Comment on "Kondo Resonance in a Mesoscopic Ring Coupled to a Quantum Dot: Exact Results for the Aharonov-Bohm-Casher Effects"

In a recent Letter [1] calculations are presented, using the Bethe ansatz, for the zero temperature persistent current in a finite ring of length L with a "side-coupled" quantum dot, in the Kondo regime. The results obtained are in disagreement with those that we obtained in Refs. [2,3] using essentially the same model.

Eckle *et al.* (EJS) argue that the persistent current for this system is that of an ideal ring, of $O(ev_F/L)$, at least when the flux is an integral number of half flux quanta and the ring is sufficiently large. In the latter papers we show that the persistent current is much different depending on whether the circumference of the ring is larger or smaller than the size of the Kondo screening cloud, ξ_K. In the limit of a large ring, we find that the persistent current vanishes more rapidly than $1/L$. Our argument for this relies on the well-known fact that the transmission amplitude at the Fermi energy, and hence the Landauer conductance at $T = 0$, vanishes due to the Kondo effect. This is a direct consequence of the renormalization of the Kondo coupling to infinity at zero temperature, leading to a $\pi/2$ phase shift in the even parity channel at the Fermi energy and hence vanishing transmission amplitude at the Fermi surface. We further argued that the persistent current should vanish when the conductance does since they are both determined by the transmission amplitude at the Fermi surface. For a nonmagnetic impurity, the persistent current was shown to be completely determined [to $O(1/L)$] by the transmission amplitude at the Fermi surface by Gogolin and Prokof'ev (GP) [4]. This is a nontrivial result since current is carried by all states below the Fermi surface but a near cancellation occurs. To $O(1/L)$ the sum of currents reduces to the integral of a perfect derivative, leaving a surface term which can be expressed in terms of the transmission amplitude at the Fermi surface. We argue that this calculation also applies to a magnetic impurity in the limit $L \gg \xi_K$, because its universal low energy properties are those of a nonmagnetic impurity with zero transmission amplitude. We checked that the persistent current is indeed such a universal property by demonstrating that weak coupling perturbation theory can be rearranged into a series in the renormalized Kondo coupling constant at scale L. Hence, for $L \ll \xi_K$ when the renormalized Kondo coupling is small, the persistent current is almost that of an idea ring up to a small reduction. This reduction increases as the renormalized Kondo coupling increases, leading finally to vanishing jL in the limit $L \gg \xi_K$. We note that the above argument is quite analogous to the Langreth proof of the Friedel sum rule for an Anderson impurity [5].

One possible reason why the opposite result to ours is obtained in [1] is the assumption of a strictly linear dispersion relation. This seems to allow the persistent current to be expressed in terms of "excess numbers" of electrons at the left and right Fermi points due to an exact cancellation of currents carried by electrons below the Fermi surface. However, nearly all models, such as the tight binding model, have a dispersion relation which is quadratic near zero wave vector. This fact plays an essential role in the GP result, since the surface term vanishes at $k = 0$ for that reason. A more subtle cancellation of current from states below the Fermi surface occurs in the GP derivation, ultimately leading to their formula for the current in terms of the transmission amplitude at the Fermi surface, rather than in terms of excess electron numbers at the Fermi surface as EJS find. We expect that the assumptions of EJS would lead to the wrong result, even for a nonmagnetic impurity. Three other groups have studied this problem, all of them agreeing with EJS [6–8], but we do not elaborate on our disagreements with those papers here. A slave-boson mean field theory was applied to the case of the embedded quantum dot in [9], obtaining agreement with our results for that related problem. We have applied this approximation to the side-coupled quantum dot obtaining results consistent with our general arguments [10]. We hope that large scale numerical simulations will eventually be able to resolve this disagreement.

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