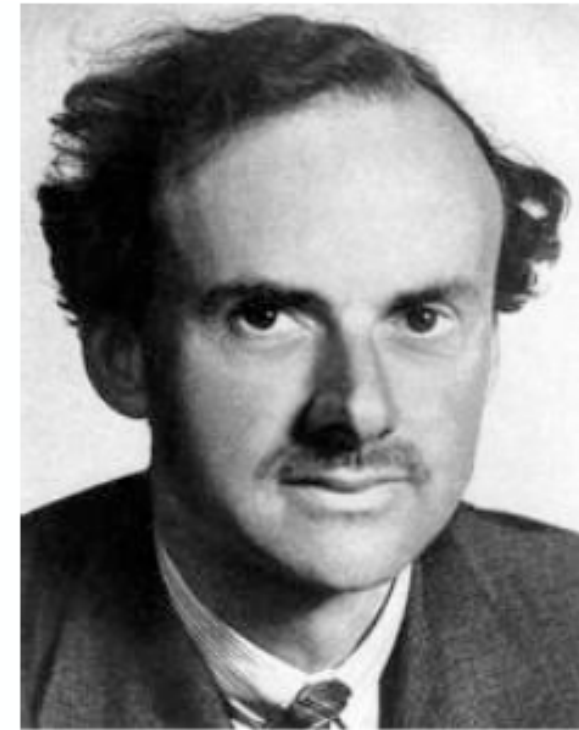
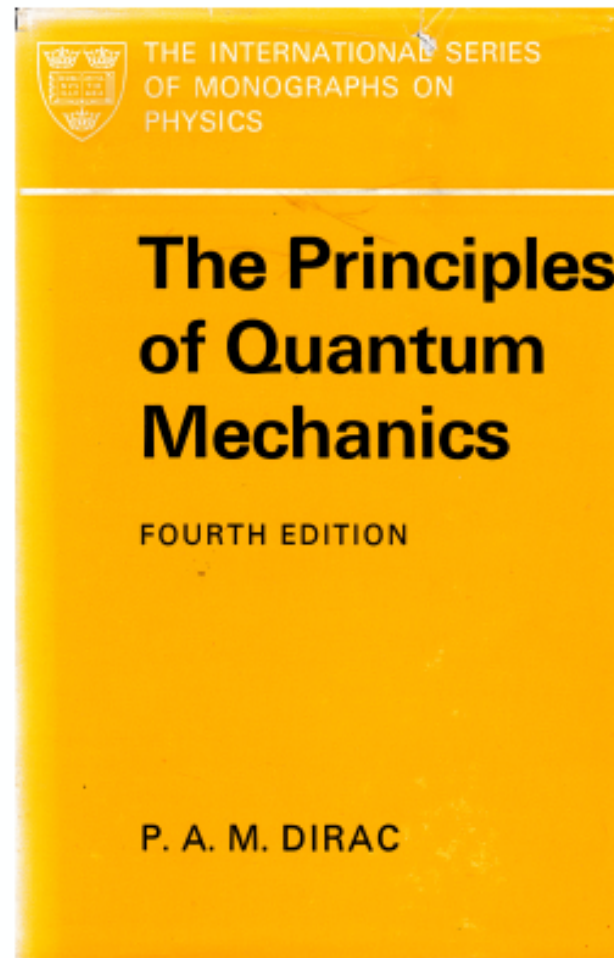




Jean Dieudonné
1906 - 1992

"When one gets to the mathematical theories which are at the basis of quantum mechanics, one realizes that the attitude of many physicists in the handling of these theories borders on the delirium... One has to wonder what remains in the mind of a student who has absorbed this unbelievable accumulation of nonsense...!"

I sin bok *The Principles of Quantum Mechanics* (1930) ställer Paul Dirac frågan (i en fotnot) om det går att formulera kvantmekaniken med hjälp av en Lagrangefunktion (*analytisk mekanik*) istället för att använda en Hamiltonfunktion (*Hamiltons mekanik*)...



$$(i\gamma \cdot \partial - m)\psi = 0$$

1933 utvecklar han detta i en artikel publicerad i *Physikalische Zeitschrift der Sovietunion*:

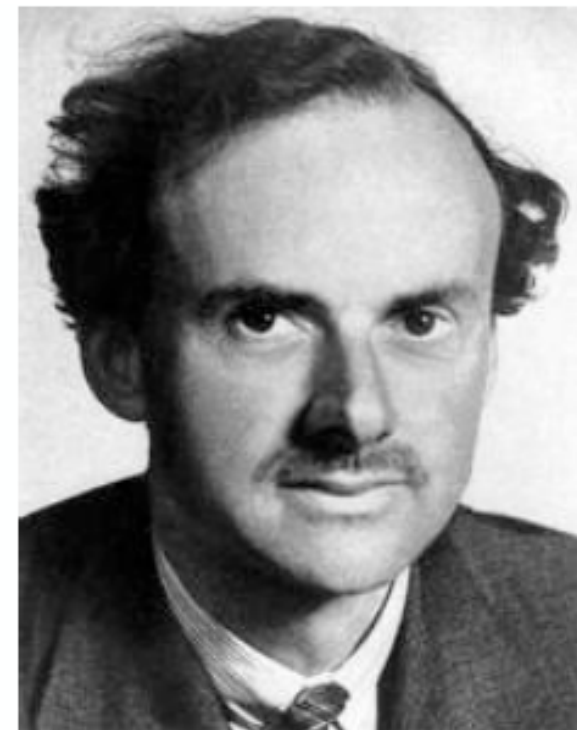
In 1933 Dirac published a paper in *Physikalische Zeitschrift der Sowjetunion* on "The Lagrangian in Quantum Mechanics." He begins by saying:

"Quantum mechanics was built up on a foundation of analogy with the Hamiltonian theory of classical mechanics. This is because the classical notion of canonical coordinates and momenta was found to be one with a very simple quantum analogue. . . .

Now there is an alternative formulation for classical dynamics, provided by the Lagrangian. This requires one to work in terms of coordinates and velocities instead of coordinates and momenta. The two formulations are, of course, closely related, but there are reasons for believing that the Lagrangian one is the more fundamental.

In the first place the Lagrangian method allows one to collect together all the equations of motion and express them as the stationary property of a certain action function. (This action function is just the time integral of the Lagrangian.) There is no corresponding action principle in terms of the coordinates and momenta of the Hamiltonian theory. [This is not true, but it doesn't matter.] Secondly the Lagrangian method can easily be expressed relativistically, on account of the action function being a relativistic invariant; while the Hamiltonian method is essentially nonrelativistic in form, since it marks out a particular time variable. . . .

For these reasons it would seem desirable to take up the question of what corresponds in the quantum theory to the Lagrangian method of the classical theory. "



$$(i\gamma \cdot \partial - m)\psi = 0$$

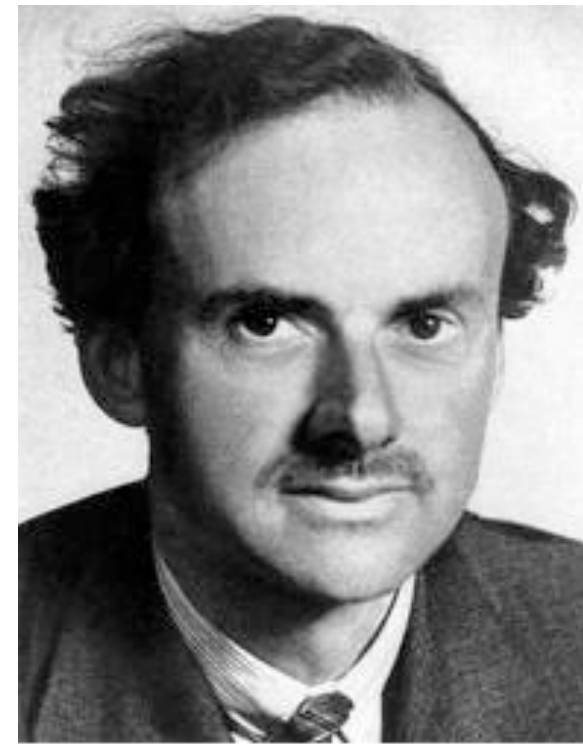
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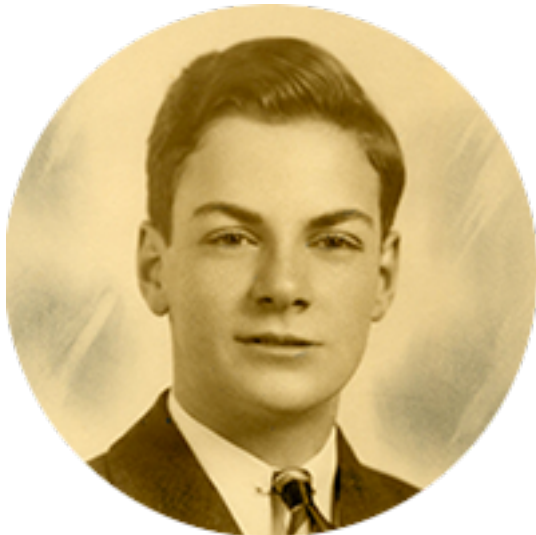
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$$(i\gamma \cdot \partial - m)\psi = 0$$



Feynman (doktorand vid Princeton University i början av 1940-talet) inspirerades av Dirac...

Ett annat sätt att göra kvantmekanik: Feynmans vägintegralformulering

... löser inte "tolkningsproblemet" av kvantmekaniken, men

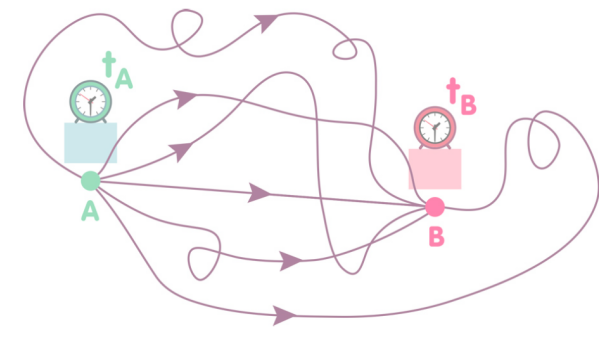
- förenklar ofta analys och räkningar
- enklare att göra konsistent med speciell relativitetsteori → kvantfältteori
- gör symmetrier och topologiska effekter transparenta
-
-
- ... och förklarar *minsta verkans princip* i klassisk fysik!



Feynman vid Nobelbanketten 1965



Feynmans vägintegralformulering av kvantmekaniken



BAKGRUNDKLASSISK MEKANIK

HAMILTON FORMULERING

$$\left\{ \begin{array}{l} \dot{x} = \frac{\partial H}{\partial p} \\ \dot{p} = -\frac{\partial H}{\partial x} \end{array} \right. \quad H = \frac{p^2}{2m} + V(x), \quad \vec{F} = -\frac{dV}{dx}$$

$$\Rightarrow \textcircled{m\ddot{x} = \vec{F}}$$

"LOKAL
FORMALISM"

$$x(t=0)$$

$$\dot{x}(t=0)$$

BAKGRUND

KLASSISK MEKANIK

HAMILTON FORMULERING

$$\begin{cases} \dot{x} = \frac{\partial H}{\partial p} \\ \dot{p} = -\frac{\partial H}{\partial x} \end{cases} \quad H = \frac{p^2}{2m} + V(x), \quad \vec{F} = -\frac{dV}{dx} \\ \Rightarrow \text{ (} m\ddot{x} = \vec{F} \text{)}$$

"LOKAL
FORMALISM"

$x(t=0)$

$\dot{x}(t=0)$

ANALYTISK MEKANIK

$$S = \int_{t=0}^{t=T} dt L(x, \dot{x}, t)$$

$$L(x, \dot{x}, t) = K - V$$

$$\delta S = 0$$

"GLOBAL
FORMALISM"

$x(t=0)$

$x(t=T)$

BAKGRUNDKLASSISK MEKANIK

HAMILTON FORMULERING

$$\begin{cases} \dot{x} = \frac{\partial H}{\partial p} \\ \dot{p} = -\frac{\partial H}{\partial x} \end{cases} \quad H = \frac{p^2}{2m} + V(x), \quad \vec{F} = -\frac{dV}{dx} \\ \Rightarrow (m\ddot{x} = \vec{F})$$

"LOKAL
FORMALISM"

$$\begin{aligned} x(t=0) \\ \dot{x}(t=0) \end{aligned}$$

ANALYTISK MEKANIK

$$S = \int_{t=0}^{t=T} dt L(x, \dot{x}, t)$$

$$L(x, \dot{x}, t) = K - V$$

$$\delta S = 0$$

"GLOBAL
FORMALISM"

$$\begin{aligned} & x(t=0) \quad \curvearrowright \quad x(t=T) \\ & \curvearrowleft \end{aligned}$$

$$\{f, g\} = \frac{\partial f}{\partial x} \frac{\partial g}{\partial p} - \frac{\partial f}{\partial p} \frac{\partial g}{\partial x}$$

$$\{x, p\} = 1$$

⇓ KANONISK
KVANTISERING

$$[\hat{x}, \hat{p}] = i\hbar$$

"KANONISK OPERATORFORMALISM"

Heisenberg, Born, Jordan, Schrödinger 1925-26

Dirac 1928

von Neumann 1933

BAKGRUND

KLASSISK MEKANIK

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"LOKAL
FORMALISM"

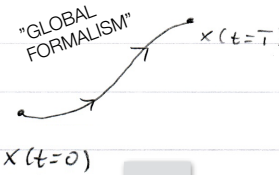
$$\begin{matrix} x(t=0) \\ \dot{x}(t=0) \end{matrix}$$

ANALYTISK MEKANIK

$$S = \int_{t=0}^{t=T} dt L(x, \dot{x}, t)$$

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$$\{f, g\} = \frac{\partial f}{\partial x} \frac{\partial g}{\partial p} - \frac{\partial f}{\partial p} \frac{\partial g}{\partial x}$$

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Dirac 1933



BAKGRUND

KLASSISK MEKANIK

HAMILTON FORMULERING

$$\begin{cases} \dot{x} = \frac{\partial H}{\partial p} \\ \dot{p} = -\frac{\partial H}{\partial x} \end{cases} \Rightarrow \begin{cases} H = \frac{p^2}{2m} + V(x), \quad \vec{F} = -\frac{dV}{dx} \\ m\ddot{x} = \vec{F} \end{cases}$$

"LOKAL
FORMALISM"

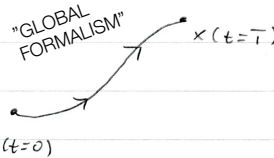
$$\begin{cases} x(t=0) \\ \dot{x}(t=0) \end{cases}$$

ANALYTISK MEKANIK

$$S = \int_{t=0}^{t=T} dt L(x, \dot{x}, t)$$

$$L(x, \dot{x}, t) = K - V$$

$$\delta S = 0$$



$$\{f, g\} = \frac{\partial f}{\partial x} \frac{\partial g}{\partial p} - \frac{\partial f}{\partial p} \frac{\partial g}{\partial x}$$

$$\{x, p\} = 1$$

⇓ KANONISK
KVANTISERING

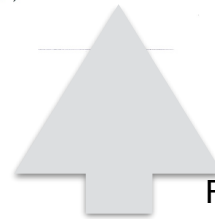
$$[\hat{x}, \hat{p}] = i\hbar$$

"KANONISK OPERATORFORMALISM"

Heisenberg, Born, Jordan, Schrödinger 1925-26

Dirac 1928

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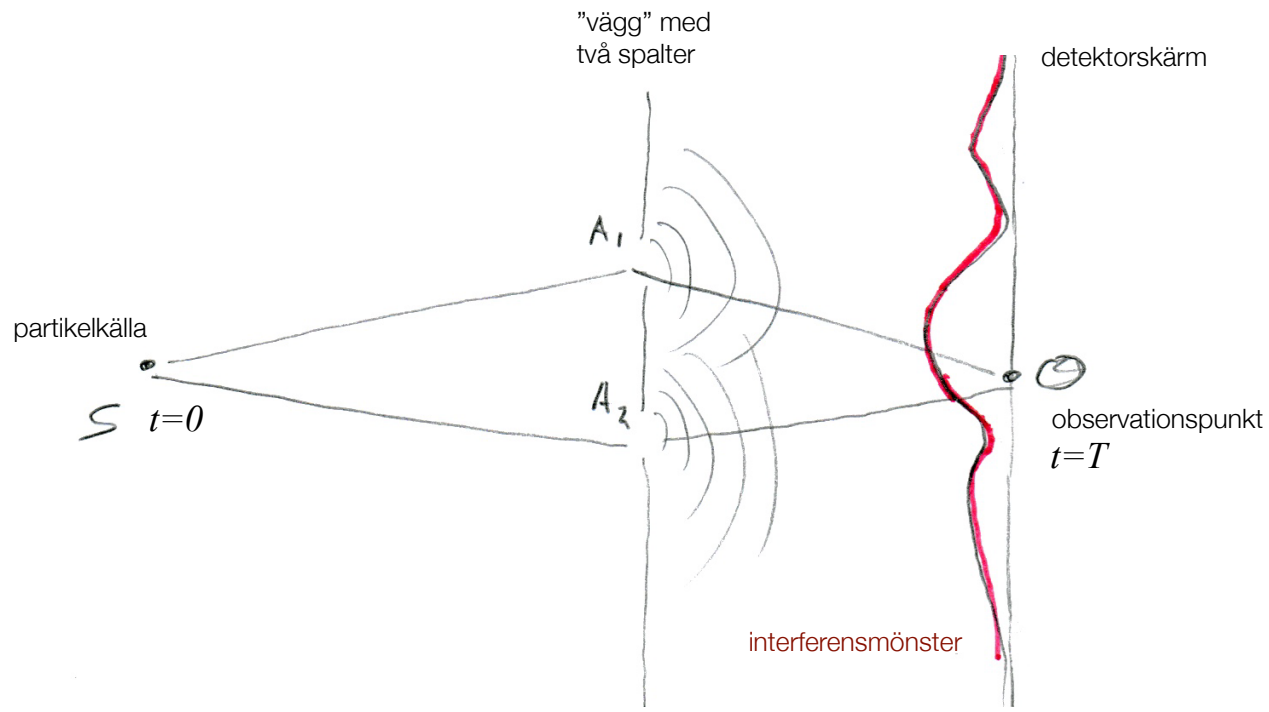
Feynman 1948

VÄG
INTEGRAL
FORMULERING
AV KVANTMEKANIKEN



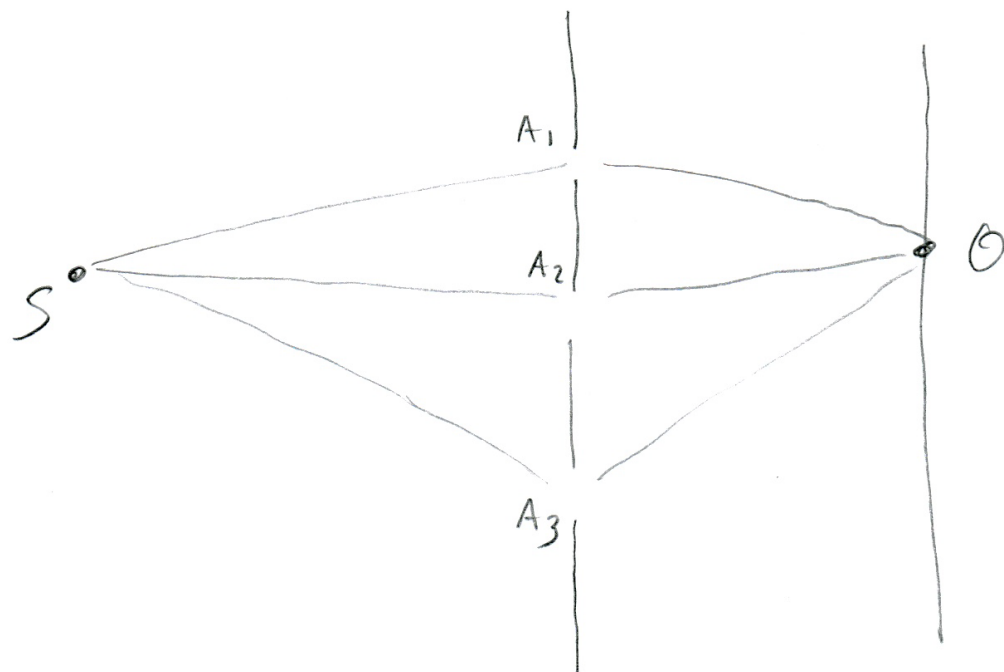
Feynmans vägintegralformulering av kvantmekaniken

motivation: dubbelspaltsexperimentet

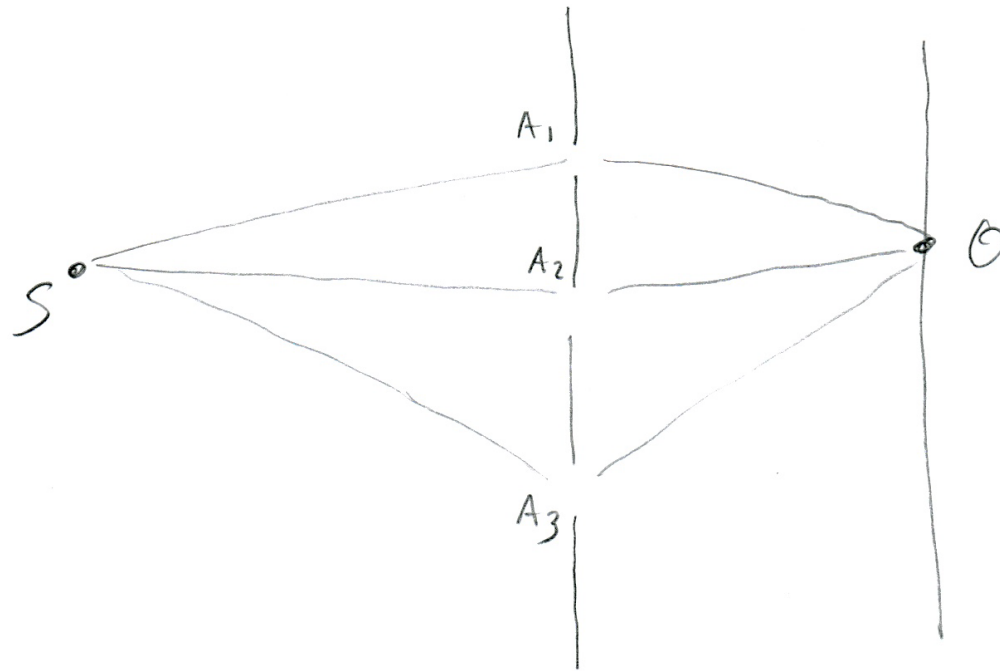


$$A_{S \rightarrow O} = A_{S \rightarrow A_1 \rightarrow O} + A_{S \rightarrow A_2 \rightarrow O}$$

$$P(O) = |A_{S \rightarrow O}|^2 = \text{sannolikheten att detektera partikeln i punkten O}$$



$$A_{S \rightarrow O} = A_{S \rightarrow A_1 \rightarrow O} + A_{S \rightarrow A_2 \rightarrow O} + A_{S \rightarrow A_3 \rightarrow O}$$



$$\mathcal{A}_{S \rightarrow O} = \mathcal{A}_{S \rightarrow A_1 \rightarrow O} + \mathcal{A}_{S \rightarrow A_2 \rightarrow O} + \mathcal{A}_{S \rightarrow A_3 \rightarrow O}$$

⋮

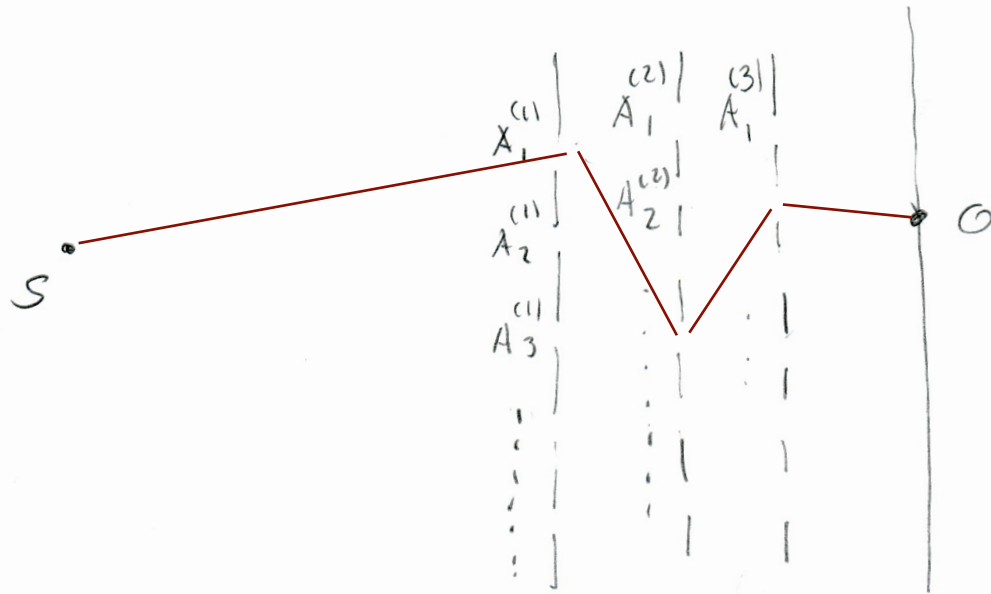
$$\mathcal{A}_{S \rightarrow O} = \sum_{i=1}^{\text{\# spalten}} \mathcal{A}_{S \rightarrow A_i \rightarrow O}$$

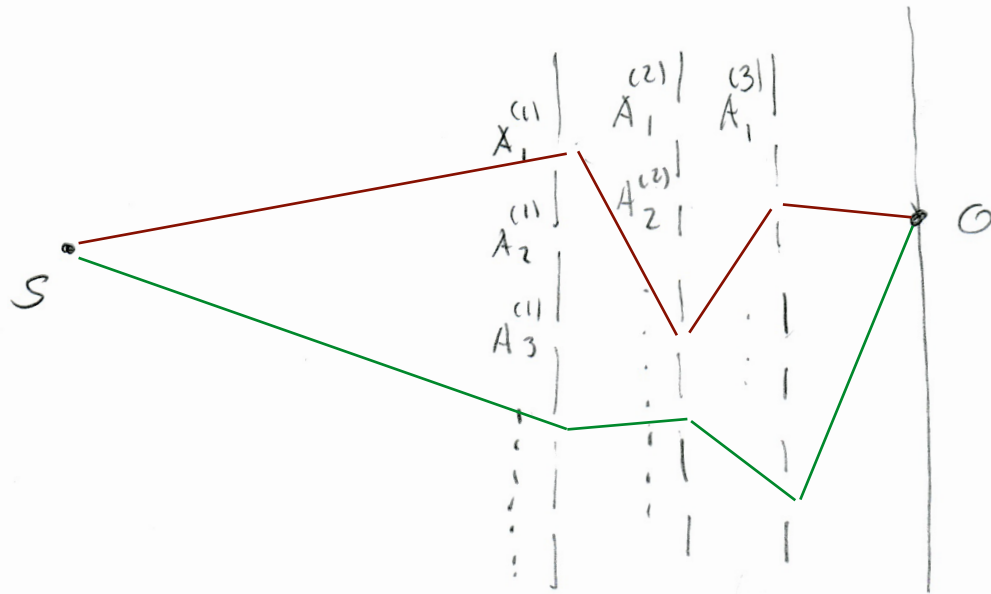
$$P(O) = |\mathcal{A}_{S \rightarrow O}|^2$$

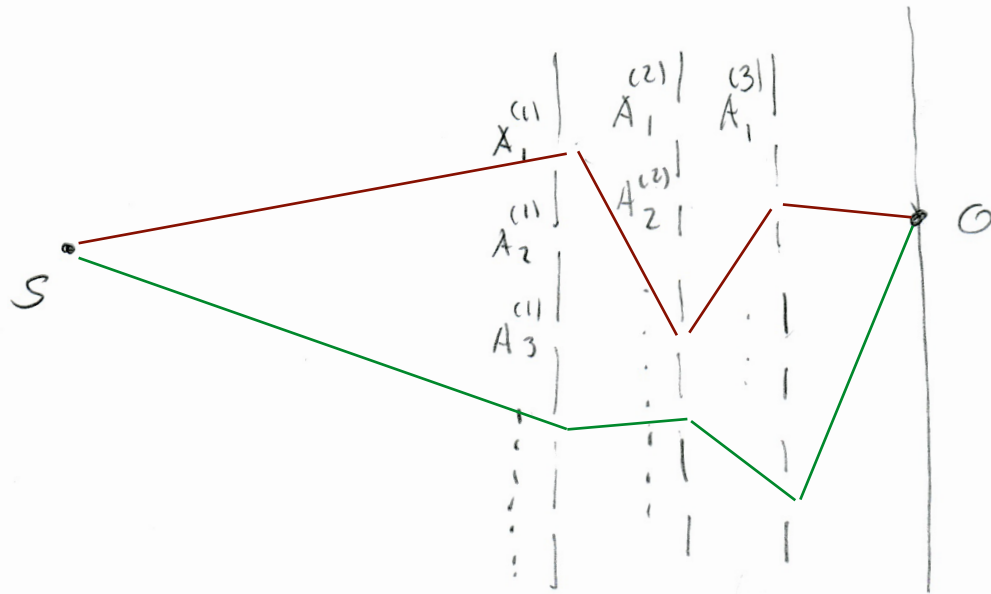
S

$$\begin{array}{c} \begin{array}{c} (1) \\ A_1 \end{array} \Bigg| \begin{array}{c} (2) \\ A_1 \end{array} \Bigg| \begin{array}{c} (3) \\ A_1 \end{array} \\ \begin{array}{c} (1) \\ A_2 \end{array} \Bigg| \begin{array}{c} (2) \\ A_2 \end{array} \Bigg| \begin{array}{c} (3) \\ \vdots \end{array} \\ \begin{array}{c} (1) \\ A_3 \end{array} \Bigg| \begin{array}{c} (2) \\ \vdots \end{array} \Bigg| \begin{array}{c} (3) \\ \vdots \end{array} \\ \vdots \Bigg| \vdots \Bigg| \vdots \\ \vdots \Bigg| \vdots \Bigg| \vdots \\ \vdots \Bigg| \vdots \Bigg| \vdots \end{array}$$



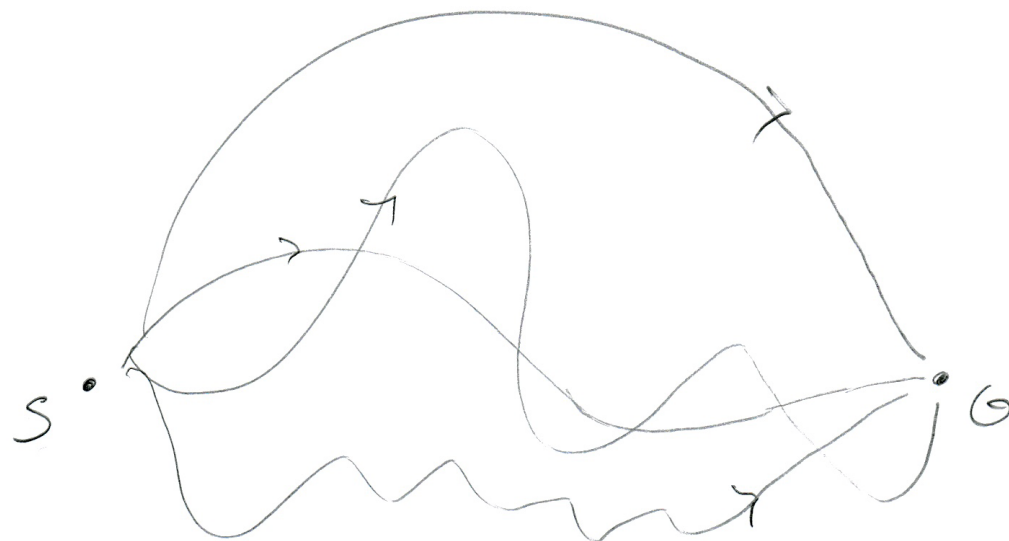






$$\mathcal{A}_{S \rightarrow O} = \sum_{i,j,k}^{\# \text{ starter}} \mathcal{A}(S \rightarrow A_i^{(1)} \rightarrow A_j^{(2)} \rightarrow A_k^{(3)} \rightarrow O)$$

$$P(O) = |\mathcal{A}_{S \rightarrow O}|^2$$

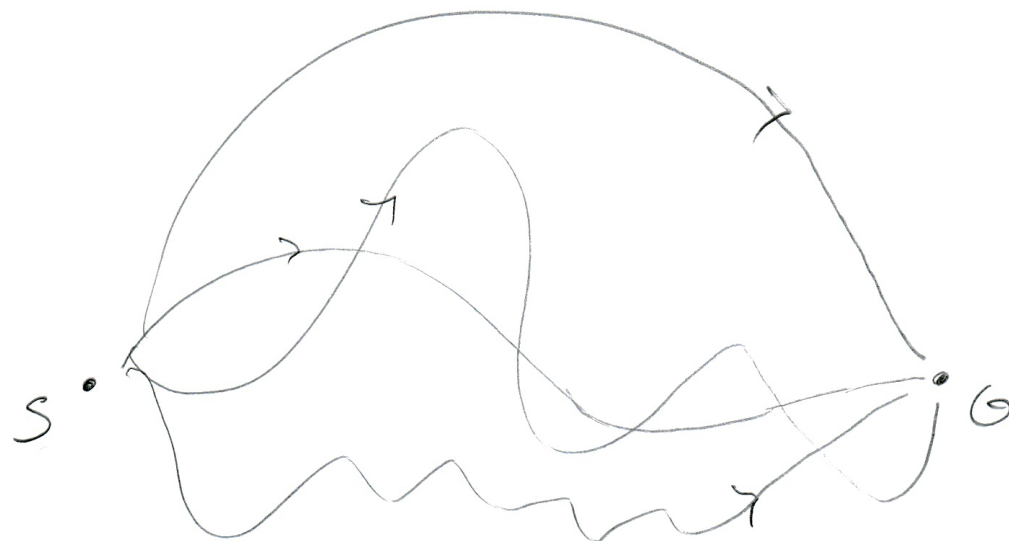


SPALTER $\rightarrow \infty$

VÄGGAR $\rightarrow \infty$



PARTIKELN "PRÖVAR" ALLA
VÄGAR FRÅN S TILL O!



SPALTER $\rightarrow \infty$

VÄGGAR $\rightarrow \infty$



PARTIKELN "PRÖVAR" ALLA
VÄGAR FRÅN S TILL O!

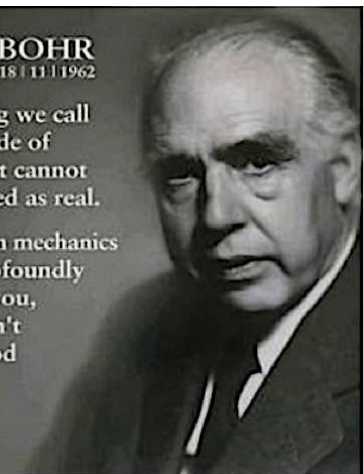
The Zen of Physics...



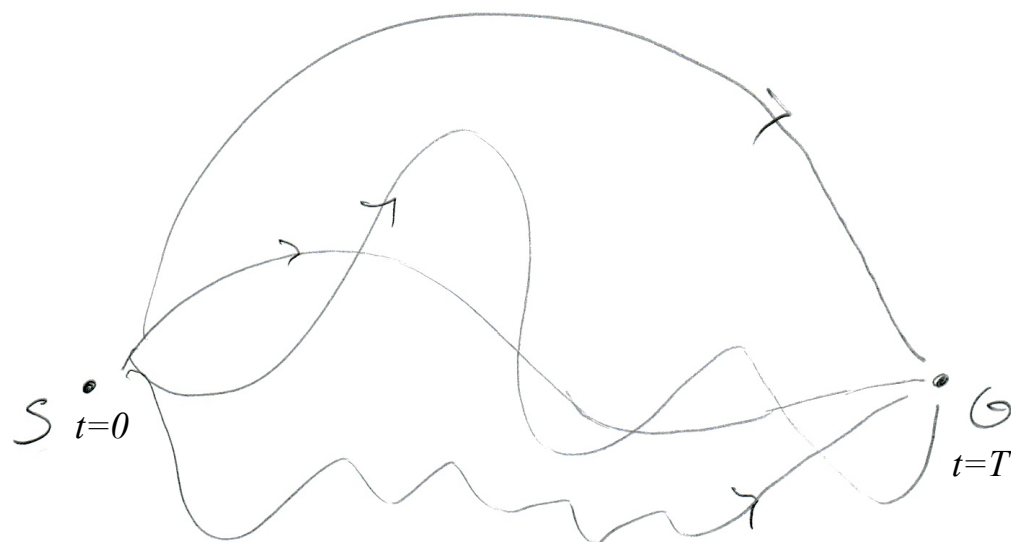
NIELS BOHR
7 | 10 | 1885 – 18 | 11 | 1962

Everything we call
real is made of
things that cannot
be regarded as real.

If quantum mechanics
hasn't profoundly
shocked you,
you haven't
understood
it yet.



The Lord Buddha: "The world is bound up by, and shrouded by a veil of delusion. It appears as real, and is regarded as if it were fine! The fool bound to his illusive acquisitions, blinded by darkness, assumes it as eternal, but for one who sees, and really understands, there is nothing real, stable, or ever same, neither here, nor there at all ..." [Udāna VII 10]



SPALTER $\rightarrow \infty$

VÄGGAR $\rightarrow \infty$

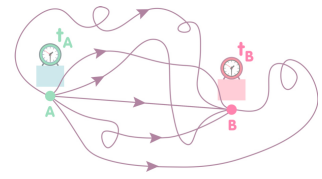


**PARTIKELN "PRÖVAR" ALLA
VÄGAR FRÅN S TILL O!**

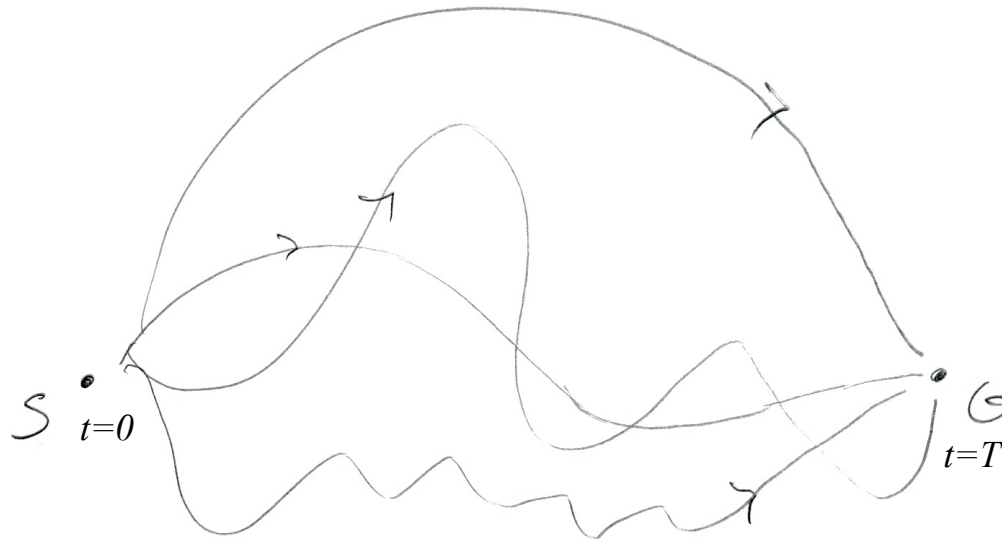
$$A_{S \rightarrow O} = \lim_{n \rightarrow \infty} \sum_{v=1}^n A(S \rightarrow O \text{ längs vägen } v \text{ under tiden } T)$$

↑
vägindex

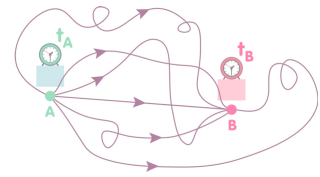
$$P(O) = |A_{S \rightarrow O}|^2$$



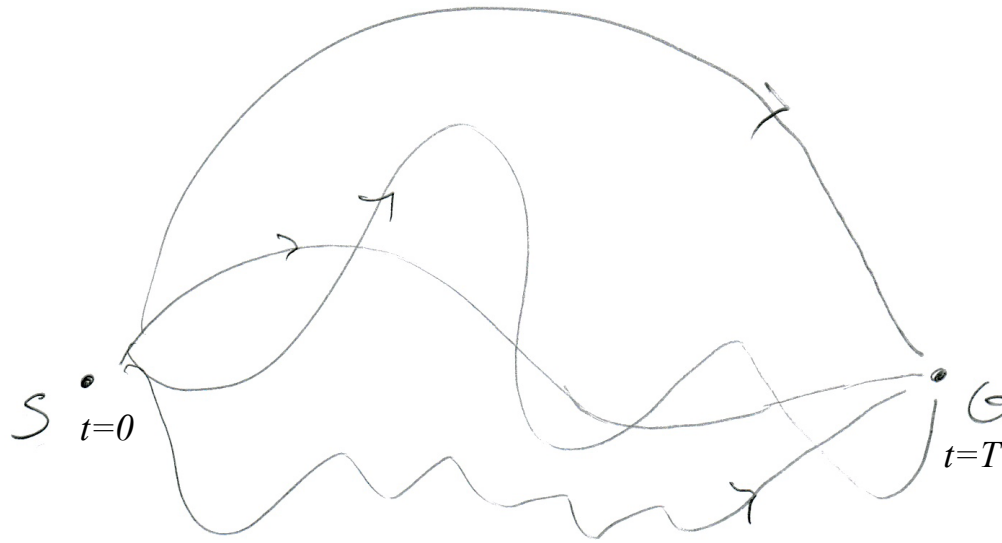
Feynmans vägintegralformulering av kvantmekaniken (forts.)



$$U_{S \rightarrow G} = \lim_{n \rightarrow \infty} \sum_{\substack{\nu=1 \\ \uparrow \\ \text{vägindex}}}^n \mathcal{A}(S \rightarrow G \text{ längs vägen} \\ \text{under tiden } T)$$



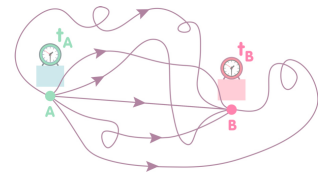
Feynmans vägintegralformulering av kvantmekaniken (forts.)



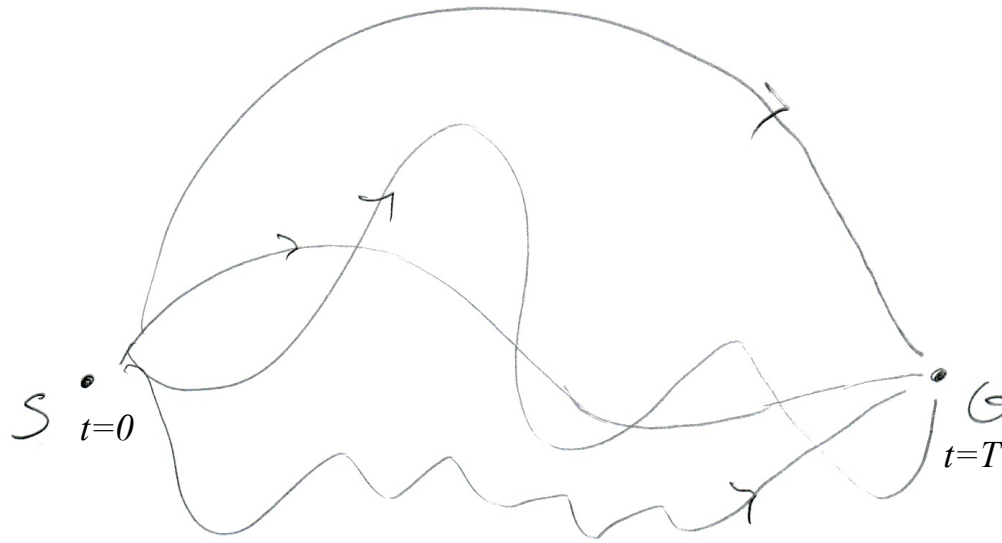
$$U_{S \rightarrow O} = \lim_{n \rightarrow \infty} \sum_{\nu=1}^n \mathcal{A}(S \rightarrow O \text{ längs vägen } \nu \text{ under tiden } T)$$

↑
vägindex

$$= \int_{\text{VÄGAR } \nu} \mathcal{A}(S \rightarrow O \text{ längs vägen } \nu \text{ under tiden } T) d\nu$$



Feynmans vägintegralformulering av kvantmekaniken (forts.)

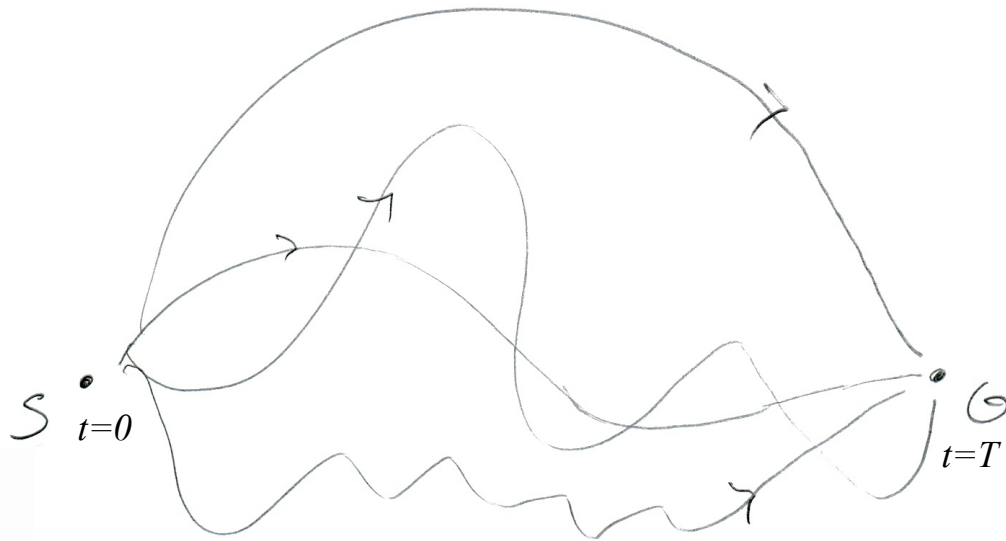


$$U_{S \rightarrow O} = \lim_{n \rightarrow \infty} \sum_{\nu=1}^n \mathcal{A}(S \rightarrow O \text{ längs vägen } \nu \text{ under tiden } T)$$

↑
vägindex

$$= \int_{\text{VÄGAR } \nu} \mathcal{A}(S \rightarrow O \text{ längs vägen } \nu \text{ under tiden } T) d\nu$$

Vi behöver formalisera detta!

Feynmans vägintegralformulering
av kvantmekaniken (forts.)

$$S = (x_i, y_i) \equiv q_i$$

$$G = (x_f, y_f) \equiv q_f$$

$$|q_i\rangle = \text{TILLSTÄNDET FÖR PARTIKERN } |q_i\rangle$$

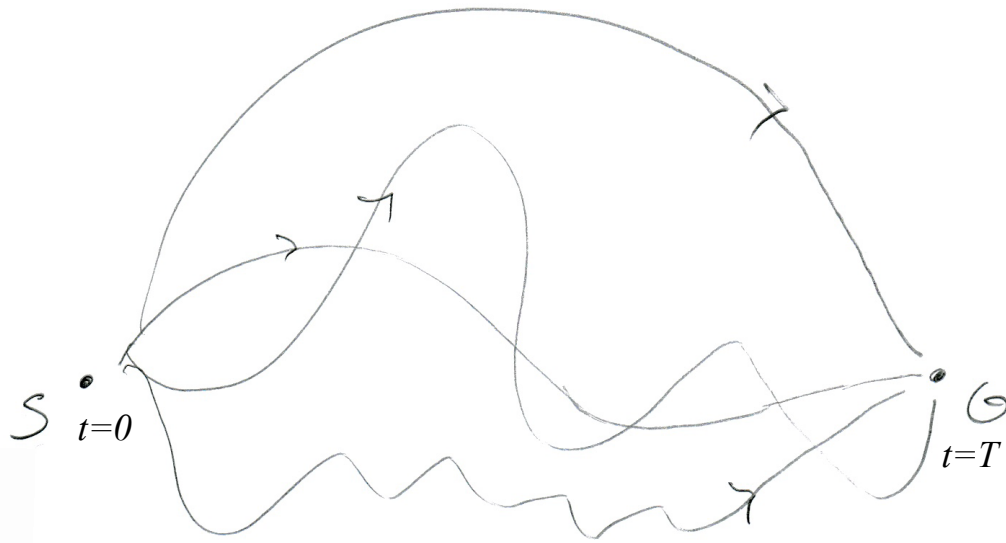
$$|q_f\rangle = \text{---"---} \quad q_f$$

 $\hbar=1$

ON BASVEKTORER

$$A_{S \rightarrow G} = \langle q_f | e^{-i\hat{H}T} | q_i \rangle$$

$$|\Psi(T)\rangle = \alpha_A |q_A\rangle + \alpha_B |q_B\rangle + \dots \alpha_F |q_F\rangle + \dots$$



$$S = (x_i, y_i) \equiv q_i$$

$$G = (x_f, y_f) \equiv q_f$$

$$|q_i\rangle = \text{TILLSTÄNDET FÖR PARTIKELN } |q_i\rangle$$

$$|q_f\rangle = \text{---"---} \quad q_f$$

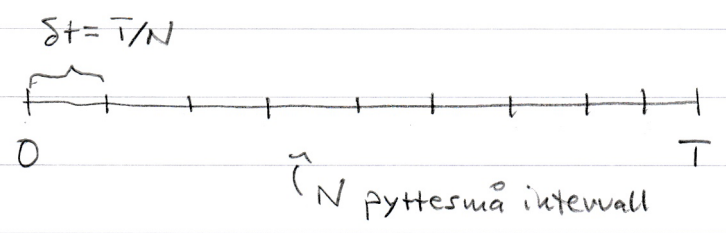
 $\hbar=1$

ON BASVEKTORER

$$A_{S \rightarrow G} = \langle q_f | \underbrace{e^{-i\hat{H}T}}_{\text{---"---}} | q_i \rangle = \alpha_f$$

$$|\Psi(T)\rangle = \alpha_A |q_A\rangle + \alpha_B |q_B\rangle + \dots + \alpha_f |q_f\rangle + \dots$$

Låt oss nu undersöka $\mathcal{A}_{S \rightarrow 0}$ närmare!



$$\mathcal{A}_{S \rightarrow 0} = \langle q_F | e^{-i\hat{H}T} | q_I \rangle$$

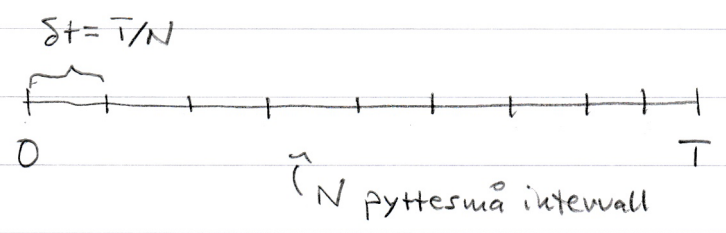
$$= \langle q_F | e^{-i\hat{H}\delta t} \mathbb{1} e^{-i\hat{H}\delta t} \mathbb{1} \dots \mathbb{1} e^{-i\hat{H}\delta t} | q_I \rangle$$

"RESOLUTION OF IDENTITY"

$$= \langle q_F | e^{-i\hat{H}\delta t} \underbrace{\int dq_{N-1} |q_{N-1}\rangle \langle q_{N-1}|}_{\mathbb{1}} e^{-i\hat{H}\delta t} \underbrace{\int dq_{N-2} |q_{N-2}\rangle \langle q_{N-2}|}_{\mathbb{1}} e^{-i\hat{H}\delta t} \dots \underbrace{\int dq_1 |q_1\rangle \langle q_1|}_{\mathbb{1}} e^{-i\hat{H}\delta t} |q_I\rangle$$

$$= \int \dots \int_{q_1}^{q_{N-1}} \underbrace{\prod_{j=1}^{N-1} dq_j}_{\mathbb{1}} \langle q_F | e^{-i\hat{H}\delta t} |q_{N-1}\rangle \langle q_{N-1} | e^{-i\hat{H}\delta t} |q_{N-2}\rangle \dots \dots \langle q_1 | e^{-i\hat{H}\delta t} |q_I\rangle \underbrace{|q_0\rangle}_{\mathbb{1}} \quad (1)$$

Låt oss nu undersöka $\mathcal{A}_{S \rightarrow 0}$ närmare!



$$\mathcal{A}_{S \rightarrow 0} = \langle q_F | e^{-i\hat{H}T} | q_I \rangle$$

$$= \langle q_F | e^{-i\hat{H}\delta t} \mathbb{1} e^{-i\hat{H}\delta t} \mathbb{1} \dots \mathbb{1} e^{-i\hat{H}\delta t} | q_I \rangle$$

"RESOLUTION OF IDENTITY"

$$= \langle q_F | e^{-i\hat{H}\delta t} \underbrace{\int dq_{N-1} |q_{N-1}\rangle \langle q_{N-1}|}_{\mathbb{1}} e^{-i\hat{H}\delta t} \underbrace{\int dq_{N-2} |q_{N-2}\rangle \langle q_{N-2}|}_{\mathbb{1}} e^{-i\hat{H}\delta t} \dots \underbrace{\int dq_1 |q_1\rangle \langle q_1|}_{\mathbb{1}} e^{-i\hat{H}\delta t} |q_I\rangle$$

alla integrationsgränser från $-\infty$ till ∞

$$= \int_{q_1} \dots \int_{q_{N-1}} \underbrace{\prod_{j=1}^{N-1} \int dq_j}_{\mathbb{1}} \langle q_F | e^{-i\hat{H}\delta t} |q_{N-1}\rangle \langle q_{N-1} | e^{-i\hat{H}\delta t} |q_{N-2}\rangle \dots \dots \langle q_1 | e^{-i\hat{H}\delta t} |q_I\rangle \underbrace{|q_0\rangle}_{\mathbb{1}} \quad (1)$$

UNDERSÖK EN AV FAKTORISERNA I INTEGRANDEN :

$\left\langle q_{j+1} \left| e^{-i\hat{K}dt} \right| q_j \right\rangle$

← välj $\hat{H} = \frac{\hat{p}^2}{2m}$ (FRI PARTIKEL) , $\hat{p} = (\hbar \hat{k})$

$\int_{-\infty}^{\infty} \delta(p) dp = 1$

$\frac{1}{2\pi\hbar} \int |p\rangle \langle p| dp = 1$

↑ NORMALISERING "resolution of identity"

$\langle q | p \rangle = e^{ipq}$

UNDERSÖK EN AV FAKTORISERNA I INTEGRANDEN :
 värd $\hat{H} = \frac{\hat{p}^2}{2m}$ (FÖR PARTIKEL) , $\hat{p} = (\hbar \hat{k})^2$

$$\langle q_{j+1} | e^{-i\hat{H}dt} | q_j \rangle$$

$$\int_{-\infty}^{\infty} |p\rangle \langle p| dp = 1$$

↑ NORMALISERING
"resolution of identity"

$$\langle q | p \rangle = e^{ipq}$$

⇓

$$\begin{aligned} \langle q_{j+1} | e^{-i\delta t \frac{\hat{p}^2}{2m}} | q_j \rangle &= \langle q_{j+1} | e^{-i\delta t \frac{\hat{p}^2}{2m}} \underbrace{\int_{-\infty}^{\infty} dp |p\rangle \langle p|}_{=1} | q_j \rangle \\ &= \int_{-\infty}^{\infty} dp \underbrace{\langle q_{j+1} | e^{-i\delta t \frac{\hat{p}^2}{2m}} | p \rangle}_{e^{-i\delta t \frac{p^2}{2m}} \langle q_{j+1} | p \rangle} \underbrace{\langle p | q_j \rangle}_{\langle q_j | p \rangle^* = e^{-ipq_j}} \end{aligned}$$

$\underbrace{e^{-i\delta t \frac{p^2}{2m}} \langle q_{j+1} | p \rangle}_{e^{ipq_{j+1}}}$

UNDERSÖK EN AV FAKTORISERNA I INTEGRANDEN :
 värd $\hat{H} = \frac{\hat{p}^2}{2m}$ (FRI PARTIKEL) , $\hat{p} = (\hbar \hat{k})^2$

$$\langle q_{j+1} | e^{-i\hat{H}dt} | q_j \rangle$$

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⇓

$$\langle q_{j+1} | e^{-i\hat{H}dt} | q_j \rangle = \langle q_{j+1} | e^{-i\hat{H}dt} \int_{-\infty}^{\infty} dp |p\rangle \langle p| q_j \rangle$$

$$= \int_{-\infty}^{\infty} dp \langle q_{j+1} | e^{-i\hat{H}dt} | p \rangle \langle p | q_j \rangle$$

$$= \int_{-\infty}^{\infty} dp e^{-i\hat{H}dt} \langle q_{j+1} | p \rangle \langle p | q_j \rangle$$

$$= \int_{-\infty}^{\infty} dp e^{-i\frac{p^2}{2m}dt} e^{ipq_{j+1}} e^{-ipq_j} = \int_{-\infty}^{\infty} dp e^{-i\frac{p^2}{2m}dt + ip(q_{j+1} - q_j)}$$

$$\text{Ty } e^{-i\hat{H}dt} |p\rangle = e^{-i\frac{p^2}{2m}dt} |p\rangle$$

$$\text{(ACHTUNG : } f(\hat{p}) |p\rangle = f(p) |p\rangle$$

BEWIS : TAYLORUNTERZUGAL !)

UNDERSÖK EN AV FAKTORISERNA I INTEGRANDEN :

välj $\hat{H} = \frac{\hat{p}^2}{2m}$ (FRI PARTIKEL), $\hat{p} = (\hbar \hat{k})^2$

$\langle q_{j+1} | e^{-i\hat{H}t} | q_j \rangle$

$\int |p\rangle \langle p| dp = 1$
 ↑ NORMALISERING "resolution of identity"

$\langle q | p \rangle = e^{ipq}$

⇓

$\langle q_{j+1} | e^{-i\hat{H}t} \frac{\hat{p}^2}{2m} | q_j \rangle = \langle q_{j+1} | e^{-i\hat{H}t} \frac{\hat{p}^2}{2m} \int dp |p\rangle \langle p| q_j \rangle$

$= \int dp \langle q_{j+1} | e^{-i\hat{H}t} \frac{\hat{p}^2}{2m} | p \rangle \langle p | q_j \rangle$
 $e^{-i\hat{H}t} \frac{\hat{p}^2}{2m} | p \rangle = e^{-i\frac{\hbar^2 k^2 t}{2m}} | p \rangle = e^{-i\frac{p^2 t}{2m}} | p \rangle$
 $\langle p | q_j \rangle = e^{-ipq_j}$

$= \int dp e^{-i\frac{p^2 t}{2m}} e^{ip(q_{j+1} - q_j)} \quad (2)$

$$\langle q_{j+1} | e^{-i\hat{H}dt} | q_j \rangle = \frac{1}{2\pi} \int dp e^{-i\delta t p^2/2m} e^{ip(q_{j+1} - q_j)} \quad (2)$$

$$\langle q_{j+1} | e^{-i\hat{H}t} | q_j \rangle = \frac{1}{2\pi i} \int dp e^{-i\hat{H}t p^2/2m} e^{ip(q_{j+1} - q_j)} \quad (2)$$

"HJÄLPRESULTAT":

KOMPLEXVÄRD

GAUSSISK INTEGRAL!

ALLMÄNT:

$$\int dx e^{\frac{1}{2}iax^2 + iJx} = \sqrt{\frac{2\pi i}{a}} e^{-iJ^2/2a}$$

IFR

$$\int dx e^{-\frac{1}{2}ax^2 + Jx} = \sqrt{\frac{2\pi}{a}} e^{J^2/2a}$$

$$a \rightarrow -ia, \quad J \rightarrow iJ$$

$$\langle q_{j+1} | e^{-i\hat{H}\delta t} | q_j \rangle = \frac{1}{2\pi i} \int dp e^{-i\delta t p^2/2m} e^{ip(q_{j+1} - q_j)} \quad (2)$$

"HJÄLPRESULTAT":

KOMPLEXVÄRD

GAUSSISKA INTEGRAL!

ALLIÄNT: $\int dx e^{\frac{1}{2}iax^2 + iJx} = \sqrt{\frac{2\pi i}{a}} e^{-iJ^2/2a}$

IFR $\int dx e^{-\frac{1}{2}ax^2 + Jx} = \sqrt{\frac{2\pi}{a}} e^{J^2/2a}$

$a \rightarrow -ia, J \rightarrow iJ$

$$J = q_{j+1} - q_j$$

$$a = -\delta t/m, x = p$$

$$\langle q_{j+1} | e^{-i\delta t \hat{p}^2/2m} | q_j \rangle = \sqrt{\frac{-2\pi i m}{\delta t}} e^{i\delta t (m/2) [(q_{j+1} - q_j)/\delta t]^2} \quad (3)$$

$$A_{S \rightarrow 0} = \langle q_F | e^{-i\hat{H}T} | q_I \rangle$$

$$= \int \dots \int_{q_1}^{q_{N-1}} dq_1 \dots dq_{N-1} \langle q_F | e^{-i\hat{H}\delta t} | q_{N-1} \rangle \langle q_{N-1} | e^{-i\hat{H}\delta t} | q_{N-2} \rangle \dots \dots \langle q_1 | e^{-i\hat{H}\delta t} | q_I \rangle \underbrace{|q_I\rangle}_{|q_0\rangle} \quad (1)$$

$$\langle q_{j+1} | e^{-i\hat{H}\delta t} | q_j \rangle = \frac{1}{2\pi\hbar} \int dp e^{-i\delta t p^2/2m} e^{ip(q_{j+1} - q_j)} \quad (2)$$

$$\langle q_{j+1} | e^{-i\delta t \hat{p}^2/2m} | q_j \rangle = \sqrt{\frac{-2\pi\hbar i m}{\delta t}} e^{i\delta t (m/2) [(q_{j+1} - q_j)/\delta t]^2} \quad (3)$$

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$$\langle q_{j+1} | e^{-i\delta t \hat{p}^2 / 2m} | q_j \rangle = \sqrt{\frac{-2\pi i m}{\delta t}} e^{i\delta t (m/2) [(q_{j+1} - q_j) / \delta t]^2} \quad (3)$$

sätt in (3) i (1):

$$\mathcal{A}_{S \rightarrow 0} = \langle q_f | e^{-i\hat{H}T} | q_i \rangle$$

$$= \left(-\frac{2\pi i m}{\delta t}\right)^{(N-1)/2} \int \prod_{j=1}^{N-1} dq_j \prod_{j=0}^{N-1} e^{i\delta t m/2 [(q_{j+1} - q_j) / \delta t]^2}$$

$$= \left(-\frac{2\pi i m}{\delta t}\right)^{(N-1)/2} \int \prod_{j=1}^{N-1} dq_j e^{i\delta t m/2 \sum_{j=0}^{N-1} [(q_{j+1} - q_j) / \delta t]^2} \quad (4)$$

$$(q_0 = q_i, q_N = q_f)$$

$$\begin{aligned}
 \mathcal{A}_{S \rightarrow 0} &= \langle q_{\text{f}} | e^{-i\hat{H}T} | q_{\text{i}} \rangle \\
 &= \left(-\frac{2\tilde{u}im}{\delta t} \right)^{(N-1)/2} \prod_{j=1}^{N-1} \int dq_j e^{i\delta t m/2 \sum_{j=0}^{N-1} [(q_{j+1} - q_j)/\delta t]^2} \\
 &= \left(-\frac{2\tilde{u}im}{\delta t} \right)^{(N-1)/2} \prod_{j=1}^{N-1} \int dq_j e^{i\delta t m/2 \sum_{j=0}^{N-1} [(q_{j+1} - q_j)/\delta t]^2} \quad (4)
 \end{aligned}$$

$(q_0 = q_{\text{i}}, q_N = q_{\text{f}})$

$$\begin{aligned}
 \mathcal{A}_{S \rightarrow 0} &= \langle q_{\text{f}} | e^{-i\tilde{H}T} | q_{\text{i}} \rangle \\
 &= \left(-\frac{2\tilde{u}im}{\delta t} \right)^{(N-1)/2} \prod_{j=1}^{N-1} \int dq_j e^{i\delta t m/2 \sum_{j=0}^{N-1} [(q_{j+1} - q_j)/\delta t]^2} \\
 &= \left(-\frac{2\tilde{u}im}{\delta t} \right)^{(N-1)/2} \prod_{j=1}^{N-1} \int dq_j e^{i\delta t m/2 \sum_{j=0}^{N-1} [(q_{j+1} - q_j)/\delta t]^2} \quad (4)
 \end{aligned}$$

($q_0 = q_{\text{i}}, q_N = q_{\text{f}}$)

Vi tar nu gränsvärdena $\delta t \rightarrow 0, N \rightarrow \infty$

$$\begin{aligned}
 \overbrace{[(q_{j+1} - q_j)/\delta t]^2}^{\delta q} &\rightarrow \dot{q}^2 \\
 \delta t \sum_{j=0}^{N-1} \dots &\rightarrow \int_0^T dt \dots
 \end{aligned}$$

"VÄGİNTEGRAL"

$$\left(-\frac{2\tilde{u}im}{\delta t} \right)^{(N-1)/2} \prod_{j=1}^{N-1} \int dq_j \dots \rightarrow \int \mathcal{D}[q(t)] \dots$$

$$\left(-\frac{2i\hbar m}{\delta t}\right)^{(N-1)/2} \prod_{j'=1}^{N-1} \int dq_{j'} \dots$$

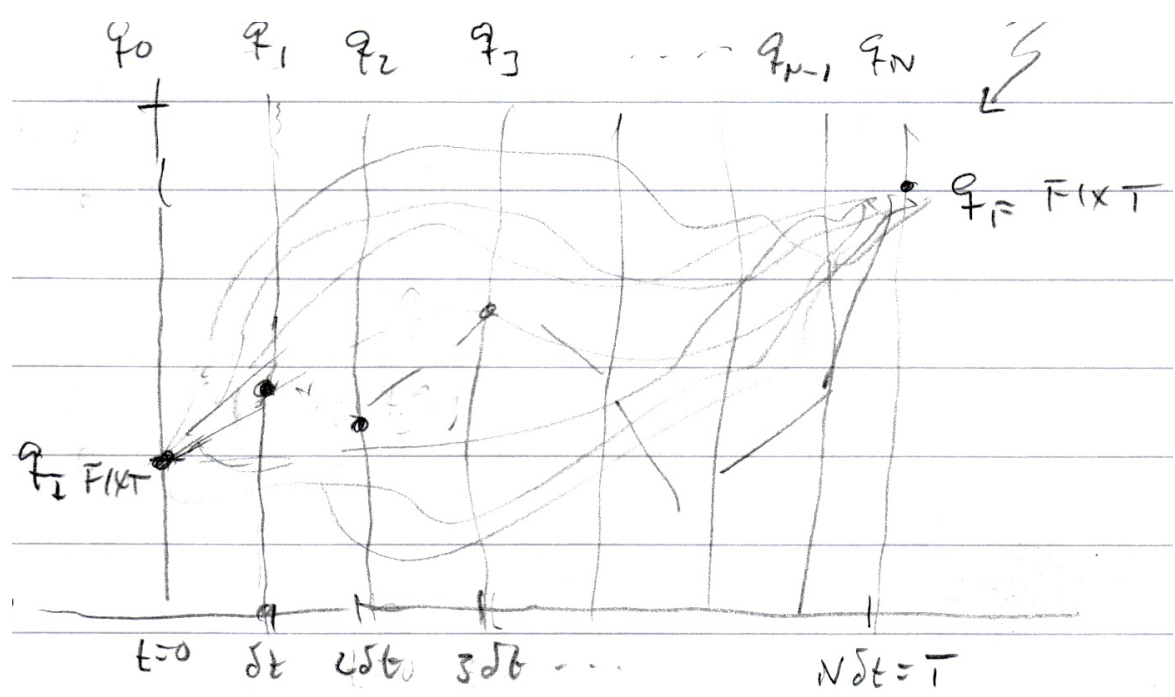
"VÄGİNTEGRAL"

$$\rightarrow \int \mathcal{D}[q(t)] \dots$$

"VÄGİNTEGRAL"

$$\left(-\frac{2\tilde{u}im}{\delta t} \right)^{(N-1)/2} \prod_{j=1}^{N-1} \int dq_j, \dots \rightarrow \int \mathcal{D}[q(t)] \dots$$

∞

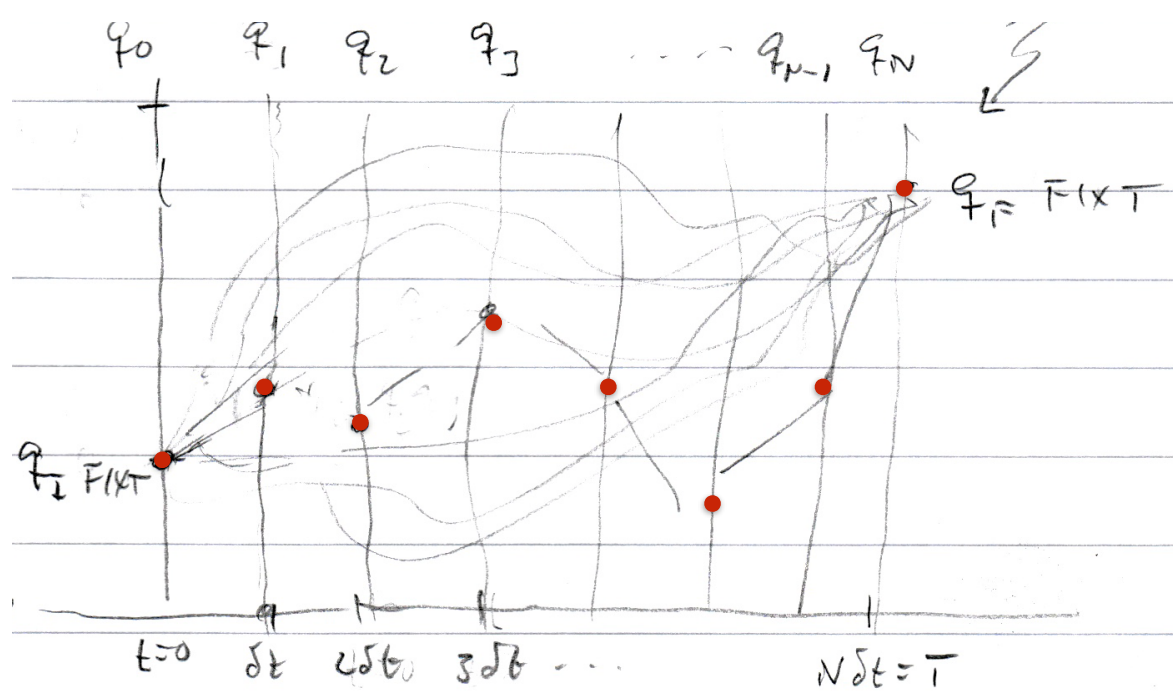


∞

"VÄGİNTEGRAL"

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∞

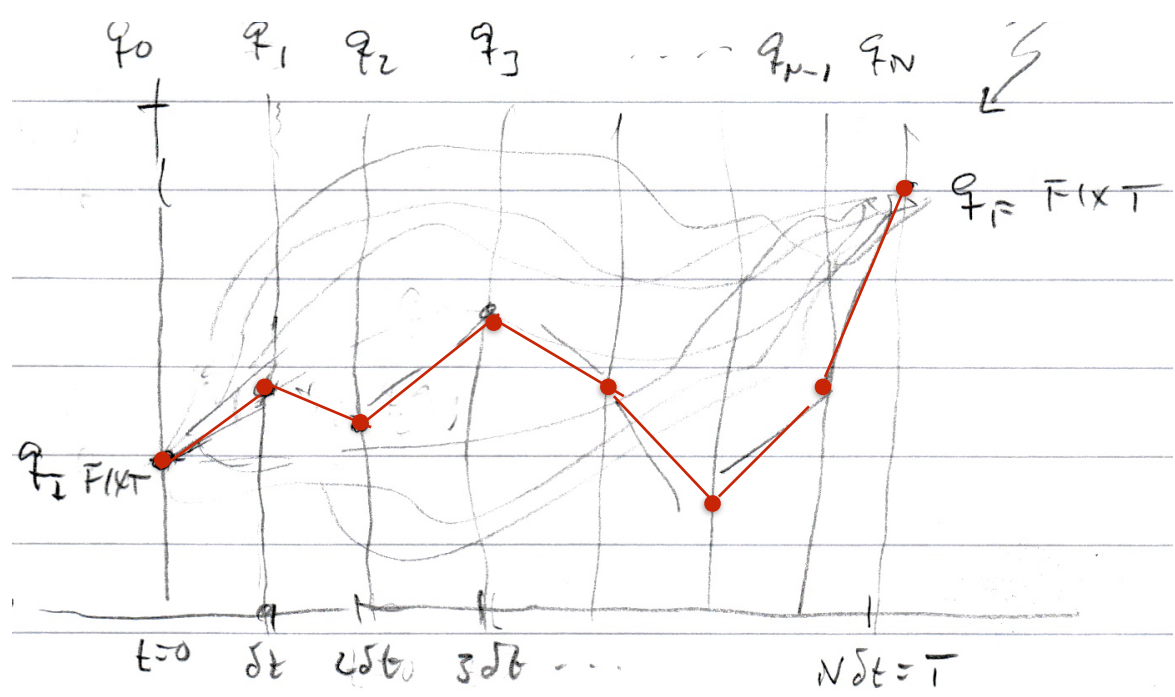


-∞

"VÄGİNTEGRAL"

$$\left(-\frac{2\tilde{u}im}{\delta t} \right)^{(N-1)/2} \prod_{j=1}^{N-1} \int dq_j, \dots \rightarrow \int \mathcal{D}[q(t)] \dots$$

∞



-∞

Efter att vi tagit alla gränsvärden...

VI KAN DÅ SKRIVA $\hat{H} = \hat{p}^2/2m$

$$A_{s \rightarrow 0} = \langle q_f | e^{-i\hat{H}T} | q_i \rangle = \int \mathcal{D}[q(t)] e^{i \int_0^T dt \frac{1}{2} m \dot{q}^2}$$

$$\mathcal{A}_{s \rightarrow 0} = \langle q_F | e^{-i\hat{H}T} | q_I \rangle = \int \mathcal{D}[q(t)] e^{i \int_0^T dt \frac{1}{2} m \dot{q}^2}$$

$\hat{H} = \hat{p}^2 / 2m$

VI HAR GJORT HÄRLEDNINGEN FÖR EN FRI PARTIKEL...

GENERALISERA TILL FALLET MED NOLLSKILD POTENTIAL

$$\mathcal{A} = \langle q_F | e^{-i\hat{H}T} | q_I \rangle = \int \mathcal{D}[q(t)] e^{i \int_0^T dt \left(\frac{1}{2} m \dot{q}^2 - V(q) \right)}$$

$\hat{H} = \hat{p}^2 / 2m + V(q)$

$L(q, \dot{q})$

$$\mathcal{A}_{s \rightarrow 0} = \langle q_F | e^{-i\hat{H}T} | q_I \rangle = \int \mathcal{D}[q(t)] e^{i \int_0^T dt \frac{1}{2} m \dot{q}^2}$$

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VI HAR GJORT HÄRLEDNINGEN FÖR EN FRI PARTIKEL...

GENERALISERA TILL FALLET MED NOLLSKILD POTENTIAL

$$\mathcal{A} = \langle q_F | e^{-i\hat{H}T} | q_I \rangle = \int \mathcal{D}[q(t)] e^{i \int_0^T dt \left(\frac{1}{2} m \dot{q}^2 - V(q) \right)}$$

$\hat{H} = \hat{p}^2 / 2m + V(q)$

Hamilton-operator

klassisk Lagrange-funktion

$$\hat{H} = \hat{p}^2 / 2m \quad ; \int_0^T dt \frac{1}{2} m \dot{q}^2$$

$$\mathcal{A}_{s \rightarrow 0} = \langle q_F | e^{-i\hat{H}T} | q_I \rangle = \int \mathcal{D}[q(t)] e^{i \int_0^T dt \frac{1}{2} m \dot{q}^2}$$

VI HAR GJORT HÄRLEDNINGEN FÖR EN FRI PARTIKEL...

GENERALISERA TILL FALLET MED NOLLSKILD POTENTIAL

$$\hat{H} = \hat{p}^2 / 2m + V(q) \quad ; \int_0^T dt (\frac{1}{2} m \dot{q}^2 - V(q))$$

$$\mathcal{A} = \langle q_F | e^{-i\hat{H}T} | q_I \rangle = \int \mathcal{D}[q(t)] e^{i \int_0^T dt L(q, \dot{q})}$$

Hamilton-operator

klassisk Lagrange-funktion

SÄTT TILLGÅNG TIL ! (SI ENHETEN)

SÄTT IN INTEGRATIONSGRÄNSER q_I

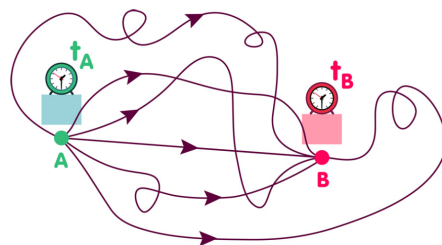
$$\langle q_F | e^{-i\hat{H}T/\hbar} | q_I \rangle = \int_{q_I} \mathcal{D}[q(t)] e^{i \int_0^T dt L(q, \dot{q}) / \hbar}$$

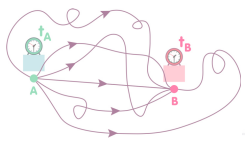
klassisk verkan S

$$= \int_{q_I}^{q_F} \mathcal{D}[q(t)] e^{iS/\hbar}$$

FEYNMANS
VÄG INTEGRAL

Feynmans vägintegralformulering av kvantmekaniken (epilog)





VI KAN DÅ SKRIVA $\hat{H} = \hat{p}^2/2m$

$$U_{s \rightarrow 0} = \langle q_F | e^{-i\hat{H}T} | q_I \rangle = \int \mathcal{D}[q(t)] e^{i \int_0^T dt \frac{1}{2} m \dot{q}^2}$$

VI HAR GJORT HÄRLEDNINGEN FÖR EN FRI PARTIKEL...

GENERALISERA TILL FALLET MED NOLLSKILD POTENTIAL

$$U = \langle q_F | e^{-i\hat{H}T} | q_I \rangle = \int \mathcal{D}[q(t)] e^{i \int_0^T dt (\frac{1}{2} m \dot{q}^2 - V(q))}$$

Hamilton-operator

klassisk Lagrange-funktion

$$\langle q_F | e^{-i\hat{H}T/\hbar} | q_I \rangle = \int_{q_I}^{q_F} \mathcal{D}[q(t)] e^{i \int_0^T dt L(q, \dot{q})/\hbar}$$

klassisk verkan S

$$= \int_{q_I}^{q_F} \mathcal{D}[q(t)] e^{iS/\hbar}$$

FEYNMANS VÄG INTEGRAL

$$\langle q_f | e^{-i\hat{H}T/\hbar} | q_i \rangle = \int_{q_i}^{q_f} \mathcal{D}[q(t)] e^{i \int_0^T dt L(q, \dot{q}) / \hbar}$$

matriselement av tidsutvecklingsoperatoren ("propagator")
 = amplituden för en partikel att gå från $|q_i\rangle$ till $|q_f\rangle$

klassisk verkan S

$$= \int_{q_i}^{q_f} \mathcal{D}[q(t)] e^{iS/\hbar}$$

FEYNMANS
 VÄRINTTEGRAL

$$\langle q_F | e^{-i\hat{H}T/\hbar} | q_I \rangle = \int_{q_I}^{q_F} \mathcal{D}[q(t)] e^{i \int_0^T dt L(q, \dot{q}) / \hbar}$$

matriselement av tidsutvecklingsoperatoren ("propagator")
 = amplituden för en partikel att gå från $|q_I\rangle$ till $|q_F\rangle$

klassisk verkan S

$$= \int_{q_I}^{q_F} \mathcal{D}[q(t)] e^{iS/\hbar}$$

FEYNMANS
VÄG INTEGRAL

Fermioniska vägintegraler måste skrivas i termer av (antikommuterande) Grassmann-variabler

rules for the integration of a Grassmann quantity:

$$\int 1 d\theta = 0$$

$$\int \theta d\theta = 1$$

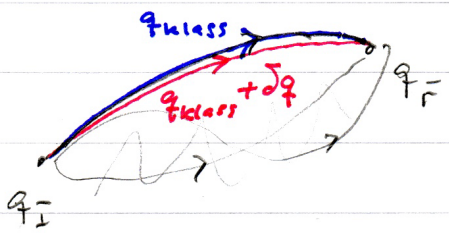
we conclude that the operations of integration and differentiation of a Grassmann number are identical

Vi kan "förstå" **minsta verkans princip** som en konsekvens
 av den klassiska gränsen $\hbar \rightarrow 0 \Rightarrow$ NÄRLIGGANDE VÄGAR
 INTERFERERAR DESTRUKTIVT P.G.A. VÅDSAMMA FÄSFLUKTUATIONER
 OM DEN KLASSISKA VERKAN INTE ÄR STATIONÄR

$$q_F = \int_{q_I} \mathcal{D}[q(t)] e^{iS/\hbar}$$

q_I \hookrightarrow FEYNMANS
VÄG INTEGRAL

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$$= \int_{q_I}^{q_F} \mathcal{D}[q(\tau)] e^{iS/\hbar}$$

FEYNMANS VÄG INTEGRAL

DEN KLASSISKA VÄGEN q_{klass} OPPFYLLER

$$\delta S[q_{klass}] = 0$$

\Downarrow

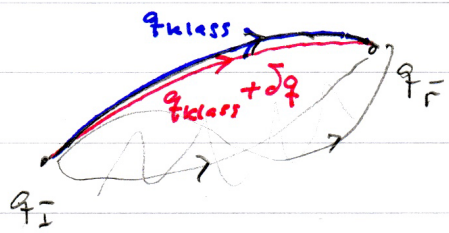
$$S[q_{klass}] - S[q_{klass} + \delta q] \approx 0$$

\Downarrow

$$e^{iS[q_{klass}]/\hbar} \approx e^{iS[q_{klass} + \delta q]/\hbar}$$

KONSTRUKTIV INTERFERENS!

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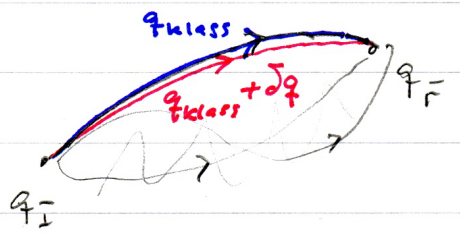
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KONSTRUKTIV INTERFERENS!

Bara den klassiska vägen "överlever" i gränsen $\hbar \rightarrow 0$.
Villkor: S stationär, dvs. minsta verkans princip är uppfylld!

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INTERFERERAR DESTRUKTIVT P.G.A. VÄLDSAMMA FASFLUKTUATIONER OM DEN KLASSISKA VERKAN INTE ÄR STATIONÄR



ett formellt argument använder stationära fasapproximationen (se AWH)

$$= \int_{q_I}^{q_F} \mathcal{D}[q(\tau)] e^{iS/\hbar}$$

Feynmans VÄG INTEGRAL

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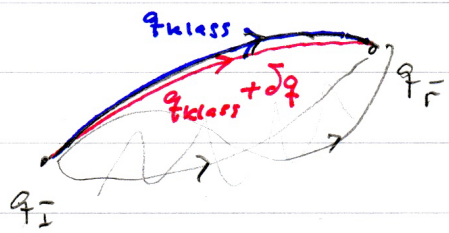
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KONSTRUKTIV INTERFERENS!

MEN ... $\hbar \neq 0$!
BOHR'S ARGUMENT: $\hbar \sim 6.6 \times 10^{-16}$ eVs ≈ 0
PÅ MAKROSKOPISKA SKALOR!

Bara den klassiska vägen "överlever" i gränsen $\hbar \rightarrow 0$.
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