

# Kondo Physics in 2D Topological Insulators with Rashba Interactions

Erik Eriksson (University of Gothenburg)  
Anders Ström (currently at TU Braunschweig)  
Girish Sharma (currently at Clemson University)  
Henrik Johannesson (University of Gothenburg)

# Outline

---

2D topological insulators... some basics

# Outline

---

2D topological insulators... some basics

**At the edge: A new kind of electron liquid**

# Outline

---

2D topological insulators... some basics

At the edge: A new kind of electron liquid

**Adding a magnetic impurity...**

# Outline

---

2D topological insulators... some basics

At the edge: A new kind of electron liquid

Adding a magnetic impurity...

**... and a Rashba spin-orbit interaction**

# Outline

---

2D topological insulators... some basics

At the edge: A new kind of electron liquid

Adding a magnetic impurity...

... and a Rashba spin-orbit interaction

**Electrical control of the Kondo effect!**

# Outline

---

2D topological insulators... some basics

At the edge: A new kind of electron liquid

Adding a magnetic impurity...

... and a Rashba spin-orbit interaction

Electrical control of the Kondo effect!

**Summary and outlook**

# Outline

---

2D topological insulators... some basics

At the edge: A new kind of electron liquid

Adding a magnetic impurity...

... and a Rashba spin-orbit interaction

Electrical control of the Kondo effect!

Summary and outlook





**Topological Phases in Condensed Matter and Cold Atom Systems**



## Topological Phases in Condensed Matter and Cold Atom Systems

”topologically ordered”

FQHE,  $Z_2$  spin liquids,...

”Thouless type”

IQHE,...

”symmetry protected”

topological insulators,...

...

”symmetry protected”

# topological insulators

*protected by time-reversal invariance*

# 2D topological insulators

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005)

B. A. Bernevig and S. C. Zhang, Phys. Rev. Lett. **96**, 106802 (2006)

B. A. Bernevig, T. A. Hughes, and S. C. Zhang, Science **314**, 1757 (2006)

*experiment:* M. König *et al.*, Science **318**, 766 (2007)

C. Brüne *et al.*, Nature Physics **8**, 486 (2012)

# 2D topological insulators

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

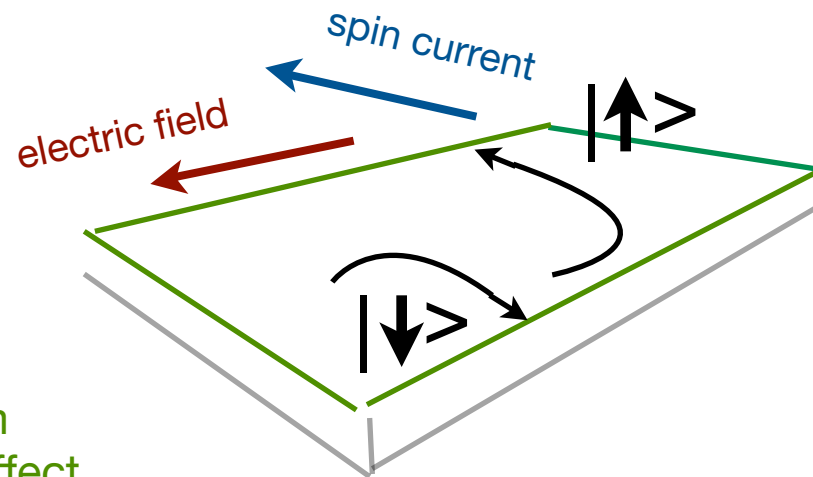
C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005)

B. A. Bernevig and S. C. Zhang, Phys. Rev. Lett. **96**, 106802 (2006)

B. A. Bernevig, T. A. Hughes, and S. C. Zhang, Science **314**, 1757 (2006)

*experiment:* M. König *et al.*, Science **318**, 766 (2007)

C. Brüne *et al.*, Nature Physics **8**, 486 (2012)



**"reference state":**  
2D bulk insulator  
showing quantum  
spin Hall (QSH) effect

$$\sigma_{xy}^s = \pm \frac{e}{2\pi}$$

# 2D topological insulators

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005)

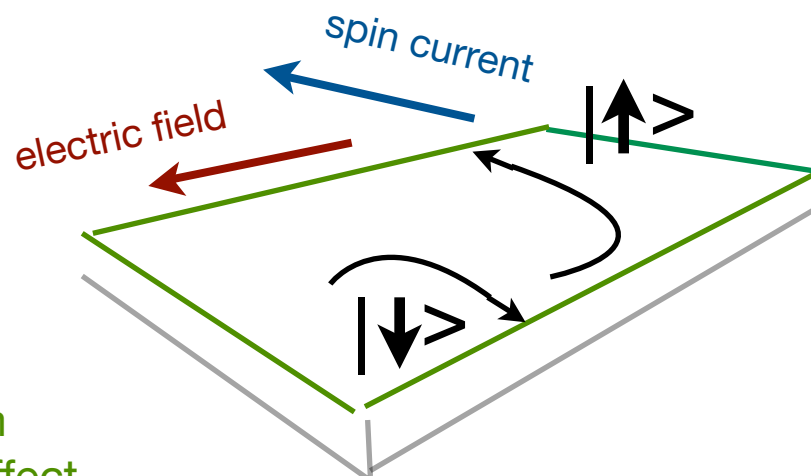
B. A. Bernevig and S. C. Zhang, Phys. Rev. Lett. **96**, 106802 (2006)

B. A. Bernevig, T. A. Hughes, and S. C. Zhang, Science **314**, 1757 (2006)

experiment: M. König *et al.*, Science **318**, 766 (2007)

C. Brüne *et al.*, Nature Physics **8**, 486 (2012)

"reference state":  
2D bulk insulator  
showing quantum  
spin Hall (QSH) effect



$$\sigma_{xy}^s = C_s \frac{e}{4\pi}, \quad C_s = \begin{cases} \pm 2 & \text{if QSH insulator} \\ 0 & \text{if ordinary insulator} \end{cases}$$

# 2D topological insulators

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005)

B. A. Bernevig and S. C. Zhang, Phys. Rev. Lett. **96**, 106802 (2006)

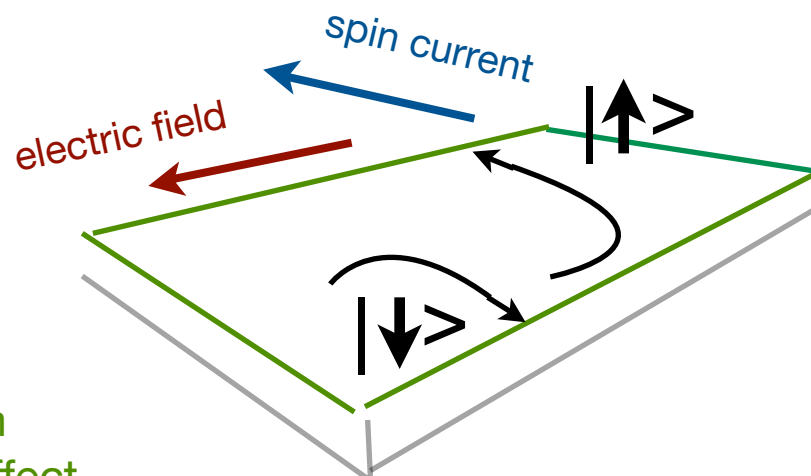
B. A. Bernevig, T. A. Hughes, and S. C. Zhang, Science **314**, 1757 (2006)

experiment: M. König *et al.*, Science **318**, 766 (2007)

C. Brüne *et al.*, Nature Physics **8**, 486 (2012)

Perturb adiabatically with a  
time-reversal invariant spin-  
nonconserving interaction

"reference state":  
2D bulk insulator  
showing quantum  
spin Hall (QSH) effect



$$\sigma_{xy}^s = C_s \frac{e}{4\pi}, \quad C_s = \begin{cases} \pm 2 & \text{if QSH insulator} \\ 0 & \text{if ordinary insulator} \end{cases}$$

# 2D topological insulators

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005)

B. A. Bernevig and S. C. Zhang, Phys. Rev. Lett. **96**, 106802 (2006)

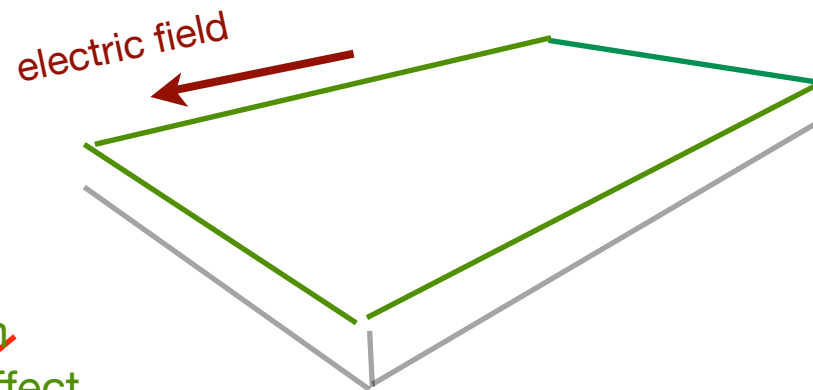
B. A. Bernevig, T. A. Hughes, and S. C. Zhang, Science **314**, 1757 (2006)

*experiment:* M. König *et al.*, Science **318**, 766 (2007)

C. Brüne *et al.*, Nature Physics **8**, 486 (2012)

Perturb adiabatically with a  
time-reversal invariant spin-  
nonconserving interaction

"reference state":  
2D bulk insulator  
showing quantum  
spin Hall (QSH) effect



$$\sigma_{xy}^s = C_s \frac{e}{4\pi}, \quad C_s = \begin{cases} \pm 2 & \text{if QSH insulator} \\ 0 & \text{if ordinary insulator} \end{cases}$$



# 2D topological insulators

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005)

B. A. Bernevig and S. C. Zhang, Phys. Rev. Lett. **96**, 106802 (2006)

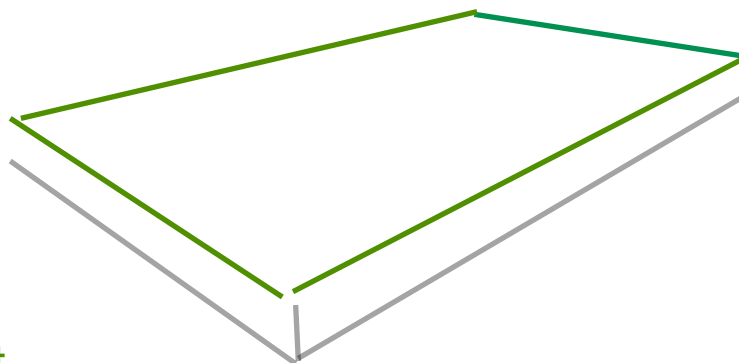
B. A. Bernevig, T. A. Hughes, and S. C. Zhang, Science **314**, 1757 (2006)

experiment: M. König *et al.*, Science **318**, 766 (2007)

C. Brüne *et al.*, Nature Physics **8**, 486 (2012)

Perturb adiabatically with a  
time-reversal invariant spin-  
nonconserving interaction

"reference state":  
2D bulk insulator  
showing quantum  
spin Hall (QSH) effect



$$\sigma_{xy}^s = C_s \frac{e}{4\pi}, \quad C_s = \begin{cases} \pm 2 & \text{if QSH insulator} \\ 0 & \text{if ordinary insulator} \end{cases}$$

$$\nu = \begin{cases} 1 & \text{"2D topological insulator"} \\ 0 & \text{ordinary insulator} \end{cases}$$

# 2D topological insulators

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005)

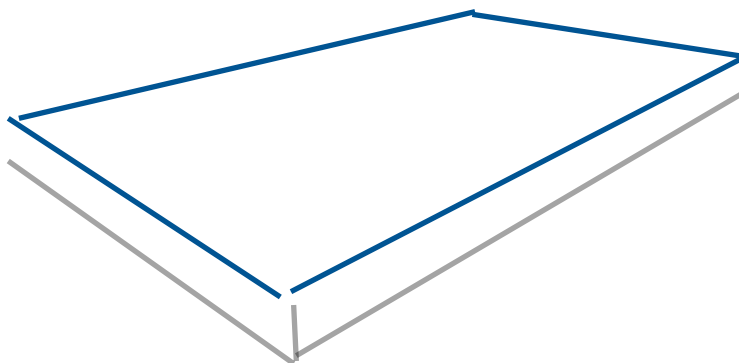
B. A. Bernevig and S. C. Zhang, Phys. Rev. Lett. **96**, 106802 (2006)

B. A. Bernevig, T. A. Hughes, and S. C. Zhang, Science **314**, 1757 (2006)

*experiment:* M. König *et al.*, Science **318**, 766 (2007)

C. Brüne *et al.*, Nature Physics **8**, 486 (2012)

**2D topological insulator**  
"adiabatically connected"  
to an ideal quantum spin  
Hall (QSH) insulator



$$\nu = \begin{cases} 1 & \text{"2D topological insulator"} \\ 0 & \text{ordinary insulator} \end{cases}$$

**$Z_2$  topological invariant**

encodes Berry curvature structure of the bulk bands

# 2D topological insulators

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

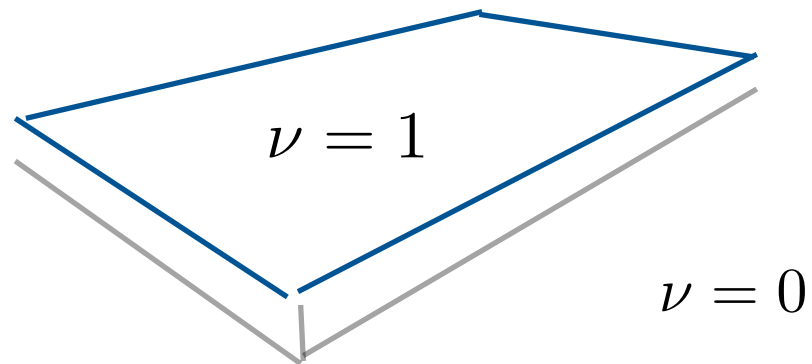
C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005)

B. A. Bernevig and S. C. Zhang, Phys. Rev. Lett. **96**, 106802 (2006)

B. A. Bernevig, T. A. Hughes, and S. C. Zhang, Science **314**, 1757 (2006)

*experiment:* M. König *et al.*, Science **318**, 766 (2007)

C. Brüne *et al.*, Nature Physics **8**, 486 (2012)



# 2D topological insulators

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

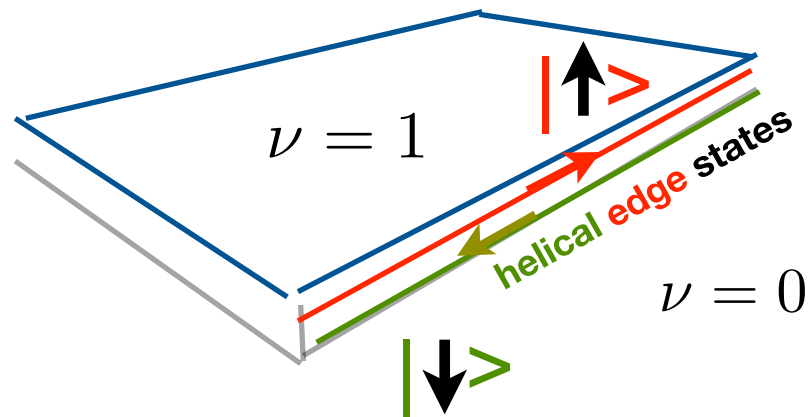
C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005)

B. A. Bernevig and S. C. Zhang, Phys. Rev. Lett. **96**, 106802 (2006)

B. A. Bernevig, T. A. Hughes, and S. C. Zhang, Science **314**, 1757 (2006)

*experiment:* M. König *et al.*, Science **318**, 766 (2007)

C. Brüne *et al.*, Nature Physics **8**, 486 (2012)



# 2D topological insulators

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

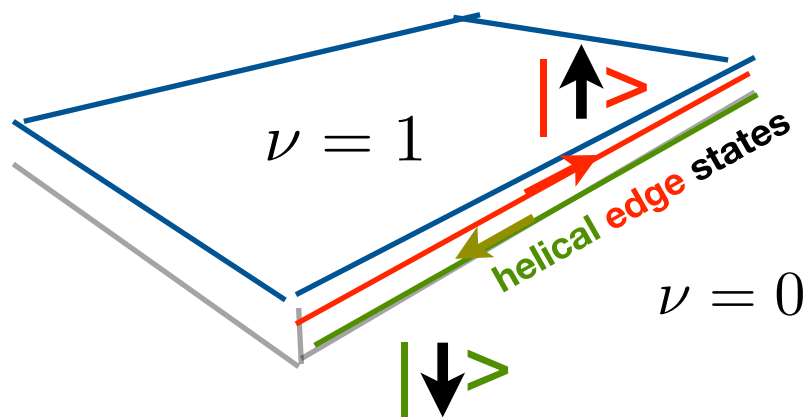
C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005)

B. A. Bernevig and S. C. Zhang, Phys. Rev. Lett. **96**, 106802 (2006)

B. A. Bernevig, T. A. Hughes, and S. C. Zhang, Science **314**, 1757 (2006)

experiment: M. König *et al.*, Science **318**, 766 (2007)

C. Brüne *et al.*, Nature Physics **8**, 486 (2012)



are **pseudospins** that keep track on the total angular momentum sectors (good quantum numbers in the presence of spin-nonconserving interactions)

# 2D topological insulators

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

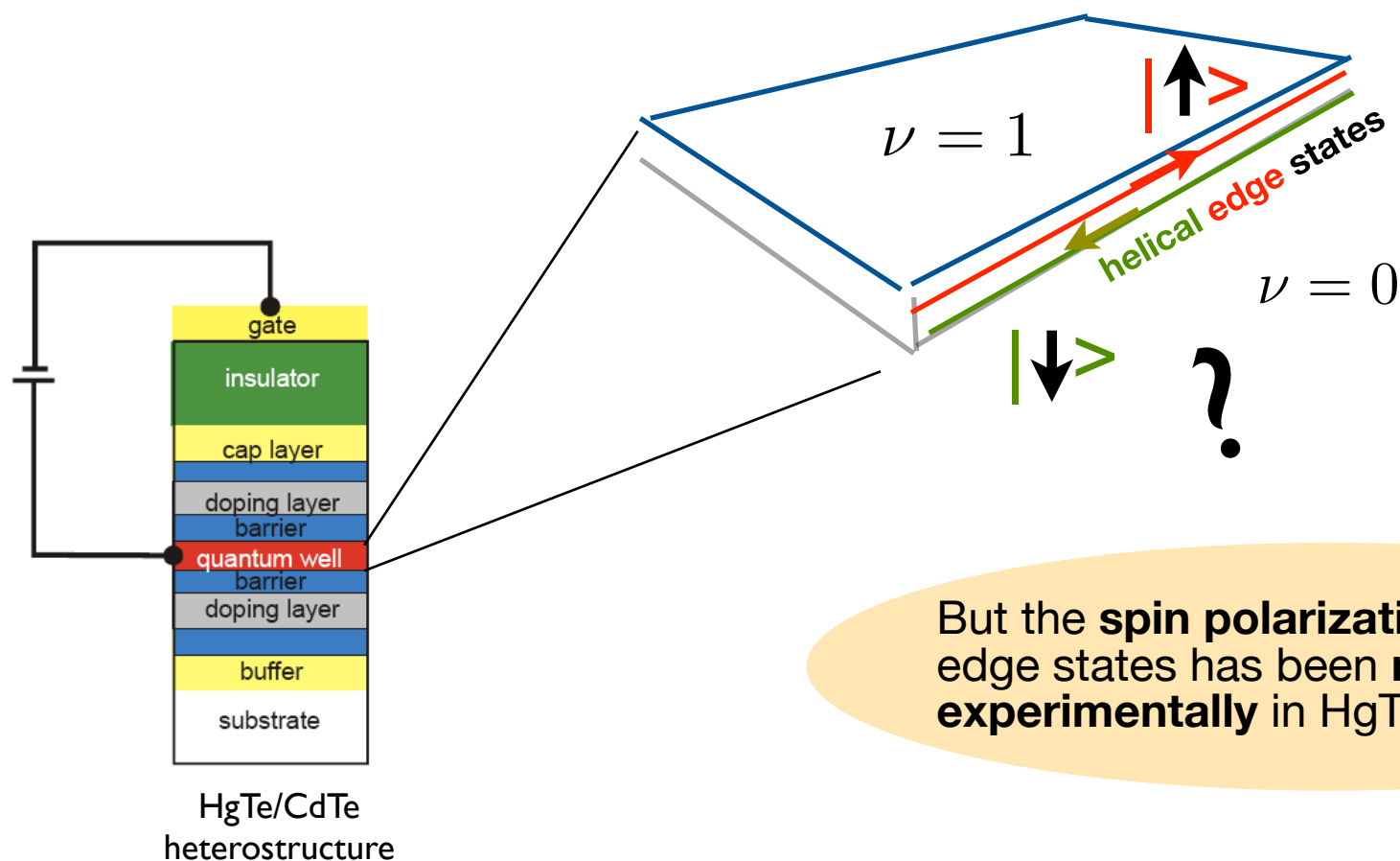
C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005)

B. A. Bernevig and S. C. Zhang, Phys. Rev. Lett. **96**, 106802 (2006)

B. A. Bernevig, T. A. Hughes, and S. C. Zhang, Science **314**, 1757 (2006)

experiment: M. König *et al.*, Science **318**, 766 (2007)

**C. Brüne *et al.*, Nature Physics **8**, 486 (2012)**



# 2D topological insulators

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988)

C. L. Kane and E. J. Mele, Phys. Rev. Lett. **95**, 226801 (2005)

B. A. Bernevig and S. C. Zhang, Phys. Rev. Lett. **96**, 106802 (2006)

**B. A. Bernevig, T. A. Hughes, and S. C. Zhang, Science 314, 1757 (2006)**

”BHZ model”

$$|\uparrow\rangle |+\rangle = \Psi_1 |E_{1+}\rangle + \Psi_2 |H_{1+}\rangle$$

$$|\downarrow\rangle |-\rangle = \hat{T} |+\rangle = -\Psi_1^* |E_{1-}\rangle - \Psi_2^* |H_{1-}\rangle$$

$$|E_{1\pm}\rangle = \alpha |\Gamma_{6, \pm \frac{1}{2}}\rangle + \beta |\Gamma_{8, \pm \frac{1}{2}}\rangle$$

$$|H_{1\pm}\rangle = \alpha |\Gamma_{8, \pm \frac{3}{2}}\rangle$$

$$\begin{aligned} \langle + | \mathbf{S} | + \rangle &= \frac{\beta}{\sqrt{3}} (\Psi_2^* \Psi_1 + \Psi_1^* \Psi_2) \hat{X} \\ &+ \frac{i\beta}{\sqrt{3}} (-\Psi_2^* \Psi_1 + \Psi_1^* \Psi_2) \hat{Y} \\ &+ (|\Psi_2|^2 [1 + |\alpha|^2] + \frac{|\Psi_1 \beta|^2}{3}) \hat{Z} \\ \langle - | \mathbf{S} | - \rangle &= -\langle + | \mathbf{S} | + \rangle \end{aligned}$$

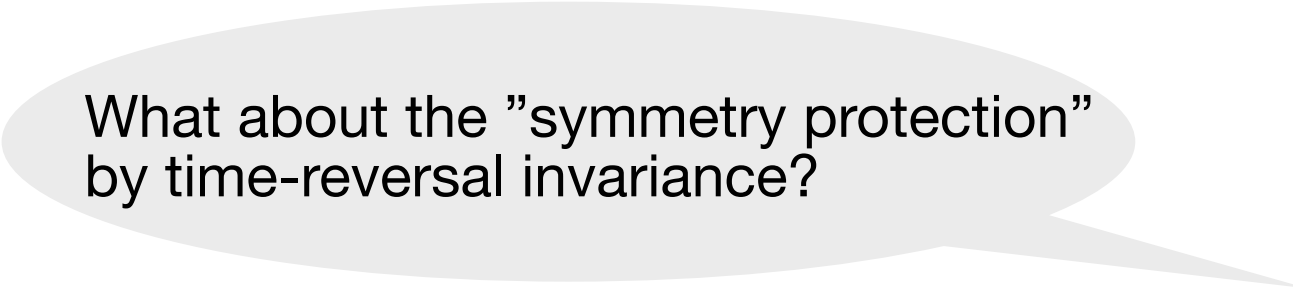


choose spin quantization axis along  $\langle + | \mathbf{S} | + \rangle$

spin-polarized helical edge states for **constant  $\Psi_{1,2}$**

*OK in a small energy range!*

P. Michetti and P. Recher, Phys. Rev. B **83**, 125420 (2011)



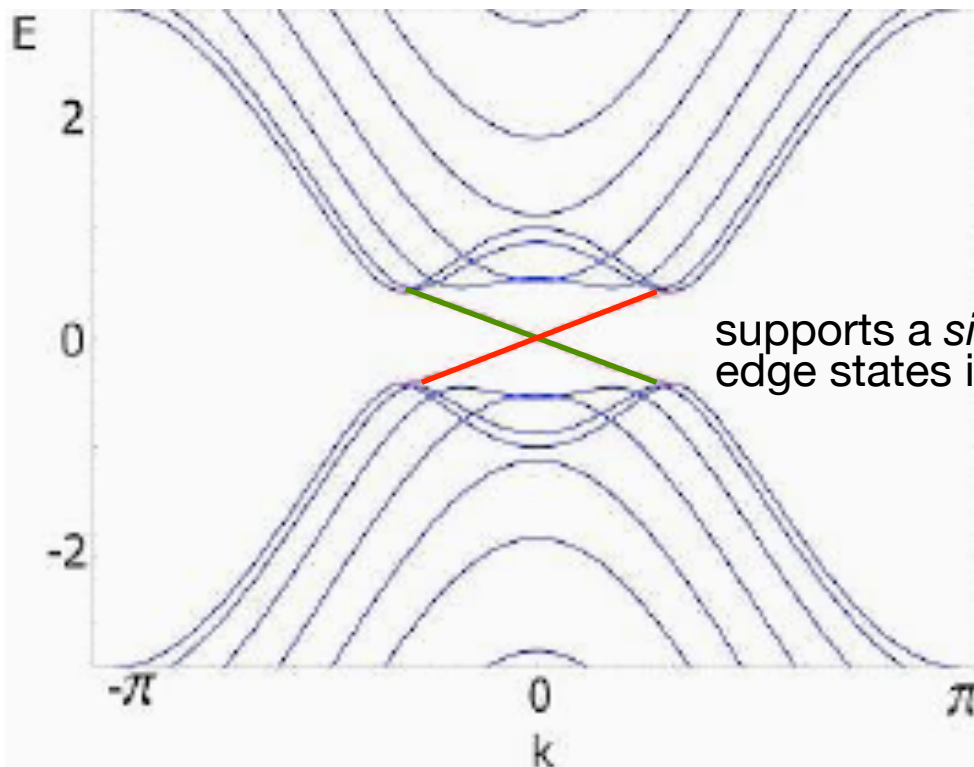
What about the "symmetry protection"  
by time-reversal invariance?



What about the "symmetry protection" by time-reversal invariance?

The helical edge states are stable against time-reversal invariant perturbations!

strong spin-orbit interactions in atomic p-orbitals create an *inverted* band gap (p-band on top of s-band)



supports a *single* Kramers pair of helical edge states inside the inverted gap

Kramers degeneracy at  $k=0$  protects the stability of the edge states

ballistic transport

$$G = \frac{2e^2}{h}$$

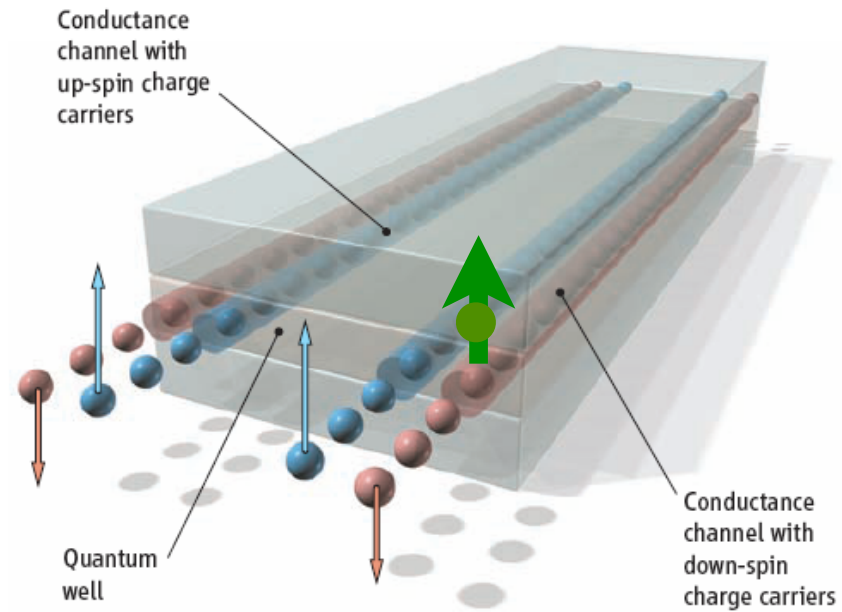
B. A. Bernevig *et al.*, PRL **95**, 066601 (2005)

**The helical edge states are stable against time-reversal invariant perturbations!**

What about the "symmetry protection" by time-reversal invariance?

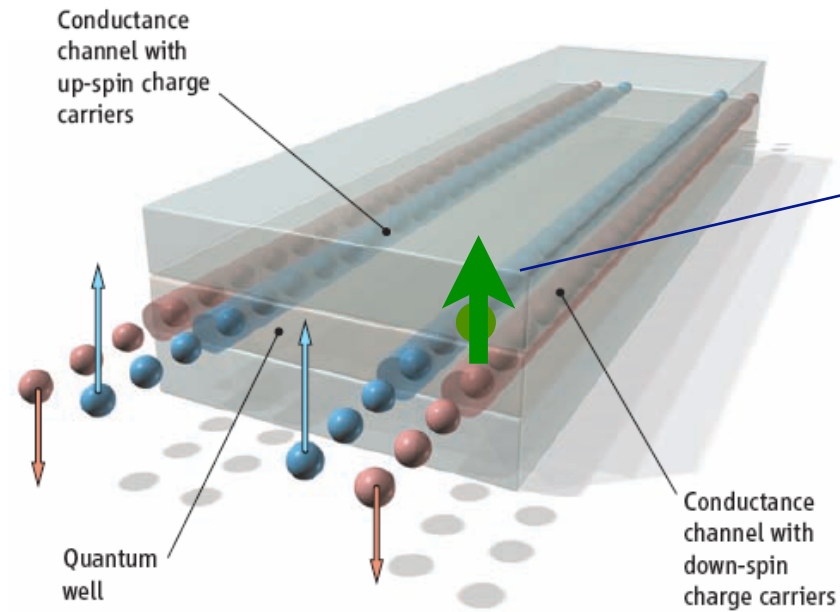
**What if time-reversal symmetry gets broken...?**

... for example, by putting in a **magnetic impurity** at the edge?



from M. König *et al.*, Science **318**, 766 (2007)

... for example, by putting in a **magnetic impurity** at the edge?



from M. König *et al.*, Science **318**, 766 (2007)

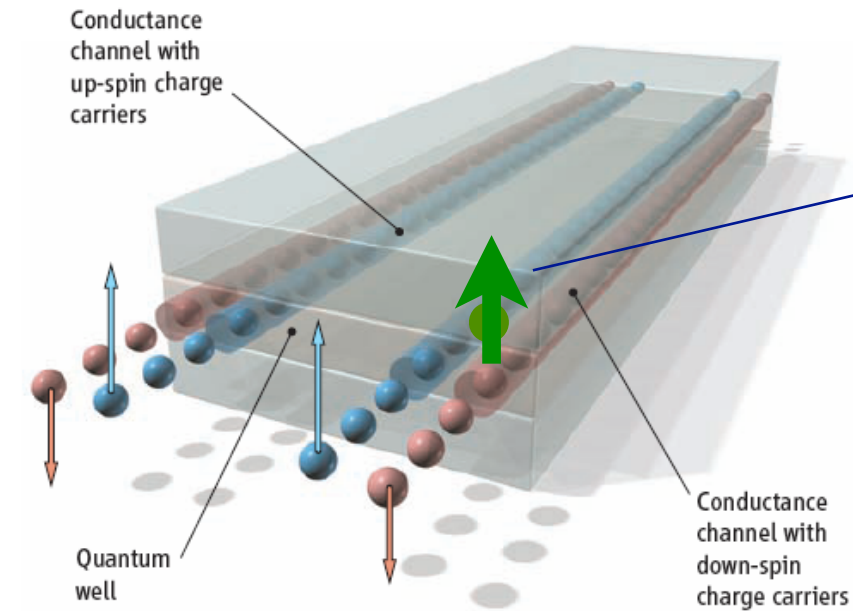
case study:  
 $\text{Mn}^{2+}$

large and positive single-ion anisotropy  $(S^z)^2$   
J. K. Furdyna, JAP **64**, R29 (1988)

$$S = 5/2 \longrightarrow S_{\text{eff}} = 1/2$$

low  $T$

... for example, by putting in a **magnetic impurity** at the edge?



from M. König *et al.*, Science **318**, 766 (2007)

case study:  
 $\text{Mn}^{2+}$

large and positive single-ion anisotropy  $(S^z)^2$

$$S = 5/2 \longrightarrow S_{\text{eff}} = 1/2$$

low  $T$

anisotropic spin exchange with the edge electrons

R. Zitko *et al.*, PRB **78**, 224404 (2008)

$$H_K = \Psi^\dagger(0) \left[ J_\perp (\sigma^+ S_{\text{eff}}^- + \sigma^- S_{\text{eff}}^+) + J_z \sigma^z S_{\text{eff}}^z \right] \Psi(0)$$

$$\Psi^T = (\psi_\uparrow, \psi_\downarrow)$$

# Adding a magnetic impurity...

The Kondo interaction is time-reversal invariant!  
Could it still cause a *spontaneous* breaking of time reversal invariance and localize the edge states?

# Adding a magnetic impurity...

## Recall the Kondo effect

One-loop RG equations:

P. W. Anderson, J. Phys. C **3**, 2436 (1970)

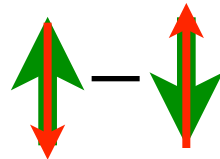
$$\begin{aligned}\frac{\partial J_{\perp}}{\partial D} &= -\nu J_{\perp} J_z + \dots \\ \frac{\partial J_z}{\partial D} &= -\nu J_{\perp}^2 + \dots\end{aligned}$$

strong-coupling physics for  $T \ll T_K$

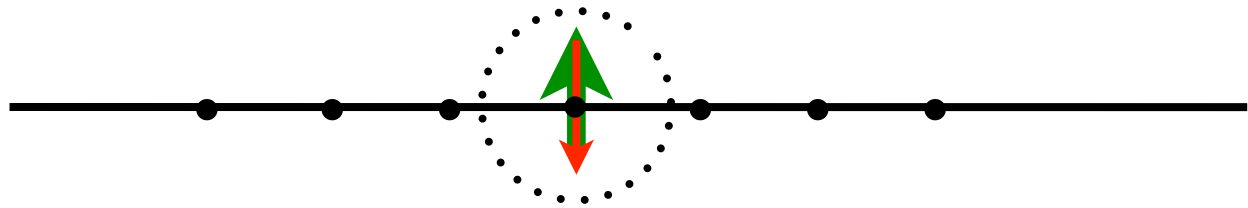
$$T_K = D_0 \exp(-\text{const.}/J_0)$$

$$J_0 \equiv \max(J_{\perp}, J_z)_{D=D_0}$$

formation of impurity-electron singlet (**"Kondo screening"**)

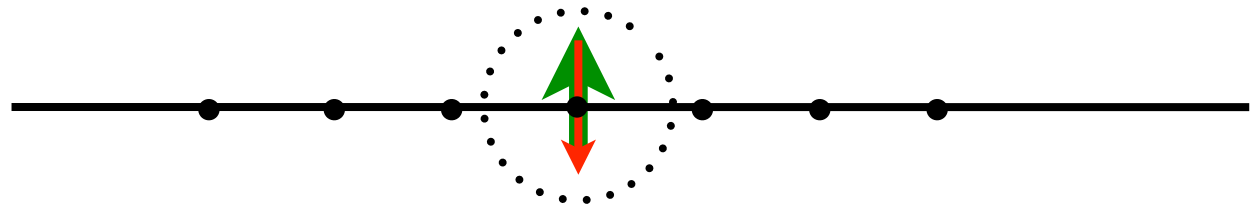


Adding a magnetic impurity...





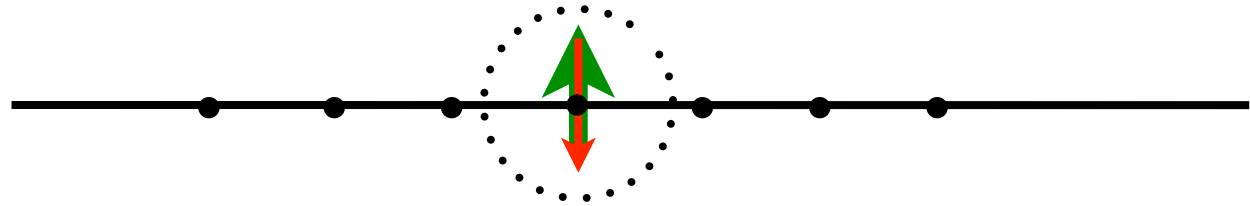
# Adding a magnetic impurity...



Pauli principle:  
punctured 1D lattice



# Adding a magnetic impurity...



*T=0 insulator!*

Pauli principle:  
punctured 1D lattice



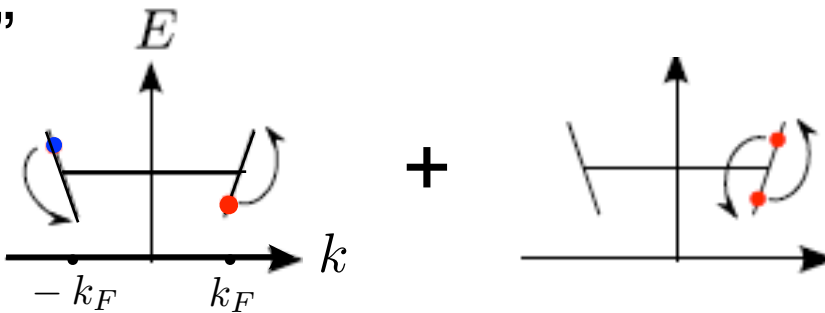
# Adding a magnetic impurity...

Does this really happen for the helical liquid?

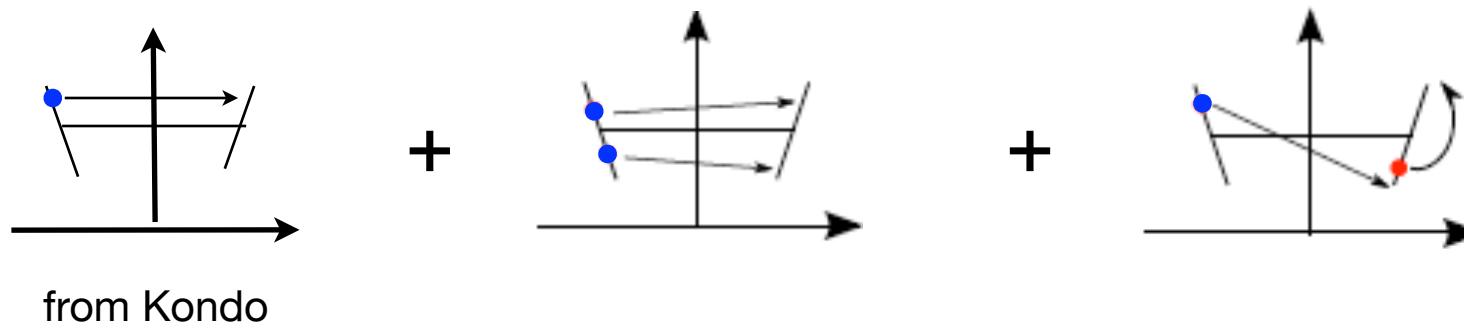
To find out, first add e-e interactions.... important in 1D!



”bulk”



local (at impurity site)



# Adding a magnetic impurity...

Adding the kinetic energy and bosonizing...

$$\begin{aligned} H = & (v/2) \int dx ((\partial_x \varphi)^2 + (\partial_x \vartheta)^2) \\ & + \frac{A}{\kappa} \cos(\sqrt{4\pi K} \varphi) + \frac{B}{\kappa} \sin(\sqrt{4\pi K} \varphi) + \frac{C}{\sqrt{K}} \partial_x \vartheta \quad \text{Kondo} \\ & + \frac{gU}{2(\pi\kappa)^2} \cos(\sqrt{16\pi K} \varphi) \quad \text{local Umklapp} \\ & + \frac{g_{ie}}{2\pi^2 \sqrt{K}} : (\partial_x^2 \vartheta) \cos(\sqrt{4\pi K} \varphi) : \quad \text{local 1-particle inelastic term} \end{aligned}$$

# Adding a magnetic impurity...

## Bosonization...

"Luttinger liquid parameter"

$$\begin{aligned} H = & (v/2) \int dx ((\partial_x \varphi)^2 + (\partial_x \vartheta)^2) \\ & + \frac{A}{\kappa} \cos(\sqrt{4\pi K} \varphi) + \frac{B}{\kappa} \sin(\sqrt{4\pi K} \varphi) + \frac{C}{\sqrt{K}} \partial_x \vartheta \\ & + \frac{gU}{2(\pi\kappa)^2} \cos(\sqrt{16\pi K} \varphi) \\ & + \frac{g_{ie}}{2\pi^2 \sqrt{K}} : (\partial_x^2 \vartheta) \cos(\sqrt{4\pi K} \varphi) : \end{aligned}$$

# Adding a magnetic impurity...

## Bosonization...

$$\begin{aligned} H = & (v/2) \int dx \left( (\partial_x \varphi)^2 + (\partial_x \vartheta)^2 \right) \\ & + \frac{A}{\kappa} \cos(\sqrt{4\pi K} \varphi) + \frac{B}{\kappa} \sin(\sqrt{4\pi K} \varphi) + \frac{C}{\sqrt{K}} \partial_x \vartheta \\ & + \frac{g_U}{2(\pi\kappa)^2} \cos(\sqrt{16\pi K} \varphi) \\ & + \frac{g_{ie}}{2\pi^2 \sqrt{K}} : (\partial_x^2 \vartheta) \cos(\sqrt{4\pi K} \varphi) : \end{aligned}$$

"Luttinger liquid parameter"

functions of Kondo couplings

# Adding a magnetic impurity...

Bosonization...

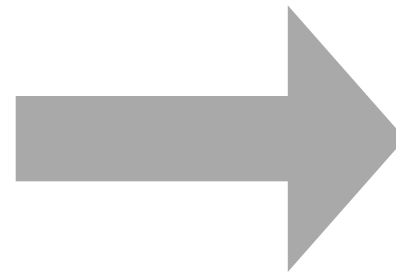
$$\begin{aligned} H = & (v/2) \int dx ((\partial_x \varphi)^2 + (\partial_x \vartheta)^2) \\ & + \frac{A}{\kappa} \cos(\sqrt{4\pi K} \varphi) + \frac{B}{\kappa} \sin(\sqrt{4\pi K} \varphi) + \frac{C}{\sqrt{K}} \partial_x \vartheta \\ & + \frac{g_U}{2(\pi\kappa)^2} \cos(\sqrt{16\pi K} \varphi) \\ & + \frac{g_{ie}}{2\pi^2 \sqrt{K}} : (\partial_x^2 \vartheta) \cos(\sqrt{4\pi K} \varphi) : \end{aligned}$$

"Luttinger liquid parameter"

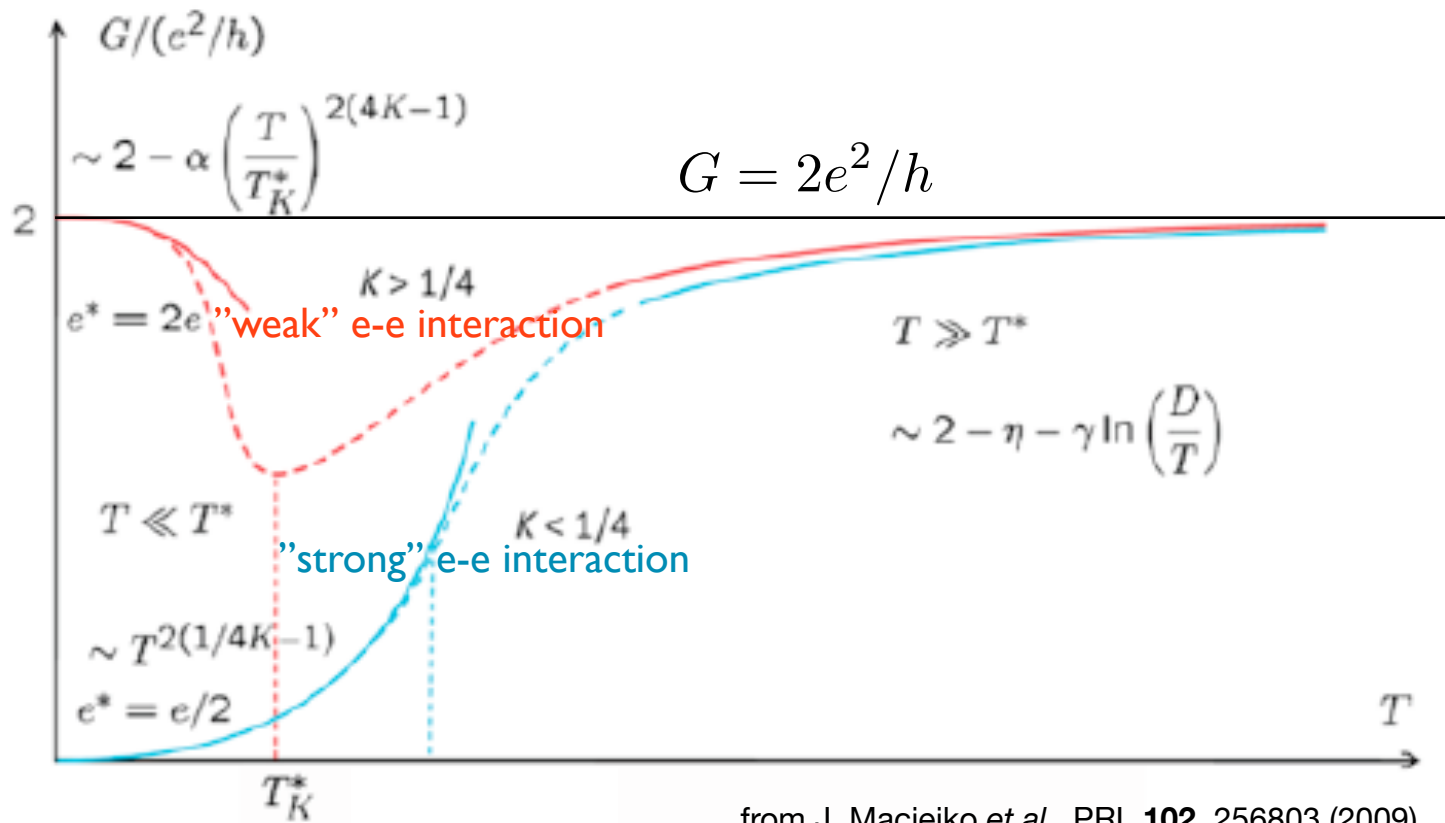
functions of Kondo couplings

...perturbative RG and linear response

J. Maciejko *et al.*, PRL **102**, 256803 (2009)



# Adding a magnetic impurity...

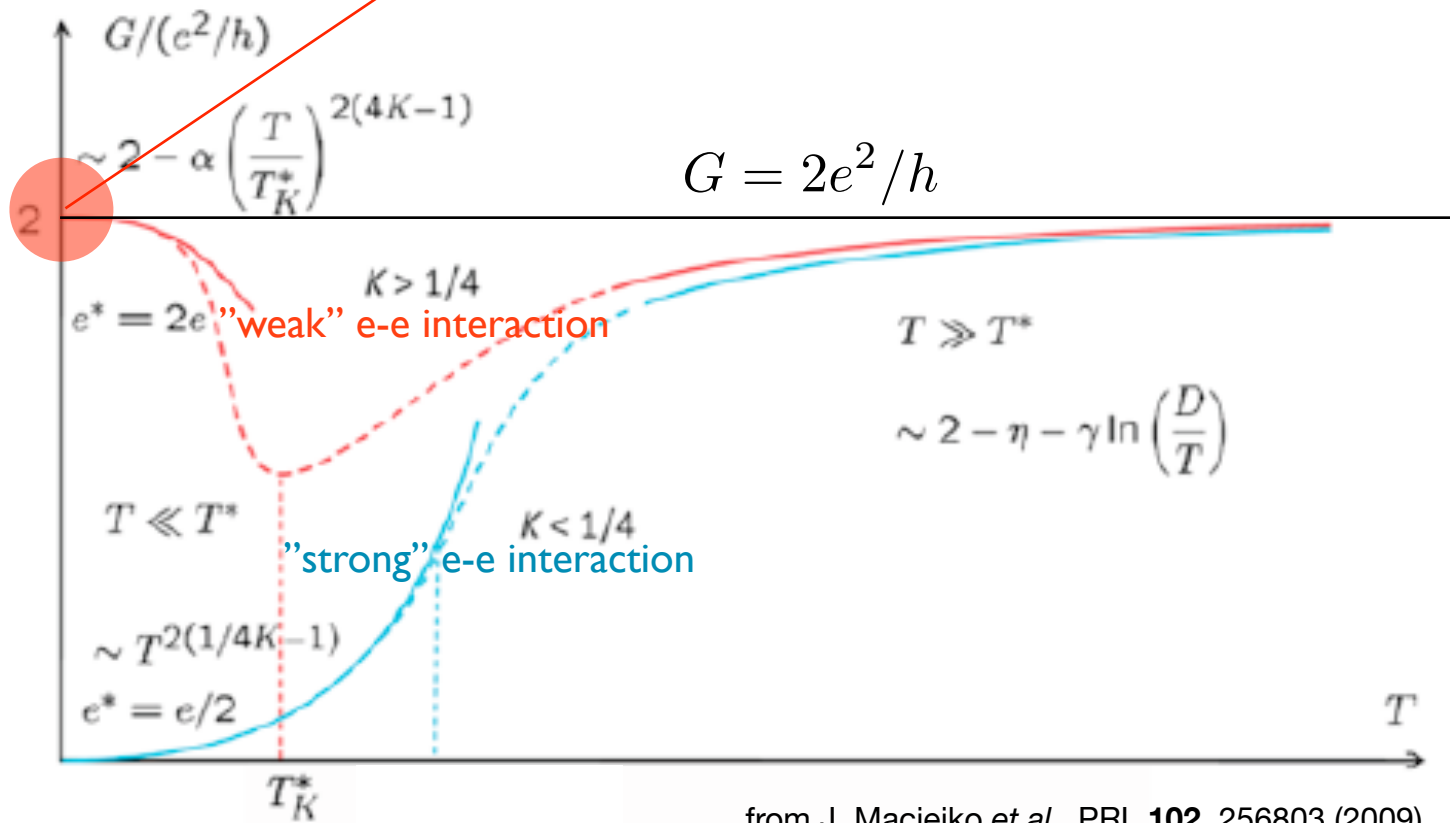


from J. Maciejko et al., PRL **102**, 256803 (2009)



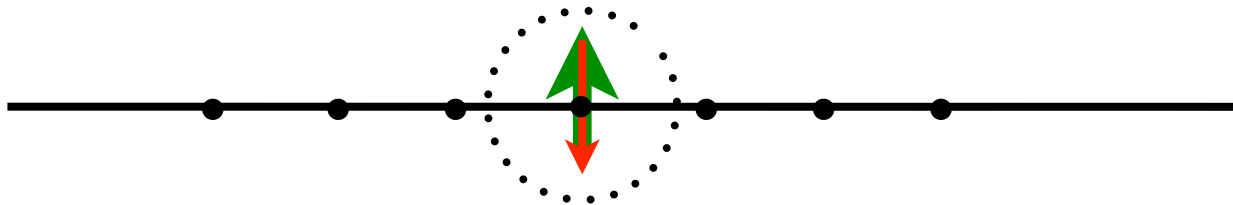
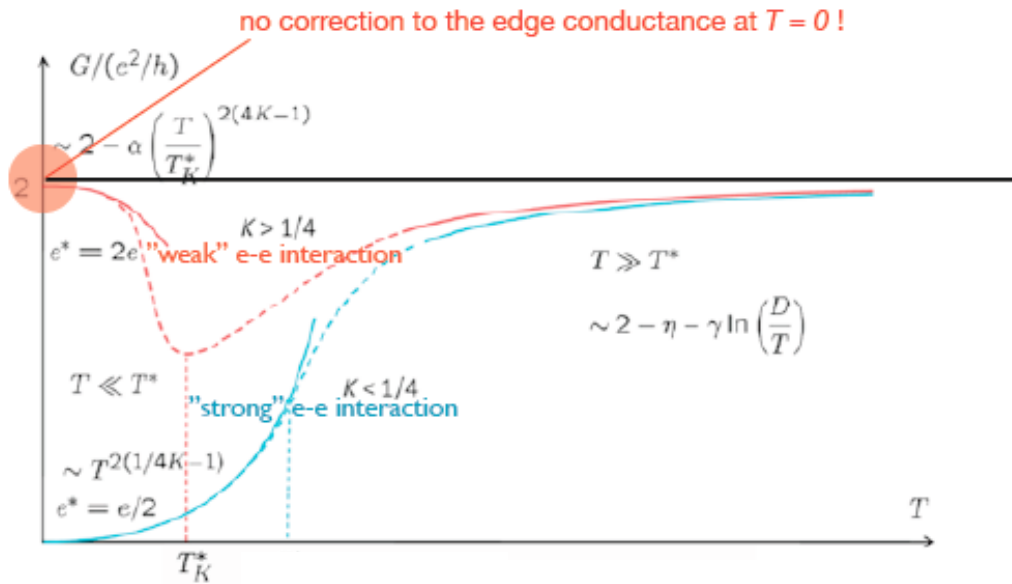
# Adding a magnetic impurity...

no correction to the edge conductance at  $T = 0$  !

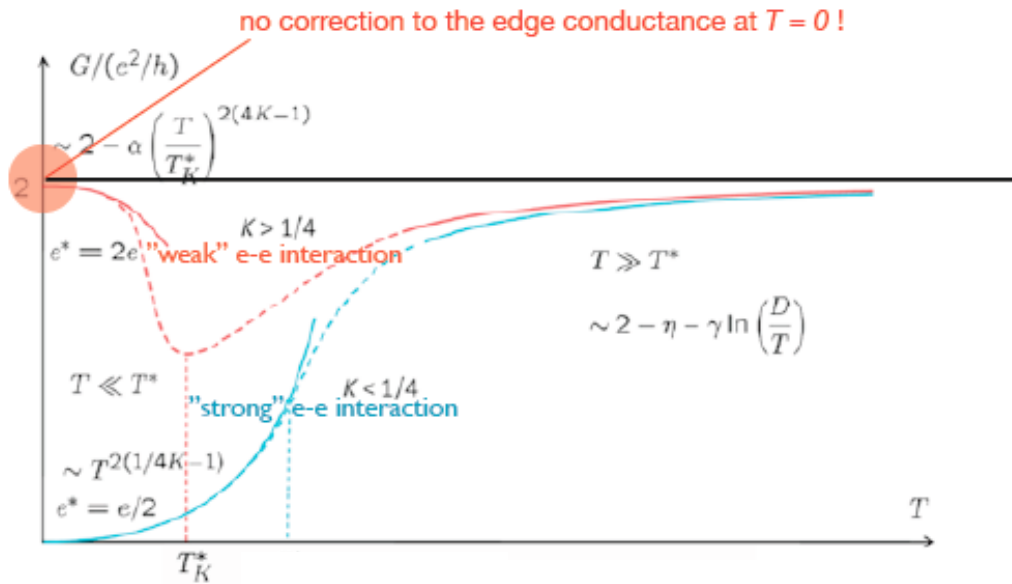


from J. Maciejko et al., PRL **102**, 256803 (2009)

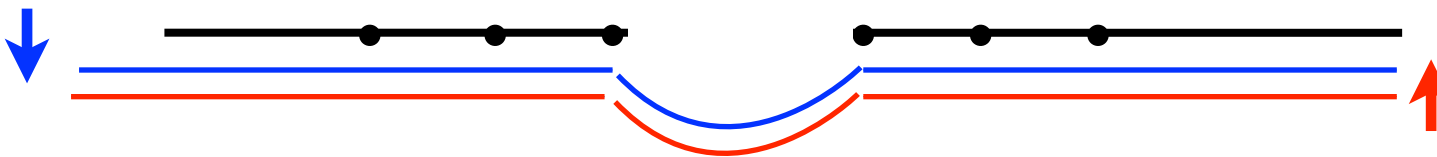
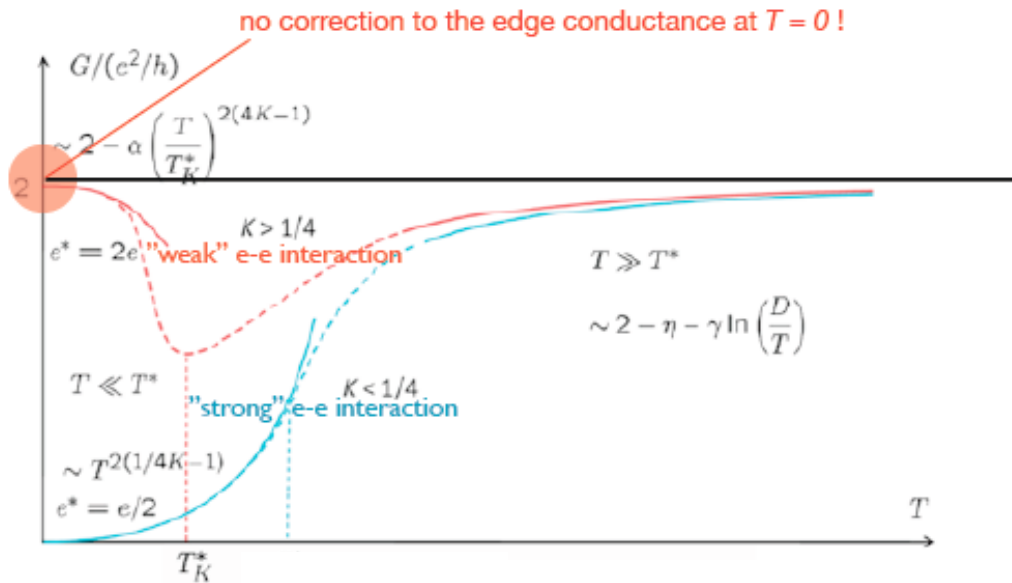
# Adding a magnetic impurity...



# Adding a magnetic impurity...



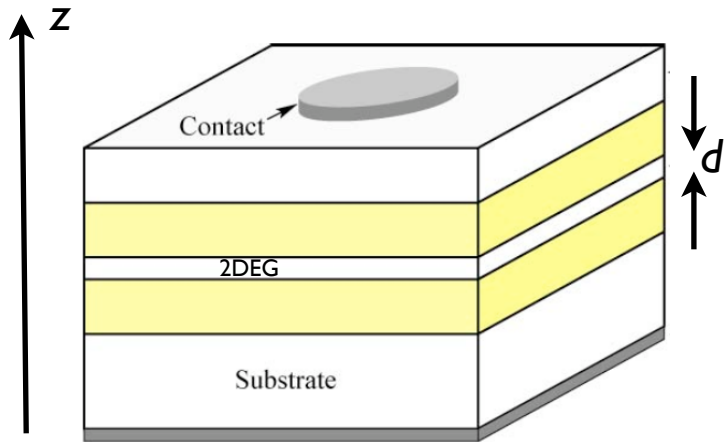
# Adding a magnetic impurity...



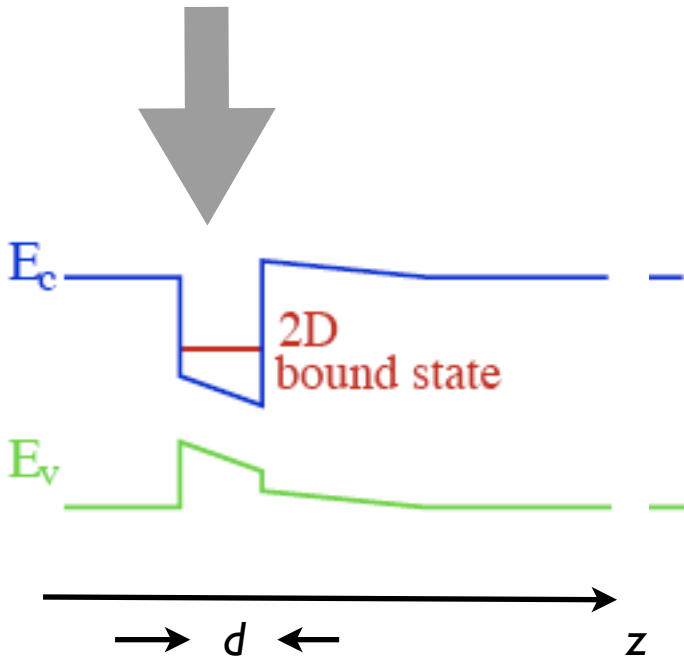
Due to its topological nature, the edge states follow the new shape of the edge.  
**Weak coupling helical edge states are robust against spontaneous breaking of time-reversal symmetry!**

**But...** one important thing is missing from the analysis!

# Rashba spin-orbit interaction!



semiconductor heterostructure

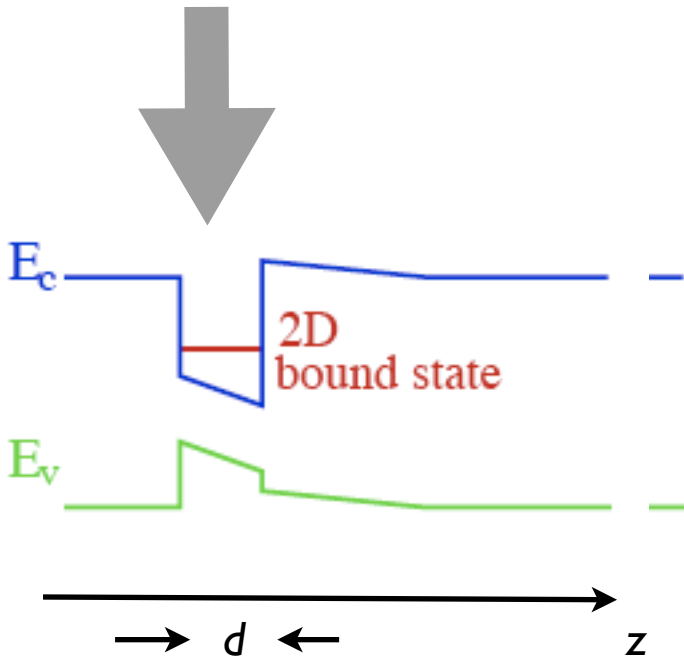
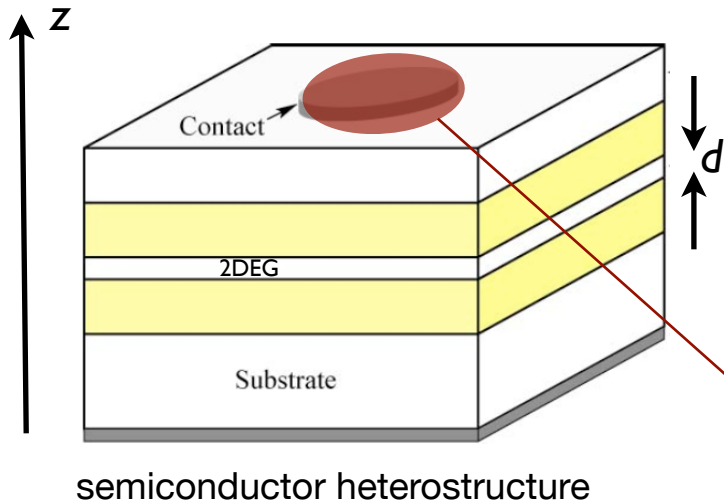


Spatial asymmetry of band edges mimics an  $E$ -field in the  $z$ -direction

$$H_R = \alpha(k_x \sigma^y - k_y \sigma^x)$$

Yu. A. Bychkov and E. I. Rashba,  
J. Phys. C **17**, 6039 (1984)

# Rashba spin-orbit interaction

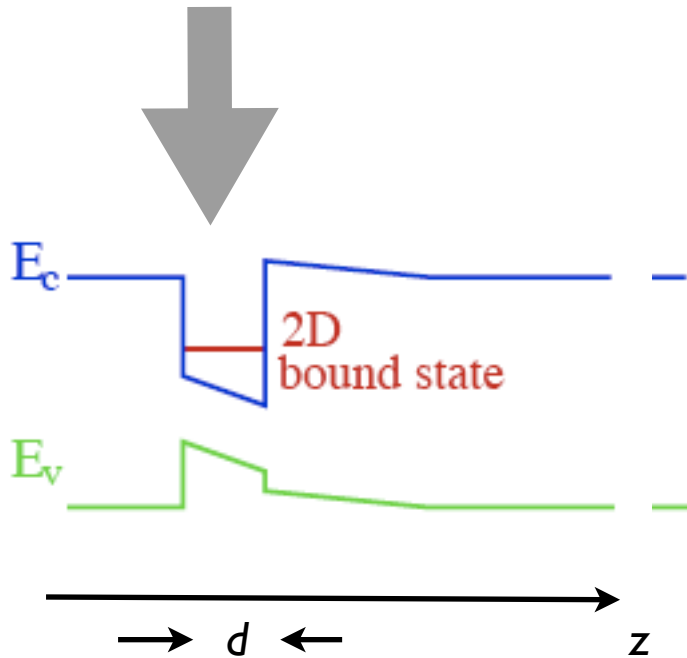
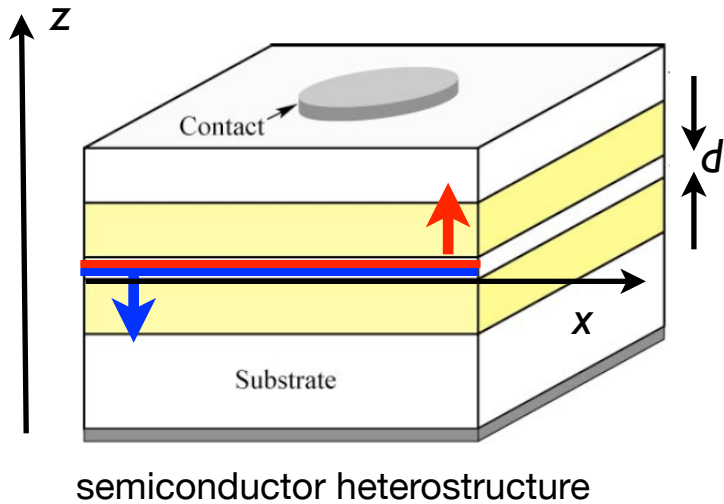


Spatial asymmetry of band edges mimics an  $E$ -field in the  $z$ -direction, **tunable by gate voltage**

$$H_R = \alpha(k_x \sigma^y - k_y \sigma^x)$$

Yu. A. Bychkov and E. I. Rashba,  
J. Phys. C **17**, 6039 (1984)

# Rashba spin-orbit interaction

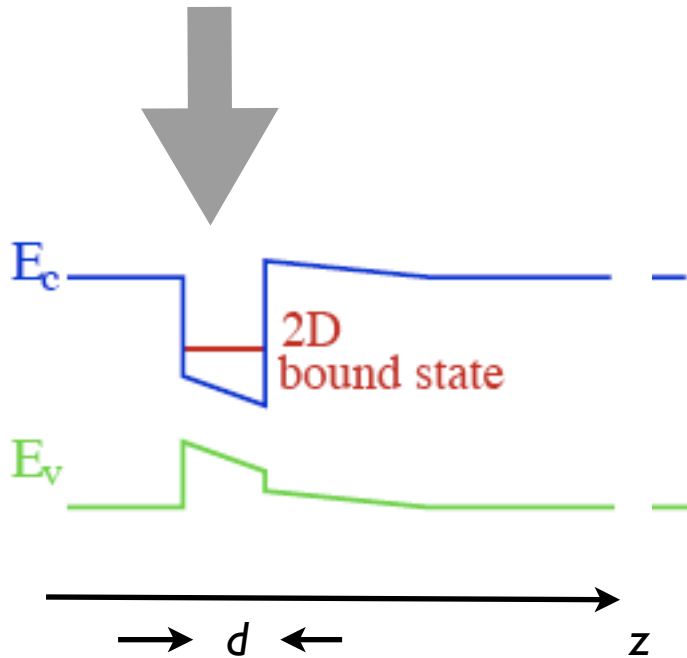
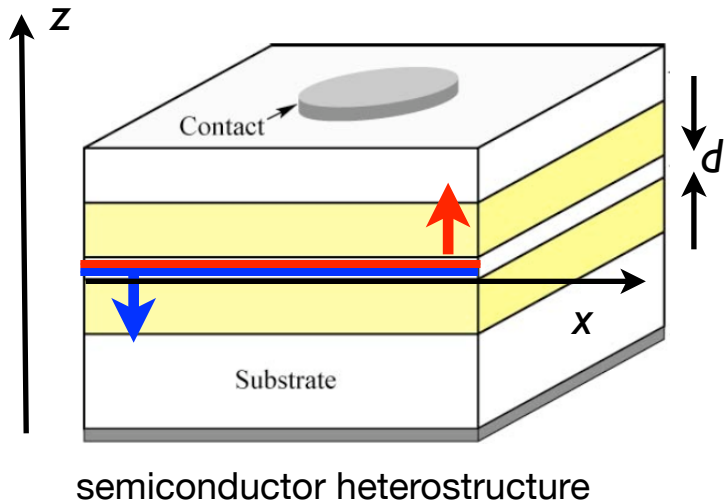


$$H_R = \alpha k_x \sigma^y$$

A schematic diagram illustrating the Rashba spin-orbit interaction. It shows a horizontal axis  $x$  with a blue arrow pointing down and a red arrow pointing up, representing the spin splitting of the bands.



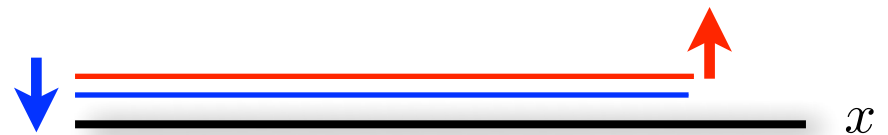
# Rashba spin-orbit interaction



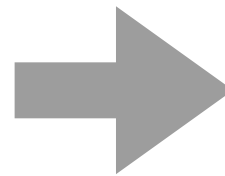
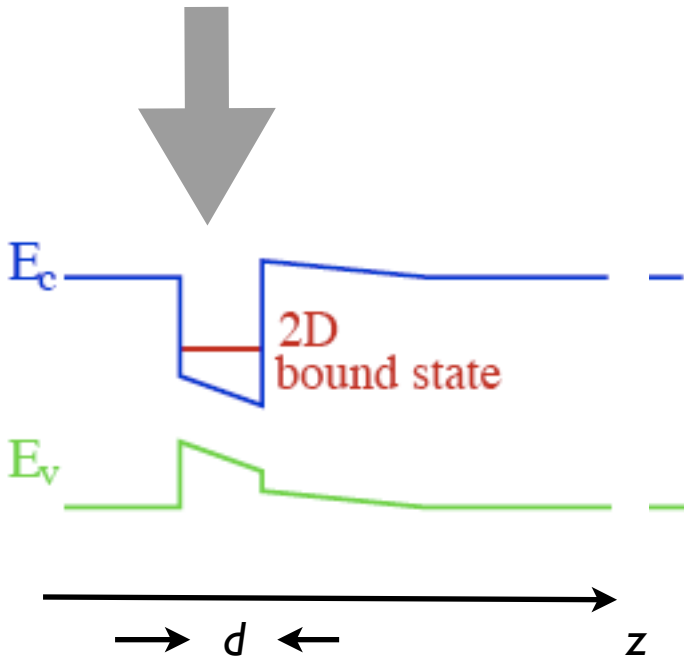
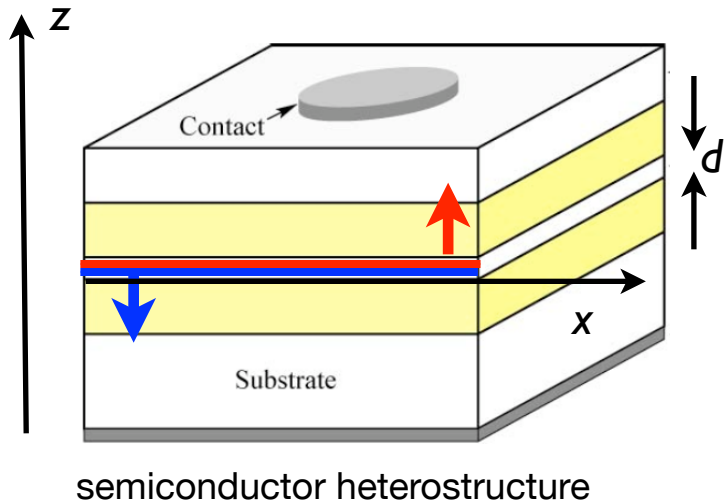
HgTe quantum well:  
weaker-amplitude  $k_x^3$  corrections

D.G. Rothe *et al.*, New J. Phys. **12**, 065012 (2010)

$$H_R = \alpha k_x \sigma^y + \dots$$



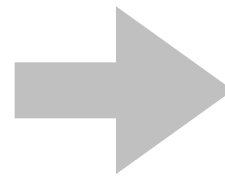
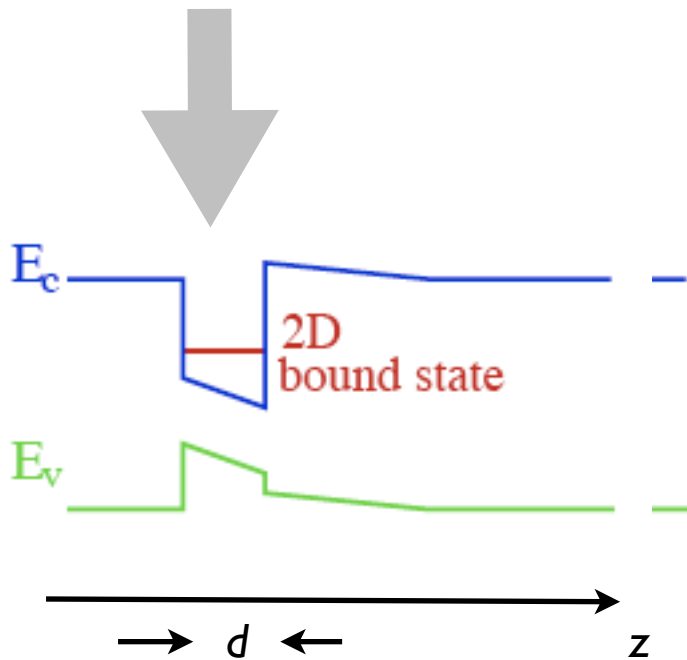
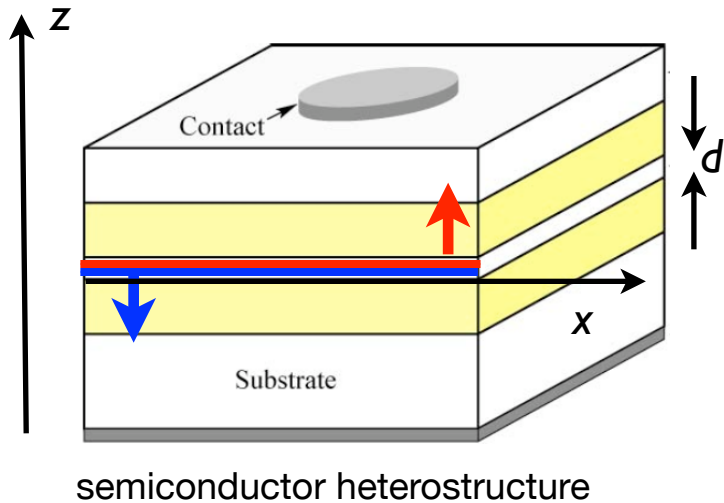
# Rashba spin-orbit interaction



$$H_R = \alpha k_x \sigma^y$$



# Rashba spin-orbit interaction



$$H_R = \alpha k_x \sigma^y$$



# Adding the Rashba interaction...

... breaks the locking of spin to momentum. However, there is still a single Kramers pair on the edge, and this is all that matters!

# Adding the Rashba interaction...

... breaks the locking of spin to momentum. However, there is still a single Kramers pair on the edge, and this is all that matters!  
*In fact, we can recover helicity by rotating the spin quantization axis:*

$$H = v_F \int dx \Psi^\dagger(x) [-i\sigma^z \partial_x] \Psi(x) + \alpha \int dx \Psi^\dagger(x) [-i\sigma^y \partial_x] \Psi(x)$$

kinetic term Rashba

$$\Psi^T = (\psi_\uparrow, \psi_\downarrow)$$

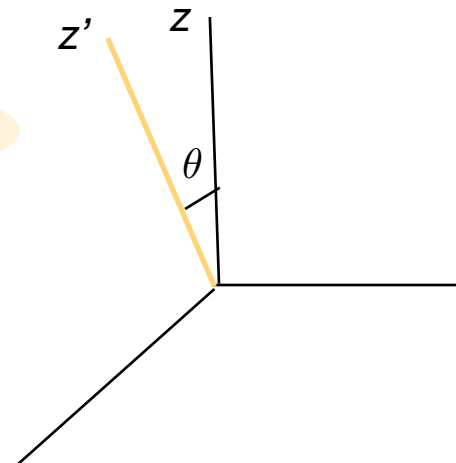


$$\Psi' = e^{-i\sigma^x \theta/2} \Psi$$

$$\cos \theta = v_F/v_\alpha \quad v_\alpha = \sqrt{v_F^2 + \alpha^2}$$

$$H' = v_\alpha \int dx \Psi'^\dagger(x) [-i\sigma^{z'} \partial_x] \Psi'(x)$$

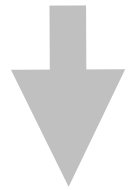
$$\Psi'^T = (\psi_{\uparrow'}, \psi_{\downarrow'})$$



# Adding the Rashba interaction...

**e-e interaction** is invariant under  $\Psi \rightarrow \Psi'$

**Kondo interaction**  $H_K = \Psi^\dagger(0) [J_\perp(\sigma^+ S_{\text{eff}}^- + \sigma^- S_{\text{eff}}^+) + J_z \sigma^z S_{\text{eff}}^z] \Psi(0)$



$$\Psi' = e^{-i\sigma^x \theta/2} \Psi$$

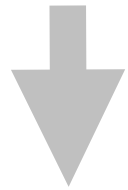
$$\mathbf{S}' = e^{-iS^x \theta/2} \mathbf{S} e^{iS^x \theta/2}$$

$$H'_K = \Psi'^\dagger(0) [J_x \sigma^x S^x + J'_y \sigma^{y'} S^{y'} + J'_z \sigma^{z'} S^{z'} + J_{\text{NC}} (\sigma^{y'} S^{z'} + \sigma^{z'} S^{y'})] \Psi'(0)$$

# Adding the Rashba interaction...

**e-e interaction** is invariant under  $\Psi \rightarrow \Psi'$

**Kondo interaction**  $H_K = \Psi^\dagger(0) [J_\perp (\sigma^+ S_{\text{eff}}^- + \sigma^- S_{\text{eff}}^+) + J_z \sigma^z S_{\text{eff}}^z] \Psi(0)$



$$\Psi' = e^{-i\sigma^x \theta/2} \Psi$$

$$\mathbf{S}' = e^{-iS^x \theta/2} \mathbf{S} e^{iS^x \theta/2}$$

$$H'_K = \Psi'^\dagger(0) [J_x \sigma^x S^x + J'_y \sigma^{y'} S^{y'} + J'_z \sigma^{z'} S^{z'} + J_{\text{NC}} (\sigma^{y'} S^{z'} + \sigma^{z'} S^{y'})] \Psi'(0)$$

XYZ Kondo

Non-Collinear term

depend on the Rashba coupling  $\alpha$   
controllable by a gate voltage

# Adding the Rashba interaction...

**e-e interaction** is invariant under  $\Psi \rightarrow \Psi'$

**Kondo interaction**  $H_K = \Psi^\dagger(0) [J_\perp (\sigma^+ S_{\text{eff}}^- + \sigma^- S_{\text{eff}}^+) + J_z \sigma^z S_{\text{eff}}^z] \Psi(0)$



$$\Psi' = e^{-i\sigma^x \theta/2} \Psi$$

$$\mathbf{S}' = e^{-iS^x \theta/2} \mathbf{S} e^{iS^x \theta/2}$$

$$H'_K = \Psi'^\dagger(0) [J_x \sigma^x S^x + J'_y \sigma^{y'} S^{y'} + J'_z \sigma^{z'} S^{z'} + J_{\text{NC}} (\sigma^{y'} S^{z'} + \sigma^{z'} S^{y'})] \Psi'(0)$$

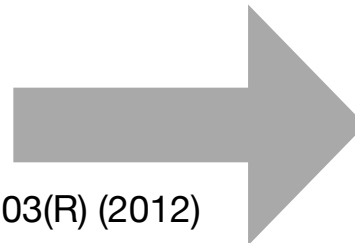
**XYZ Kondo**

**Non-Collinear term**

**depend on the Rashba coupling**  $\alpha$   
*controllable by a gate voltage*

...bosonization and perturbative RG

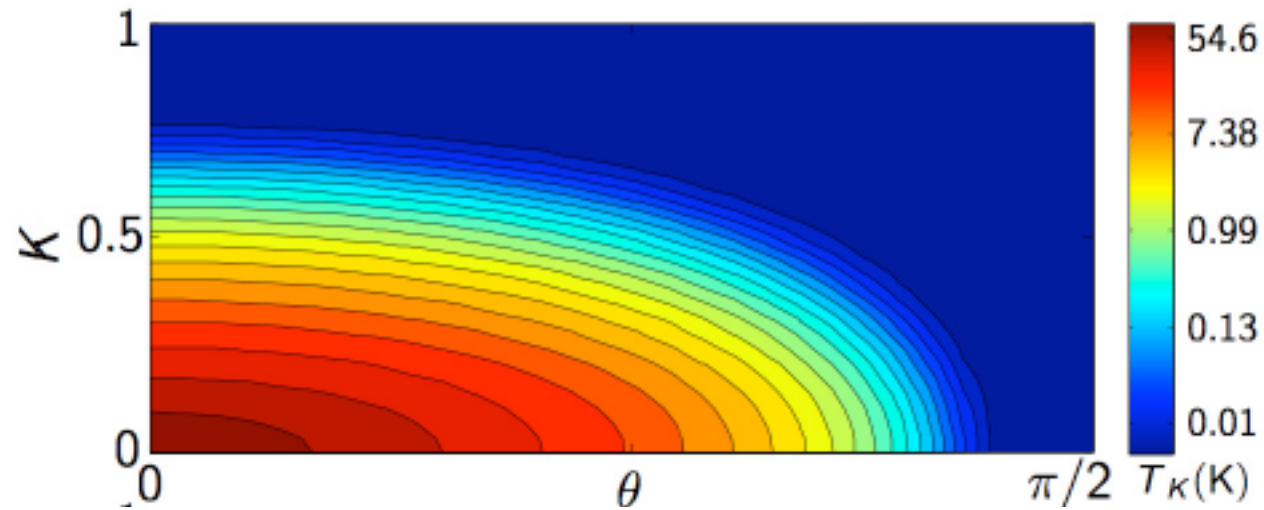
E. Eriksson, A. Ström, G. Sharma, H.J., PRB **86**, 161103(R) (2012)





# Electrical control of the Kondo temperature

via the "Rashba angle"  $\theta \sim$  gate voltage

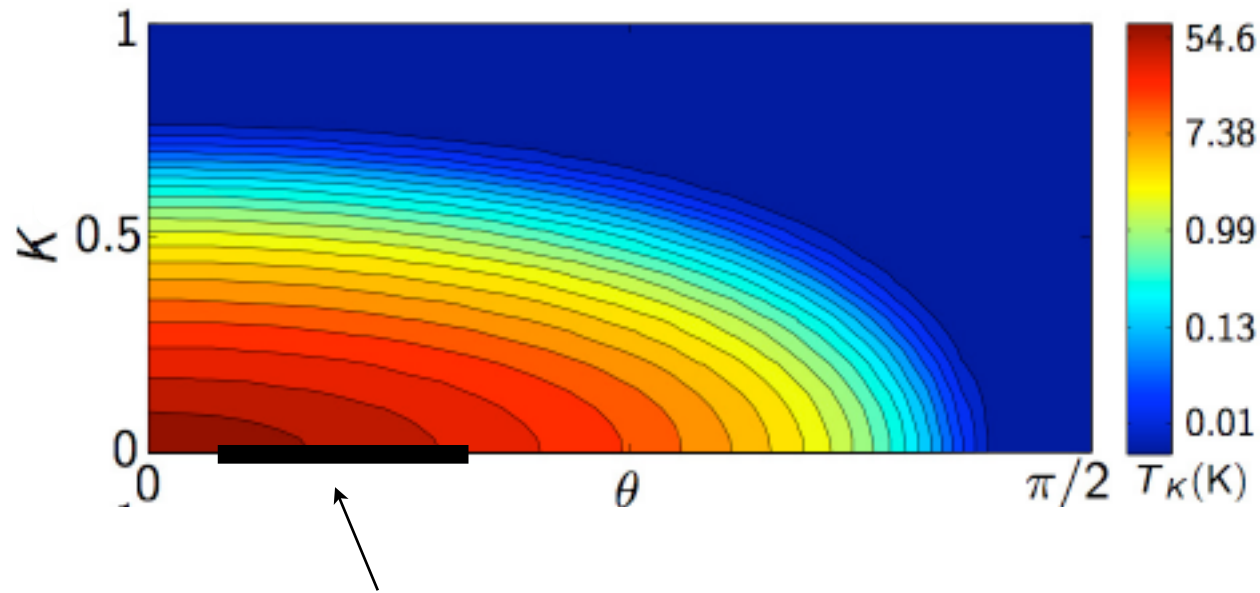


easy-plane Kondo

$$J_x = J_y = 20 \text{ meV}, J_z = 10 \text{ meV}$$

# Electrical control of the Kondo temperature

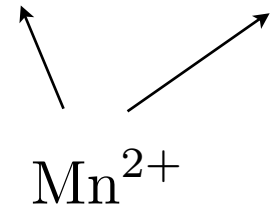
via the "Rashba angle"  $\theta \sim$  gate voltage



experimentally probed range in a HgTe quantum well  
by tuning the bias of a top gate from -2V to 2V  
J. Hinz *et al.*, Semicond. Sci. Technol. **21**, 501 (2006)

easy-plane Kondo

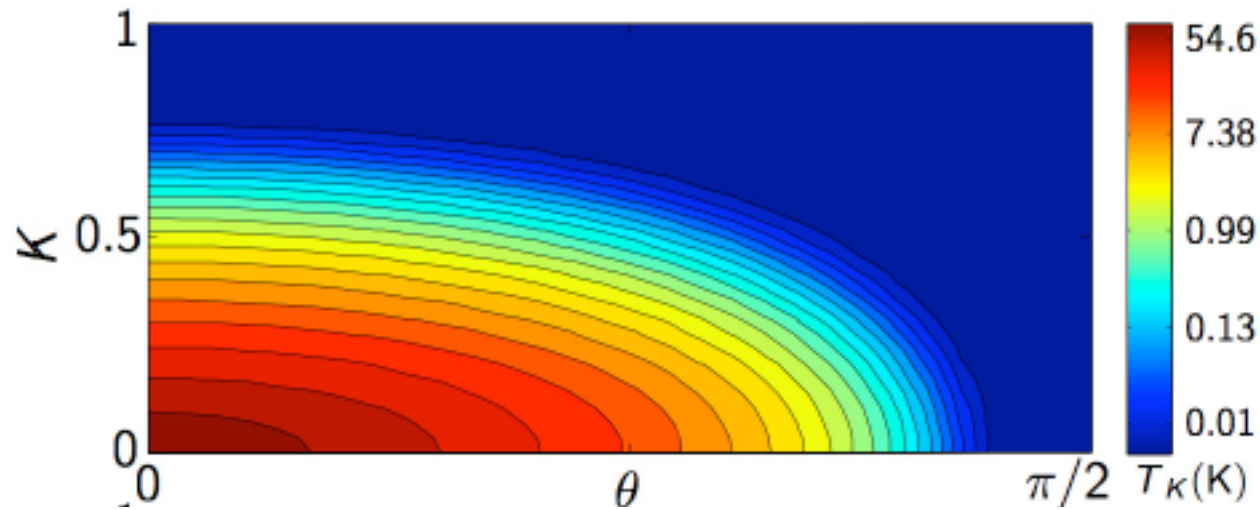
$$J_x = J_y = 20 \text{ meV}, J_z = 10 \text{ meV}$$



J.K. Furdyna, J. Appl. Phys. **64**, R29 (1988)

# Electrical control of the Kondo temperature

via the "Rashba angle"  $\theta \sim$  gate voltage



easy-plane Kondo

$$J_x = J_y = 20 \text{ meV}, J_z = 10 \text{ meV}$$

Note: Kondo temperatures modified by spin-orbit interactions or spin-dependent hopping have been proposed also for ordinary (non-helical) conduction electrons:

M. Pletyukhov and D. Schuricht, PRB **84**, 041309(R) (2011)

X.-Y. Feng and F.-C. Zhang, J. Phys.: Cond. Matt. **23**, 105602 (2011)

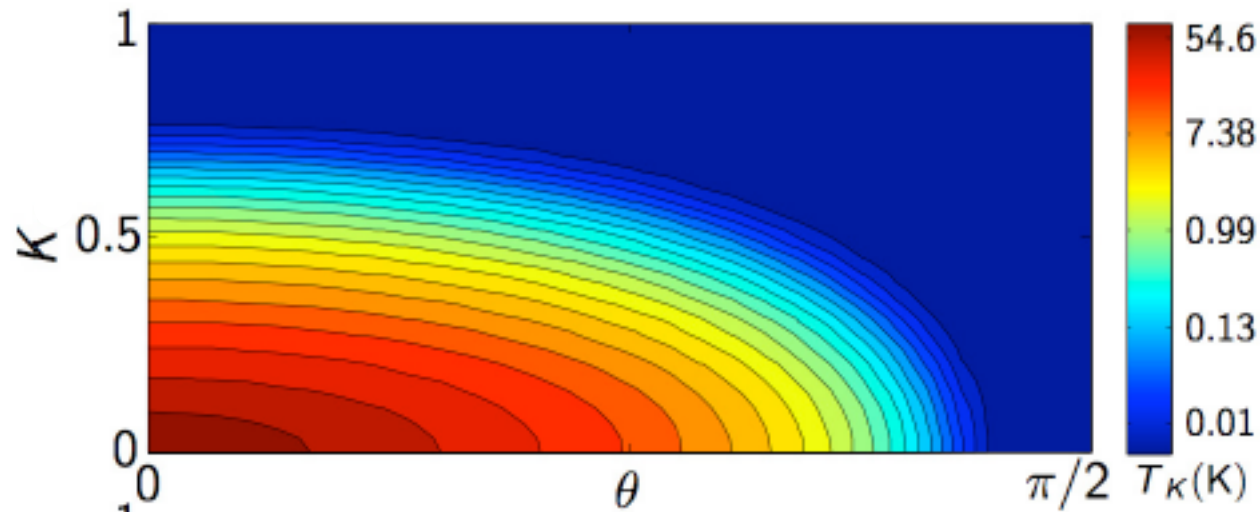
R. Zitko and J. Bonca, PRB **84**, 193411 (2011)

M. Zarea, S. E. Ulloa, and N. Sandler, PRL **108**, 046601 (2012)

L. Isaev, L. Agterberg, and I. Vekhter, PRB **85**, 081107 (2012)

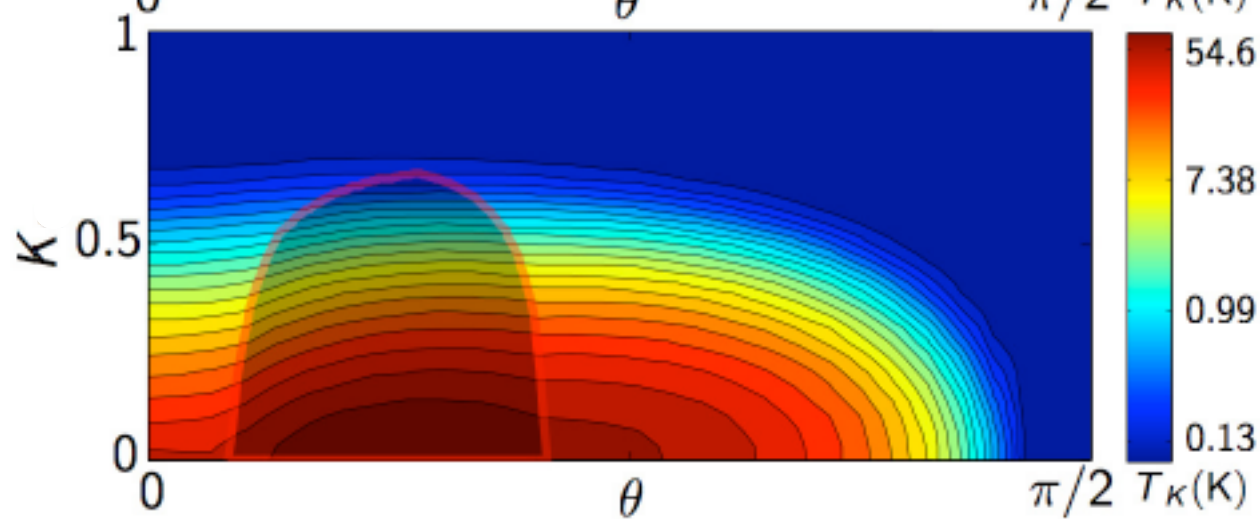
# Electrical control of the Kondo temperature

via the "Rashba angle"  $\theta \sim$  gate voltage



easy-plane Kondo

$$J_x = J_y = 20 \text{ meV}, J_z = 10 \text{ meV}$$

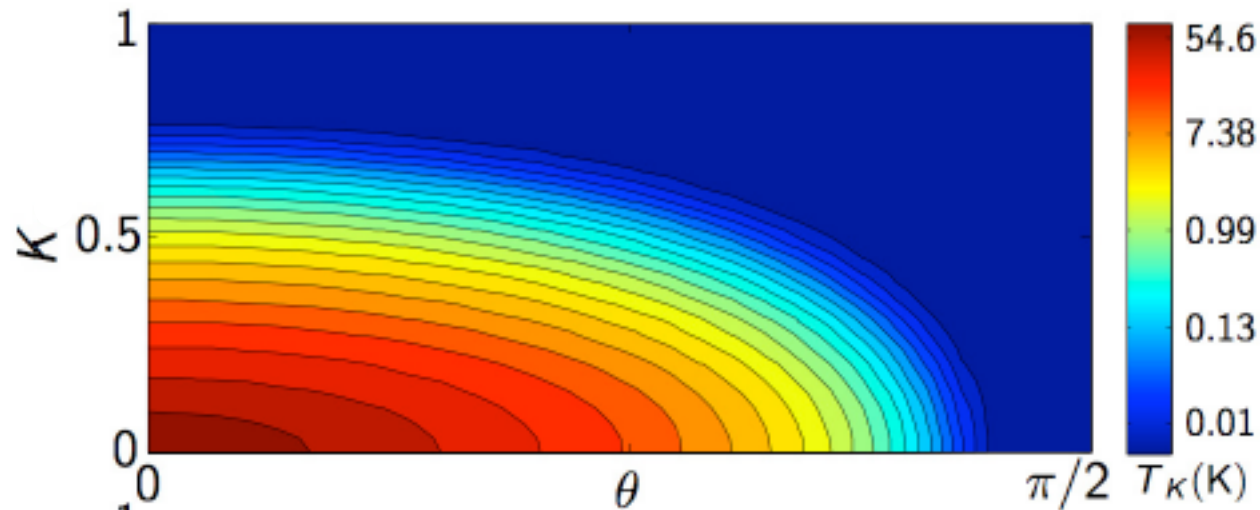


easy-axis Kondo

$$J_x = J_y = 5 \text{ meV}, J_z = 50 \text{ meV}$$

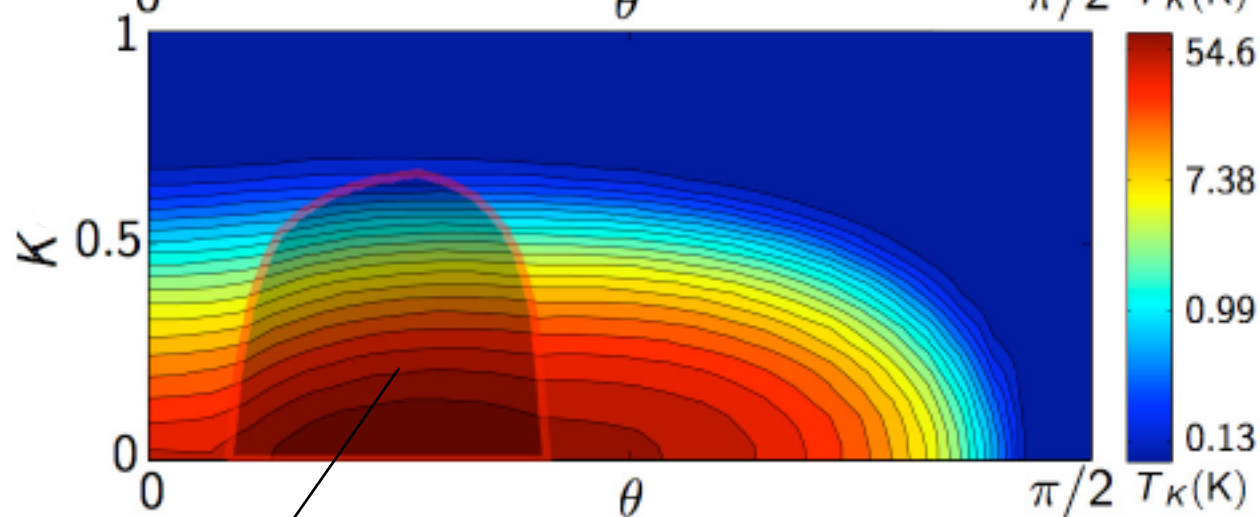
# Electrical control of the Kondo temperature

via the "Rashba angle"  $\theta \sim$  gate voltage



easy-plane Kondo

$$J_x = J_y = 20 \text{ meV}, J_z = 10 \text{ meV}$$



easy-axis Kondo

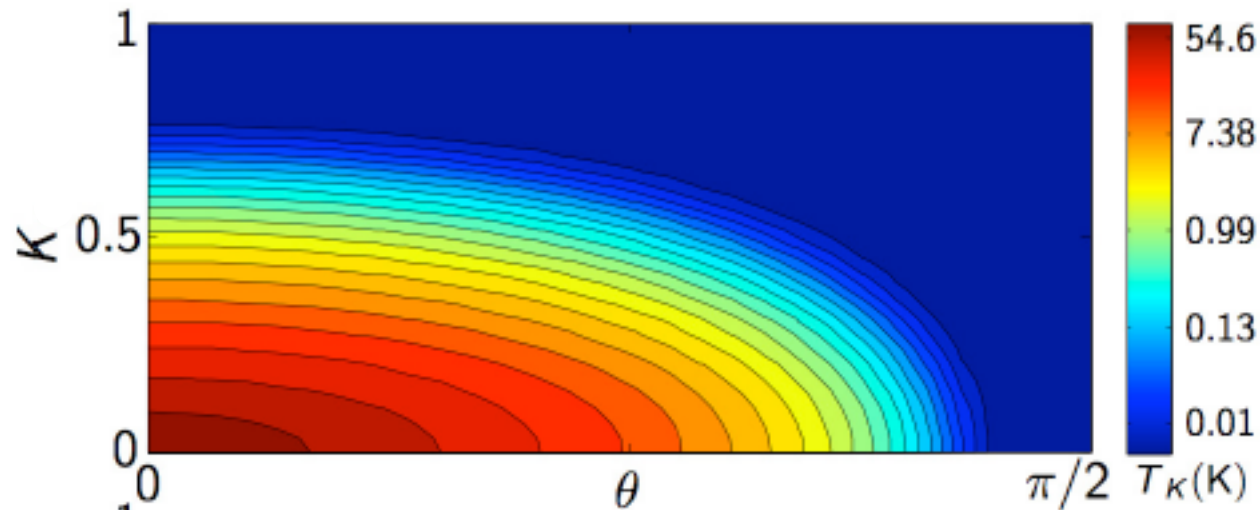
$$J_x = J_y = 5 \text{ meV}, J_z = 50 \text{ meV}$$

Region where  $J_{NC}$  dominates the RG flow

→ obstruction of Kondo screening!

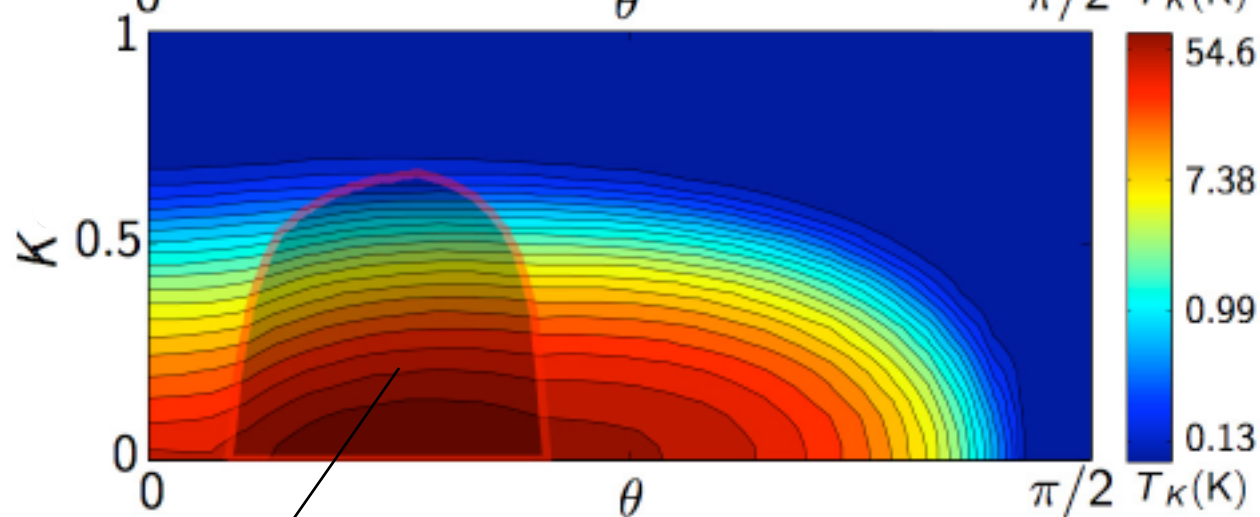
# Electrical control of the Kondo temperature

via the "Rashba angle"  $\theta \sim$  gate voltage



easy-plane Kondo

$$J_x = J_y = 20 \text{ meV}, J_z = 10 \text{ meV}$$



easy-axis Kondo

$$J_x = J_y = 5 \text{ meV}, J_z = 50 \text{ meV}$$

Region where  $J_{\text{NC}}$  dominates the RG flow  
 → obstruction of Kondo screening!

challenges "conventional wisdom" that Kondo screening cannot be killed off by time-reversal invariant perturbations!

# Low-temperature transport, $T \ll T_K$ (away from the "dome")

"weak" e-e interaction

$$G = \frac{2e^2}{h} - \delta G$$

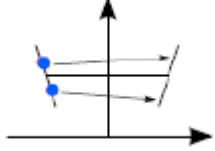
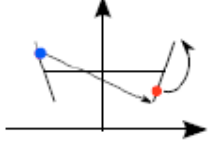
$\delta G$

$\swarrow$

$\searrow$

$\begin{matrix} 1/4 < K < 2/3 \\ K > 2/3 \end{matrix}$

$\begin{matrix} \sim (T/T_K)^{8K-2} \\ \sim (T/T_K)^{2K+2} \end{matrix}$

"strong" e-e interaction  $K < 1/4$

no signature of Rashba

$$G \sim (T/T_K)^{2(1/4K-1)} \text{ from instanton processes}$$

J. Maciejko *et al.*, PRL **102**, 256803 (2009)

J. Maciejko, PRB B **85**, 245108 (2012)

# ”High-temperature” transport, $T \gg T_K$

$$\delta I = I - I_0$$

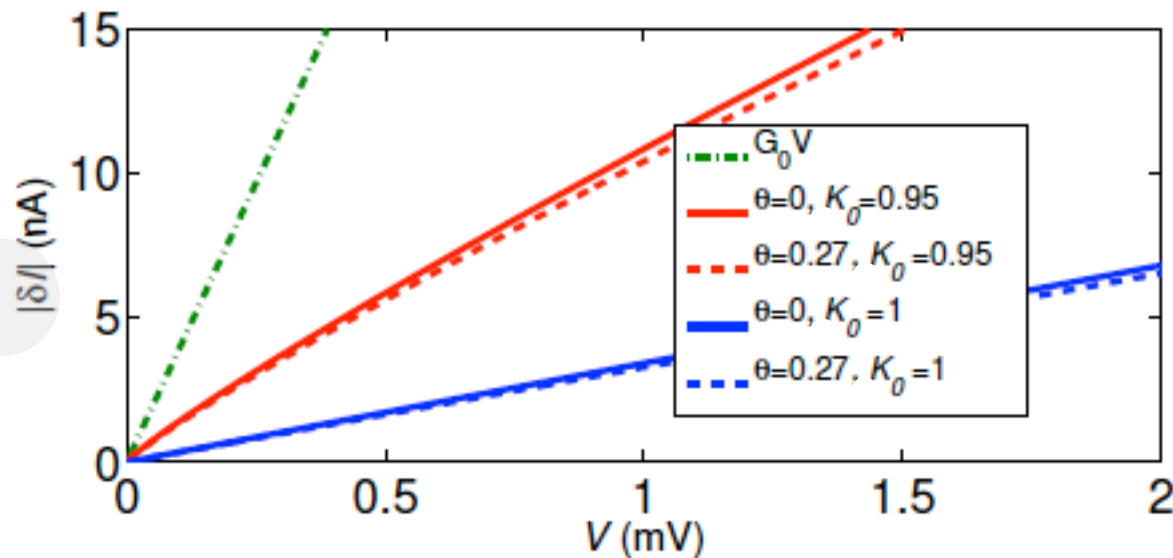
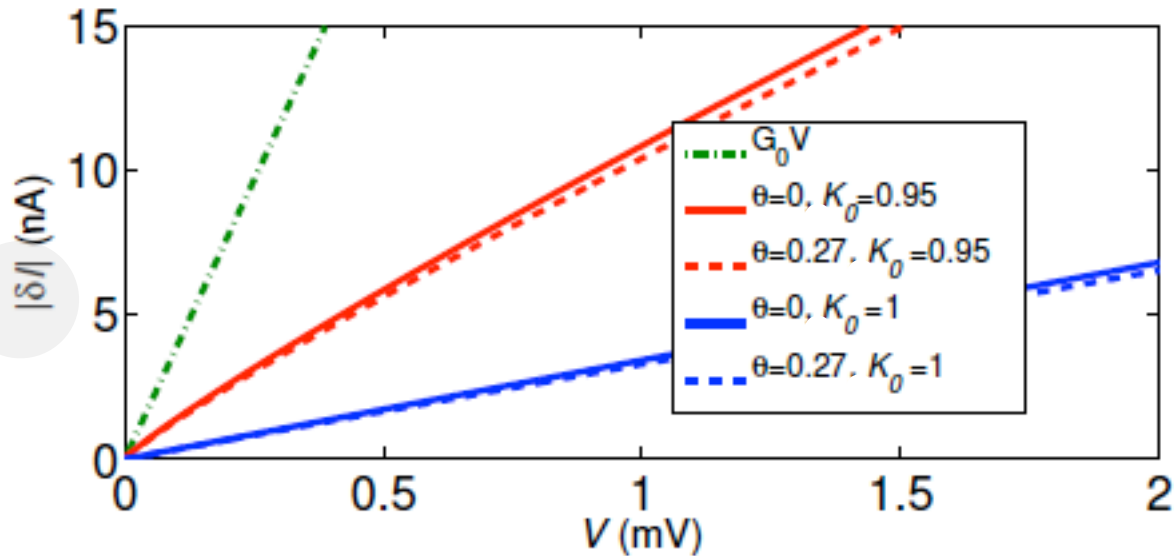


FIG. 2: The RG-improved current correction (12) at  $T = 30$  mK as a function of applied voltage, for different values of  $K_0$  and  $\theta$ . The dashed lines represent  $\theta \approx 0.27$ , corresponding to  $\hbar\alpha = 10^{-10}$  eVm.  $J_x = J_y = 3J_z = 30$  meV,  $a_0 = 0.5$  nm,  $v_F = 5 \times 10^5$  m/s, and  $D = 300$  meV.



# ”High-temperature” transport, $T \gg T_K$

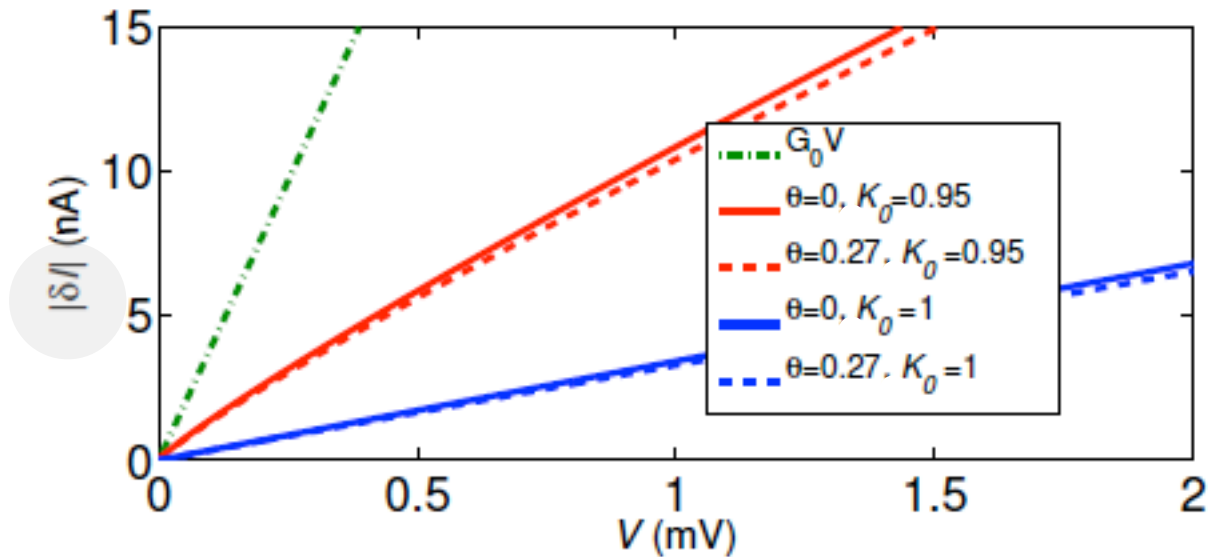
$$\delta I = I - I_0$$



The Rashba-driven correction for a fixed voltage increases with the e-e interaction. Estimated values of  $K$  for the HgTe quantum wells used in the Würzburg experiments range from  $K = 0.55$  to  $K = 0.98$ .

# ”High-temperature” transport, $T \gg T_K$

$$\delta I = I - I_0$$



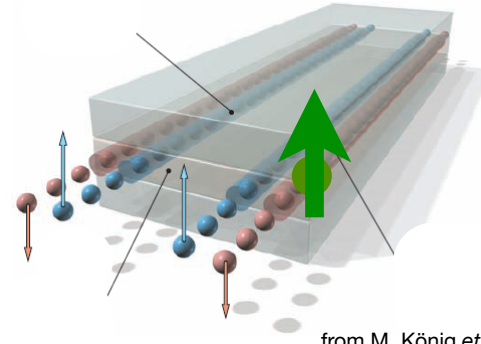
For more results on transport, see

E. Eriksson *et al.*, PRB **86**, 161103(R) (2012);

PRB **87**, 079902(E) (2013)

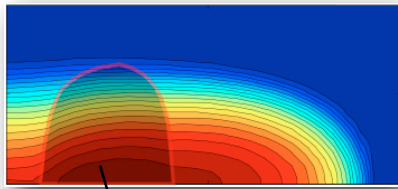
# Summary

E. Eriksson *et al.*, PRB **86**, 161103(R) (2012)  
PRB **87**, 079902(E) (2013)

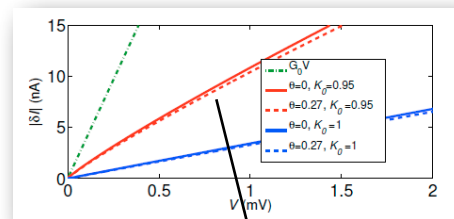


from M. König *et al.*, Science **318**, 766 (2007)

## Magnetic impurity in a helical edge liquid: Rashba coupling allows electrical "control" of Kondo temperature and IV-characteristics



blocking of Kondo screening  
(one-loop RG)



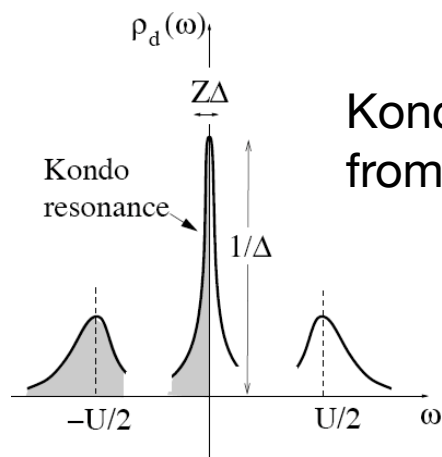
Rashba-induced  
impurity correction,  
accessible in experiment?

### More results:

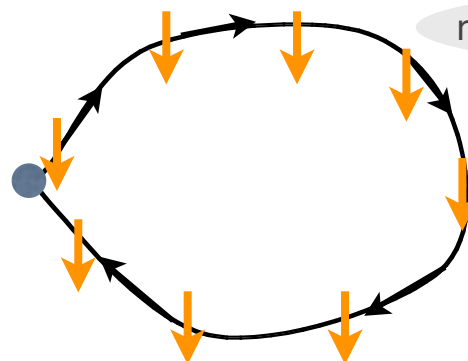
Current fluctuations, thermal transport, effects from Dresselhaus interactions, ...  
E. Eriksson, PRB **87**, 235414 (2013)

Addendum...

# Conventional picture...

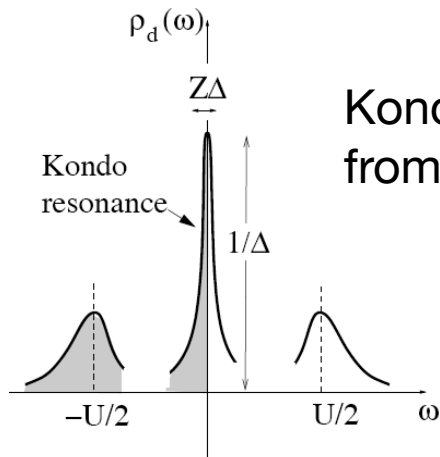


Kondo resonance at the Fermi level  
from divergence in the electron self-energy

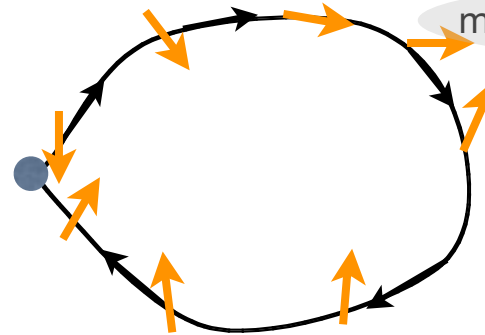


multiple electron-impurity scattering

# Conventional picture...



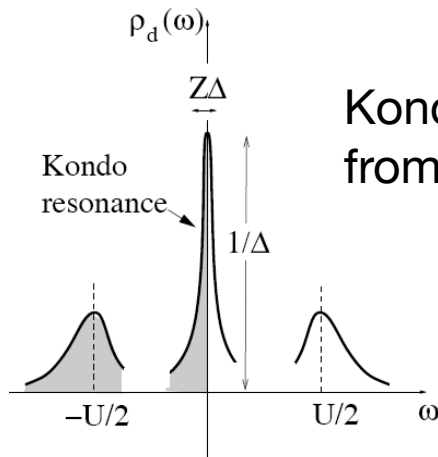
Kondo resonance at the Fermi level  
from divergence in the electron self-energy



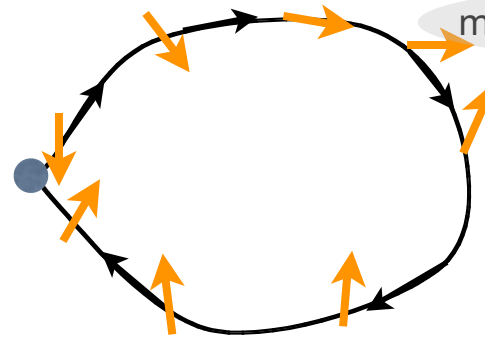
multiple electron-impurity scattering

**non-conservation of electron spin in  
the presence of spin-orbit interactions**

## Conventional picture...



Kondo resonance at the Fermi level  
from divergence in the electron self-energy



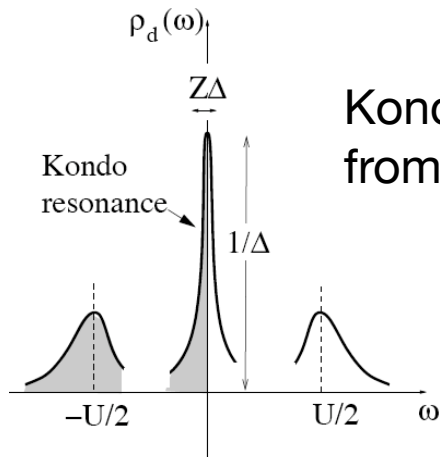
multiple electron-impurity scattering

**non-conservation of electron spin in  
the presence of spin-orbit interactions!**

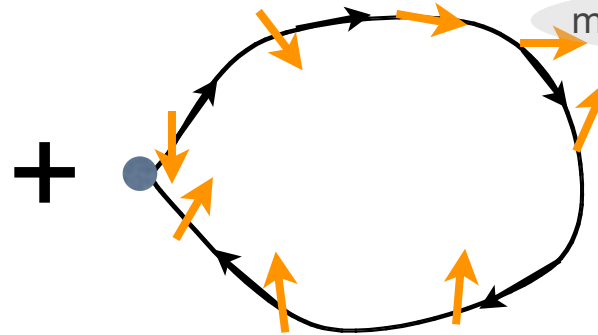
But,... experiments show that the Kondo  
effect is insensitive to spin-orbit scattering...

G. Bergmann, PRL 57, 1460 (1986)

# Conventional picture...

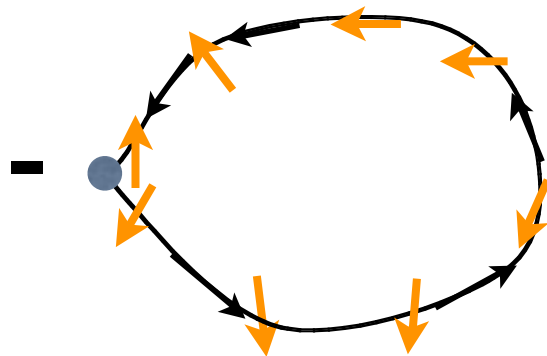


Kondo resonance at the Fermi level  
from divergence in the electron self-energy



multiple electron-impurity scattering

**non-conservation of electron spin in the presence of spin-orbit interactions!**



But,... experiments show that the Kondo effect is insensitive to spin-orbit scattering...

G. Bergmann, PRL **57**, 1460 (1986)

...protected by time-reversal invariance

= 0

Y. Meir and N.S. Wingreen, PRB **50**, 4947 (1994)