Coherence and Correlations in Quantum Systems
Evora, Portugal, October 11, 2012

Entanglement Probe of Two-Impurity Kondo Physics in a Spin Chain

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in collaboration with

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UNIVERSITY OF GOTHENBURG
THE WOLFSON FOUNDATION
PICC
EXCELLENCE IN SCIENCE
VETENSKAPSRÅDET

Outline

Background...

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Basics on two-impurity Kondo model

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Realization in the lab: double-quantum dot systems

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Applications: two-qubit gates for quantum computing?

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A new view through quantum entanglement...

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N*egativity* and *von Neumann entropy* from DMRG

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Results: Entanglement structure, Kondo cloud, and more...

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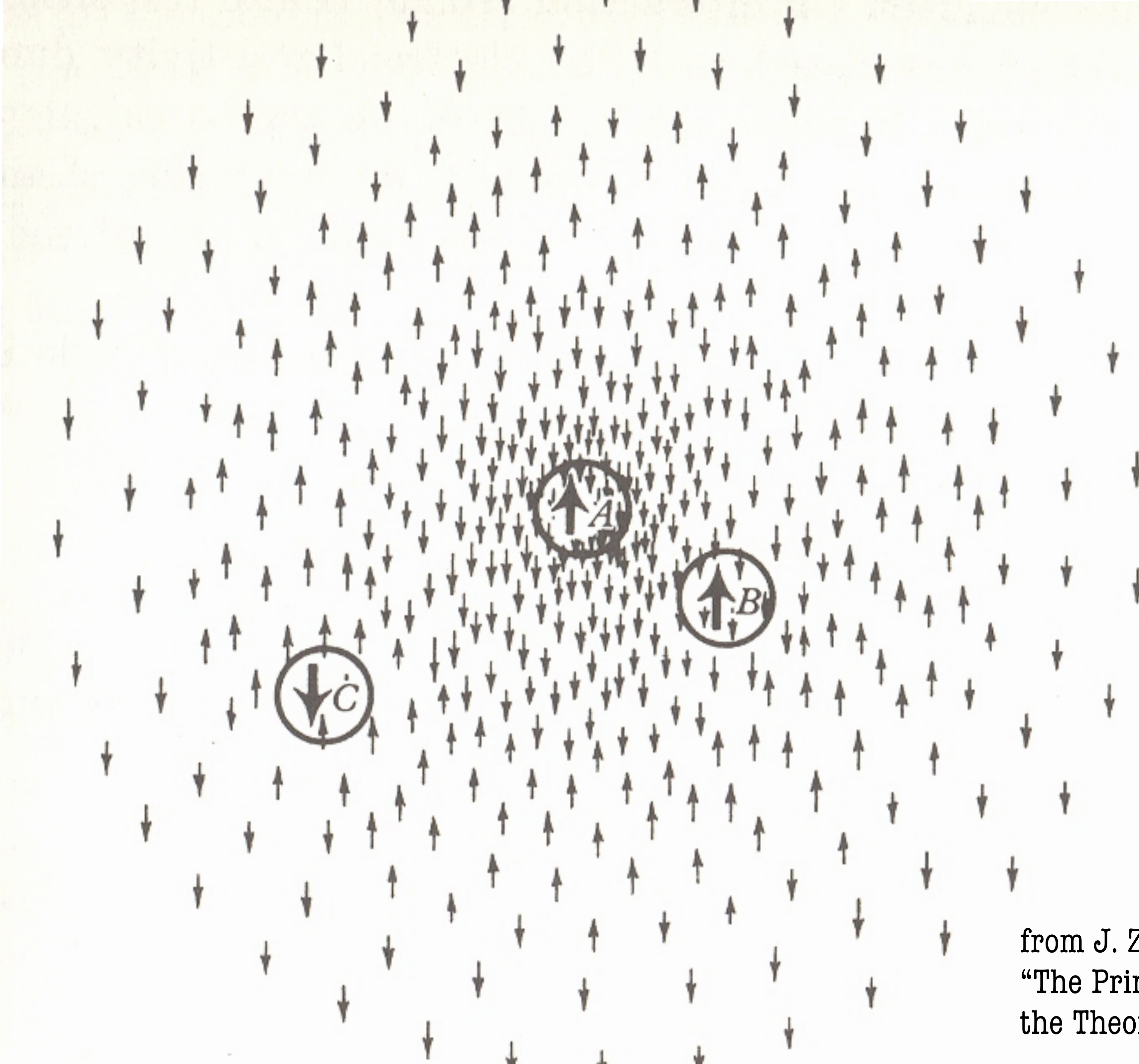
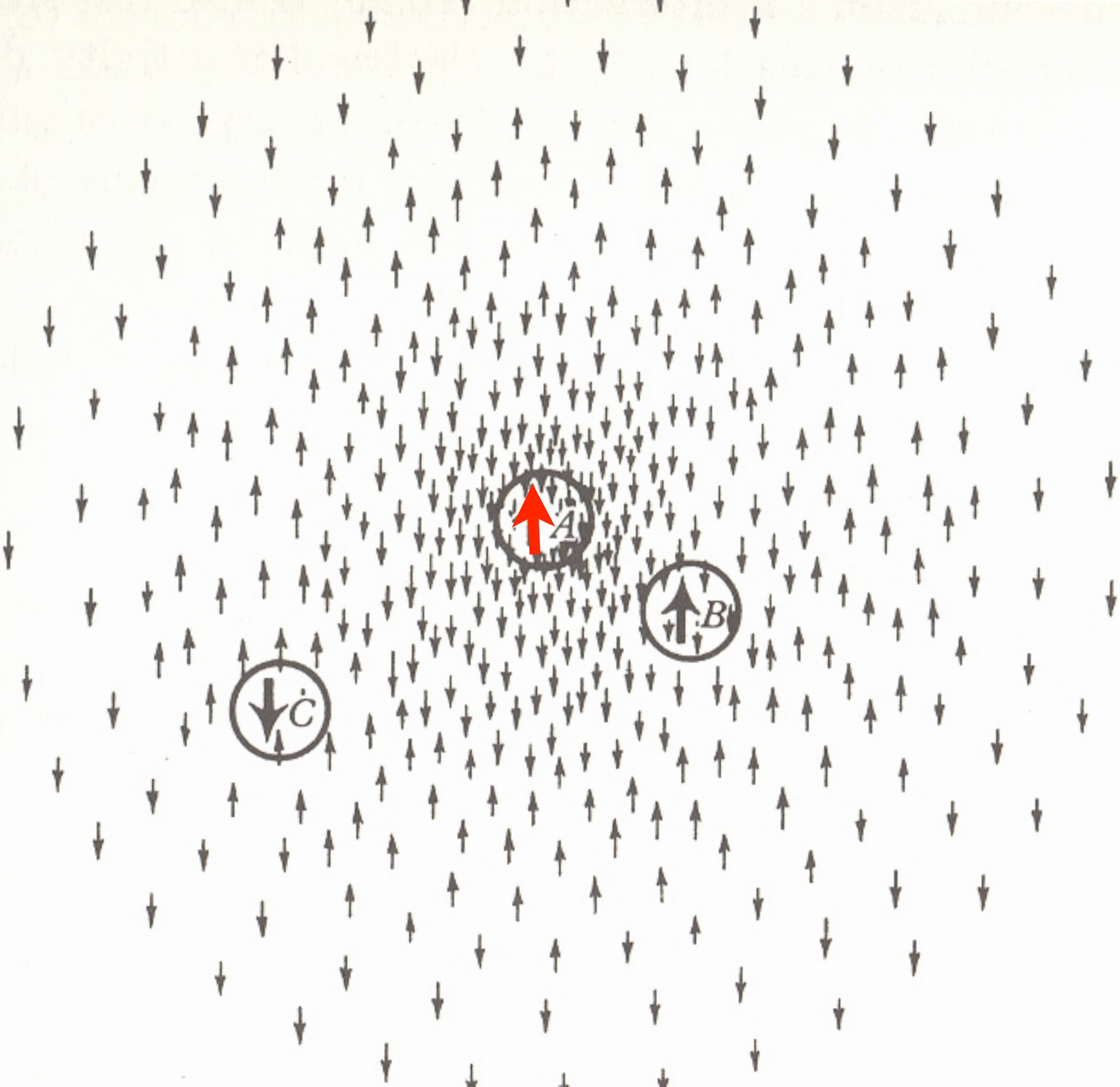
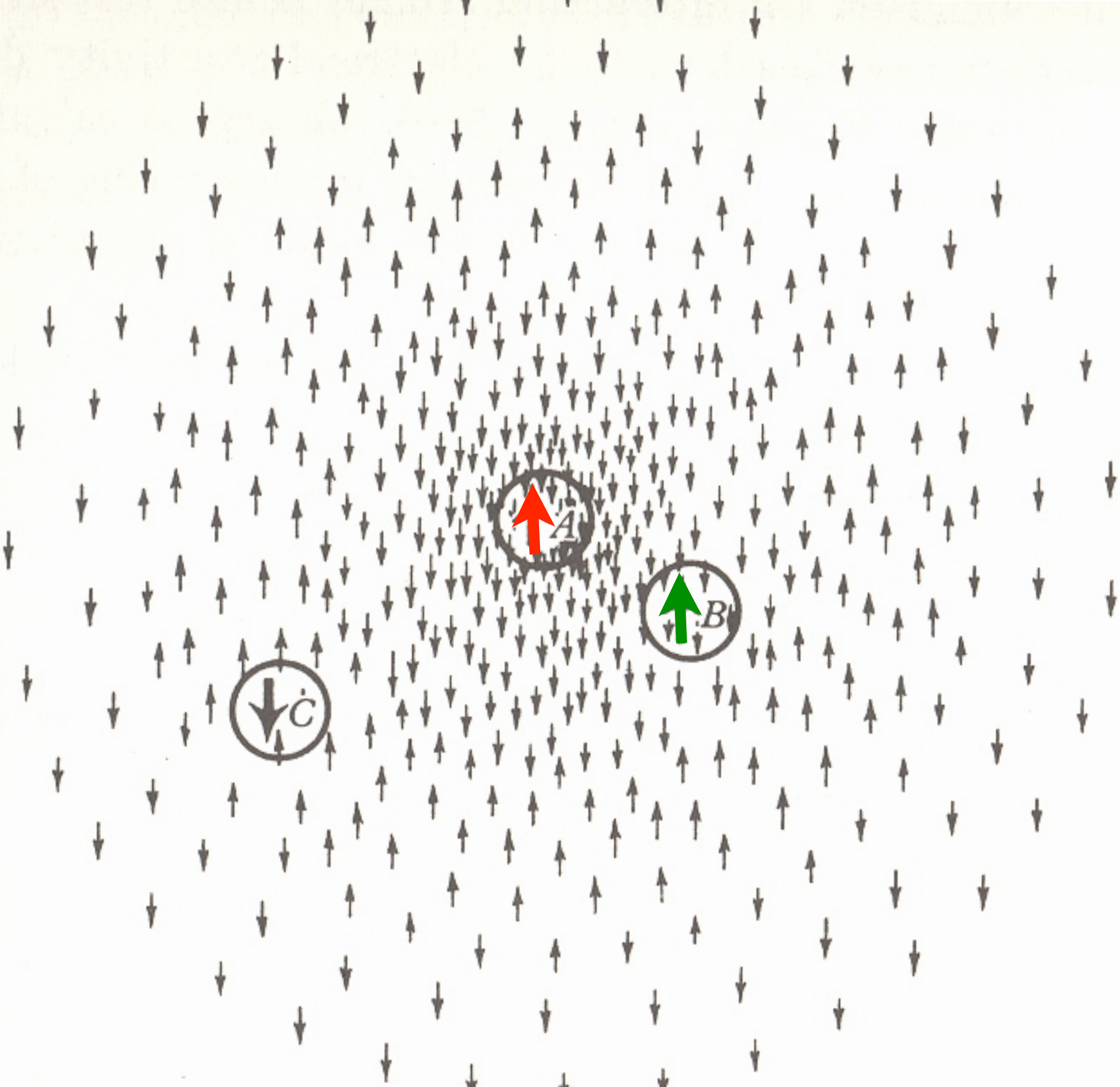
- Spin chain modeling
- N*egativity* and *von Neumann entropy* from DMRG
- Results: Entanglement structure, Kondo cloud, and more...



from J. Ziman,
“The Principles of
the Theory of Solids”


A
B
C


A
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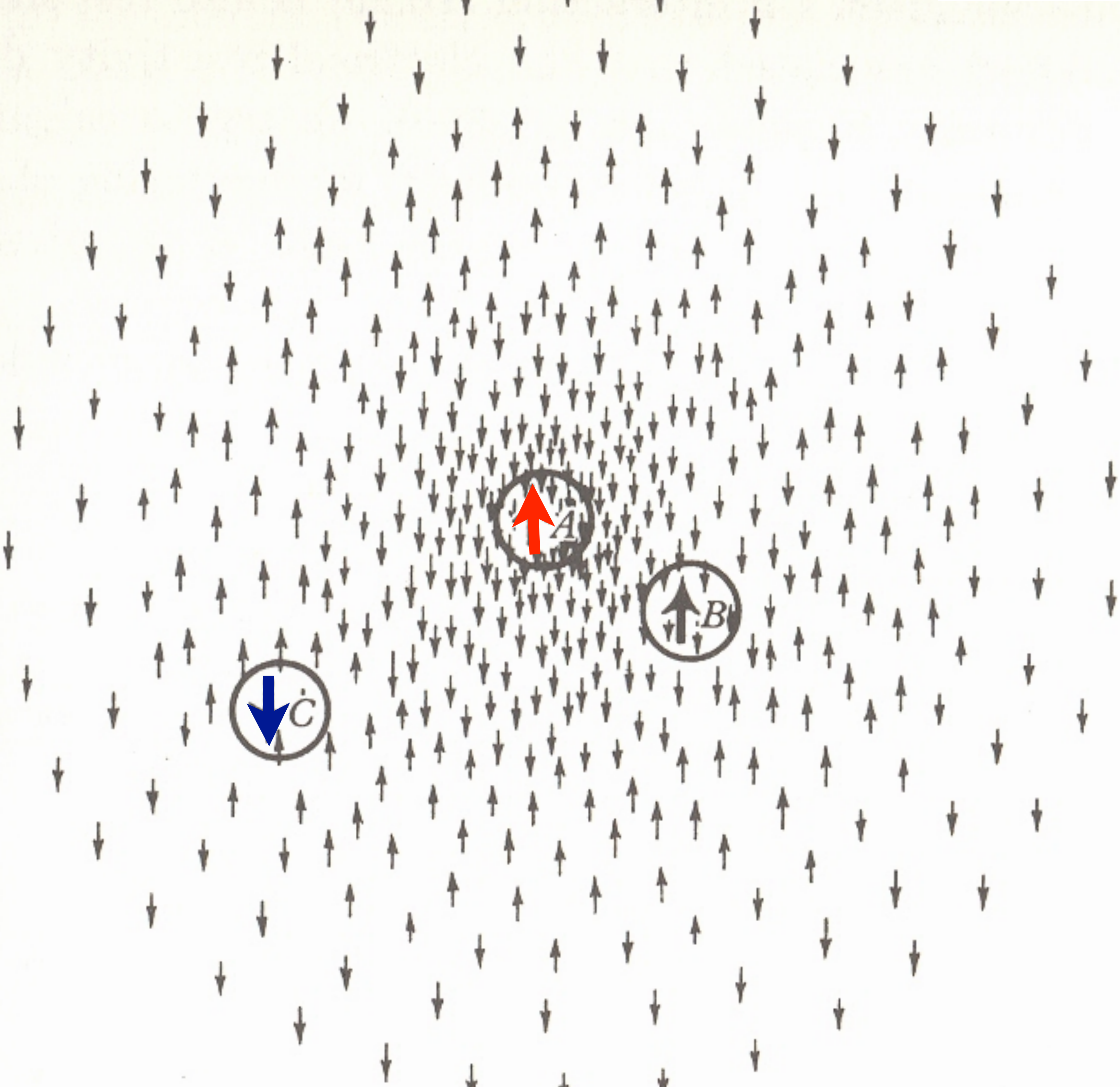
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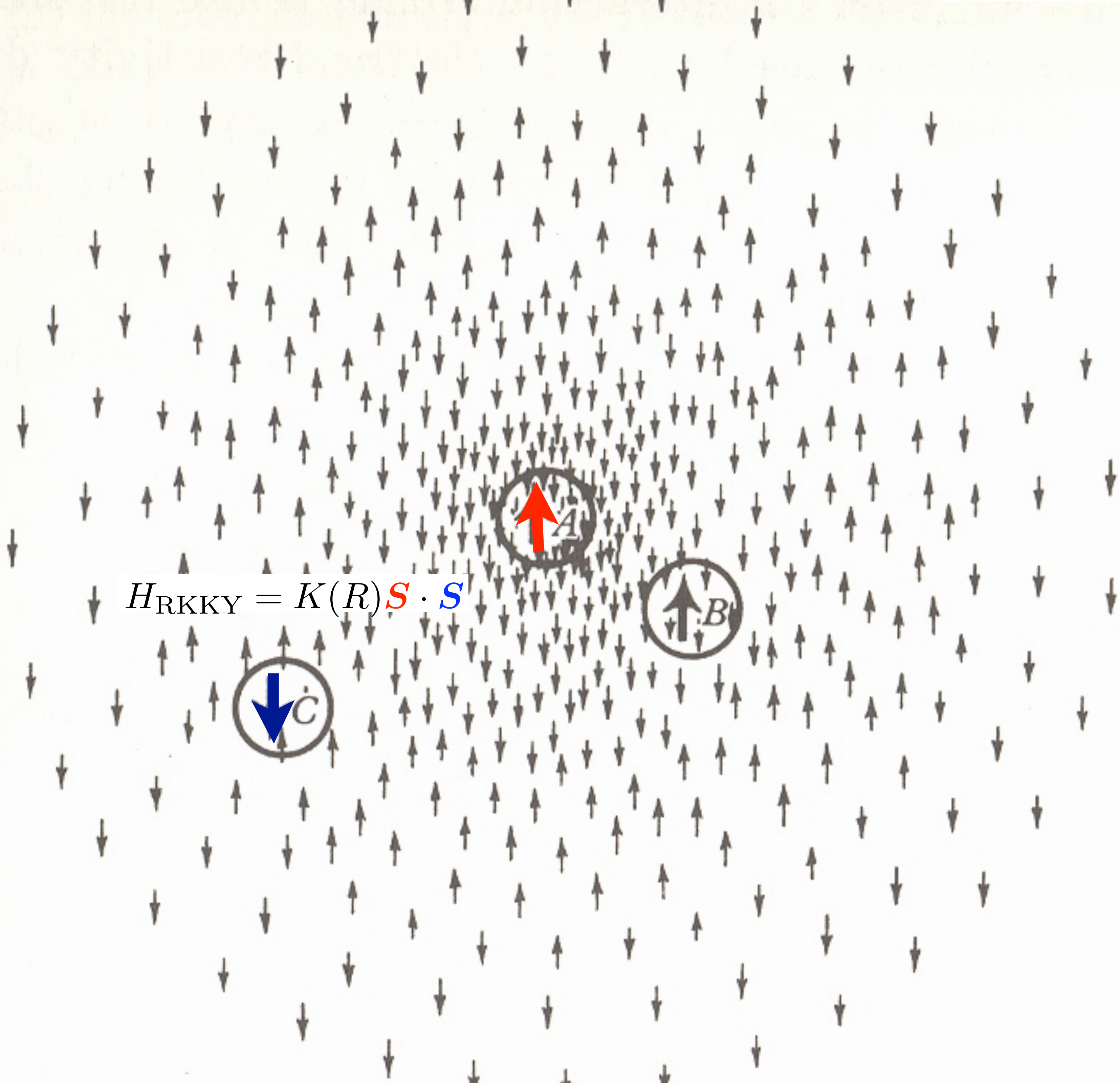
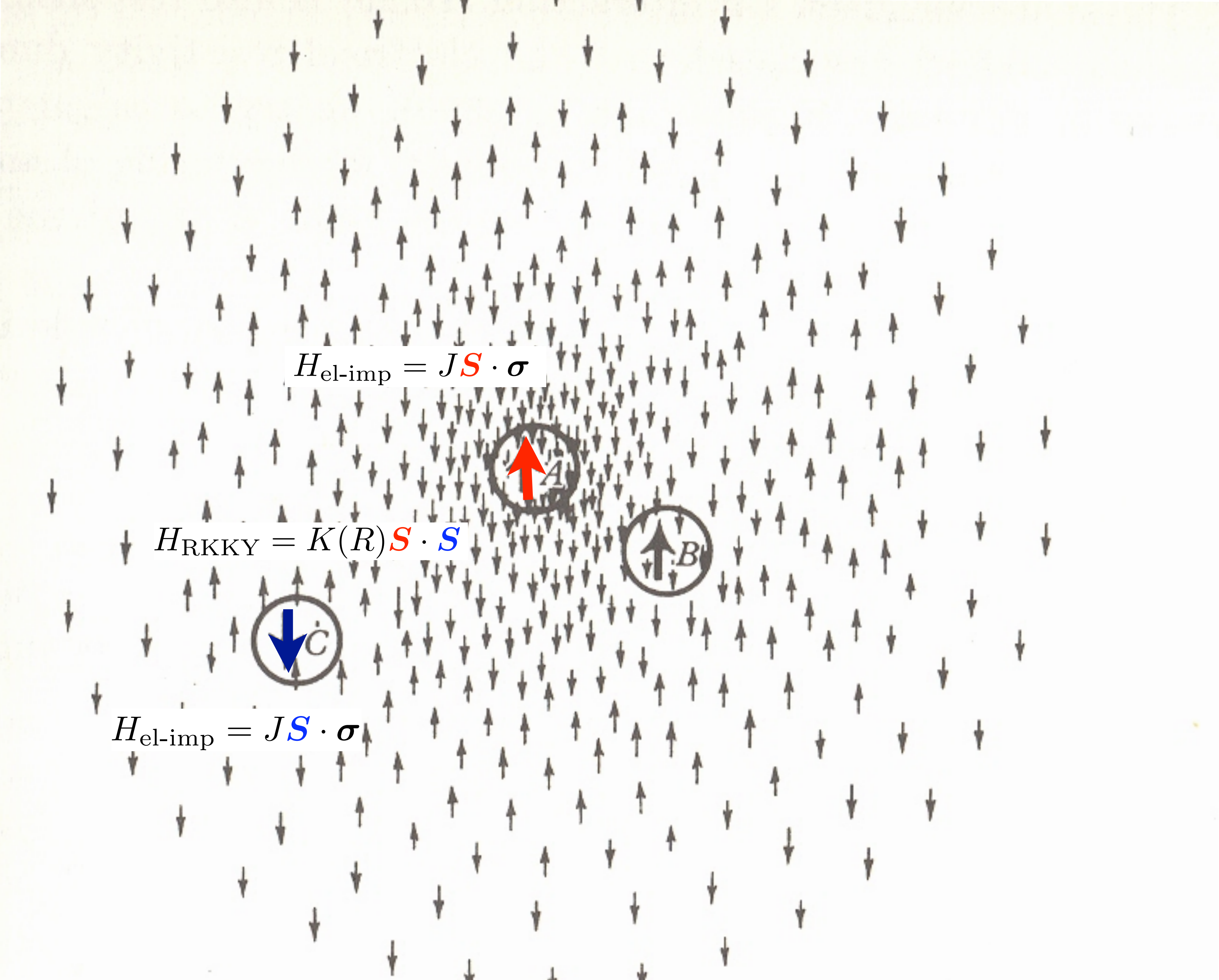

$H_{\mathrm{RKKY}} = K(R)\boldsymbol{S} \cdot \boldsymbol{S}$
A
B
C


$H_{\text{el-imp}} = J\boldsymbol{S} \cdot \boldsymbol{\sigma}$
A
B
$H_{\text{RKKY}} = K(R)\boldsymbol{S} \cdot \boldsymbol{S}$
C
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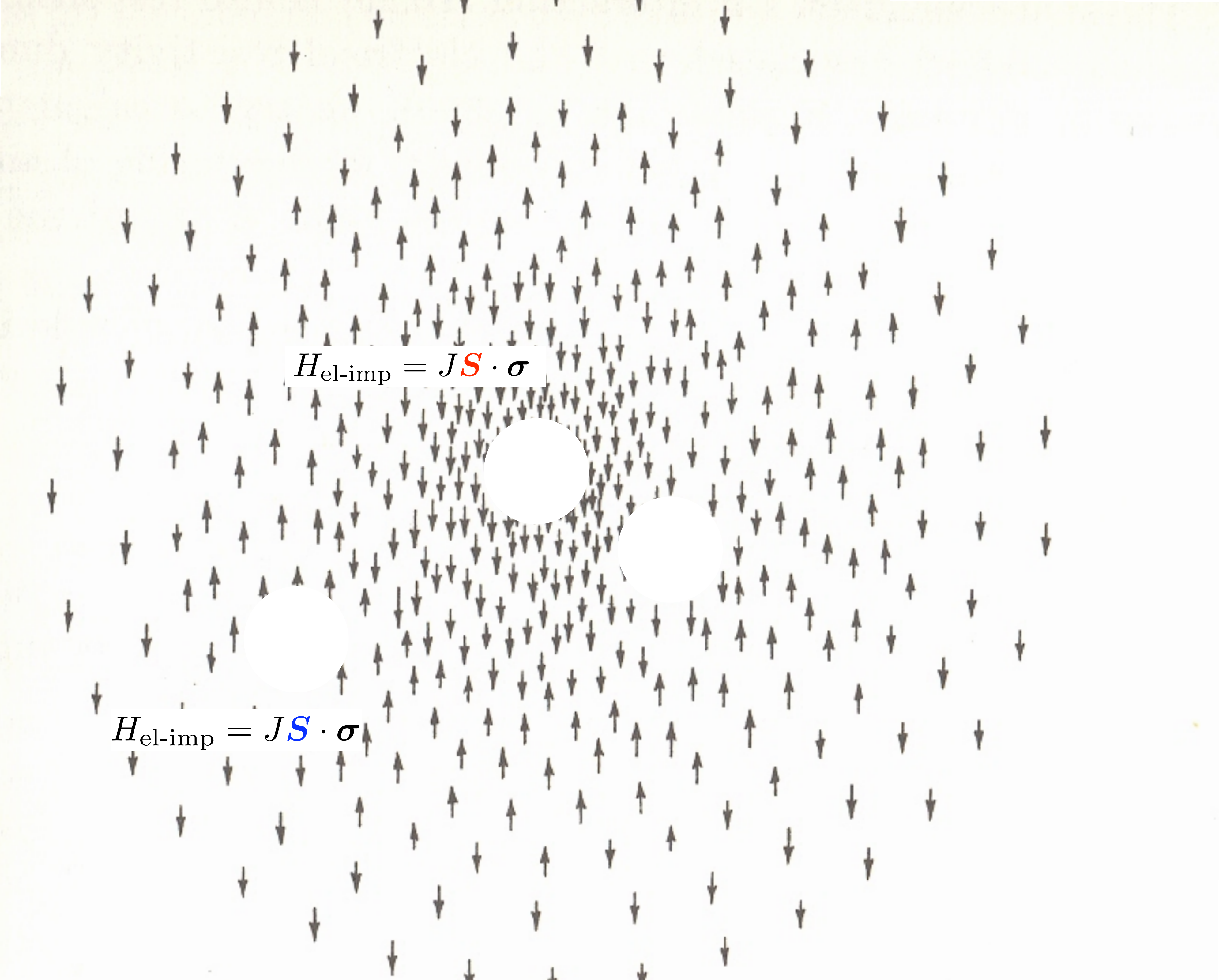
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$$H = H_{\mathrm{kin}} + J\boldsymbol{S}_1 \cdot \boldsymbol{\sigma} + J\boldsymbol{S}_2 \cdot \boldsymbol{\sigma} + K(R)\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

VOLUME 47, NUMBER 10 PHYSICAL REVIEW LETTERS 7 SEPTEMBER 1981

Two-Impurity Kondo Problem

C. Jayaprakash
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J. W. Wilkins
Nordisk Institut for Teoretisk Atomfsyik, DK-2100 Copenhagen Ø, Denmark, and Department of Physics, Cornell University, Ithaca, New York 14853

(Received 28 May 1981)

The two-impurity Kondo problem is studied by use of perturbative scaling techniques. The physics is determined by the interplay between the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction between the two impurity spins and the Kondo effect. In particular, for a strong ferromagnetic RKKY interaction the susceptibility exhibits three structures as the temperature is lowered, corresponding to the ferromagnetic locking together of the two impurity spins followed by a two-stage freezing out of their local moments by the conduction electrons due to the Kondo effect.

competition between RKKY-
interaction and Kondo screening

$$H = H_{\mathrm{kin}} + J\boldsymbol{S}_1 \cdot \boldsymbol{\sigma} + J\boldsymbol{S}_2 \cdot \boldsymbol{\sigma} + K(R)\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

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competition between RKKY-interaction and Kondo screening!

RKKY-coupled spin-singlet,
no Kondo screening

$K(R) \to -\infty$ $K(R) \to \infty$

$$H = H_{\mathrm{kin}} + J\boldsymbol{S}_1 \cdot \boldsymbol{\sigma} + J\boldsymbol{S}_2 \cdot \boldsymbol{\sigma} + K(R)\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

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competition between RKKY-interaction and Kondo screening!

RKKY-coupled spin-triplet,
Kondo screened by conduction electrons

RKKY-coupled spin-singlet,
no Kondo screening

$K(R) \to -\infty$ $K(R) \to \infty$

$$\delta = \pi/2$$

P. Nozières and A. Blandin,
J. Phys. (Paris) **41**, 193 (1980)

$$\delta = 0$$

RKKY-coupled spin-triplet,
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RKKY-coupled spin-singlet,
no Kondo screening

$K(R) \to -\infty$

$K(R) \to \infty$

particle-hole symmetry → $\delta = 0$ *or* $\delta = \pi/2$

A. Millis et al.
Field Theories in Condensed Matter Physics
ed. Z. Tesanovic, 1990

$$\delta = \pi/2$$

RKKY-coupled spin-triplet,
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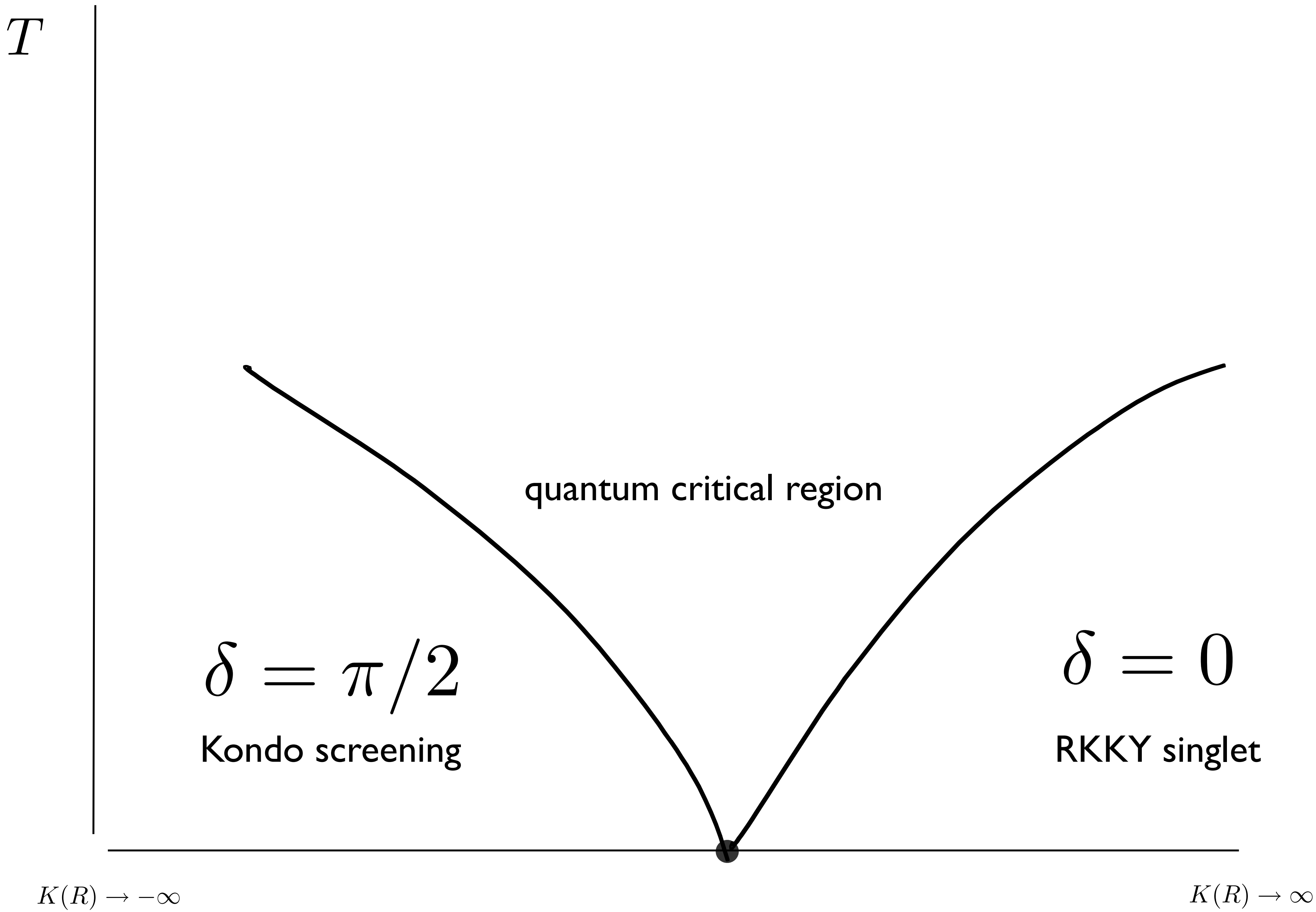
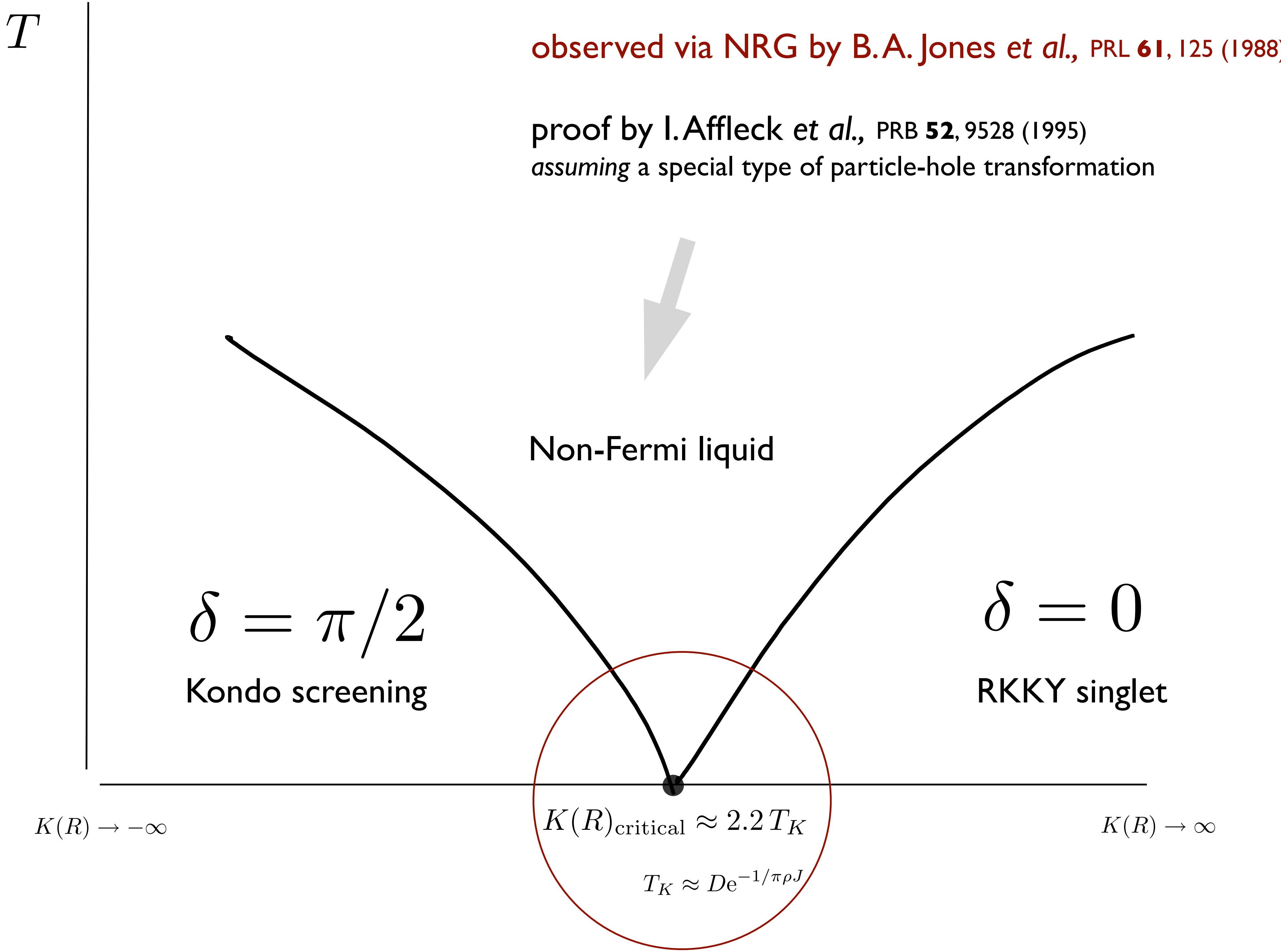
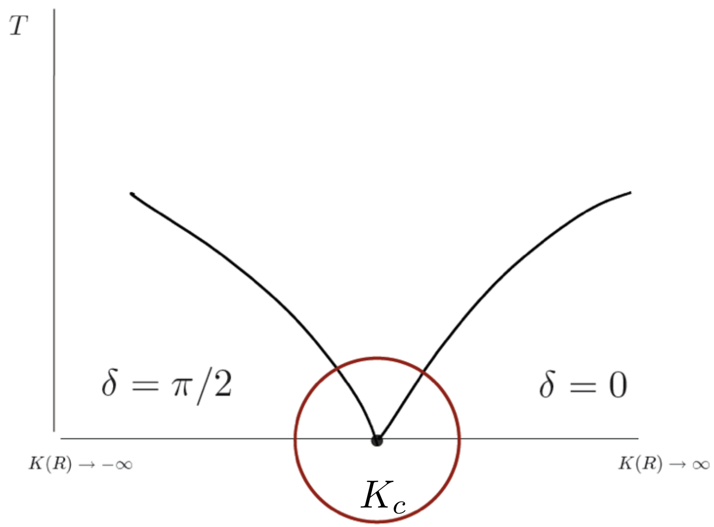


T
quantum critical region
$\delta = \pi/2$
Kondo screening
$\delta = 0$
RKKY singlet
$K(R) \to -\infty$
$K(R) \to \infty$


T
observed via NRG by B.A. Jones *et al.*, PRL **61**, 125 (1988)
proof by I. Affleck *et al.*, PRB **52**, 9528 (1995)
assuming a special type of particle-hole transformation
Non-Fermi liquid
$\delta = \pi/2$
Kondo screening
$\delta = 0$
RKKY singlet
$K(R) \to -\infty$
$K(R)_{\rm critical} \approx 2.2\, T_K$
$K(R) \to \infty$
$T_K \approx D e^{-1/\pi\rho J}$

At the quantum critical point...



$$\frac{C_{\mathrm{imp}}}{T} = \gamma \underset{T \to 0}{\longrightarrow} \frac{T_K}{(K - K_c)^2}$$

$$S_{\mathrm{imp}} \equiv \lim_{T\to 0} \lim_{L\to\infty} [S(L,T) - S_0(L,T)] = \ln \sqrt{2}$$

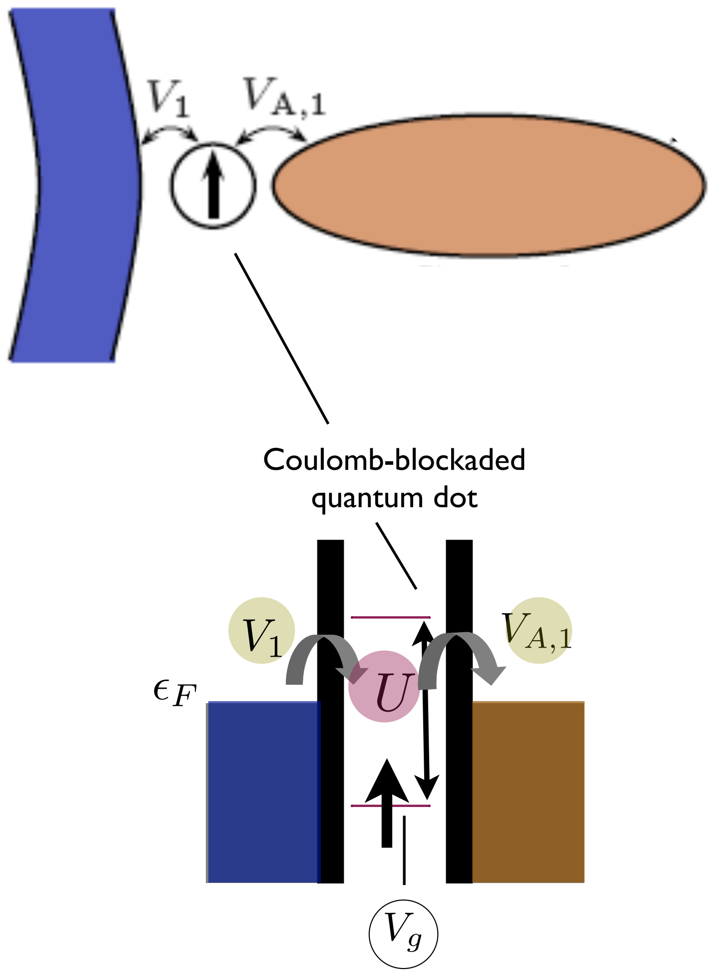
I. Affleck *et al.*, PRB **52**, 9528 (1995)

"fractional ground state degeneracy" $g^A = \sqrt{2}$

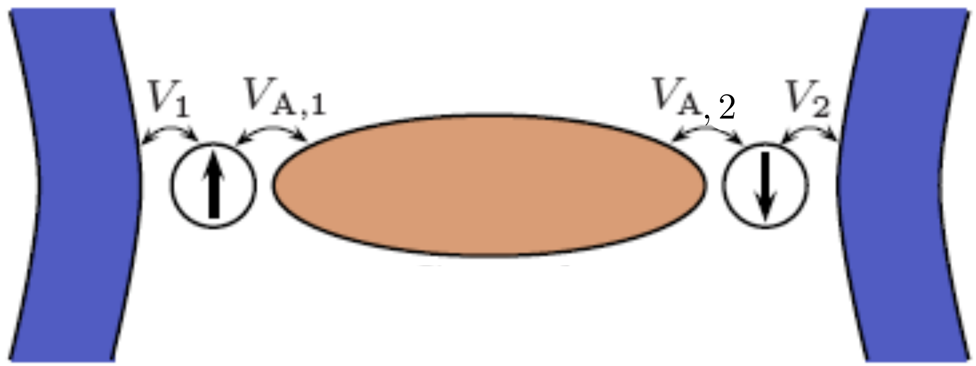
$$G \approx G_0(1 - \lambda_1 T^{1/2})$$

G. Zaránd *et al.*, PRL **97**, 166802 (2006)

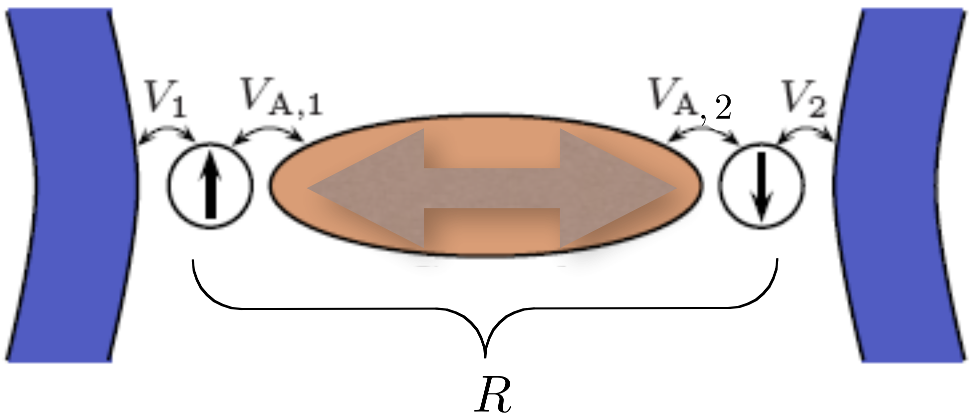
Realization in double quantum-dot systems



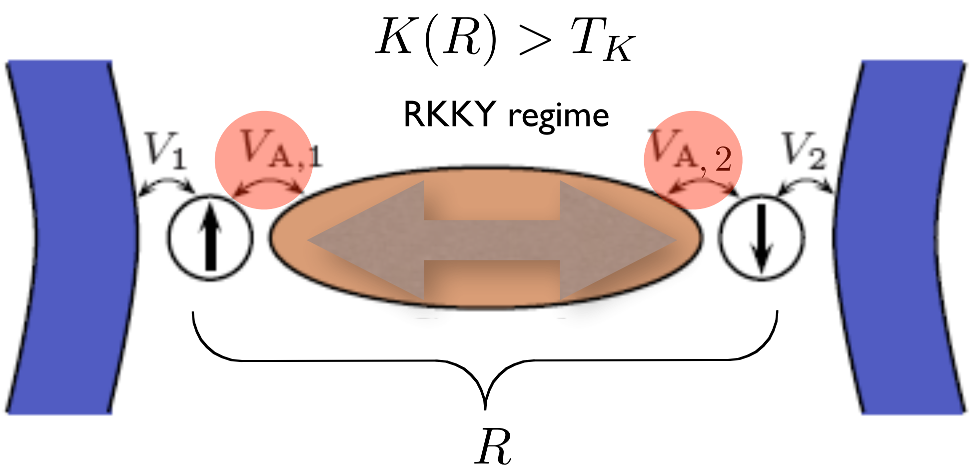
spin exchange $J \propto V^2/U$



V_1
$V_{A,1}$
$V_{A,2}$
V_2

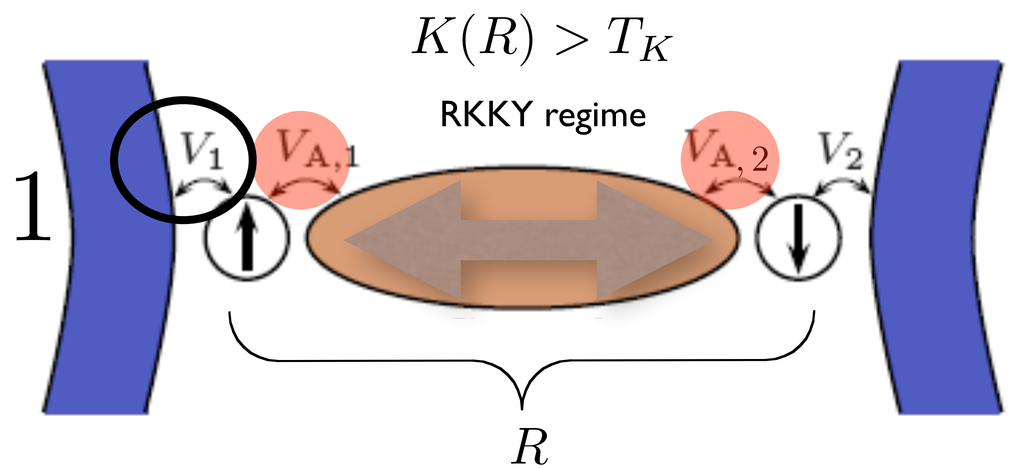


RKKY coupling $K(R) \propto (J^2/R^2)\cos(k_F R)$

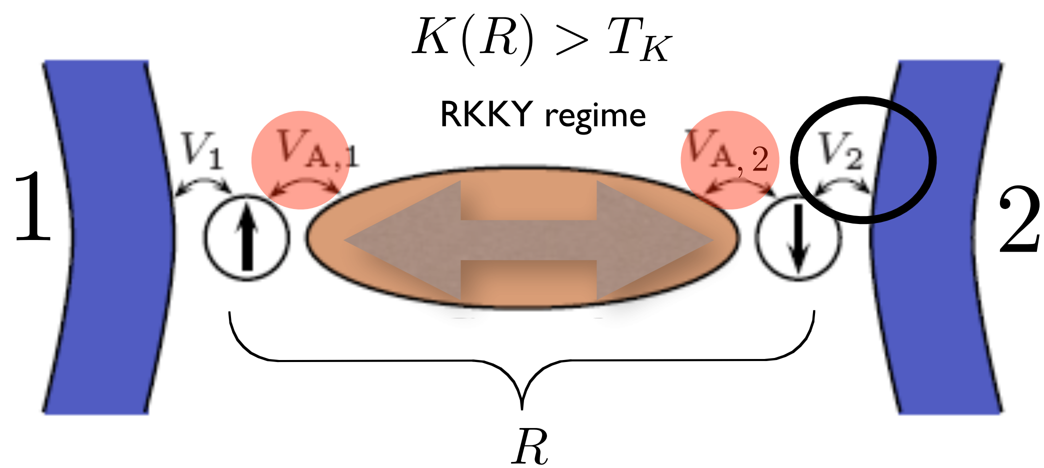


RKKY coupling $K(R) \propto (J^2/R^2)\cos(k_F R)$

Kondo temperature $T_K \propto D\exp(-1/\pi\rho J)$



$$H_{\mathrm{int}} = J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S}_2 \cdot \boldsymbol{\sigma}_2 + K(R) \boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$



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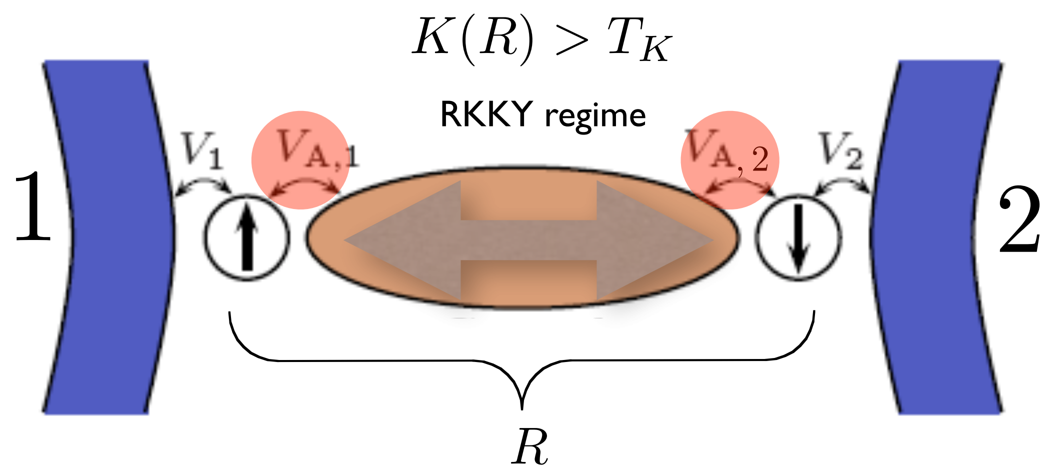

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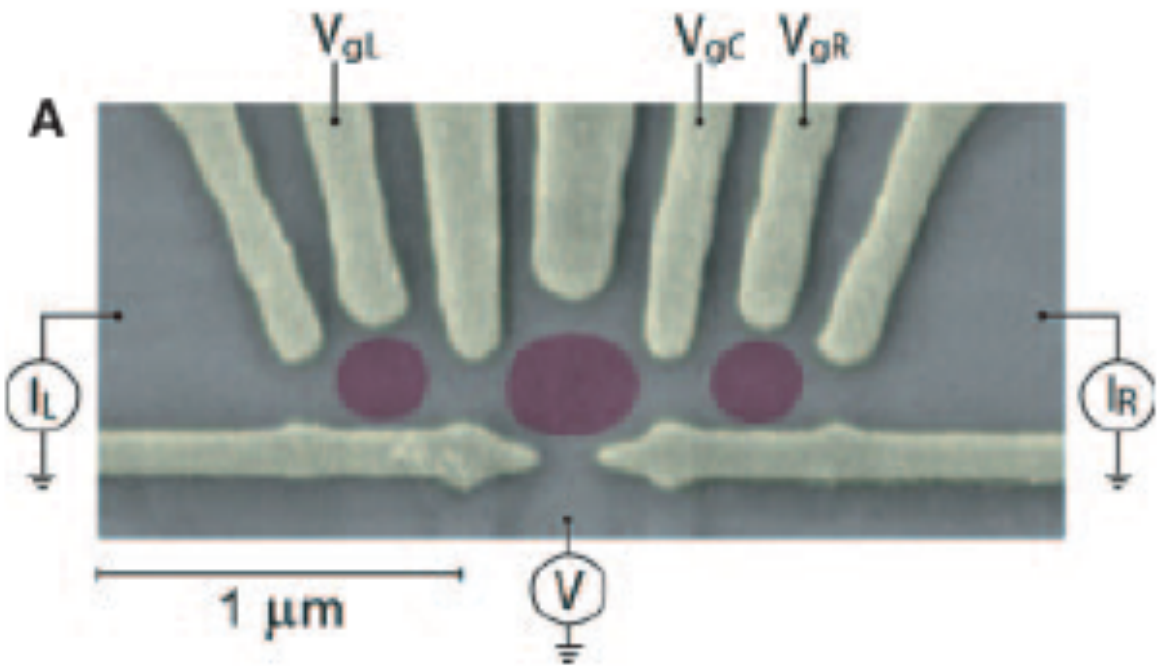
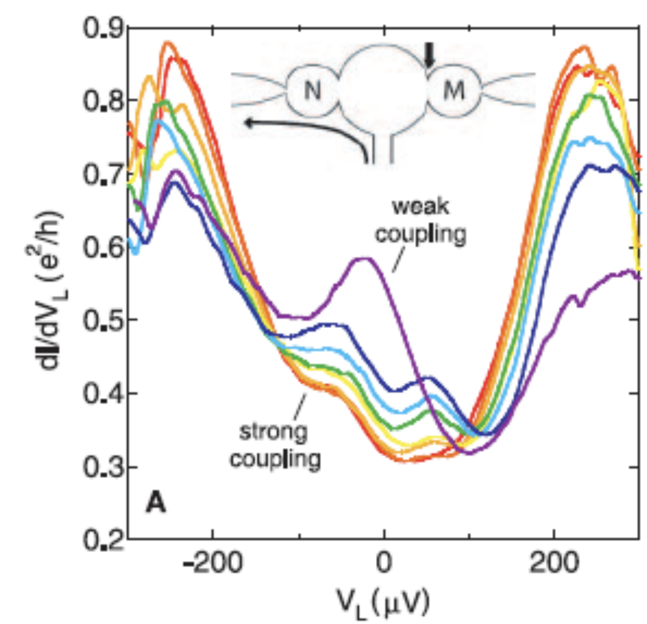
No transfer of electrons between 1 and 2: quantum critical point $K_c \approx 2.2 T_K$ is stable against electron-hole symmetry breaking *and* breaking of parity

G. Zaránd *et al.*, PRL **97**, 166802 (2006)

Realization in double quantum-dot systems

N. J. Craig *et al.*, Science **304**, 565 (2004)





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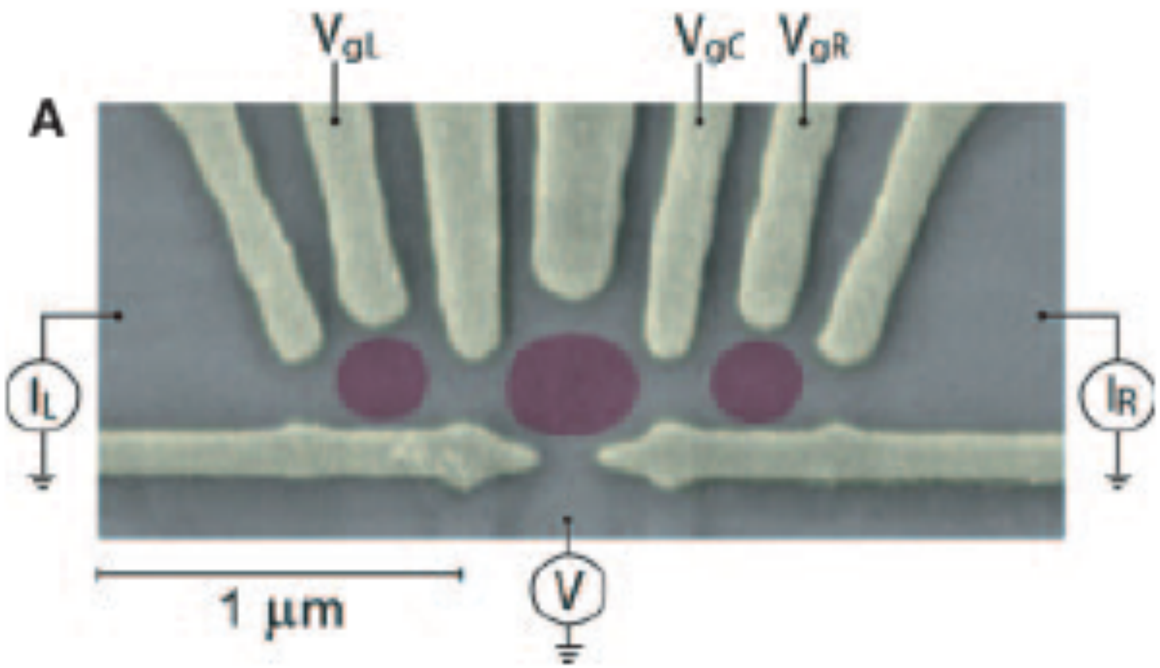
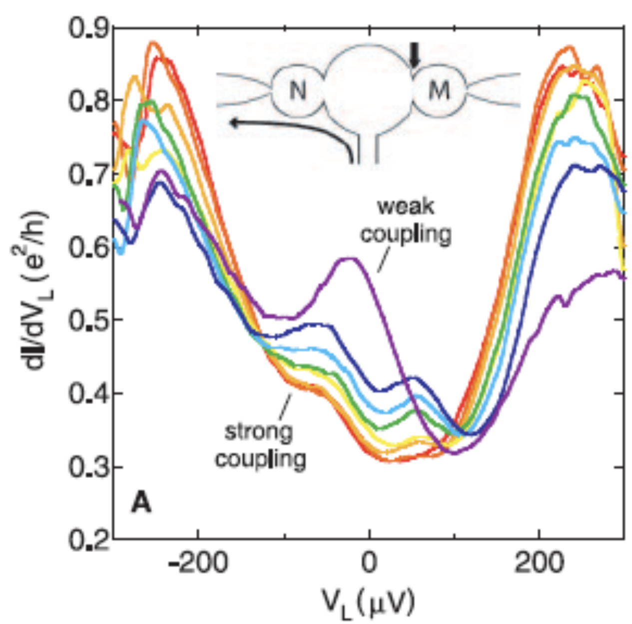
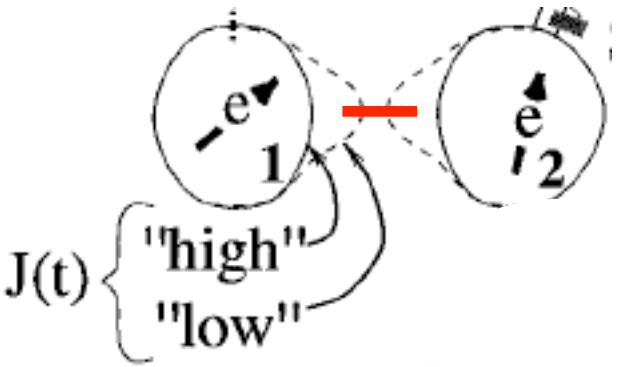




Nota Bene:
The central dot supports both RKKY and Kondo screening.
This experiment does ***not*** probe quantum criticality.
Instead, **important for proving gate-controlled RKKY!**

Possible application
”Long-distance” control of two-qubit gates...



$$H_s(t) = J(t)\vec{S}_1 \cdot \vec{S}_2$$

$$U_s(t) = T\exp\{-i\int_0^t H_s(t')dt'\}$$

$$U_s(\tau_s = \pi\hbar/J_0) = U_{sw}$$

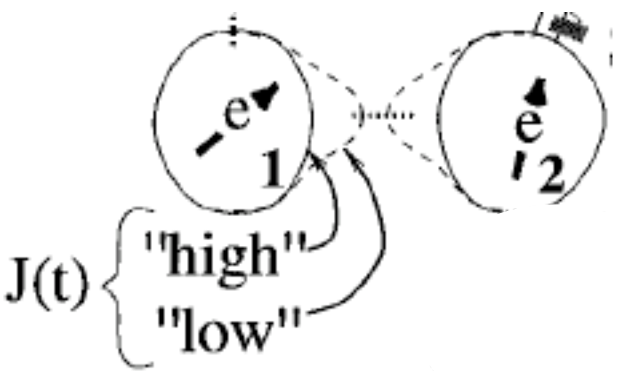
$$U_{sw} \mid ij\,\rangle = \mid ji\,\rangle, \quad i,j = \uparrow,\downarrow$$

Loss–DiVincenzo proposal for
spin-based quantum computing

PRA **57**, 120 (1998)

Possible application

”Long-distance” control of two-qubit gates...



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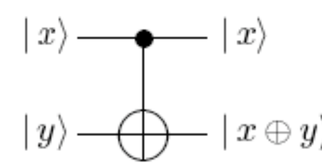
$$U_{sw} \,|\, ij \,\rangle = |\, ji \,\rangle, \quad i,j = \uparrow, \downarrow$$

$$U_{\mathrm{CNOT}} = e^{i\frac{\pi}{2}S_1^z} e^{-i\frac{\pi}{2}S_2^z} U_{sw}^{\frac{1}{2}} e^{i\pi S_1^z} U_{sw}^{\frac{1}{2}}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



U_{CNOT} + “single-qubit gates” (single-spin rotations)

→ “universal” quantum computing

”Long-distance” control of two-qubit gates...?

”Long-distance” control using RKKY

N. J. Craig *et al.*, Science **304**, 565 (2004)
M.G. Vavilov and L.I. Glazman, PRL **94**, 086805 (2005)

”Long-distance” control using RKKY

What about *spin decoherence* caused by the conduction electrons via RKKY?

GaAs/AlGaAs

$$T \approx 10\ \mathrm{mK}$$
$$R \approx 10\ \mathrm{nm}$$
$$K_0(R) \approx 5\ \mu\mathrm{eV}$$
$$\tau_{\mathrm{dec}} \approx 60\ \mathrm{ns}$$
$$\tau_{\mathrm{swap}} \approx 0.3\ \mathrm{ns}$$

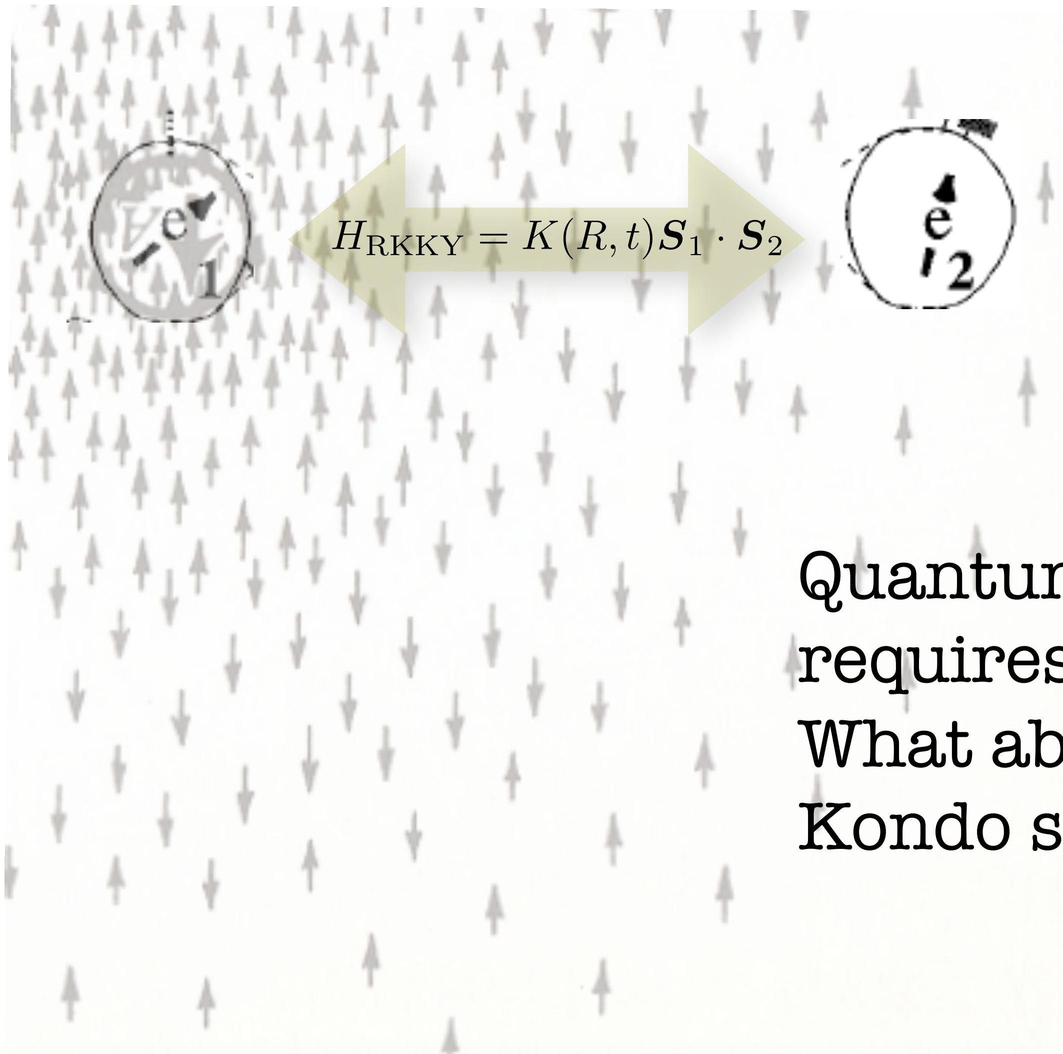
↓

$\sim$ 200 coherent $\sqrt{\mathrm{SWAP}}$ operations

N. J. Craig *et al.*, Science **304**, 565 (2004)
M.G. Vavilov and L.I. Glazman, PRL **94**, 086805 (2005)

Y. Rikitake and H. Imamura, PRB **72**, 033308 (2005)

”Long-distance” control using RKKY



Quantum information processing requires **entanglement** of qubits. What about the competition from Kondo screening?

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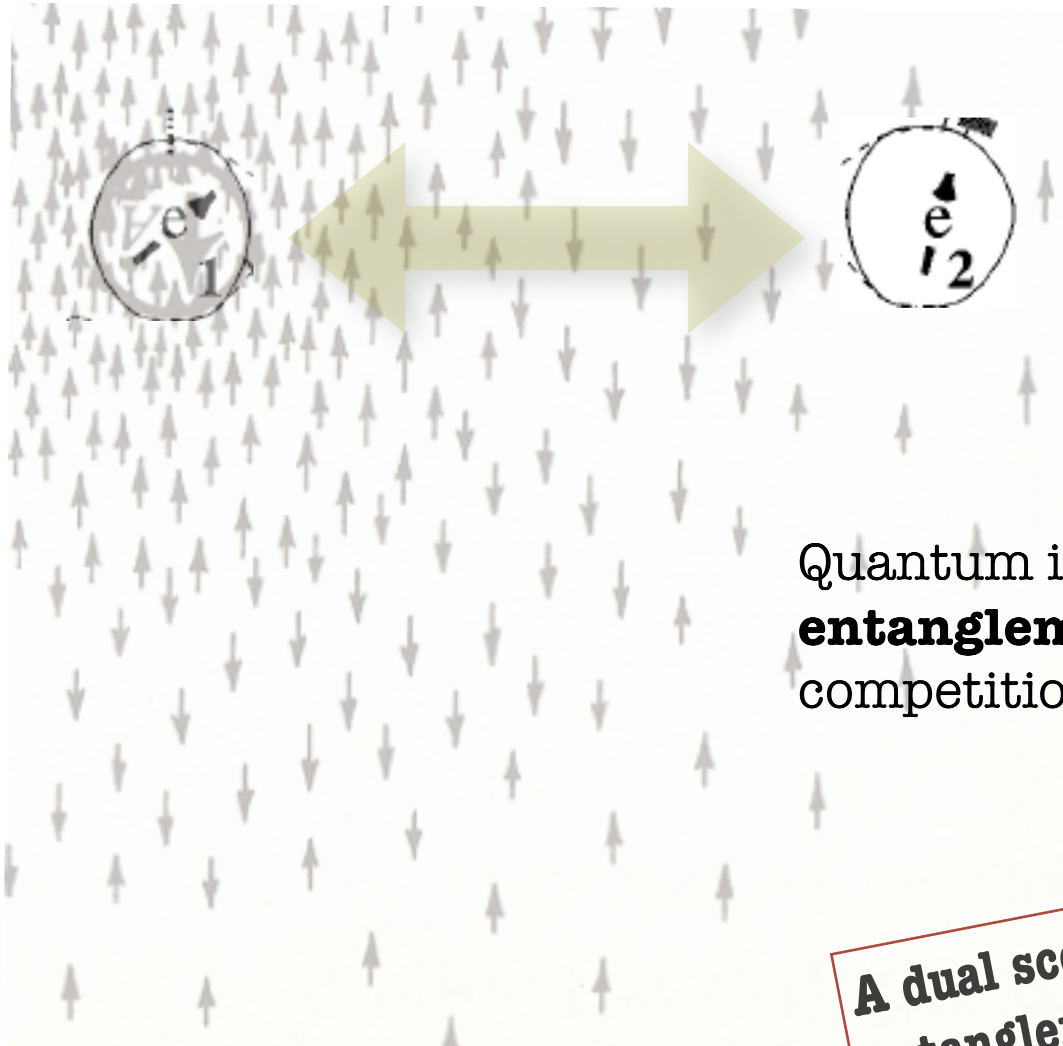

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A dual scenario: nonlocal control of two-qubit entanglement from Kondo screening + quantum quench... P. Sodano, A. Bayat, and S. Bose, PRB **81**, 100412 (2010)

N. J. Craig *et al.*, Science **304**, 565 (2004)
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Quantum entanglement probe of

the two-impurity Kondo groundstates for different RKKY couplings

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How is entanglement distributed between impurities and electrons?

Is there an RKKY threshold for impurity-bulk decoupling?

How does entanglement behave at the quantum critical point?

What can we learn about the *Kondo screening cloud?*

and more....

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How is entanglement distributed between impurities and electrons?

How to make the impurities decouple from the bulk?

How does entanglement behave at the quantum critical point?

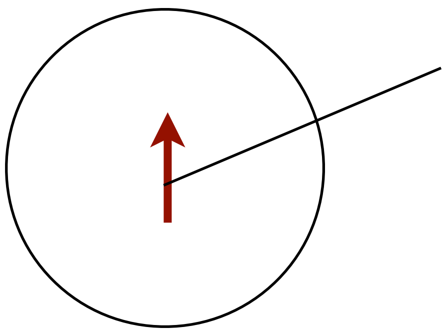
What can we learn about the *Kondo screening cloud?*

Method: DMRG computation of *negativity* and *von Neumann entropy* via a spin chain emulation of the two-impurity Kondo model

Spin chain modeling useful for DMRG and intuition!

start with the *single-impurity* Kondo model

$$H = \int \mathrm{d}^3\boldsymbol{r}[\psi^\dagger(-\nabla^2/2m)\psi + J_K\delta^3(0)\psi^\dagger\boldsymbol{\sigma}\psi\cdot\boldsymbol{S}_{\mathrm{imp}}]$$



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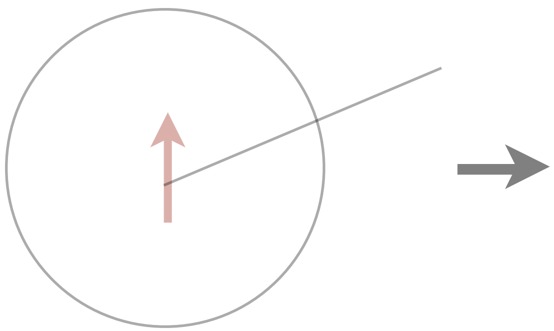
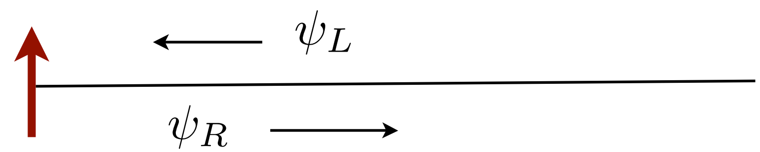
start with the *single-impurity* Kondo model

$$H = \frac{v_F}{2\pi}\int_0^\infty \mathrm{d}x\,(i\psi_L^\dagger\partial_x\psi_L - i\psi_R^\dagger\partial_x\psi_R) + v_F\lambda\psi_L^\dagger(0)\boldsymbol{\sigma}\psi_L(0)\cdot\boldsymbol{S}_{\mathrm{imp}}$$





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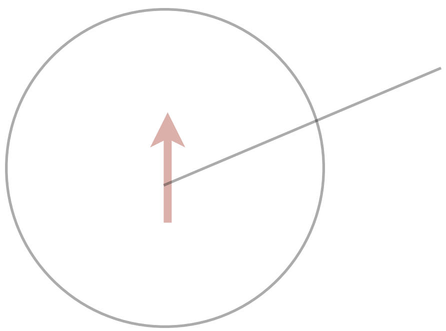
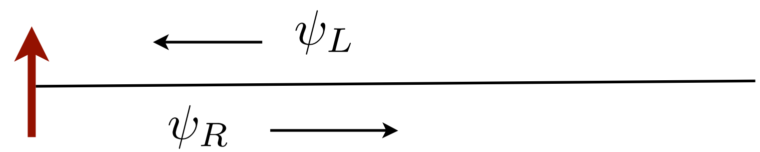


$$H = \frac{v_F}{2\pi}\int_0^\infty \mathrm{d}x\,(i\psi_L^\dagger \partial_x \psi_L - i\psi_R^\dagger \partial_x \psi_R) + v_F \lambda \psi_L^\dagger(0)\boldsymbol{\sigma}\psi_L(0)\cdot \boldsymbol{S}_{\mathrm{imp}}$$

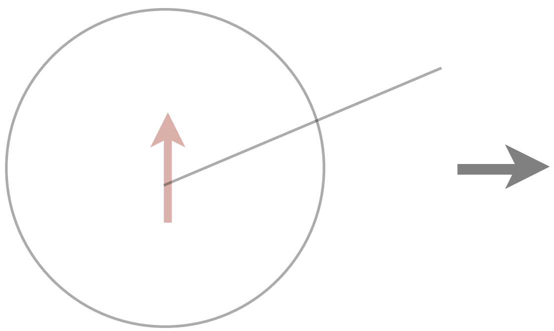


$$H = H_{\mathrm{spin}} + \cancel{H_{\mathrm{charge}}}$$

$$H_{\mathrm{spin}} = \frac{v_F}{6\pi}\int_0^\infty \mathrm{d}x(\boldsymbol{J}_L\cdot\boldsymbol{J}_L + \boldsymbol{J}_R\cdot\boldsymbol{J}_R) + v_F\lambda \boldsymbol{J}_L(0)\cdot\boldsymbol{S}_{\mathrm{imp}}$$

Spin chain modeling useful for DMRG and intuition...

start with the *single-impurity* Kondo model



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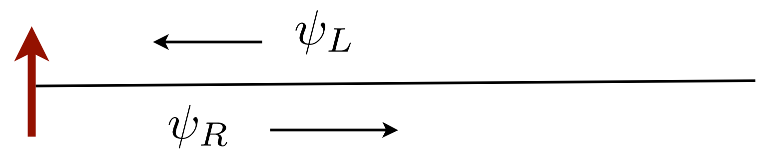

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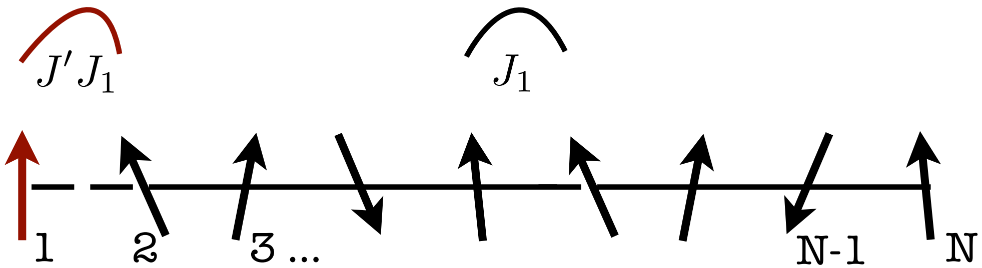
This is the low-energy effective theory
of a spin chain with a weak boundary link!



$$H = J_1 \sum_{i=2}^{N-1} \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + J' J_1 \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 \qquad J' < 1$$

Jordan-Wigner + bosonization

$$\boldsymbol{S}_i = \boldsymbol{J}_L(ai) + \boldsymbol{J}_R(ai) + (-1)^i \text{constant} \cdot \boldsymbol{n}(ai)$$

a = lattice spacing

+ continuum limit

$$H_{\text{spin}} = \frac{v_F}{6\pi} \int_0^\infty \mathrm{d}x (\boldsymbol{J}_L \cdot \boldsymbol{J}_L + \boldsymbol{J}_R \cdot \boldsymbol{J}_R) + v_F \lambda \boldsymbol{J}_L(0) \cdot \boldsymbol{S}_{\text{imp}}$$

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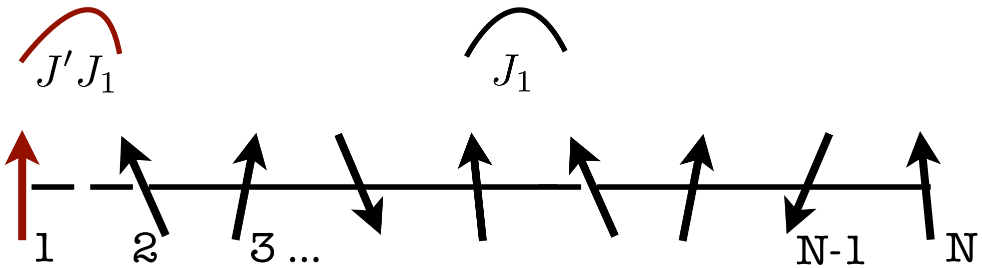

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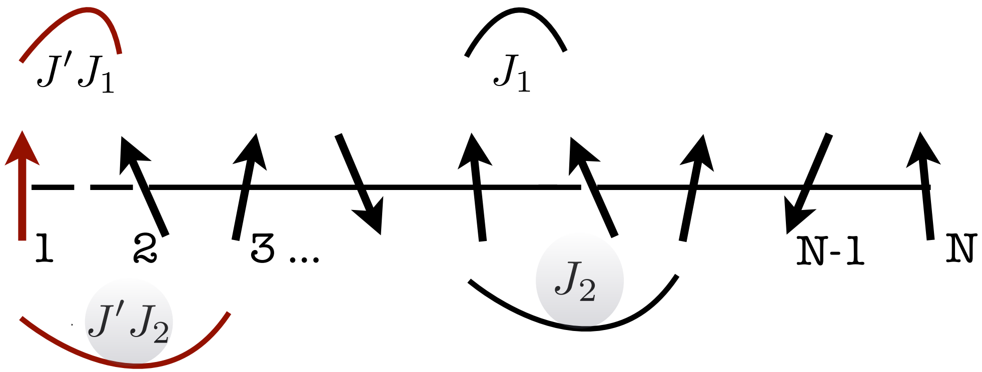
$$+ \; \textit{marginally irrelevant} \; + \; \textit{irrelevant} \text{ terms}$$



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Jordan-Wigner + bosonization

$$\boldsymbol{S}_i = \boldsymbol{J}_L(ai) + \boldsymbol{J}_R(ai) + (-1)^i \text{constant} \cdot \boldsymbol{n}(ai)$$

+ continuum limit

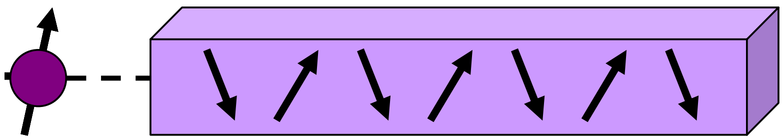
$$J_2 = 0.2412\, J_1$$

$$H_{\text{spin}} = \frac{v_F}{6\pi} \int_0^\infty \mathrm{d}x (\boldsymbol{J}_L \cdot \boldsymbol{J}_L + \boldsymbol{J}_R \cdot \boldsymbol{J}_R) + v_F \lambda \boldsymbol{J}_L(0) \cdot \boldsymbol{S}_{\text{imp}}$$

$$+\ \textit{marginally irrelevant}\ +\ \textit{irrelevant}\ \text{terms}$$

S. Eggert, PRB **54**, 9616(R) (1996)

’Kondo spin chain’



$$H = \sum_{j=1}^{2} J_j \left(J' \boldsymbol{S}_1 \cdot \boldsymbol{S}_{j+1} + \sum_{i=2}^{N-j} \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+j} \right)$$

same low-energy physics as the (spin sector) of the single-impurity Kondo model

S. Rommer and S. Eggert, PRB **62**, 4370 (2000)
N. Laflorencie *et al.*, J. Stat. Mech. P02007 (2008)

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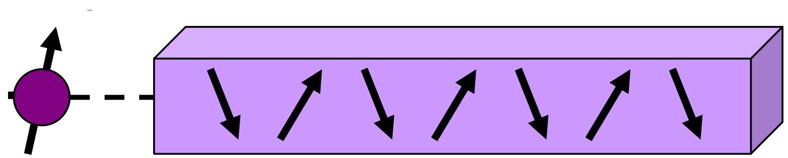

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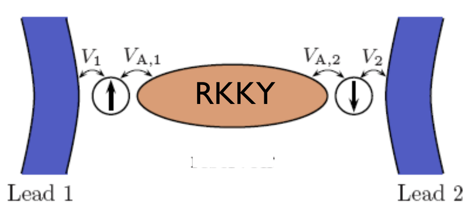
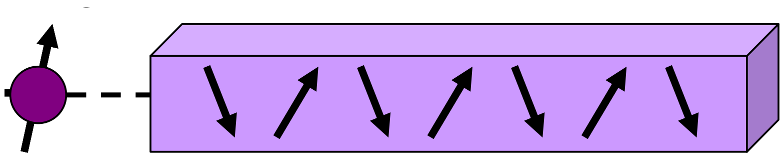
Implication for two-impurity Kondo model?



’Kondo spin chain’



$$H=\sum_{j=1}^{2} J_j\left(J'\boldsymbol{S}_1\cdot\boldsymbol{S}_{j+1}+\sum_{i=2}^{N-j}\boldsymbol{S}_i\cdot\boldsymbol{S}_{i+j}\right)$$

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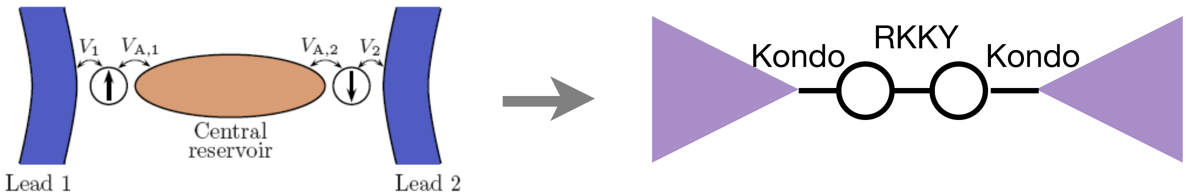

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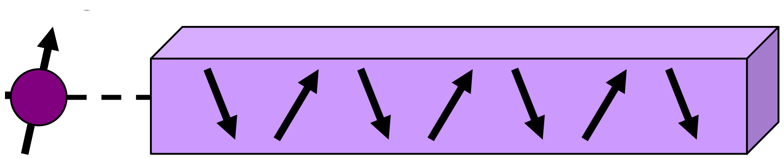

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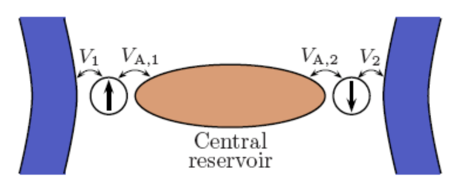
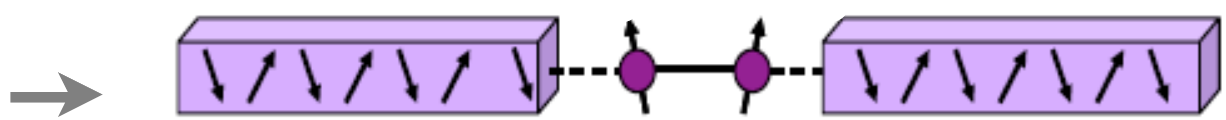
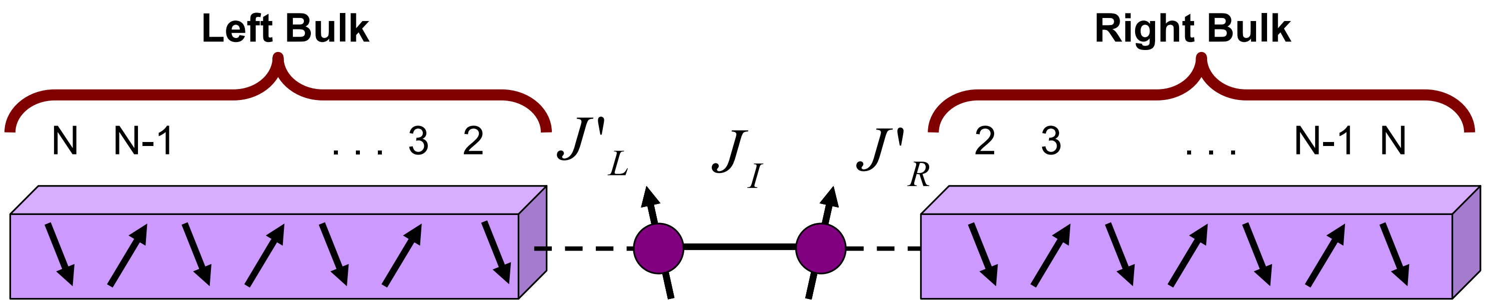




two RKKY-coupled ’Kondo spin chains’
(a.k.a. *‘two-impurity Kondo chain’*)

‘Two-impurity Kondo spin chain’



$$H = \sum_{k=L,R} H_k + H_I$$

$$H_k = \sum_{j=1}^{2} J_j \left(J'_k \boldsymbol{S}_1^k \cdot \boldsymbol{S}_{j+1}^k + \sum_{i=2}^{N-j} \boldsymbol{S}_i^k \cdot \boldsymbol{S}_{i+j}^k \right) \qquad J'_L = J'_R < 1$$

$$H_I = J_I J_1 \boldsymbol{S}_1^L \cdot \boldsymbol{S}_1^R$$

Well adapted for DMRG... and entanglement probes!

Entanglement probes of a quantum many-particle groundstate $|\Psi\rangle$

Entanglement probes of a quantum many-particle groundstate $|\Psi\rangle$


John von Neumann in the 1940s

von Neumann entropy

$$S_A = -\mathrm{Tr}\rho_A \log_2 \rho_A$$

$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$$

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John von Neumann in the 1940s

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What about

?

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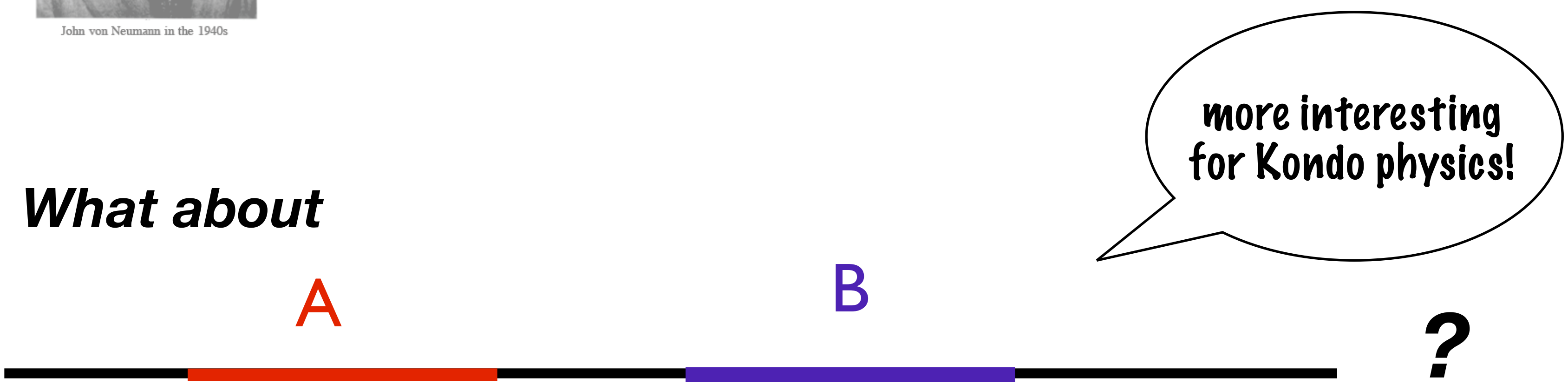

Computable entanglement measure for this case: *Negativity*

G. Vidal and R. F. Werner,
PRA **65**, 032314 (2002)

$$\mathcal{N}(\rho_{AB}) = \sum_i |a_i| - 1$$

a_i eigenvalues of $\rho_{AB}^{T^A}$

$\rho_{AB} = \mathrm{Tr}_{(AB)^c} |\Psi\rangle\langle\Psi|$

Computable entanglement measure for this case: *Negativity*


$$\mathcal{N}(\rho_{AB}) = \sum_i |a_i| - 1$$

a_i negative eigenvalues of $\rho_{AB}^{T_A}$

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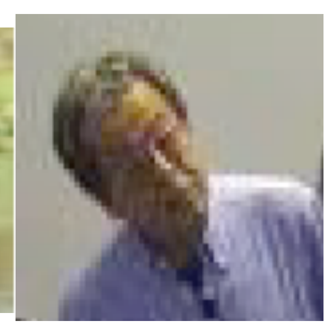
First-time (?) **application** in condensed matter:
Entanglement probe of (single-impurity) Kondo chain

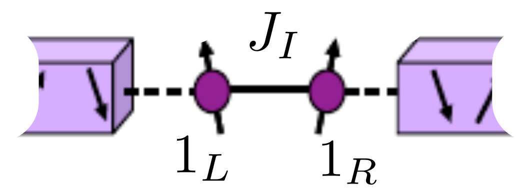

A. Bayat, P. Sodano, and S. Bose, PRB **81**, 064429 (2010)

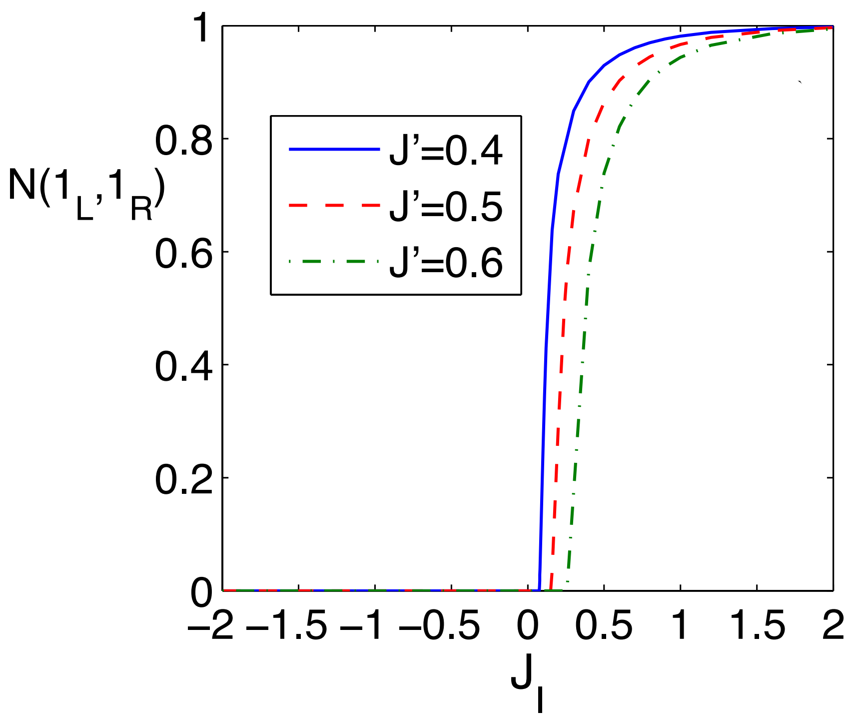
Using negativity (and von Neumann entropy!) to explore the entanglement properties of the **two-impurity Kondo chain** with DMRG...

What do we learn?

negativity for two qubits in a Werner state = concurrence

- **Entanglement between the two impurities**





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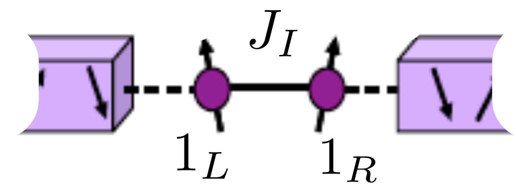

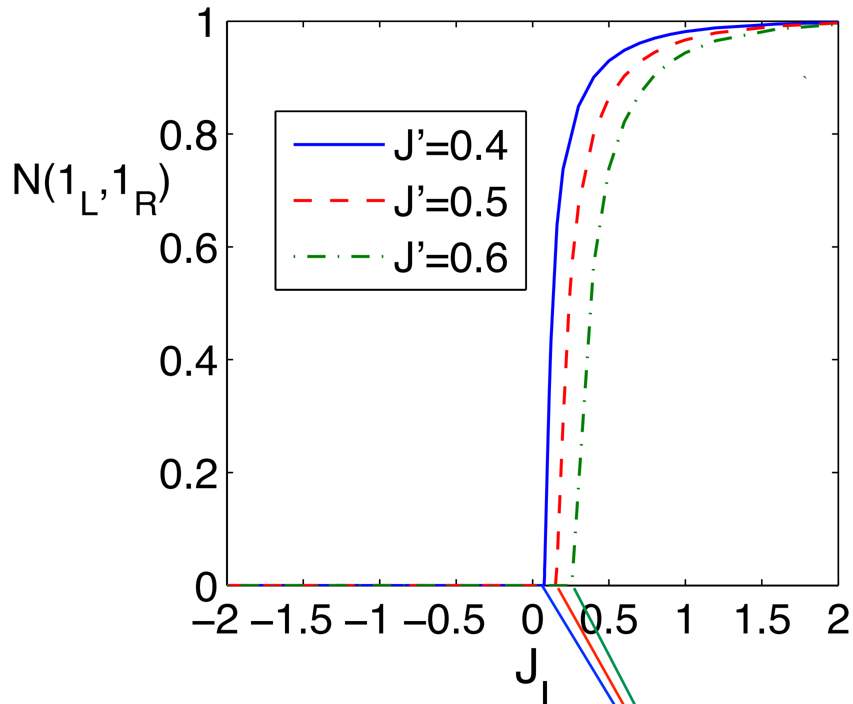

J_I^c "quantum critical point...!"

S.Y. Cho and R. H. McKenzie, PRA **73**, 012109 (2006)

negativity for two qubits in a Werner state = concurrence

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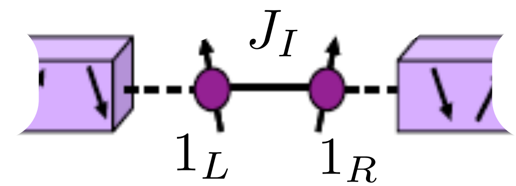
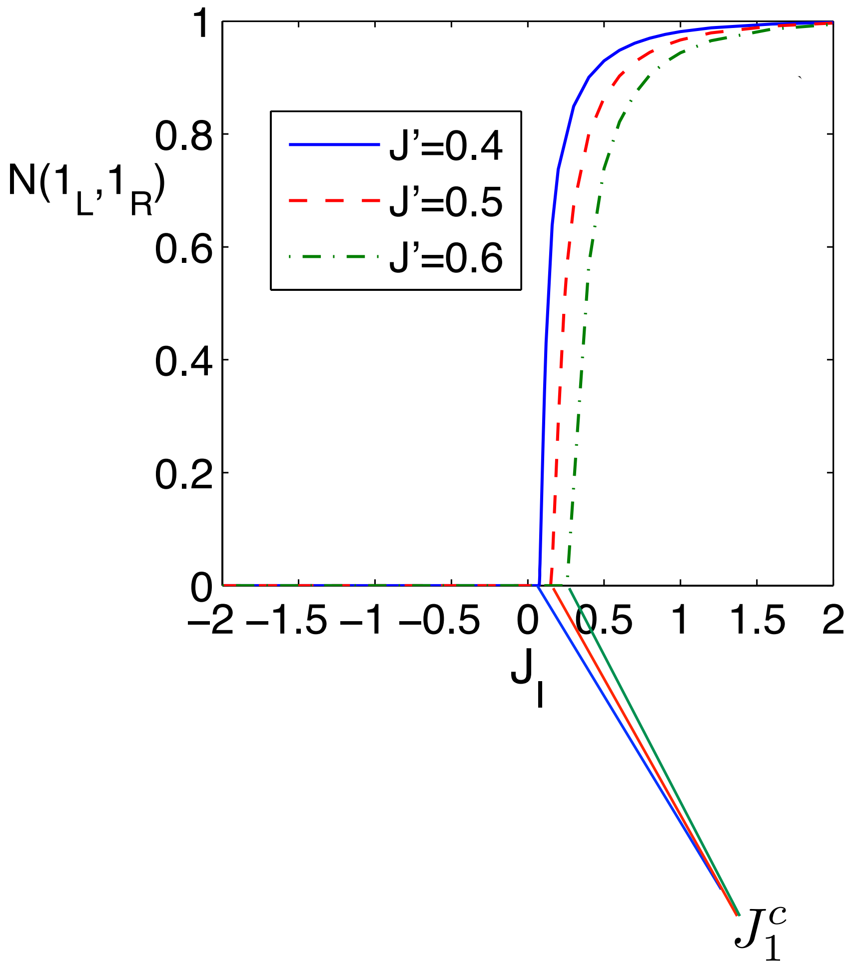
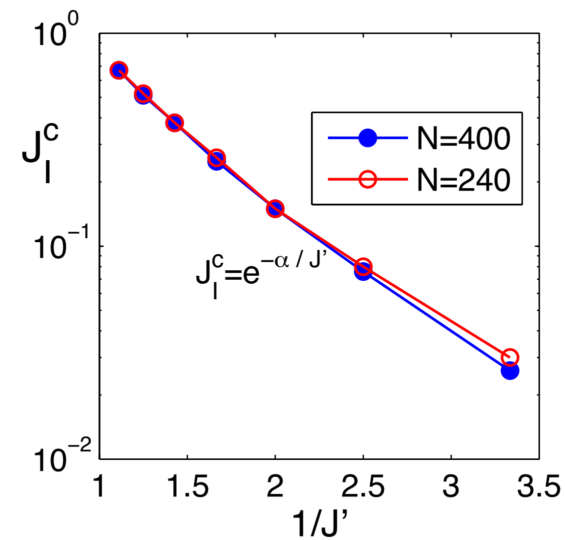


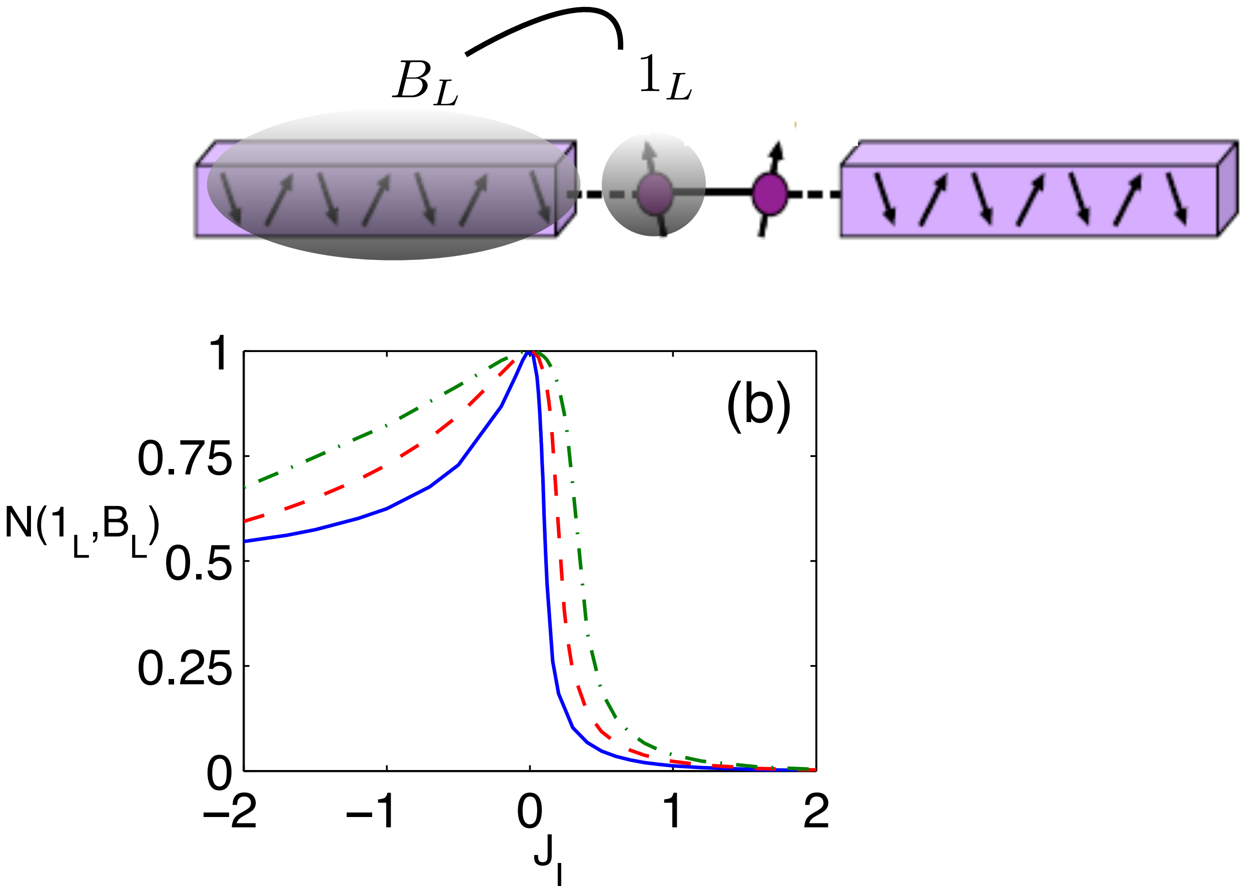




agrees with numerical RG results

B. A. Jones and C. M. Varma, PRB **40**, 324 (1989)

- **Signature of the quantum phase transition at J_1^c**



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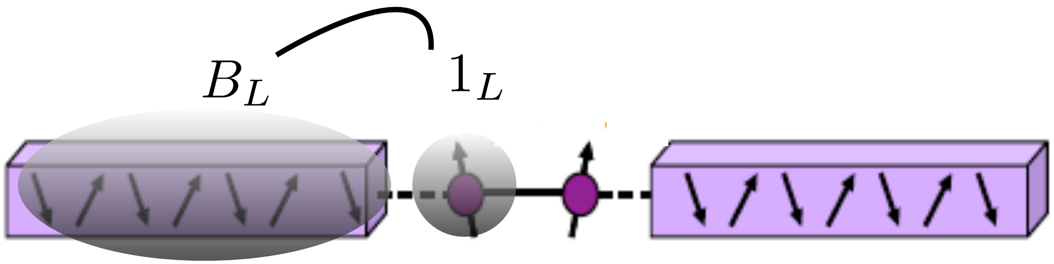

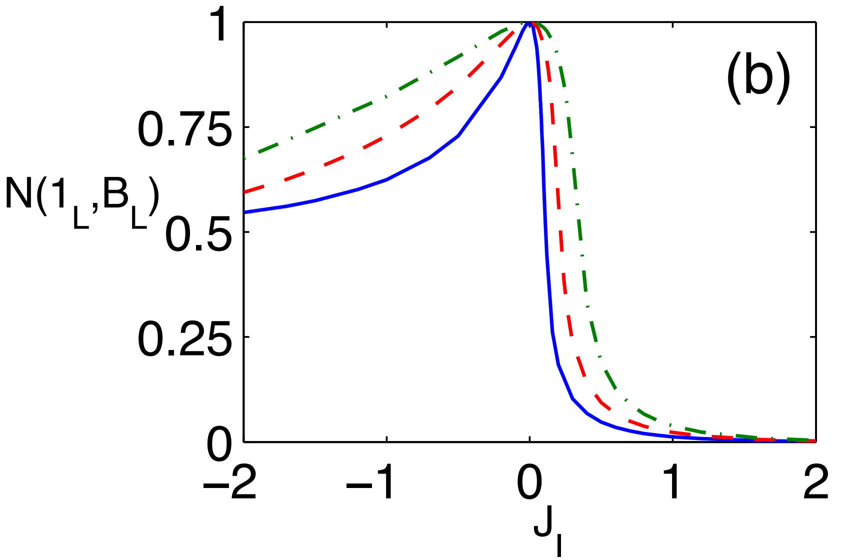

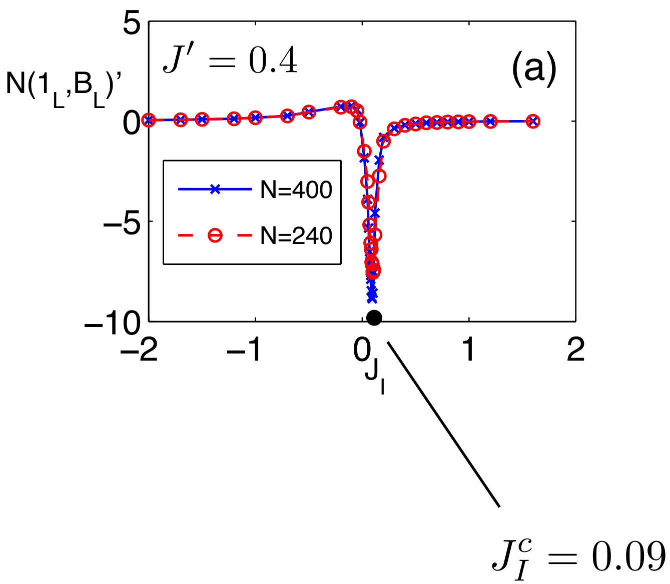

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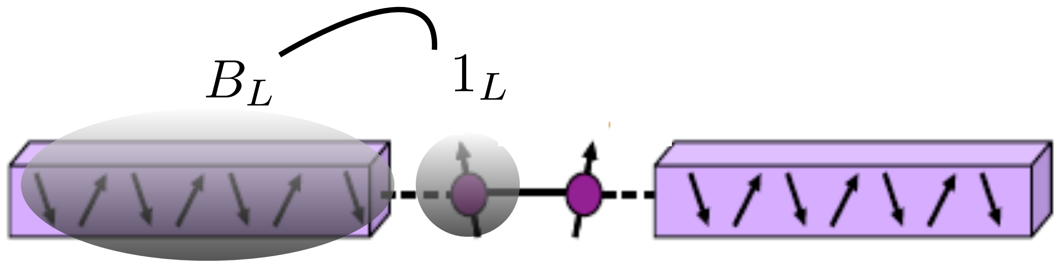
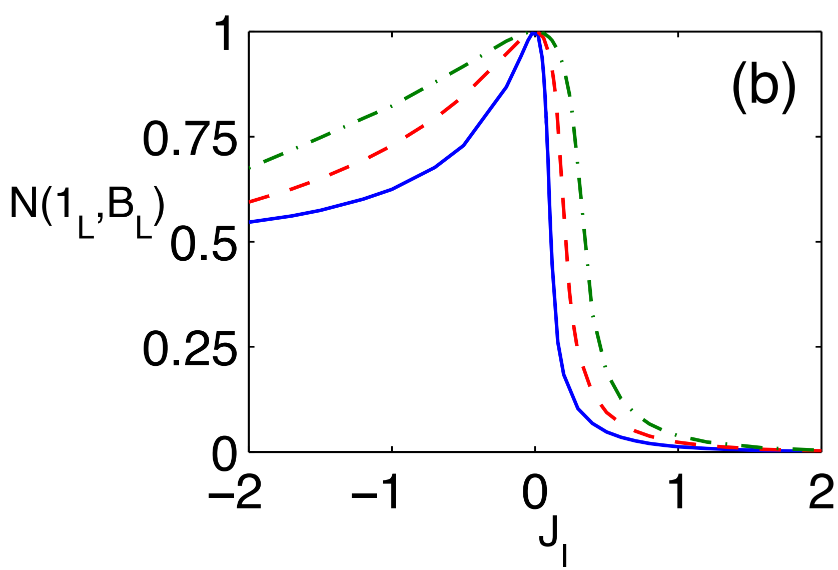
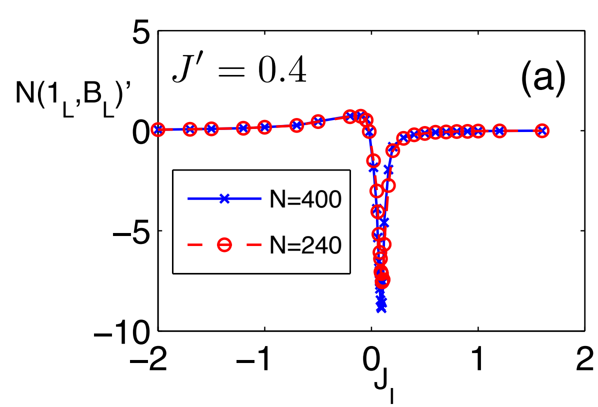




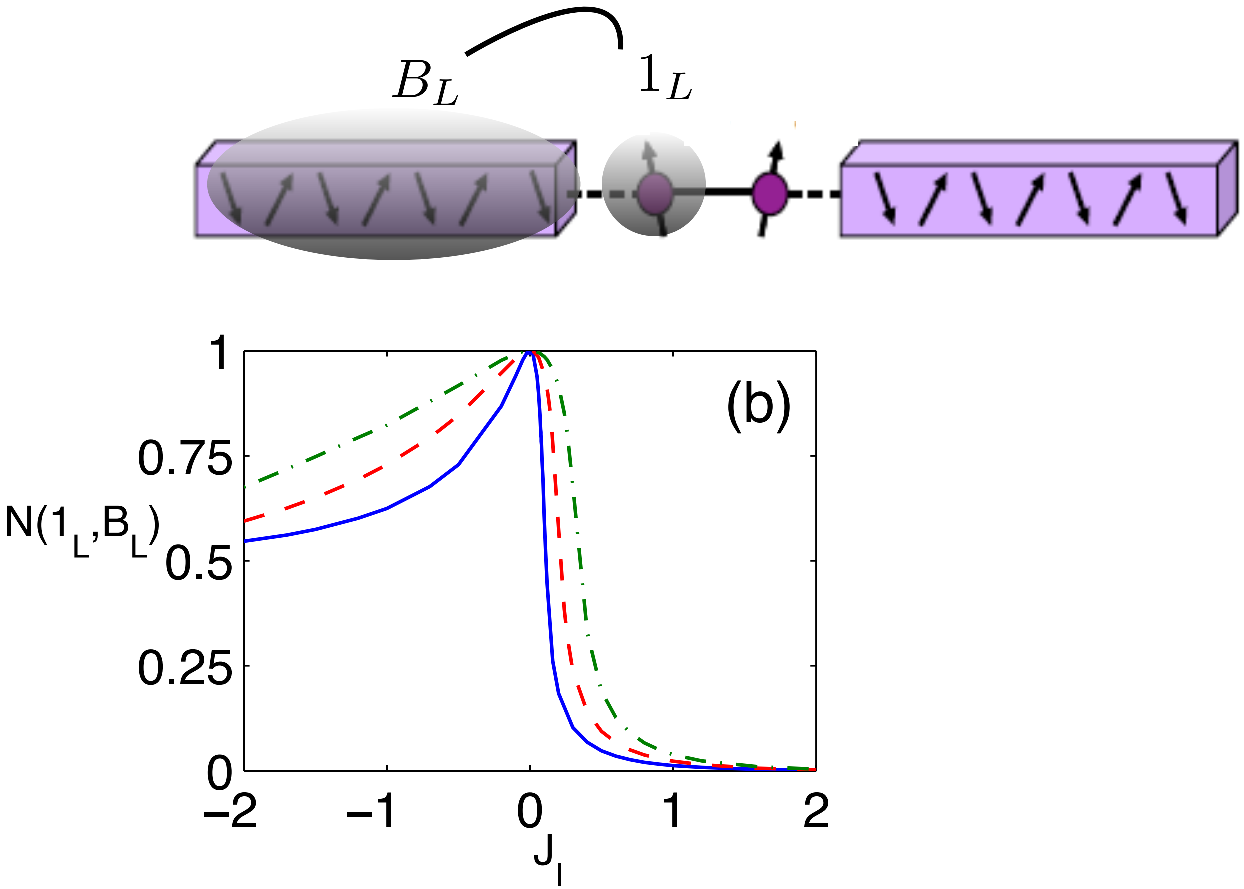


Discontinuity ***[divergence]*** in [the derivative of] an entanglement measure with respect to a control parameter

→ 1st order ***[2nd order]*** *quantum phase transition*

L.-A. Wu et al., PRL **93**, 250404 (2004)

- **Entanglement structure**



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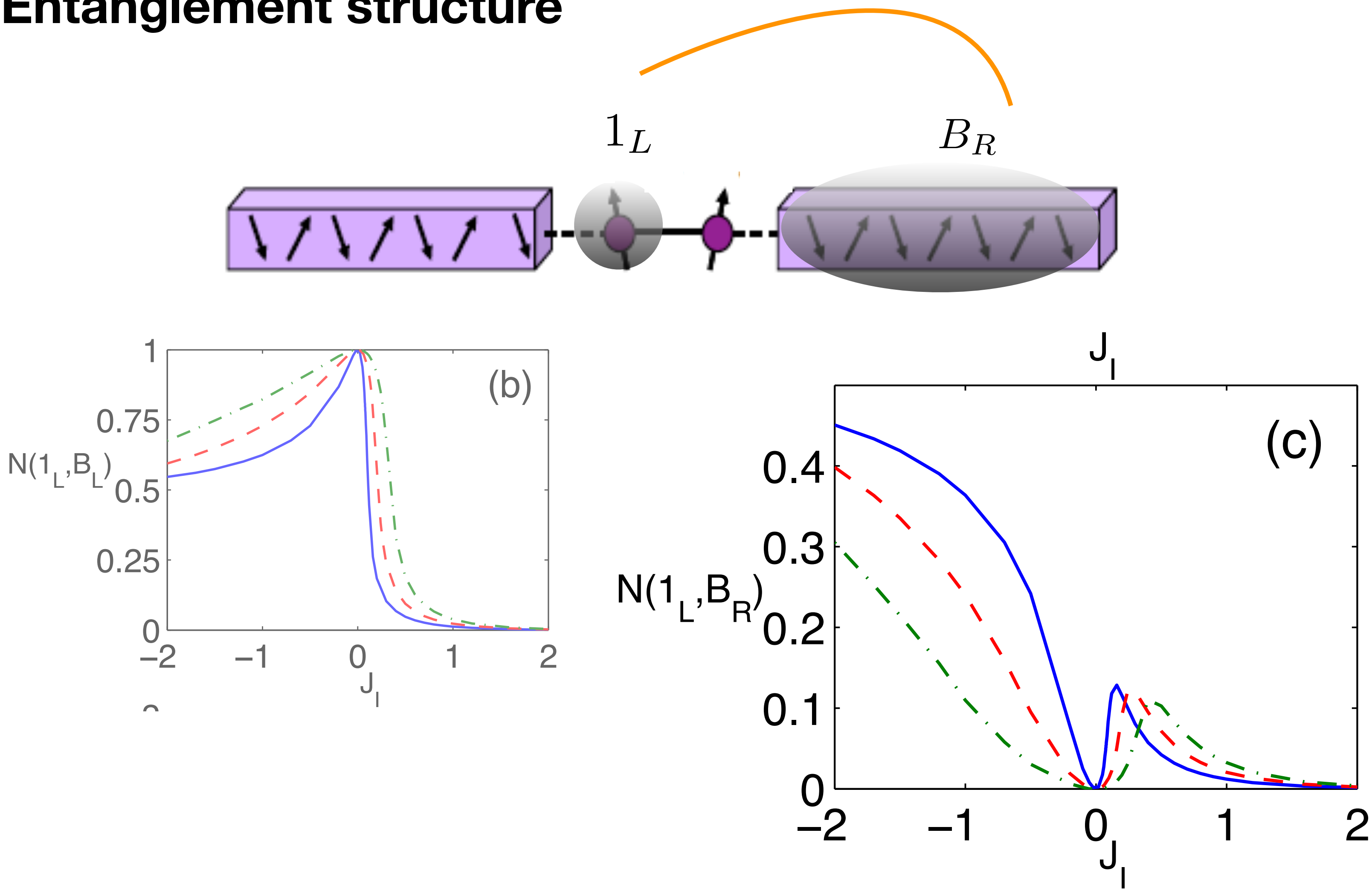

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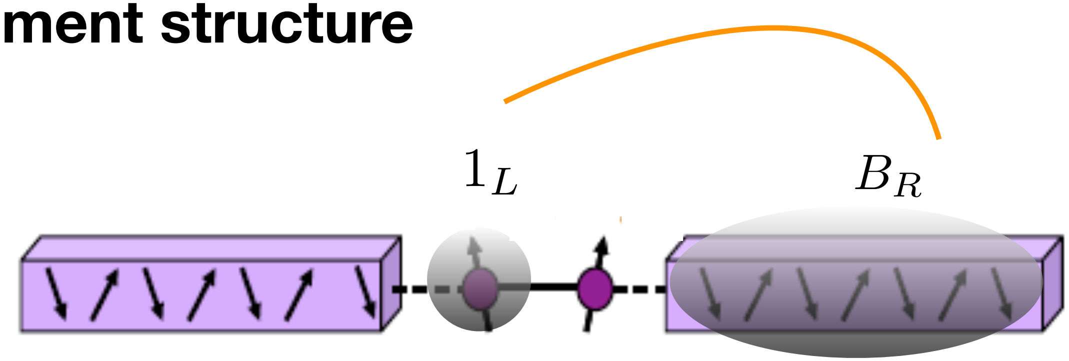
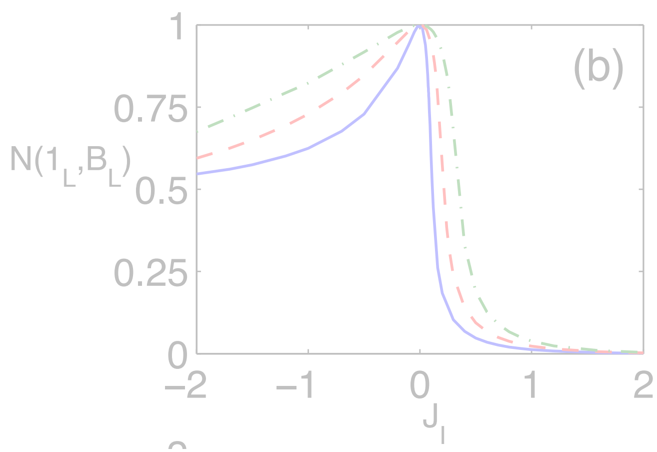
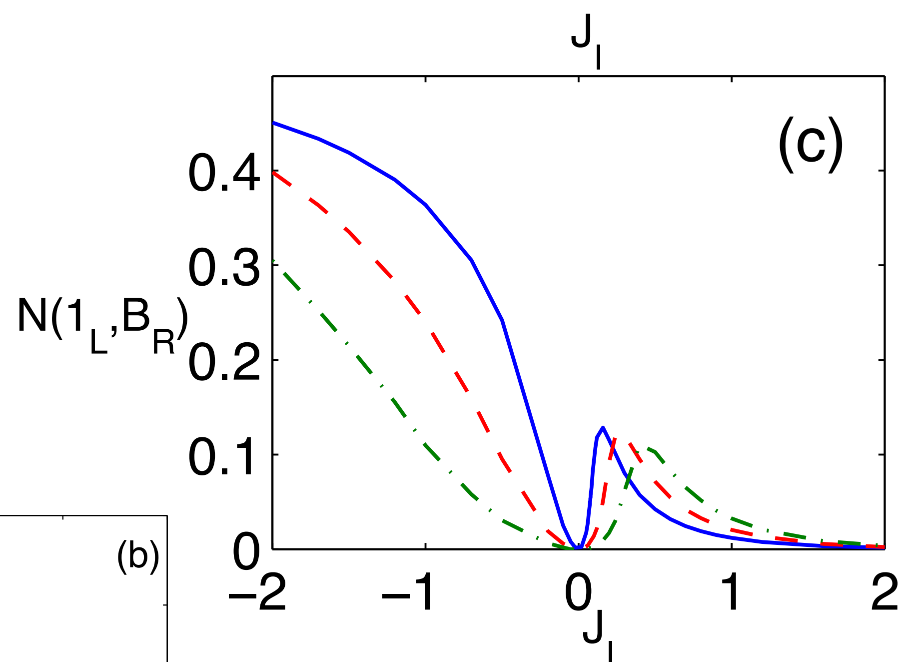
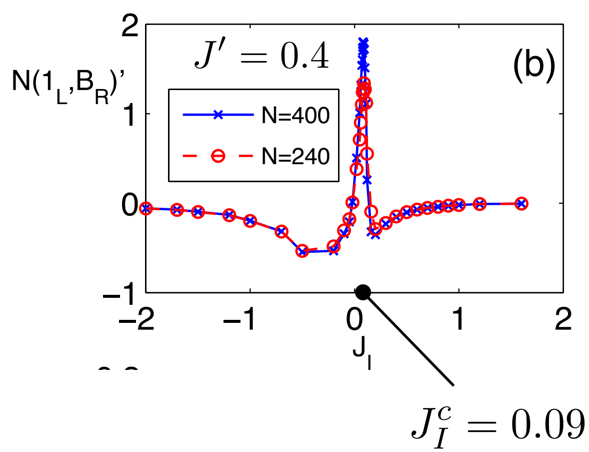








- **Kondo screening cloud**

Background: **single-impurity Kondo model...**

One-loop RG equations: $\frac{\mathrm{d}\lambda}{\mathrm{d}\ln D} = -\nu\lambda^2 + ...$

↓

effective scale-dependent coupling λ becomes of $\mathcal{O}(1)$ at

$$T_K = D_0 \exp(-\mathrm{const.}/\lambda_0)$$

- **Kondo screening cloud**

Background: **single-impurity Kondo model...**

1-loop RG equations: $$\frac{\mathrm{d}\lambda}{\mathrm{d}\ln D} = -\nu\lambda^2 + \dots$$

↓

effective scale-dependent coupling λ becomes of $\mathcal{O}(1)$ at

$$T_K = D_0 \exp(-\mathrm{const.}/\lambda_0)$$

dynamically generated energy scale
KONDO TEMPERATURE

↓

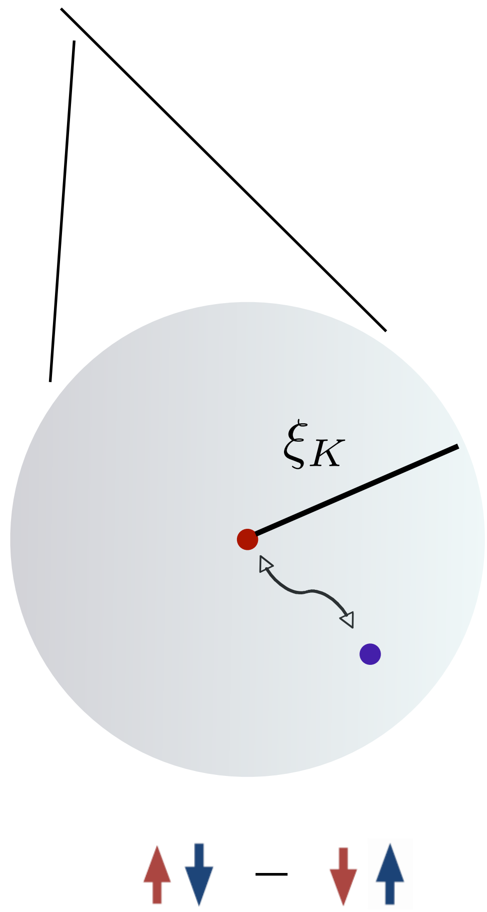
$$\xi_K = \frac{v_F}{T_K}$$

dynamically generated length scale:
KONDO SCREENING LENGTH

- **Kondo screening cloud**

Heuristics: **single-impurity Kondo model...**

$$H_{\mathrm{int}} = \lambda \psi^\dagger(0) \boldsymbol{\sigma} \psi(0) \cdot \boldsymbol{S}_{\mathrm{imp}}$$



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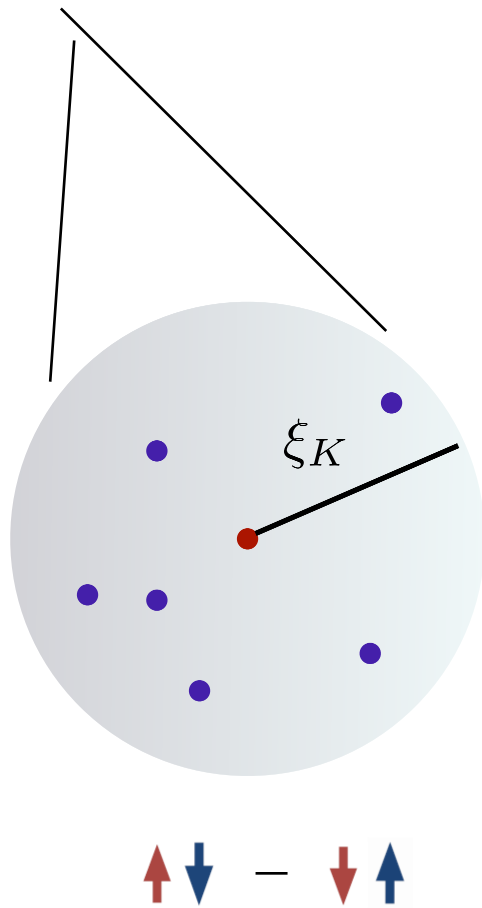

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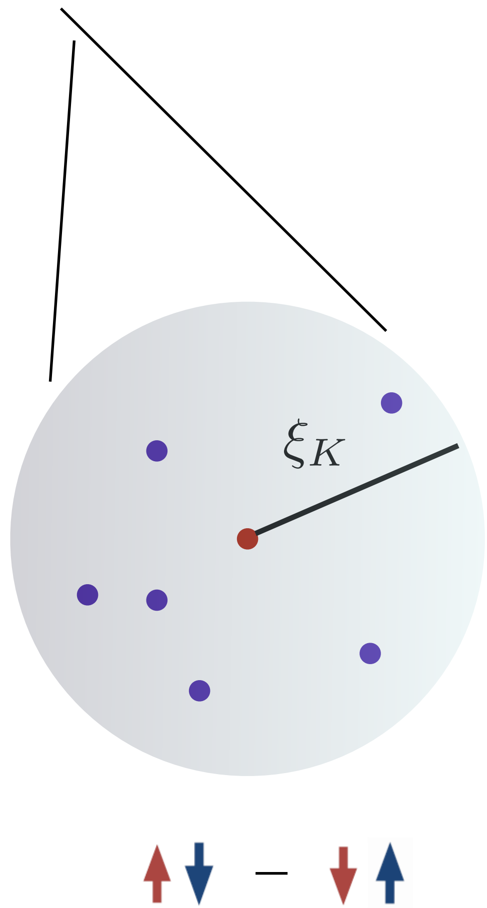


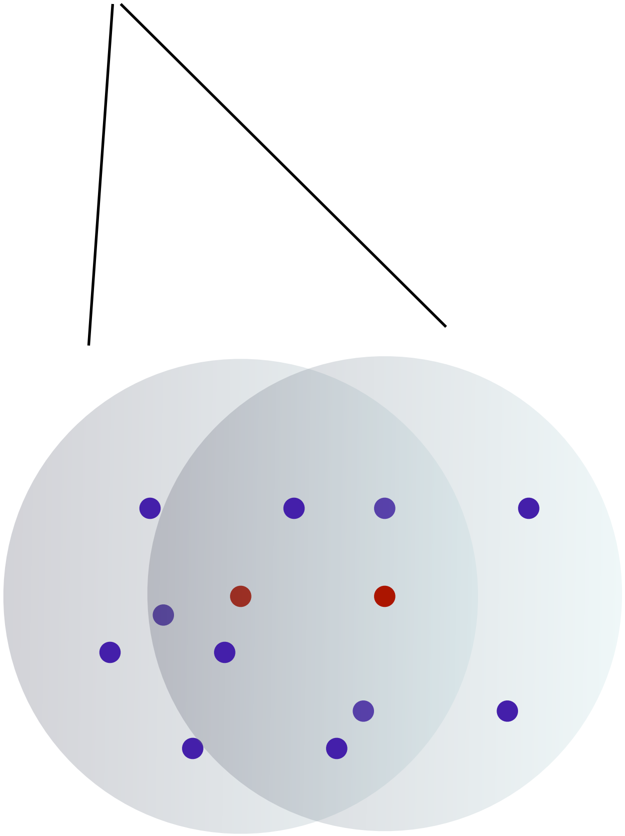
Theoretical construct....
...so far no experimental signal...

Does the Kondo cloud really exist?

- **Kondo screening cloud**

... and what about the **two-impurity Kondo model?**

$$H_{\rm int} = \lambda\psi^\dagger(\boldsymbol{r}_1)\boldsymbol{\sigma}\psi(\boldsymbol{r}_1)\cdot\boldsymbol{S}_1 + \lambda\psi^\dagger(\boldsymbol{r}_1)\boldsymbol{\sigma}\psi(\boldsymbol{r}_1)\cdot\boldsymbol{S}_1 + K(\boldsymbol{r}_1-\boldsymbol{r}_2)\boldsymbol{S}_1\cdot\boldsymbol{S}_2$$



- **Kondo screening cloud**

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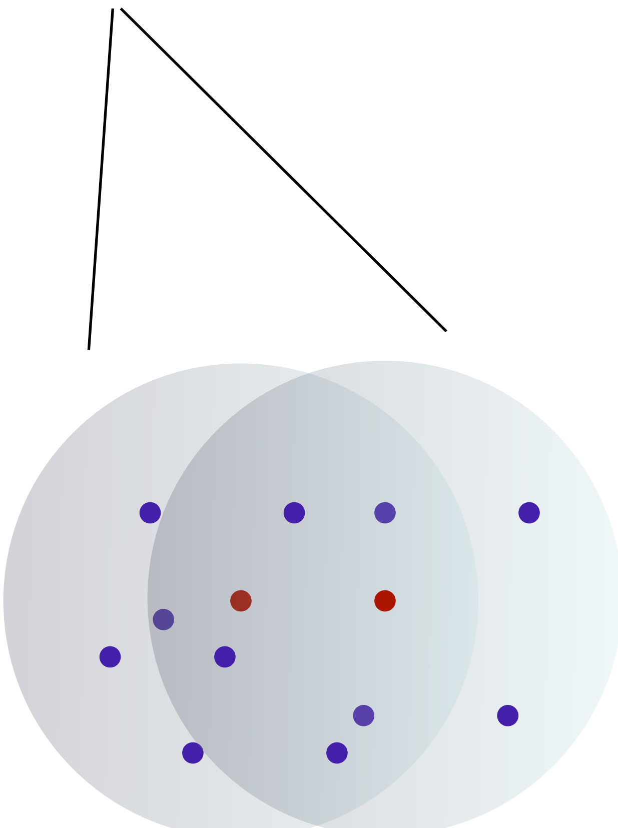
$$H_{\rm int} = \lambda \psi^\dagger(\boldsymbol{r}_1)\boldsymbol{\sigma}\psi(\boldsymbol{r}_1)\cdot\boldsymbol{S}_1 + \lambda \psi^\dagger(\boldsymbol{r}_1)\boldsymbol{\sigma}\psi(\boldsymbol{r}_1)\cdot\boldsymbol{S}_1 + K(\boldsymbol{r}_1-\boldsymbol{r}_2)\boldsymbol{S}_1\cdot\boldsymbol{S}_2$$



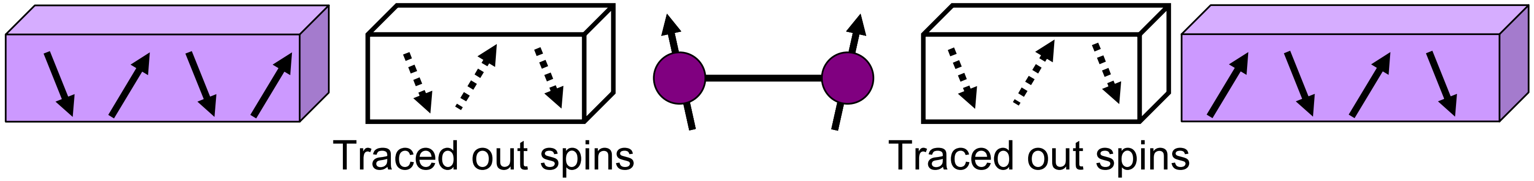
‘Constructive’ approach:

Define the screening length as an *entanglement length.*

A. Bayat, P. Sodano, and S. Bose, PRB **81**, 064429 (2010)

- **Kondo screening cloud**

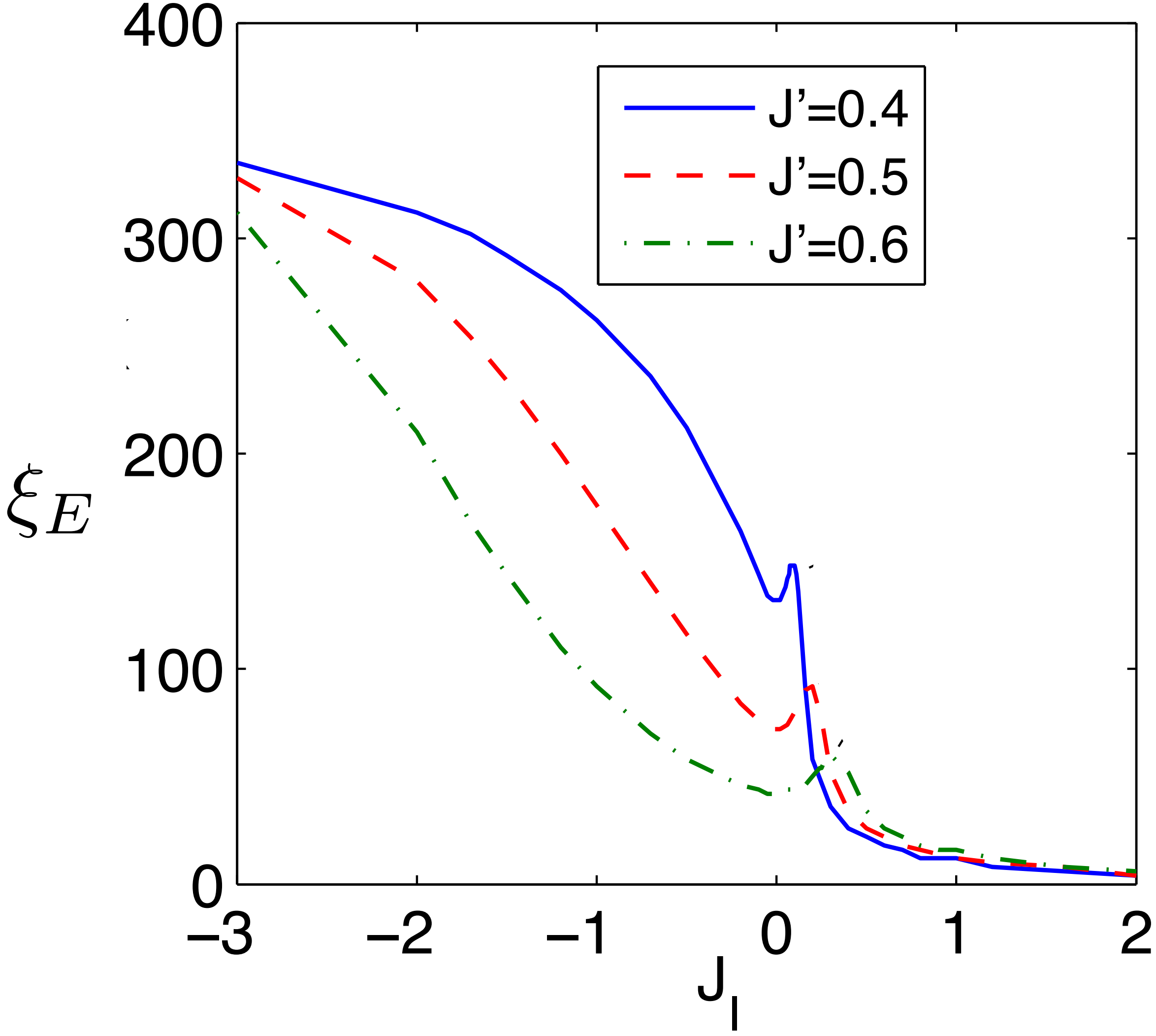
Entanglement probe:



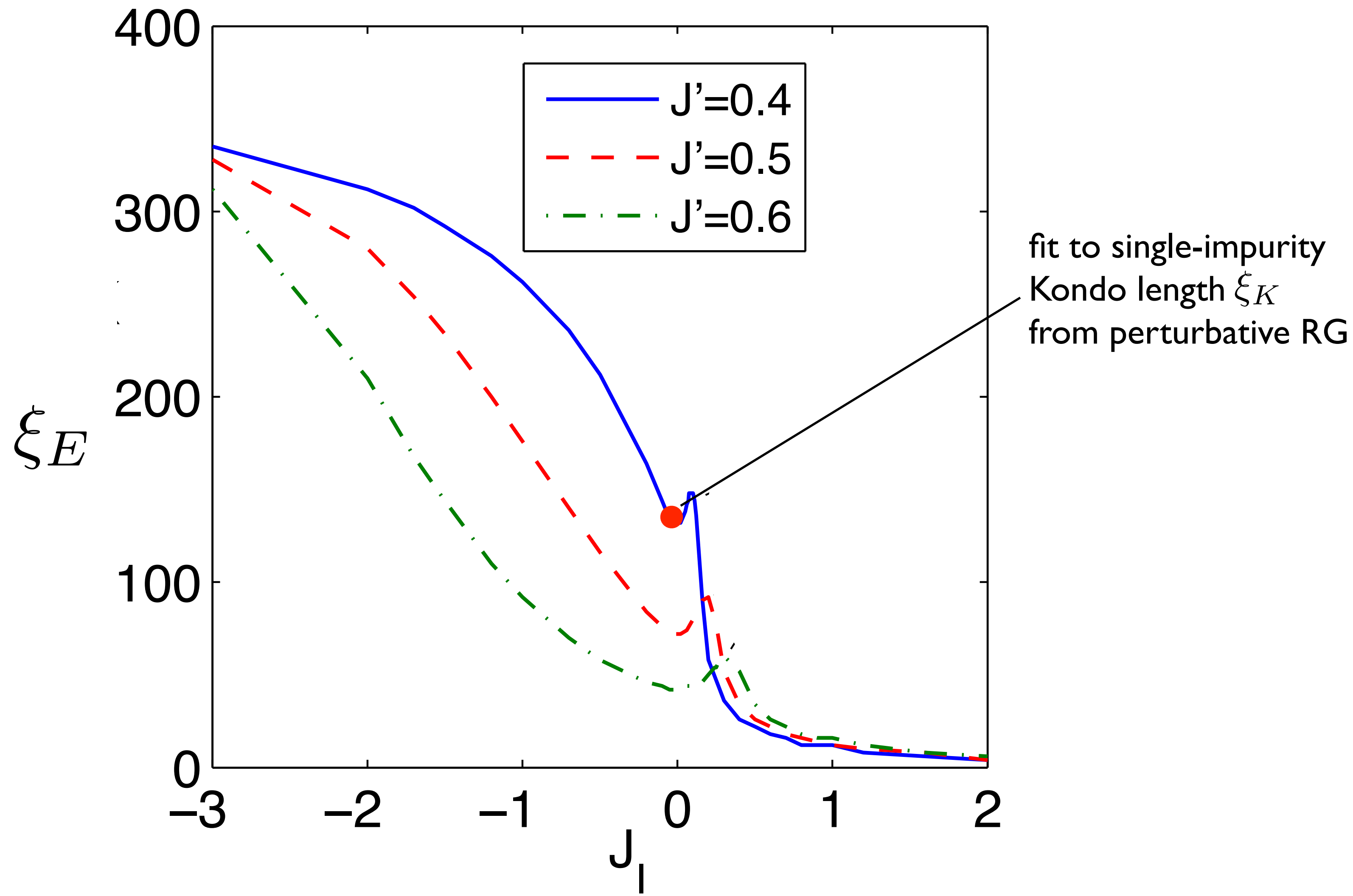
- Trace out, compute the negativity between the impurities and the rest of the system

- Define $\xi_E \equiv$ length beyond which the negativity is smaller than some cutoff (here, = 0.01)

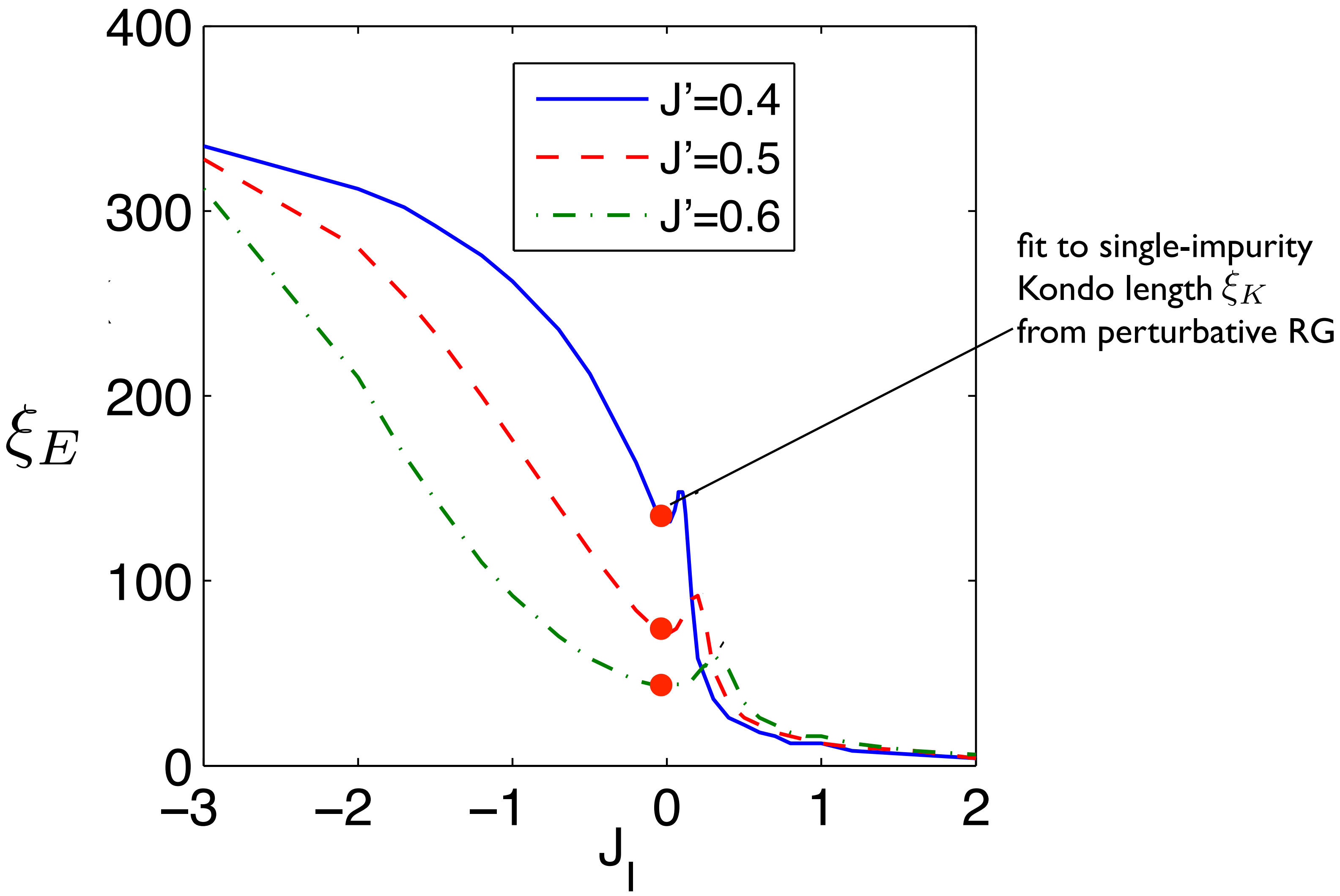
- **Kondo screening cloud**



- **Kondo screening cloud**



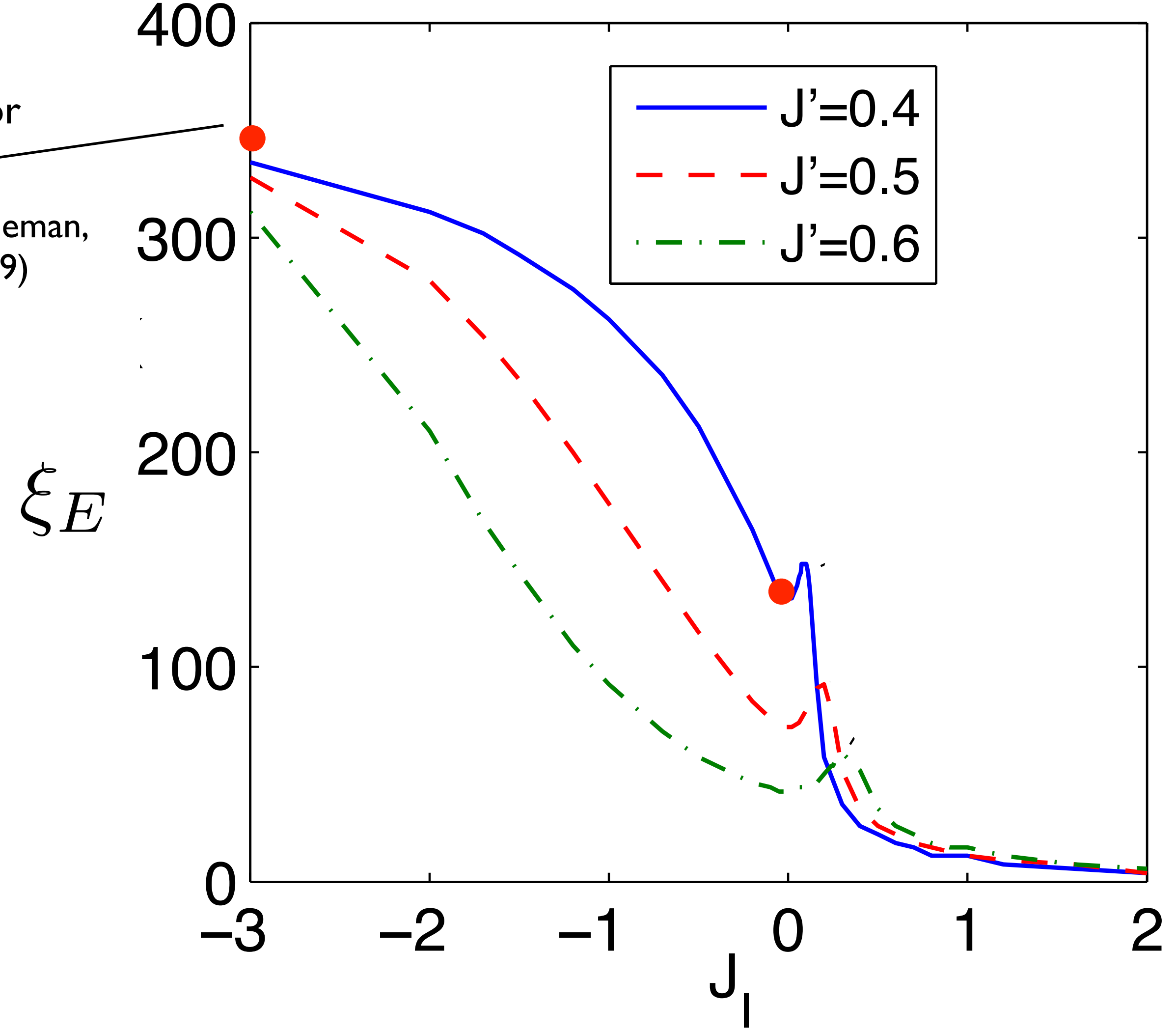
- **Kondo screening cloud**



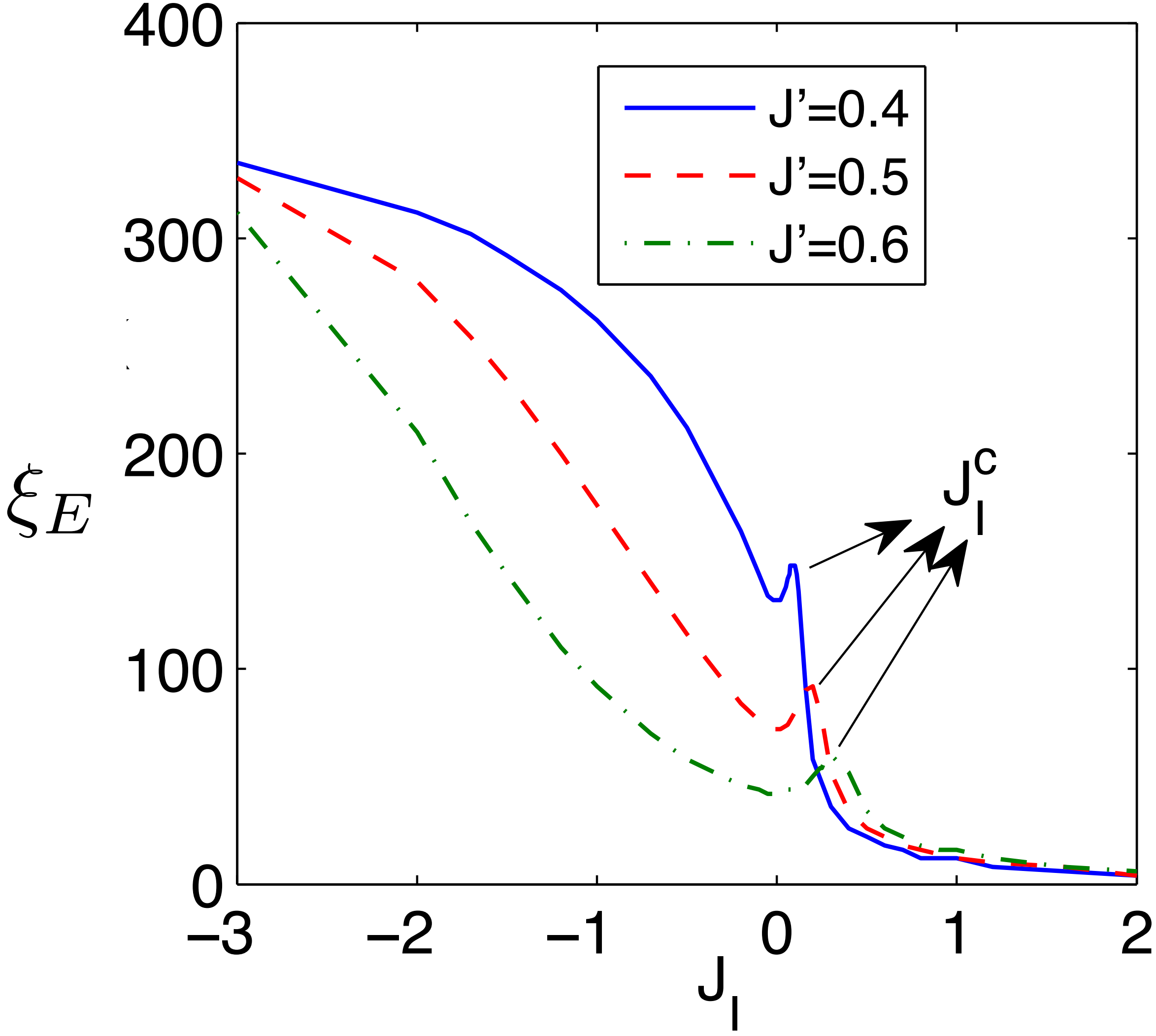
- **Kondo screening cloud**

spin-1 two-channel
Kondo length ξ_K for
$J_I \to -\infty$

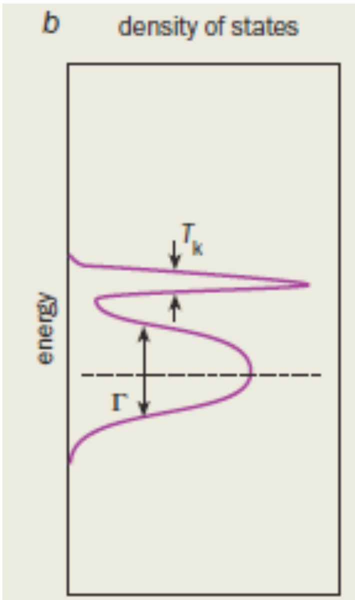
Nevidomskyy and Coleman,
PRL 103, 147205 (2009)



- **Kondo screening cloud**



- **Kondo resonance narrowing**



Experiments indicate that the width of the Kondo resonance narrows with larger impurity spin (at exact screening)

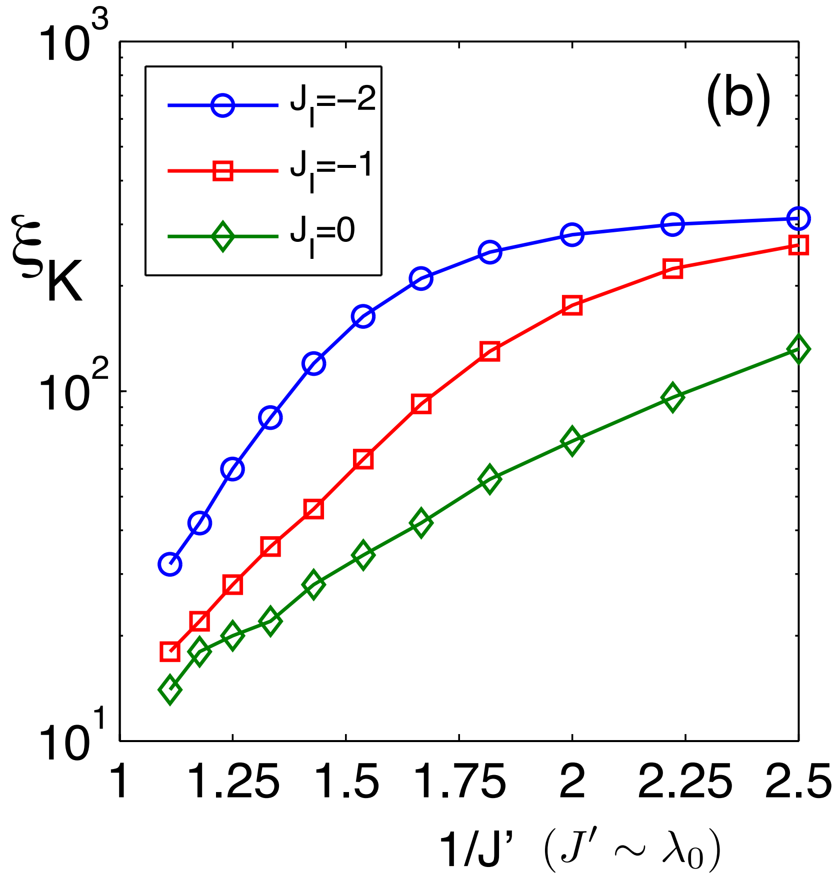
M.D. Daybell and W.A. Steyert, RMP **40**, 380 (1968)

Perturbative scaling theory suggests

$$T_K = D_0 \exp(-2S \times \text{const.}/\lambda_0)$$

A. H. Nevidomskyy and P. Coleman, PRL 103, 147205 (2009)

- **Kondo resonance narrowing**



$$\xi_K \sim \exp(\alpha(J_I)/J')$$

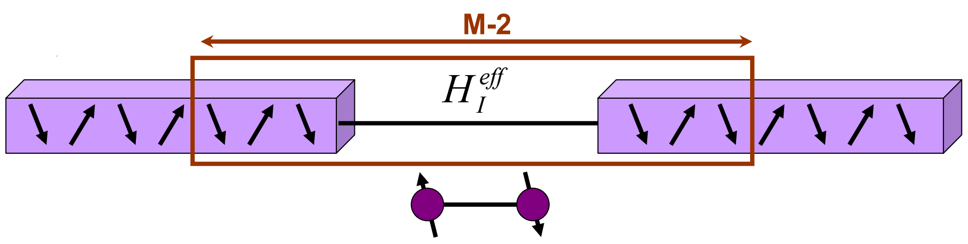
J_I	-3.00	-2.50	-2.00	-1.50	-1.00	-0.50	0.00
$\alpha(J_I)$	3.2175	3.1407	2.9403	2.6838	2.3106	2.0718	1.7686

→ $\alpha(-\infty) \approx 2.5\alpha(0)$

Nevidomskyy-Coleman predicts

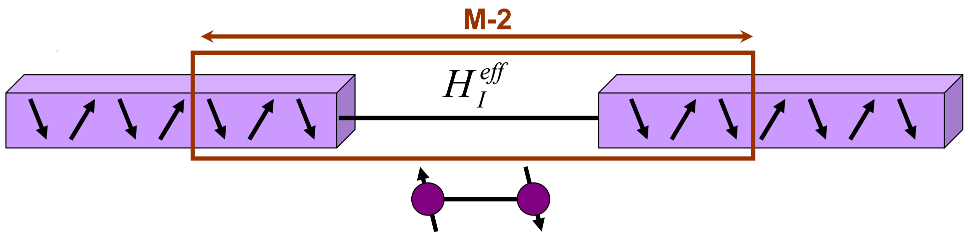
$$\alpha(-\infty) = 2\alpha(0)$$

- **Effective decoupling of impurities**



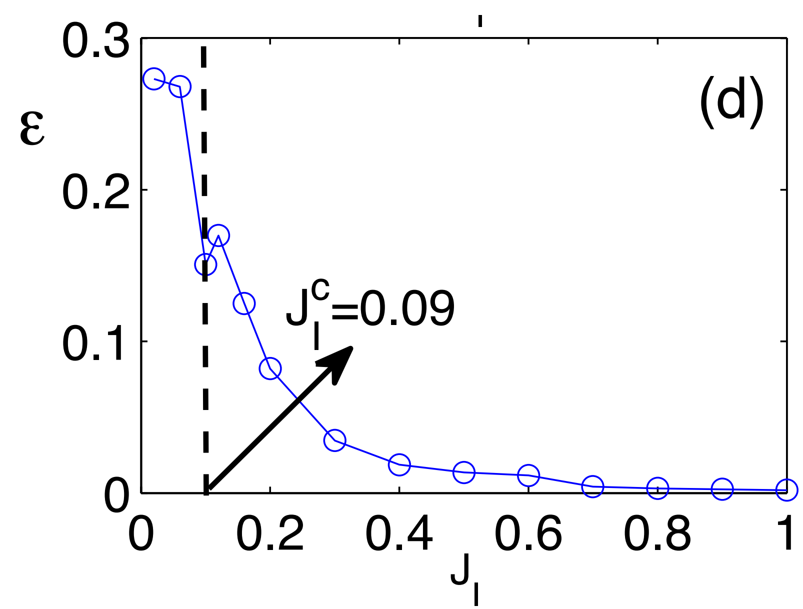
$$
\begin{aligned}
H_I^{eff} &= \frac{J'^2 J_1}{2J_I} \boldsymbol{S}_2^L \cdot \boldsymbol{S}_2^R + \frac{J'^2 J_2}{2J_I J_1} \boldsymbol{S}_3^L \cdot \boldsymbol{S}_3^R \\
&+ \frac{J'^2 J_2}{2J_I} (\boldsymbol{S}_2^L \cdot \boldsymbol{S}_3^L + \boldsymbol{S}_2^R \cdot \boldsymbol{S}_3^R - \boldsymbol{S}_2^R \cdot \boldsymbol{S}_3^L - \boldsymbol{S}_2^L \cdot \boldsymbol{S}_3^R)
\end{aligned}
$$

- **Effective decoupling of impurities**



$$H_I^{eff} = \frac{J'^2 J_1}{2J_I} \boldsymbol{S}_2^L \cdot \boldsymbol{S}_2^R + \frac{J'^2 J_2}{2J_I J_1} \boldsymbol{S}_3^L \cdot \boldsymbol{S}_3^R + \frac{J'^2 J_2}{2J_I} (\boldsymbol{S}_2^L \cdot \boldsymbol{S}_3^L + \boldsymbol{S}_2^R \cdot \boldsymbol{S}_3^R - \boldsymbol{S}_2^R \cdot \boldsymbol{S}_3^L - \boldsymbol{S}_2^L \cdot \boldsymbol{S}_3^R)$$

$$\varepsilon(J', J_I) = \frac{1}{N-4} \sum_{M=4}^{N} |S(\rho(M)) - S(\rho_{\text{eff}}(M-2))|$$



effective impurity-bulk decoupling
already for intermediate values of RKKY

Summary

Two-impurity Kondo model, DMRG entanglement probe

A. Bayat, S. Bose, P. Sodano, H.J., PRL **109**, 066403 (2012)

- nonperturbative diagnostic of Kondo-RKKY quantum phase transition
- Kondo cloud reconstruction for *any* (subcritical) RKKY and Kondo couplings
- test of *Kondo resonance narrowing*
- quantitative measure of *impurity-bulk decoupling*

- entanglement spectrum *work in progress*
- negativity scaling *work in progress*