EUROPHYSICS LETTERS 10 July 1996

Europhys. Lett., **35** (2), pp. 151-152 (1996)

Comment

Comment on "Quantum coherence in an exactly solvable one-dimensional model with defects"

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(received 1 December 1995; accepted 23 May 1996)

PACS. 71.27+a – Strongly correlated electron systems.
PACS. 71.30+h – Metal-insulator transitions.
PACS. 75.10Jm – Quantized spin models.

Abstract. – In a recent paper by Schmitteckert *et al.*, an exactly solvable XXZ chain with defects was studied. We here point out that, for this type of model, the critical exponents for the asymptotic correlation functions are insensitive to the presence of defects. Also, the defects do not enhance the persistent current of the equivalent fermionic model, contrary to a claim by Schmitteckert *et al.*

In a recent letter [1], Schmitteckert *et al.* study the properties of an exactly solvable XXZ model with defects. The requirement of integrability implies a special structure of the Hamiltonian describing the interaction of host spins with defects: Each defect couples not only to nearest-neighbour, but to next-nearest-neighbour spins as well. Such exactly solvable quantum models are well-known [2]. Schmitteckert *et al.* investigate the finite-size corrections to the ground-state energy in the presence of a magnetic flux, or equivalently, with twisted boundary conditions. The authors conclude that the energy spectrum is independent of the spatial distribution of defects, whereas the finite-size corrections have a nonlinear dependence on the defect density.

Here we want to point out that i) the independence of the spectrum from defect distribution is a well-known feature for this type of model [2], and ii) that the nonlinear dependence on the defect density stems from the presence of a nonuniversal parameter (playing the role of a *Fermi velocity* in the equivalent fermionic model). This, however, does *not* influence the conformal dimensions governing the critical correlation functions, which are the same for the model with and without defects. In addition, we correct an error in the analysis in [1].

The *Bethe ansatz* equations (BAE) used in [1] have the form

$$\left[\frac{\cosh(\lambda_j - i\eta)}{\cosh(\lambda_j + i\eta)}\right]^{M-r} \prod_{l=1}^{r} \frac{\cosh(\lambda_j + \nu_{n_l} - i\eta)}{\cosh(\lambda_j + \nu_{n_l} + i\eta)} = \exp[i\phi] \prod_{k=1,k\neq j}^{N} \frac{\sinh(\lambda_j - \lambda_k - 2i\eta)}{\sinh(\lambda_j - \lambda_k + 2i\eta)}\,, \qquad (1)$$

where M, r, and N are the number of lattice sites, defects, and down-spins (or fermions in the equivalent model of spinless fermions), respectively, η is the nearest-neighbour coupling,

ν_{n_l} is the strength of a defect at site n_l, λ_j are "rapidities" which parameterize eigenfunctions and eigenvalues, and ϕ is proportional to an external flux [3]. Equations (1) have the same structure as the BAE used in [2], [4] (with defect spin $S = 1/2$) and in [5] (with two coupled chains with unequal number of spins). Thus, the thermodynamic limit of the ground-state energy of [1] coincides with that of ref. [2], [4], [5] (taking into account the choice of the ν_{n_l} distribution in [1]).

As is well known [6], the finite-size corrections from the BAE (1) determine the conformal dimensions $\Delta^{\pm}$ which govern the asymptotic correlation functions. Specifically, $\Delta^{\pm}$ depend only on a universal quantity, the *dressed charge* Z, as

$$2\Delta^{\pm} = 2I^{\pm} + (d/2Z(Q))^2 + (Z(Q)l)^2 \pm dl\,. \tag{2}$$

Here d measures the change of particle number in the excited state with respect to the ground state, l is the total number of transitions from the right to the left Fermi branches, and $I^{\pm}$ label excitations with momenta in the vicinity of $\pm k_{\mathrm{F}}$ [2], [6]. The dressed charge $Z(Q)$ is obtained from eqs. (1) via the integral equation

$$Z(\lambda) - (2\pi)^{-1}\int_{-Q}^{Q} K(\lambda-\mu)Z(\mu)\mathrm{d}\mu = 1\,, \tag{3}$$

where *only the rhs* of eqs. (1) determines the kernel K of eq. (3). As the defects change only the lhs of eqs. (1) (the driving term), the dressed charge and *the resulting conformal dimensions are the same for the model with and without defects.* The same holds for exactly solvable multichain systems [5]. (Note that for defects with different spin S the ground state is formed by Dirac seas of strings of length 1 and $2S$ [7], and one therefore has a dressed charge matrix instead of a one-parameter dressed charge.)

The nonlinear dependence of finite-size corrections on the defect concentration x, observed in [1], is a simple consequence of the rescaling of the Fermi velocity v_{F} with x: $v_{\mathrm{F}} = \epsilon'(Q)/2\pi\rho(Q)$, where ϵ and ρ are defined in eqs. (5)-(8) in [1], with $\rho(Q)$ having a nonlinear dependence on x. (For example, for the nonmagnetic case, $Q=\infty$, and $Z^2 = \pi/2(\pi - 2\eta)$ [6].) This does *not* affect the critical behaviour of the theory.

We finally point out that there is *no* enhancement of persistent currents in the system with defects (as compared to the case with no defects). The enhancement observed in [1] is an artifact produced by the increase of magnetization (or density of fermions) when taking the limit $|\nu_n| \equiv \nu \to \infty$, for all n. Thus, this limit does *not* correspond to a nonmagnetic situation (in the thermodynamic sense).

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