Simons Center for Geometry and Physics
Stony Brook University, October 14 2016

Entanglement Probe of Two-Impurity Kondo Physics

in collaboration with

Abolfazl Bayat (UCL)
Sougato Bose (UCL)
Pasquale Sodano (IIP, Natal)

UNIVERSITY OF GOTHENBURG
CSRC
UCL
International Institute of Physics
Federal University of Rio Grande do Norte

Outline

Background...

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Basics on two-impurity Kondo model

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Experimental realizations & theoretical challenges

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A view from quantum entanglement...

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Spin chain modeling

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- Kondo screening length as an entanglement length

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- An order parameter for impurity quantum phase transitions
- Summary & outlook

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Kondo screening length as an entanglement length

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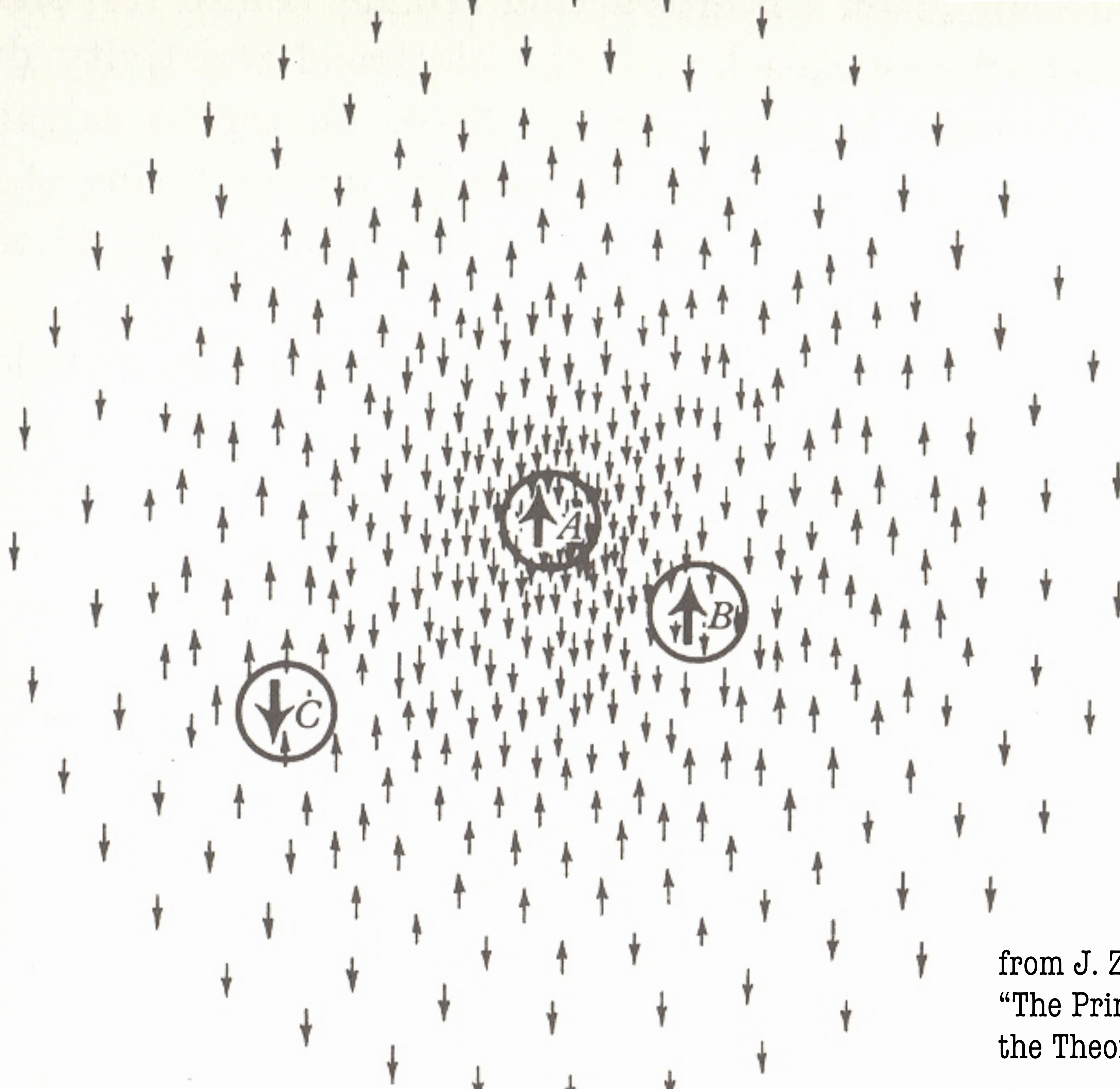
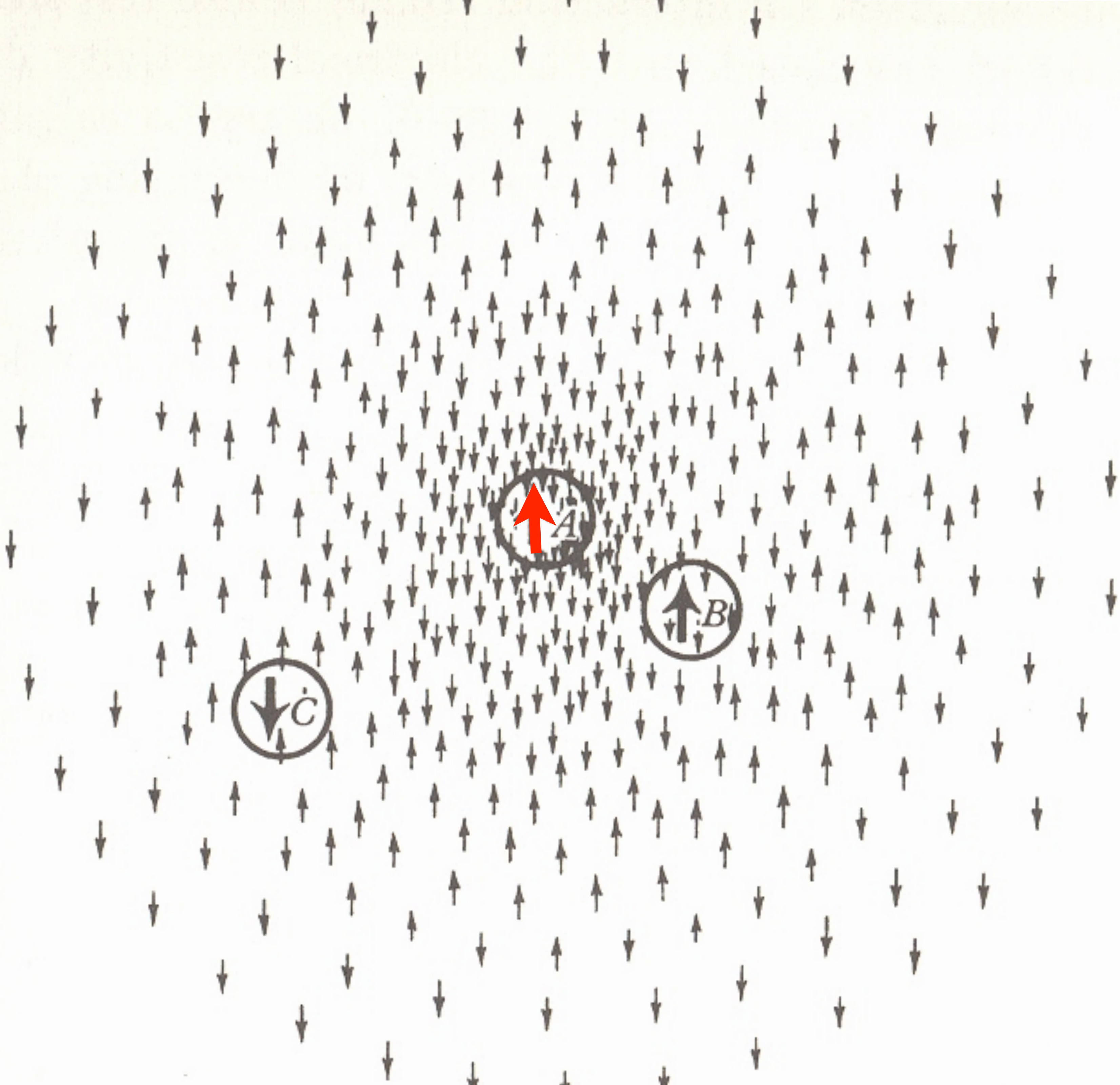
Summary & outlook



from J. Ziman,
“The Principles of
the Theory of Solids”


A
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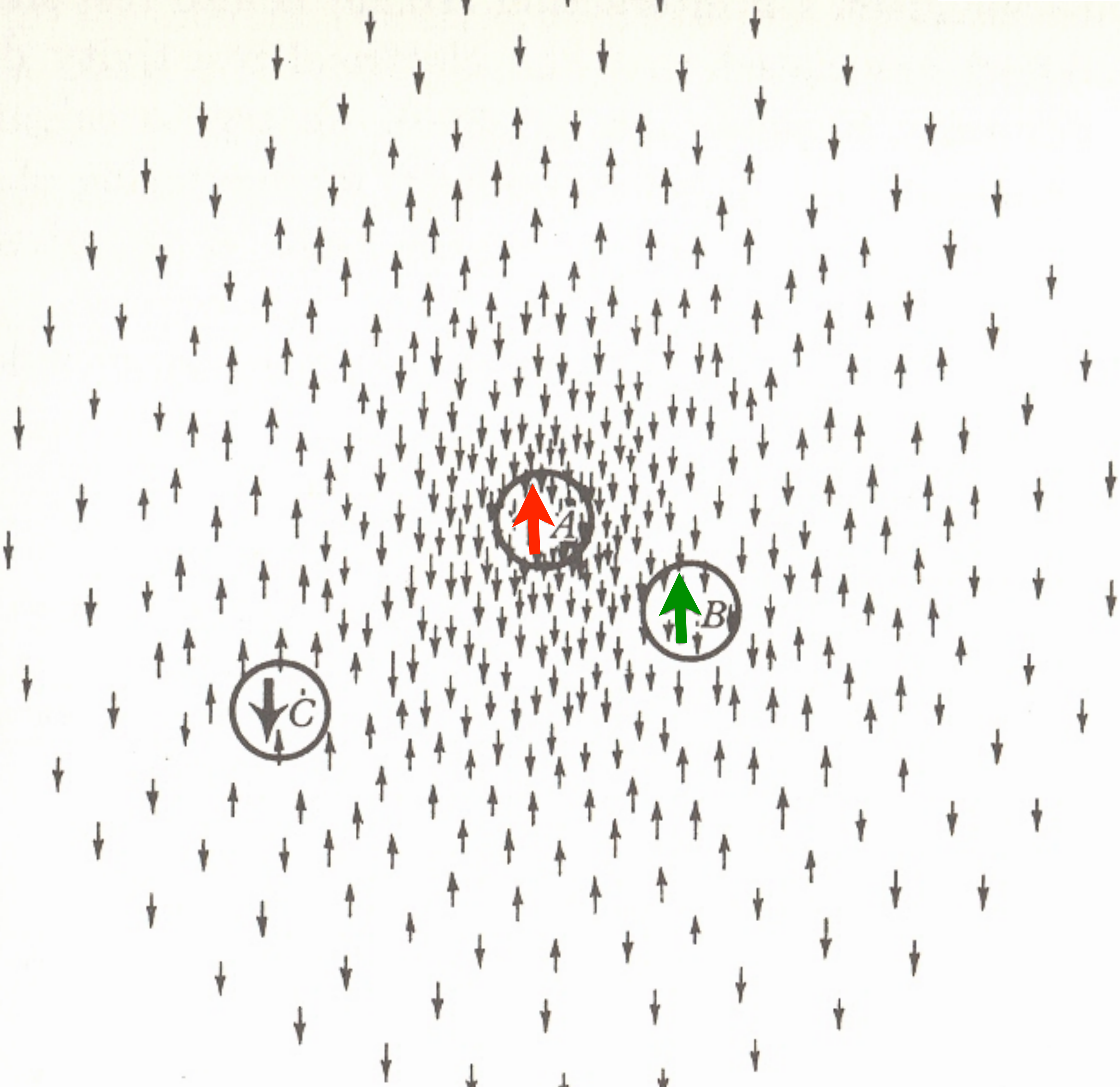
C



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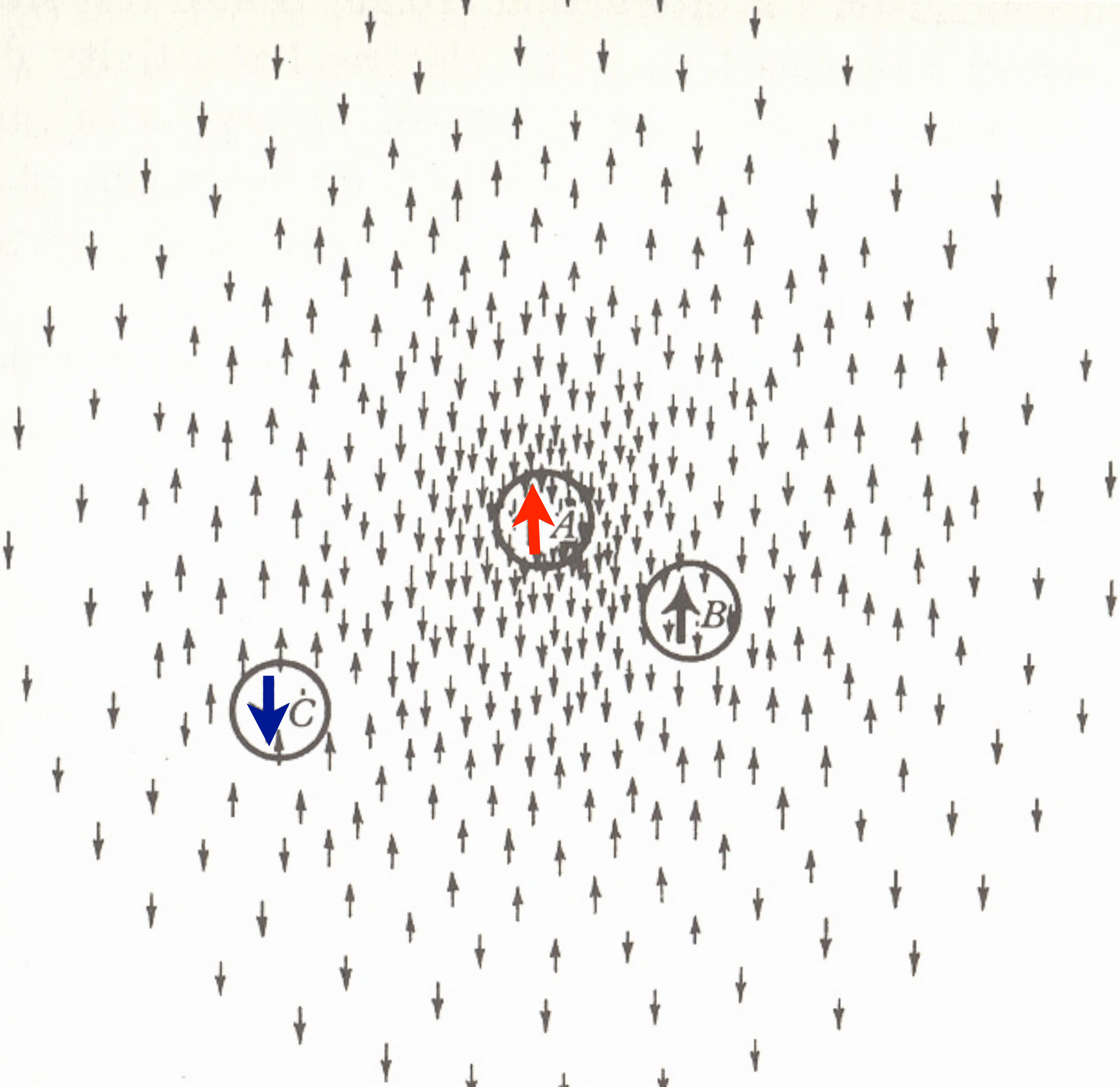
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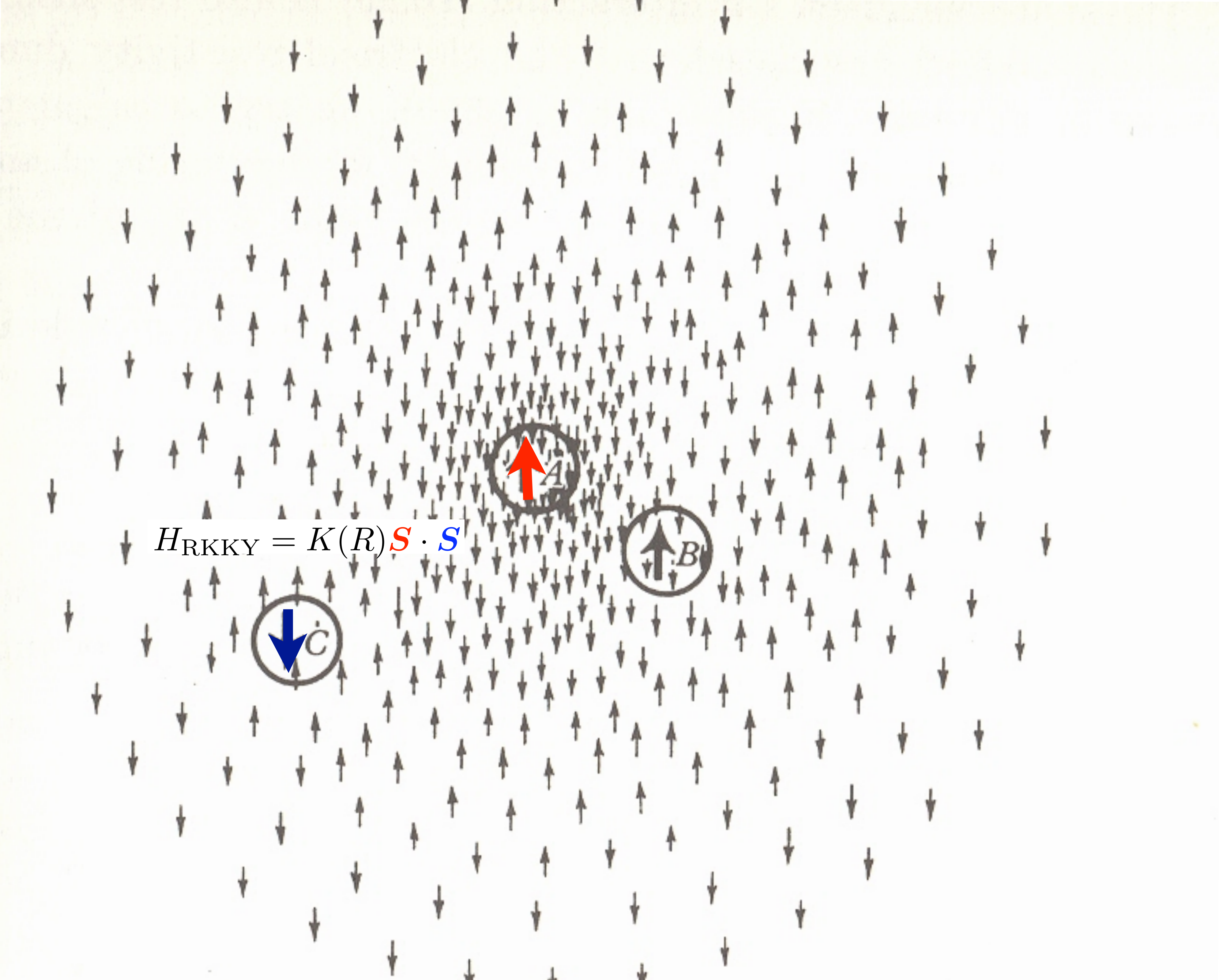
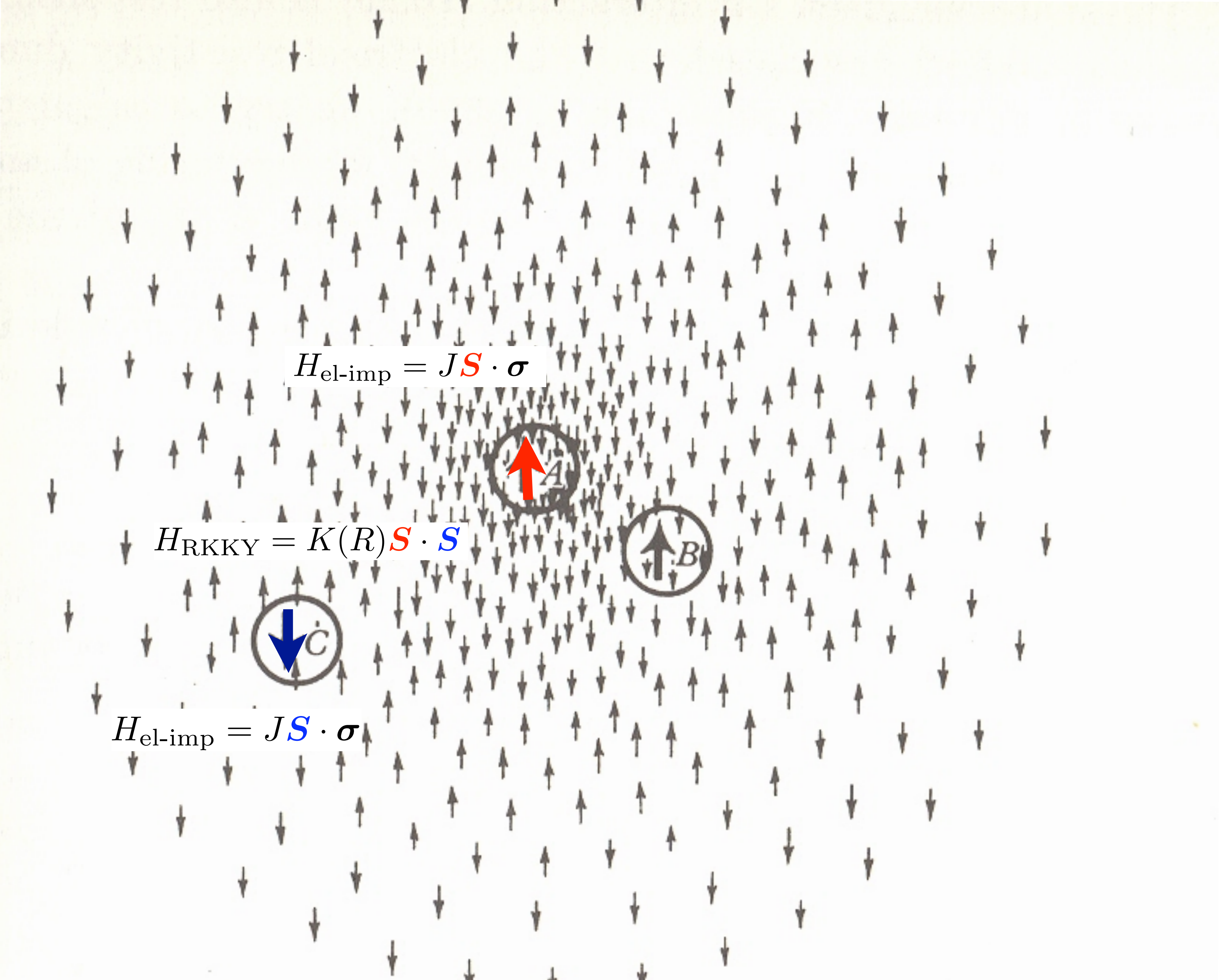

A
B
C
$H_{\mathrm{RKKY}} = K(R)\boldsymbol{S}\cdot\boldsymbol{S}$


$H_{\text{el-imp}} = J\boldsymbol{S} \cdot \boldsymbol{\sigma}$
A
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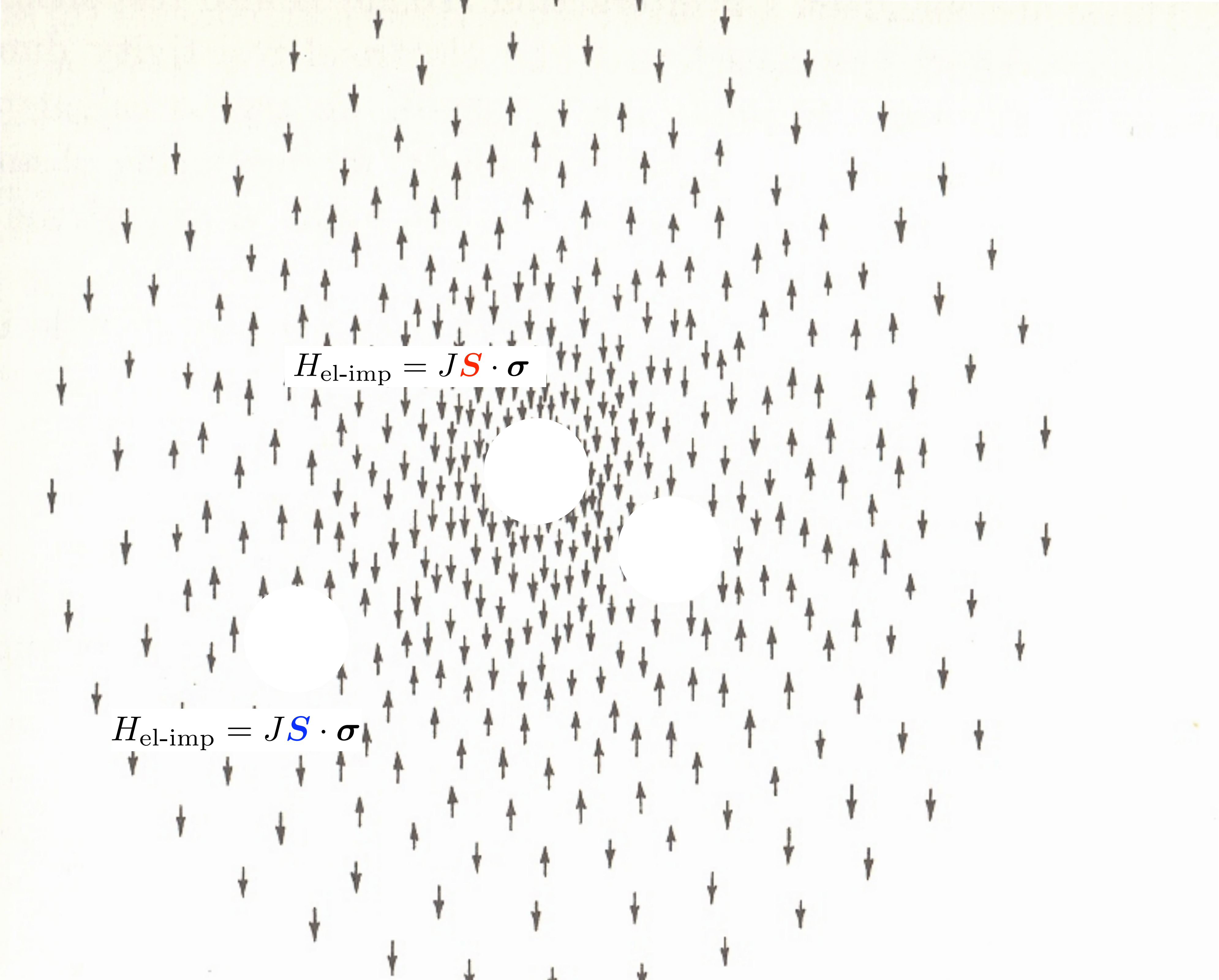
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$$H = H_{\rm kin} + J\boldsymbol{S}_1 \cdot \boldsymbol{\sigma} + J\boldsymbol{S}_2 \cdot \boldsymbol{\sigma} + K(R)\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

VOLUME 47, NUMBER 10 PHYSICAL REVIEW LETTERS 7 SEPTEMBER 1981

Two-Impurity Kondo Problem

C. Jayaprakash
Nordisk Institut for Teoretisk Atomfsyik, DK-2100 Copenhagen Ø, Denmark, and Department of Physics, Cornell University, Ithaca, New York 14853

and

H. R. Krishna-murthy
Nordisk Institut for Teoretisk Atomfsyik, DK-2100 Copenhagen Ø, Denmark, and Department of Physics, Indian Institute of Science, Bangalore, India

and

J. W. Wilkins
Nordisk Institut for Teoretisk Atomfsyik, DK-2100 Copenhagen Ø, Denmark, and Department of Physics, Cornell University, Ithaca, New York 14853
(Received 28 May 1981)

The two-impurity Kondo problem is studied by use of perturbative scaling techniques. The physics is determined by the interplay between the Ruderman-Kittel-Kasuya-Yosida (RKKY) interaction between the two impurity spins and the Kondo effect. In particular, for a strong ferromagnetic RKKY interaction the susceptibility exhibits three structures as the temperature is lowered, corresponding to the ferromagnetic locking together of the two impurity spins followed by a two-stage freezing out of their local moments by the conduction electrons due to the Kondo effect.

competition between RKKY-
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RKKY-coupled spin-singlet,
no Kondo screening

$K(R) \to -\infty$ $K(R) \to \infty$

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Kondo screened by conduction electrons

RKKY-coupled spin-singlet,
no Kondo screening

$K(R) \to -\infty$ $K(R) \to \infty$

$$\delta = \pi/2$$

P. Nozières and A. Blandin, J. Phys. (Paris) **41**, 193 (1980)

$$\delta = 0$$

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$K(R) \to -\infty$

$K(R) \to \infty$

particle-hole symmetry → $\delta = 0$ *or* $\delta = \pi/2$

A. Millis *et al.*

Field Theories in Condensed Matter Physics

ed. Z. Tesanovic, 1990

$$\delta = \pi/2$$

RKKY-coupled spin-triplet,
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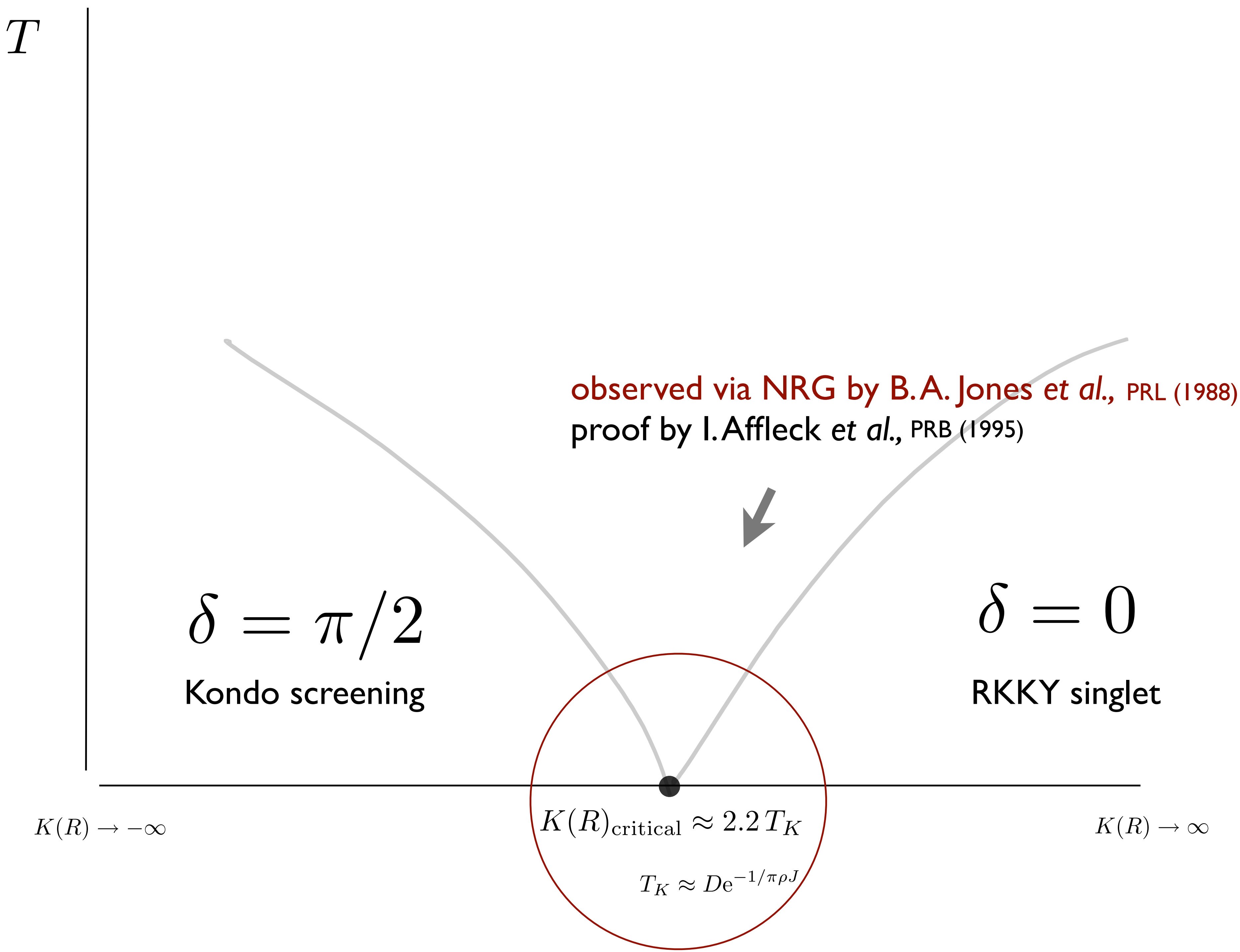
T
observed via NRG by B.A. Jones *et al.*, PRL (1988)
proof by I. Affleck *et al.*, PRB (1995)
$\delta = \pi/2$
Kondo screening
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RKKY singlet
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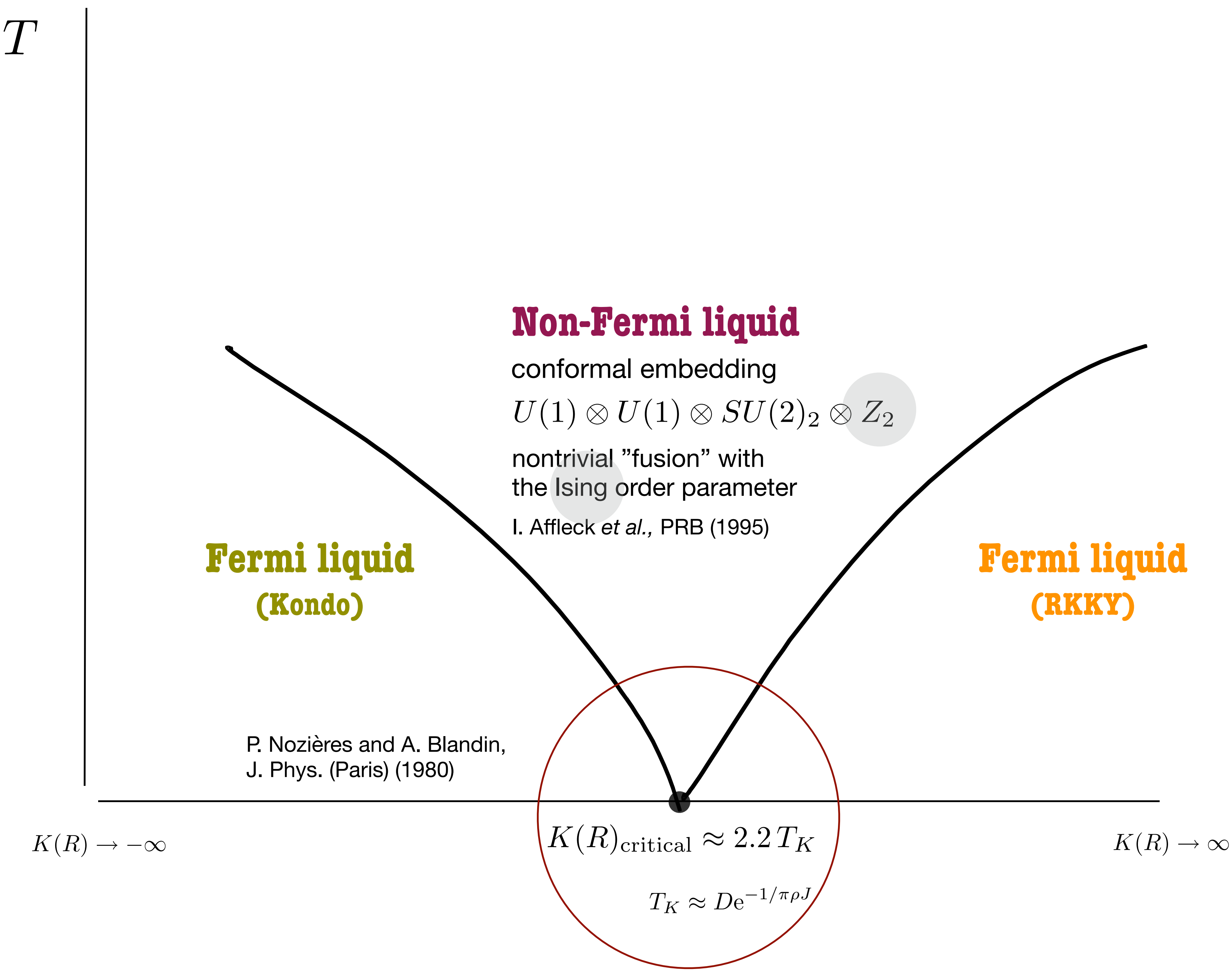
$K(R)_{\text{critical}} \approx 2.2\, T_K$
$K(R) \to \infty$
$T_K \approx D e^{-1/\pi\rho J}$



T
Non-Fermi liquid
conformal embedding
$U(1) \otimes U(1) \otimes SU(2)_2 \otimes Z_2$
nontrivial ”fusion” with
the Ising order parameter
I. Affleck *et al.*, PRB (1995)
Fermi liquid
(Kondo)
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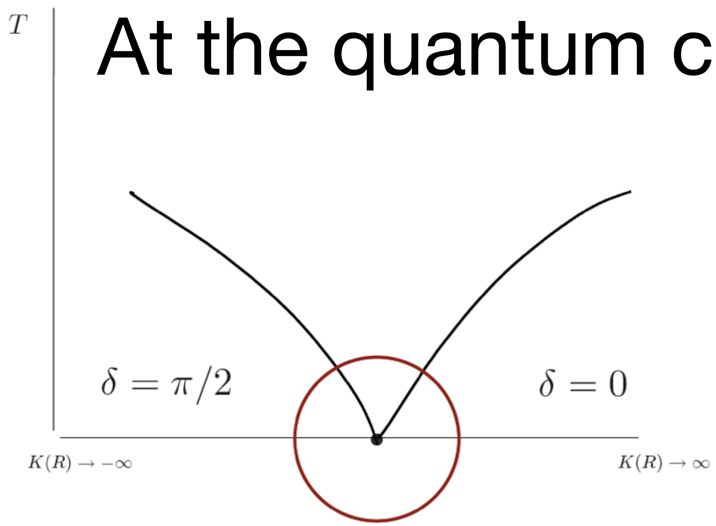
At the quantum critical point...



Thermodynamics (impurity contribution)

I. Affleck *et al.*, PRB **52**, 9528 (1995)

$$\frac{C_{\text{imp}}}{T} = \gamma \longrightarrow \frac{T_K}{(K - K_c)^2}$$

$$\chi_{\text{imp}} = \text{constant}$$

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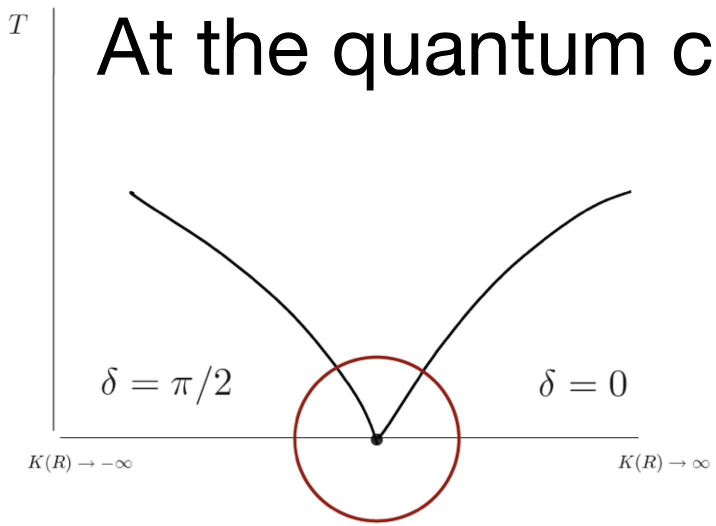

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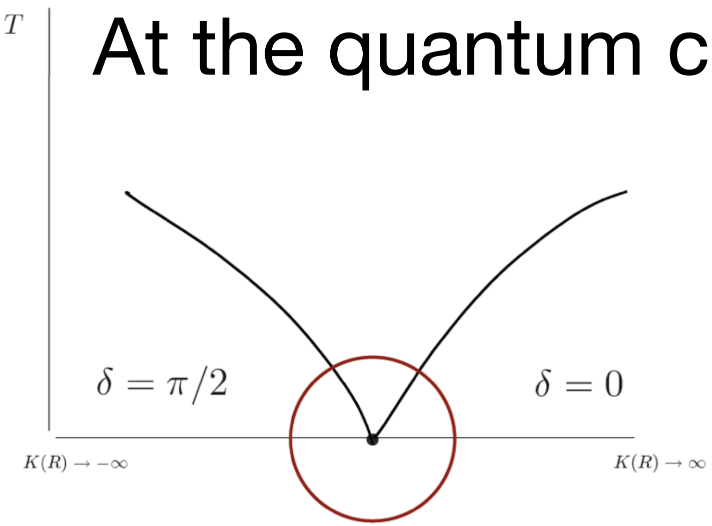
At the quantum critical point...



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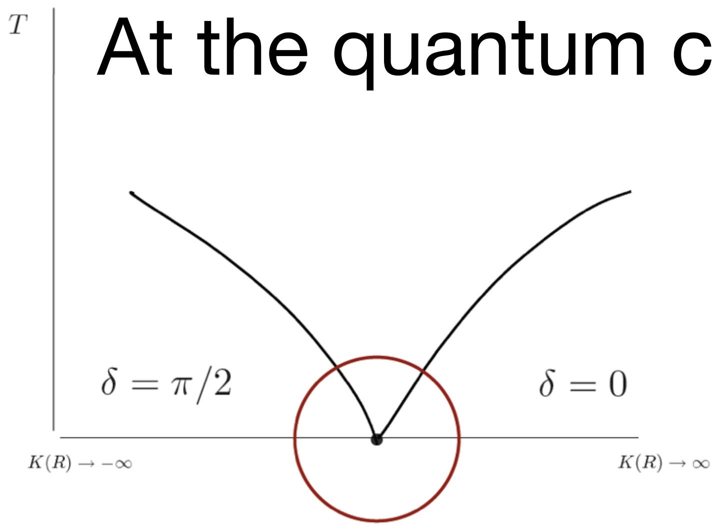
"fractional ground state degeneracy"

At the quantum critical point...



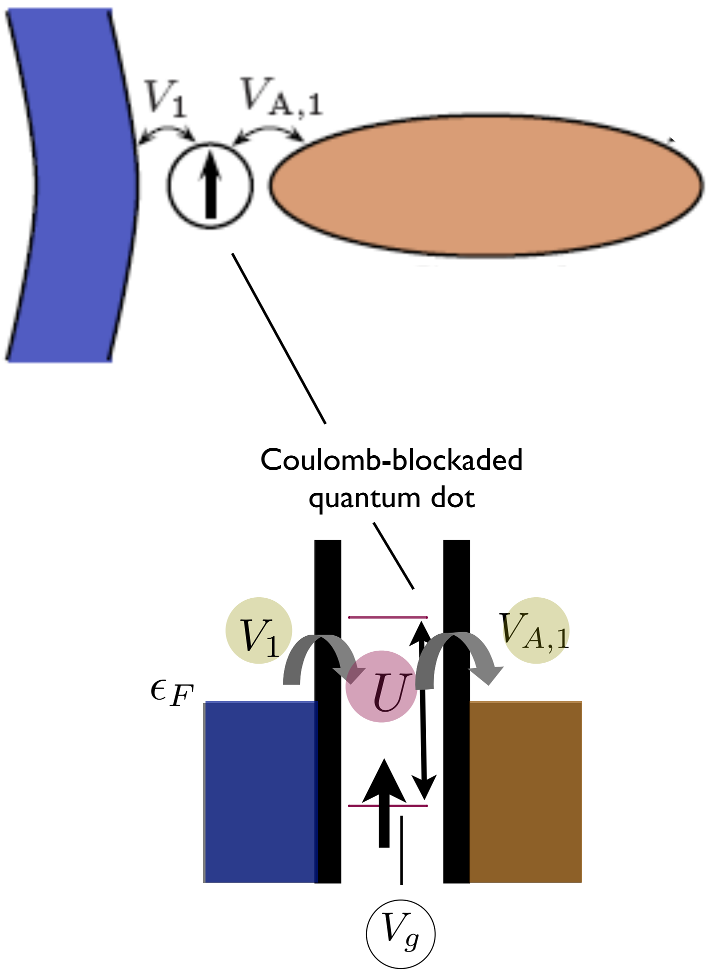
Thermodynamics (impurity contribution)

Difficult to study in the lab!

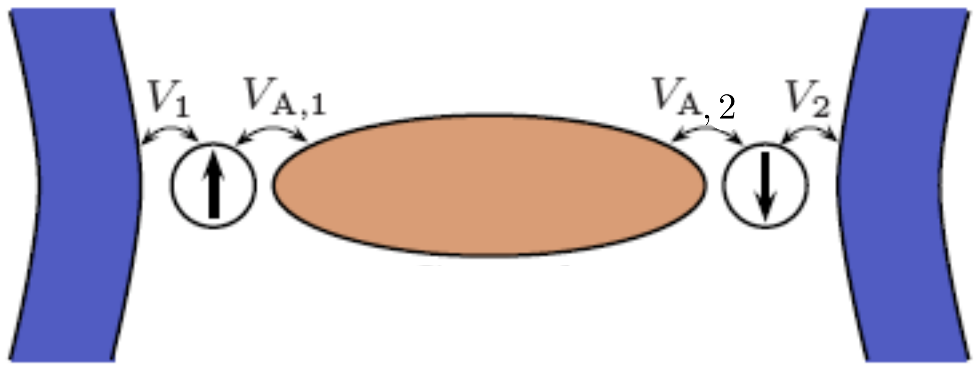
Experimentalists prefer to measure transport properties (like conductance)

Need a well-controlled realization of the two-impurity Kondo model!

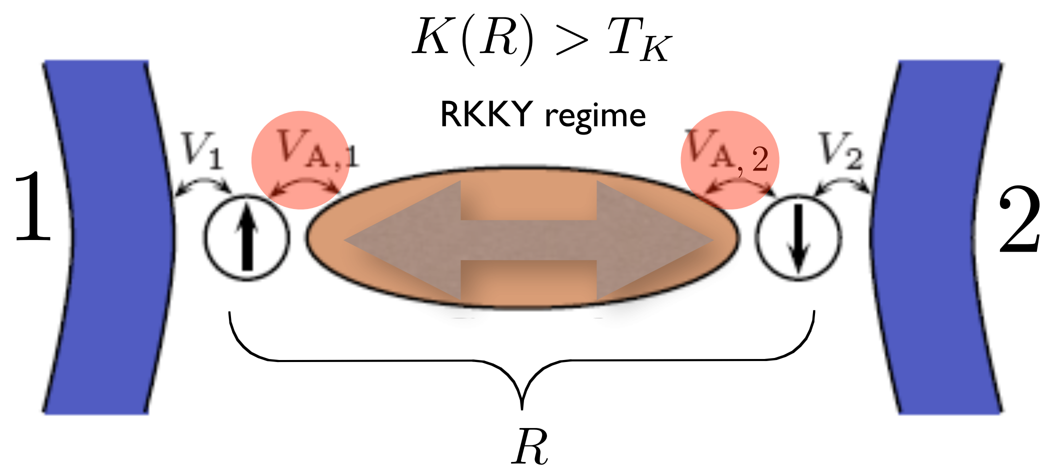
Realization in double quantum-dot systems



spin exchange $J \propto V^2 / U$



V_1
$V_{\mathrm{A},1}$
$V_{\mathrm{A},2}$
V_2

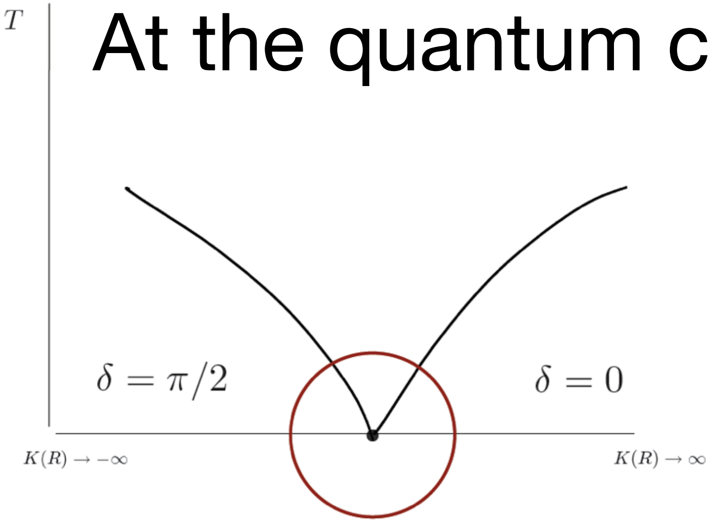


$$H_{\rm int} = J_1 \boldsymbol{S}_1 \cdot \boldsymbol{\sigma}_1 + J_2 \boldsymbol{S}_2 \cdot \boldsymbol{\sigma}_2 + K(R) \boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

No transfer of electrons between 1 and 2: quantum critical point $K_c \approx 2.2 T_K$ is stable against electron-hole symmetry breaking *and* breaking of parity.

G. Zaránd *et al.*, PRL **97**, 166802 (2006)

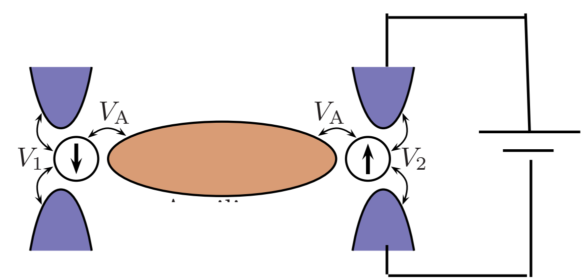
At the quantum critical point...



Transport

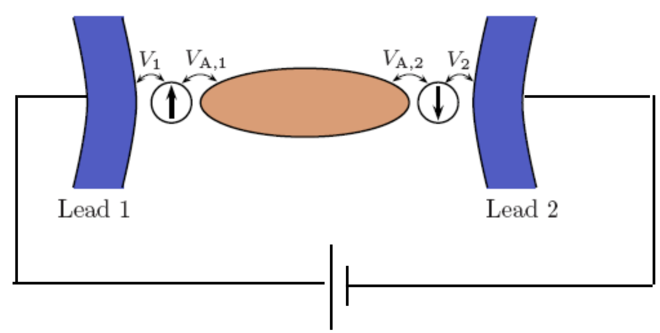
$$G \approx G_0(1 - \lambda_1 T^{1/2}), \quad T > T_1^*$$

G. Zaránd *et al.*, PRL **97**, 166802 (2006)



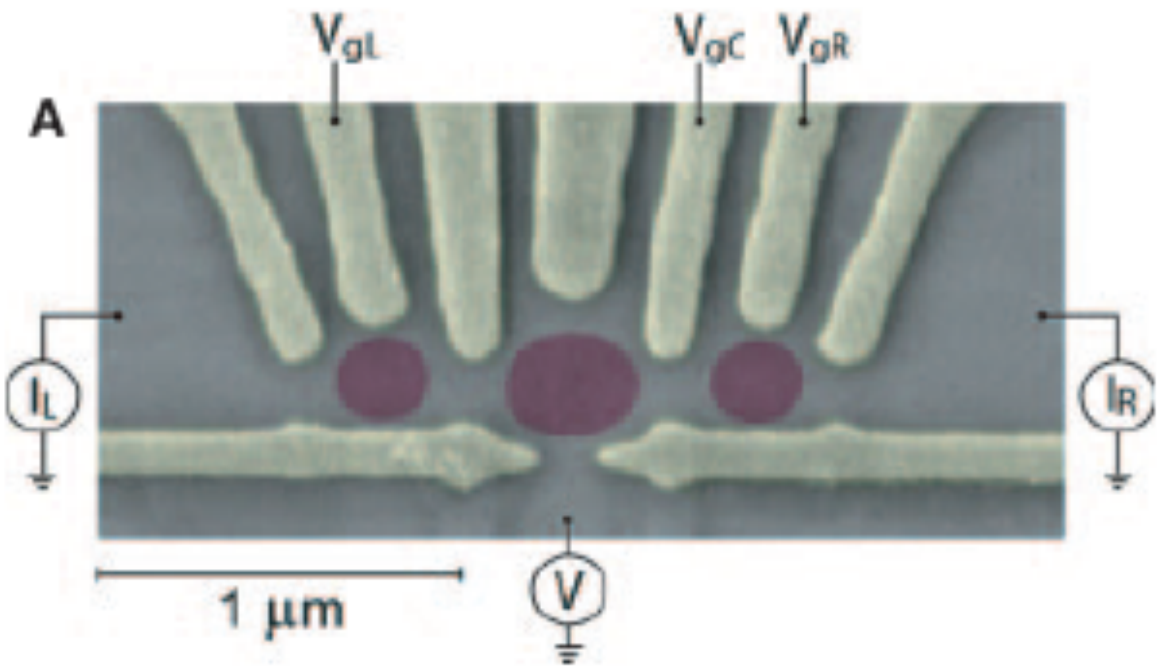
$$G \approx G_0(1 - \lambda_2 T^2), \quad T > T_2^*$$

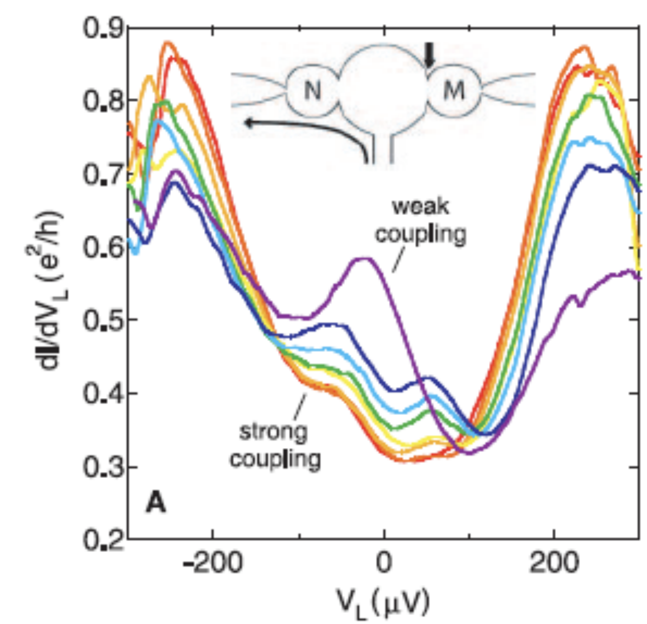
E. Sela and I. Affleck, PRL **102**, 047201(2009)



Realization in double quantum-dot systems

N. J. Craig *et al.*, Science **304**, 565 (2004)





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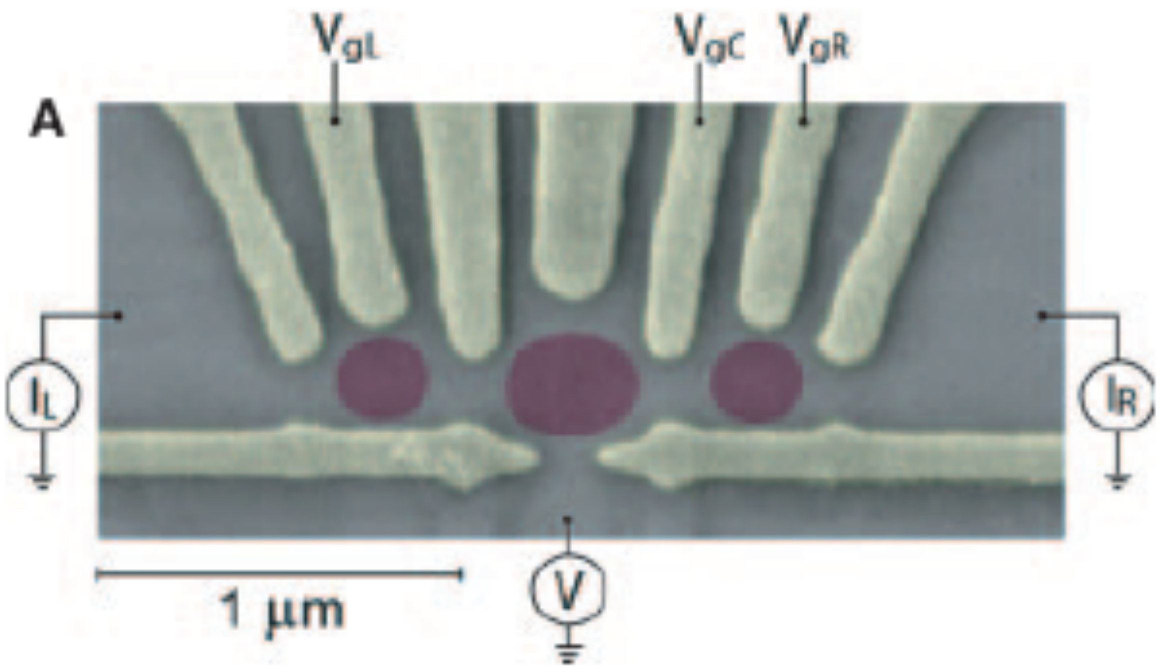
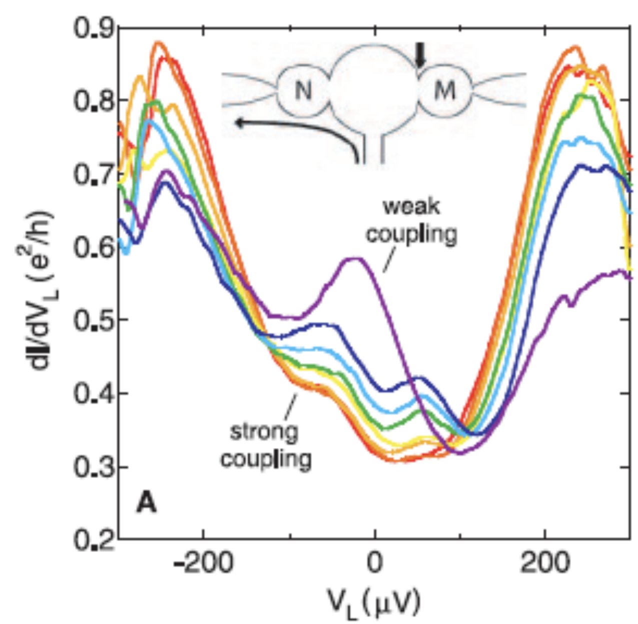


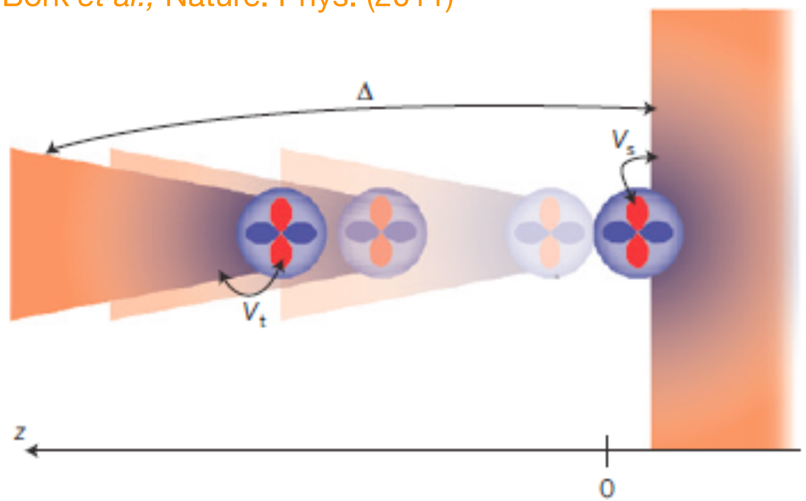


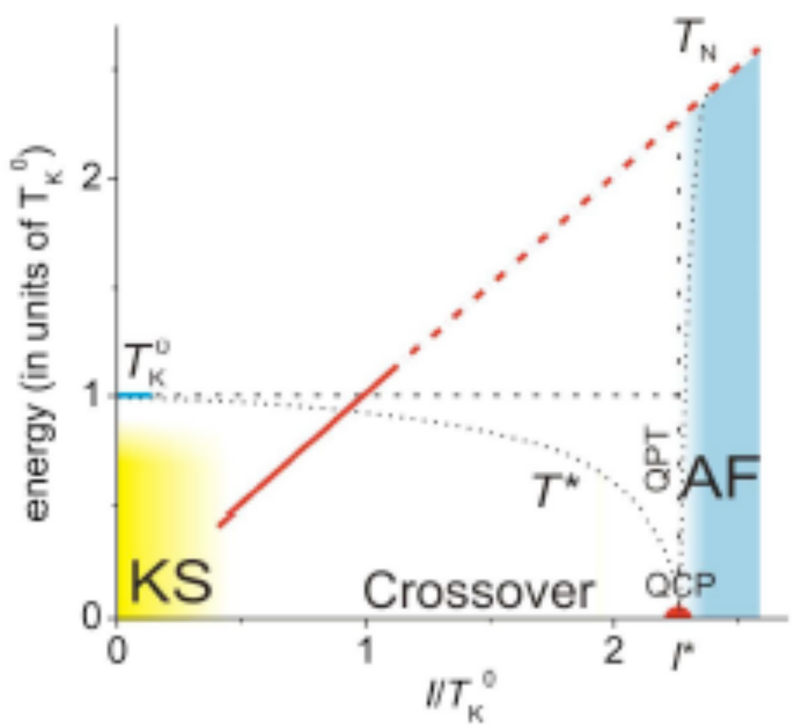
Nota Bene:
The central dot supports both RKKY *and* Kondo screening.
This experiment does ***not*** probe quantum criticality.

Other realizations of the two-impurity Kondo model:

STM – adatom setup

J. Bork *et al.*, Nature. Phys. (2011)

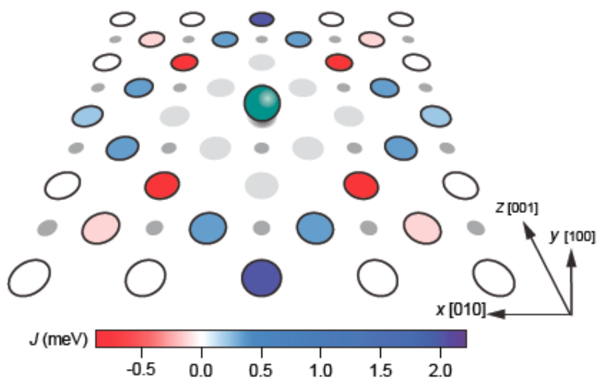


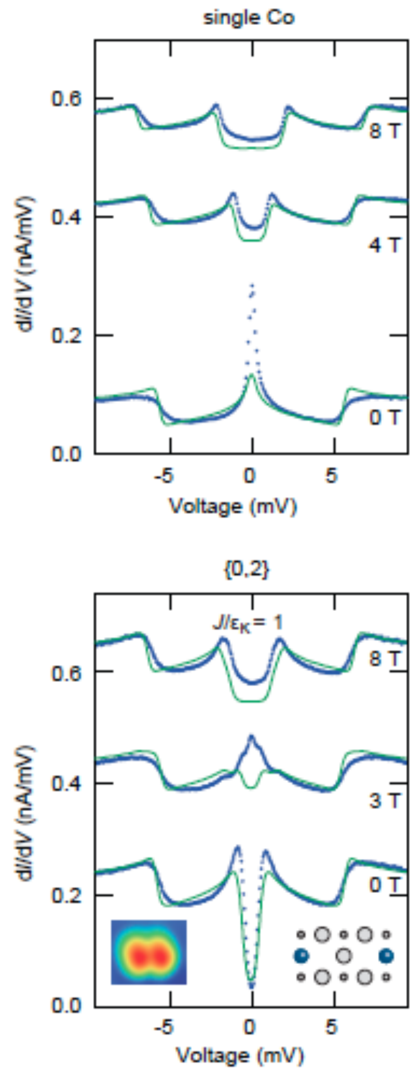


Other realizations of the two-impurity Kondo model:

STM spectra on adatom–adatom setup

A. Spinelli *et al.*, Nature Comm. **6**, 10046 (2015)





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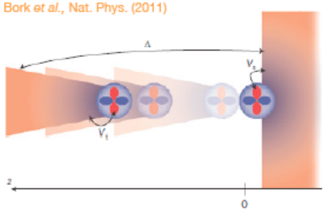

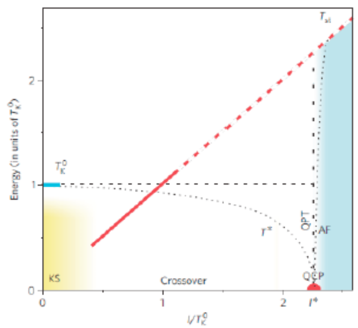

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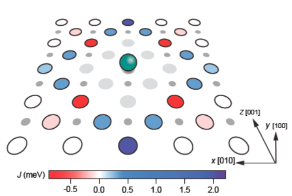

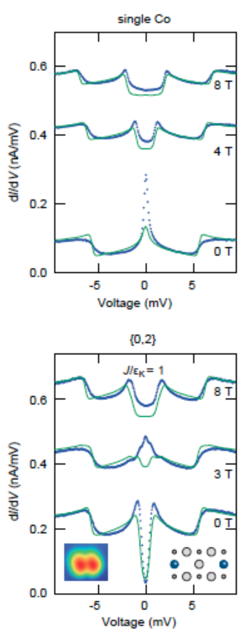

Experimental challenges...!

Experimental challenges...!

... and theoretical puzzles

Experimental challenges...

... and theoretical puzzles

Kondo screening cloud?

What is it? How to identify it?

Experimental challenges...

... and theoretical puzzles

Kondo screening cloud?

What is it? How to identify it?

How to characterize the Kondo-RKKY phase transition?

Is there an order parameter? Critical scaling?

Experimental challenges...
... and theoretical puzzles

Kondo screening cloud?

What is it? How to identify it?

How to characterize the phase transition?

Is there an order parameter? Critical scaling?

A new vista by probing the ground state entanglement:

A. Bayat, S. Bose, P. Sodano, H. J., PRL **109**, 066403 (2012)

A. Bayat, H. J., S. Bose, P. Sodano, Nature Comm. **5**, 3784 (2014)

Kondo screening cloud

Background: **single-impurity Kondo model...**

One-loop RG equations: $\frac{\mathrm{d}\lambda}{\mathrm{d}\ln D} = -\nu\lambda^2 + ...$

P. W. Anderson, J. Phys. C (1970)

↓

effective scale-dependent coupling λ becomes of $\mathcal{O}(1)$ at

$$T_K = D_0 \exp(-\mathrm{const.}/\lambda_0)$$

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$$T_K = D_0 \exp(-\mathrm{const.}/\lambda_0)$$

dynamically generated energy scale
KONDO TEMPERATURE

↓

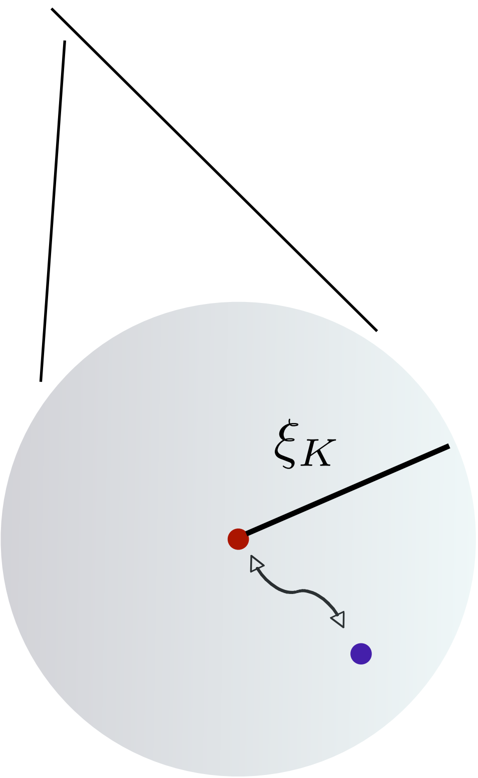
$$\xi_K = \frac{v_F}{T_K}$$

dynamically generated length scale:
KONDO SCREENING LENGTH

Kondo screening cloud

Heuristics: **single-impurity Kondo model...**

$$H_{\rm int} = \lambda \psi^\dagger(0)\boldsymbol{\sigma}\psi(0) \cdot \boldsymbol{S}_{\rm imp}$$



$\uparrow\downarrow - \downarrow\uparrow$

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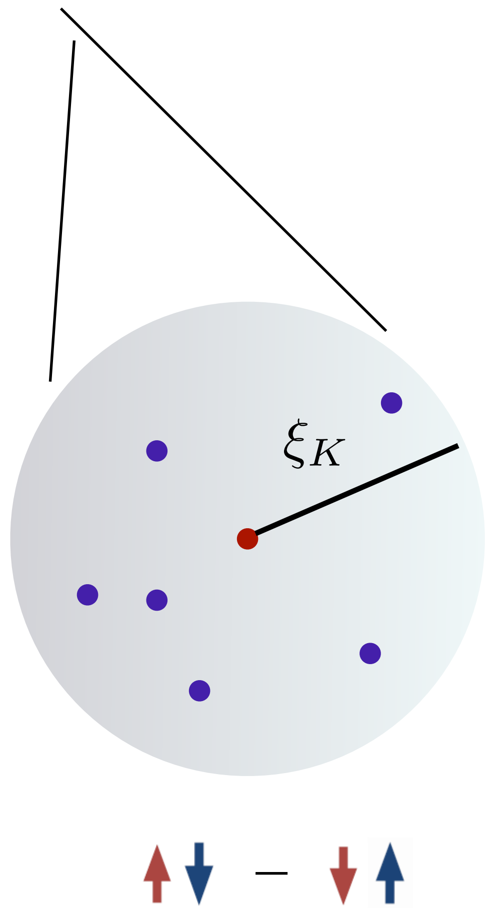

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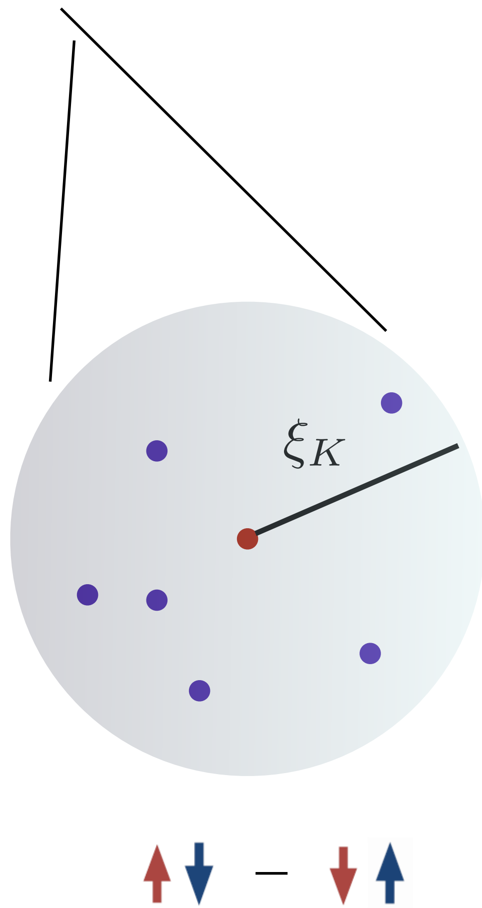

Expectation/suggestions from theory...

recent: J. Park *et al.*, PRL **110**, 246603 (2013)

...so far no experimental signal...

Does the Kondo cloud really exist?

Kondo screening cloud

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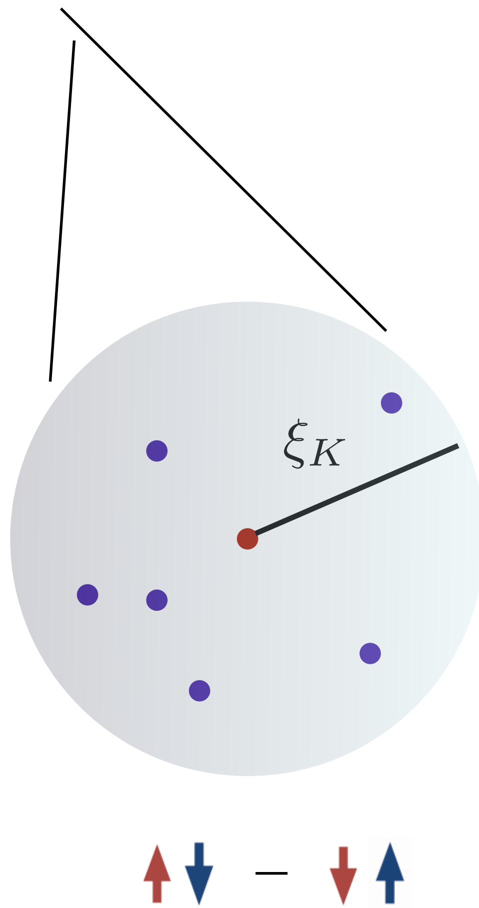
$$H_{\text{int}} = \lambda \psi^\dagger(0)\boldsymbol{\sigma}\psi(0) \cdot \boldsymbol{S}_{\text{imp}}$$



'Constructive' approach:

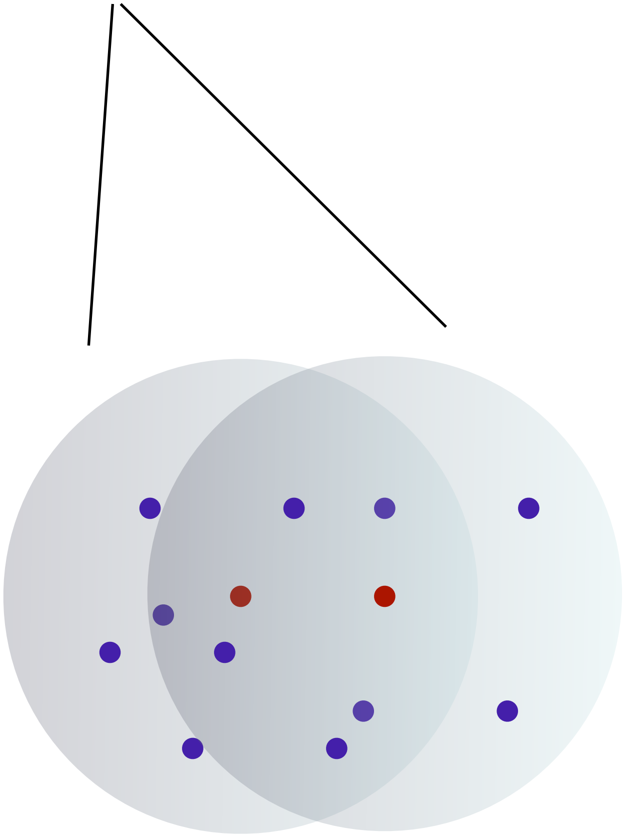
Think of the screening length as an entanglement length!

A. Bayat, P. Sodano, and S. Bose, PRB **81**, 064429 (2010)

Kondo screening cloud

... How does this play out with the **two-impurity Kondo model?**

$$H_{\text{int}} = \lambda \psi^\dagger(\boldsymbol{r}_1)\boldsymbol{\sigma}\psi(\boldsymbol{r}_1)\cdot\boldsymbol{S}_1 + \lambda \psi^\dagger(\boldsymbol{r}_1)\boldsymbol{\sigma}\psi(\boldsymbol{r}_1)\cdot\boldsymbol{S}_1 + K(\boldsymbol{r}_1 - \boldsymbol{r}_2)\boldsymbol{S}_1\cdot\boldsymbol{S}_2$$



Entanglement probe!

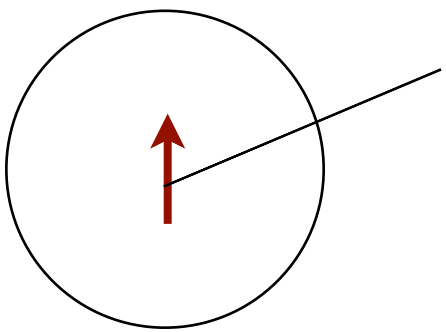
Computational method:

DMRG on a two-impurity Kondo spin chain

Two-impurity Kondo spin chain

start with the *single-impurity* Kondo model

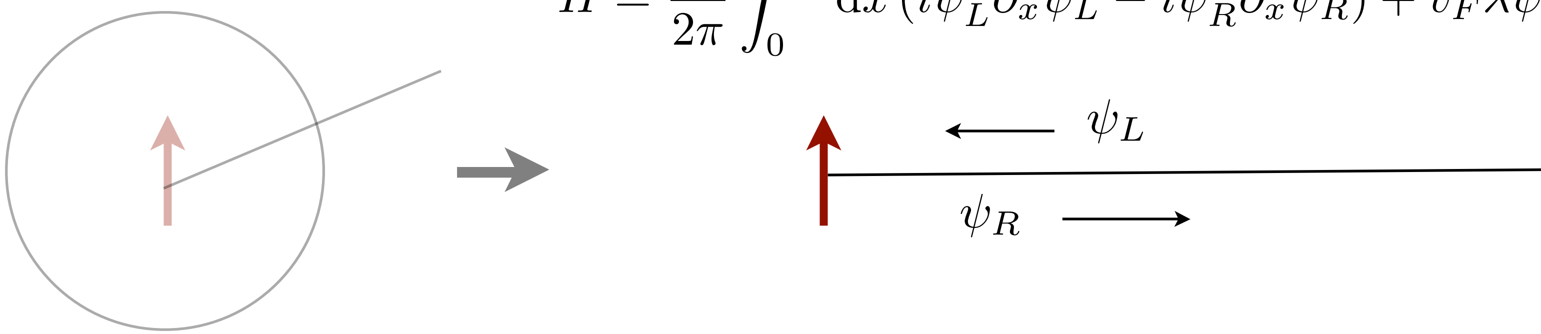
$$H = \int \mathrm{d}^3\boldsymbol{r}[\psi^\dagger(-\nabla^2/2m)\psi + J_K\delta^3(0)\psi^\dagger\boldsymbol{\sigma}\psi\cdot\boldsymbol{S}_{\mathrm{imp}}]$$



Two-impurity Kondo spin chain

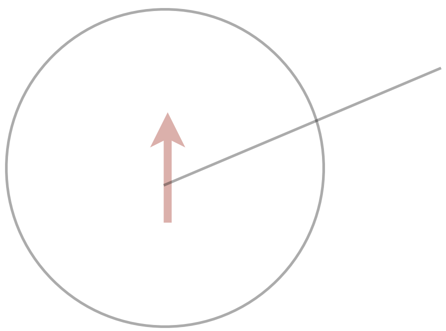
start with the *single-impurity* Kondo model

$$H = \frac{v_F}{2\pi}\int_0^\infty \mathrm{d}x\,\left(i\psi_L^\dagger\partial_x\psi_L - i\psi_R^\dagger\partial_x\psi_R\right) + v_F\lambda\psi_L^\dagger(0)\boldsymbol{\sigma}\psi_L(0)\cdot\boldsymbol{S}_{\mathrm{imp}}$$

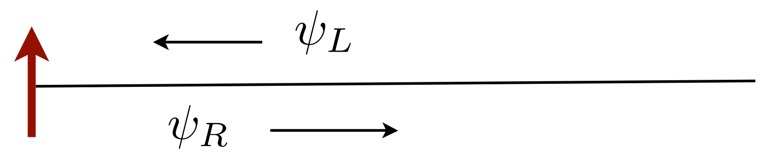


Two-impurity Kondo spin chain

start with the *single-impurity* Kondo model



$$H = \frac{v_F}{2\pi}\int_0^\infty \mathrm{d}x\,\left(i\psi_L^\dagger\partial_x\psi_L - i\psi_R^\dagger\partial_x\psi_R\right) + v_F\lambda\psi_L^\dagger(0)\boldsymbol{\sigma}\psi_L(0)\cdot\boldsymbol{S}_{\mathrm{imp}}$$

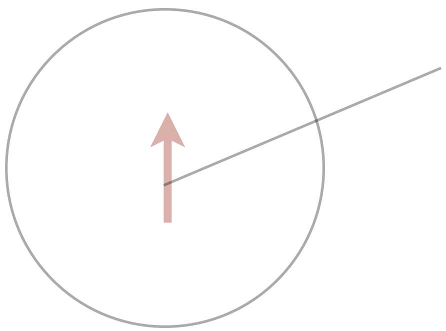


$$H = H_{\mathrm{spin}} + \cancel{H_{\mathrm{charge}}}$$

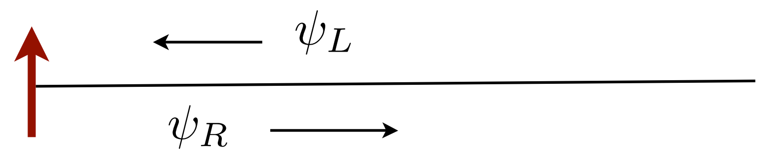
$$H_{\mathrm{spin}} = \frac{v_F}{6\pi}\int_0^\infty \mathrm{d}x(\boldsymbol{J}_L\cdot\boldsymbol{J}_L + \boldsymbol{J}_R\cdot\boldsymbol{J}_R) + v_F\lambda\boldsymbol{J}_L(0)\cdot\boldsymbol{S}_{\mathrm{imp}}$$

Two-impurity Kondo spin chain

start with the *single-impurity* Kondo model



$$H = \frac{v_F}{2\pi}\int_0^\infty \mathrm{d}x\,\left(i\psi_L^\dagger \partial_x \psi_L - i\psi_R^\dagger \partial_x \psi_R\right) + v_F \lambda \psi_L^\dagger(0)\boldsymbol{\sigma}\psi_L(0)\cdot \boldsymbol{S}_{\text{imp}}$$

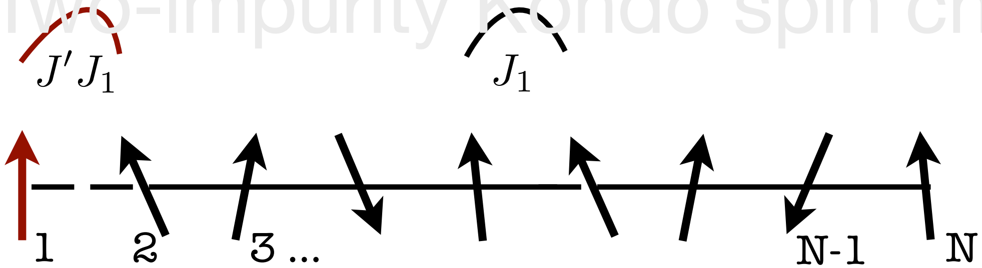


$$H = H_{\text{spin}} + H_{\text{charge}}$$

$$H_{\text{spin}} = \frac{v_F}{6\pi}\int_0^\infty \mathrm{d}x(\boldsymbol{J}_L\cdot\boldsymbol{J}_L + \boldsymbol{J}_R\cdot\boldsymbol{J}_R) + v_F\lambda \boldsymbol{J}_L(0)\cdot\boldsymbol{S}_{\text{imp}}$$

This is the low-energy effective theory
of a spin chain with a weak boundary link!

Two-impurity Kondo spin chain



$$H = J_1 \sum_{i=2}^{N-1} \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + J' J_1 \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 \qquad J' < 1$$

Non-Abelian bosonization:

$$\boldsymbol{S}_i = \boldsymbol{J}_L(ai) + \boldsymbol{J}_R(ai) + (-1)^i \text{constant} \cdot \boldsymbol{n}(ai)$$

a = lattice spacing

+ continuum limit

$$H_{\text{spin}} = \frac{v_F}{6\pi} \int_0^\infty \mathrm{d}x (\boldsymbol{J}_L \cdot \boldsymbol{J}_L + \boldsymbol{J}_R \cdot \boldsymbol{J}_R) + v_F \lambda \boldsymbol{J}_L(0) \cdot \boldsymbol{S}_{\text{imp}}$$

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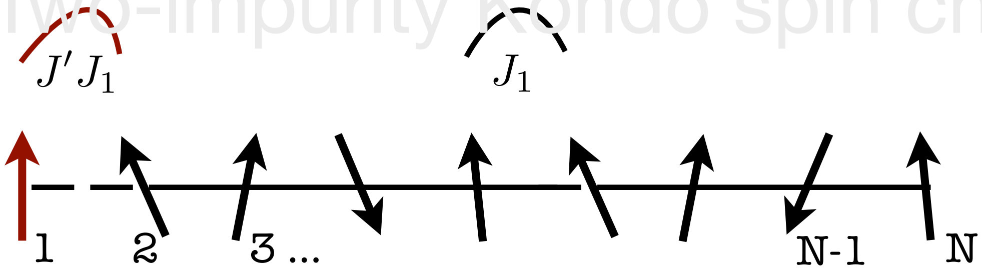
Two-impurity Kondo spin chain



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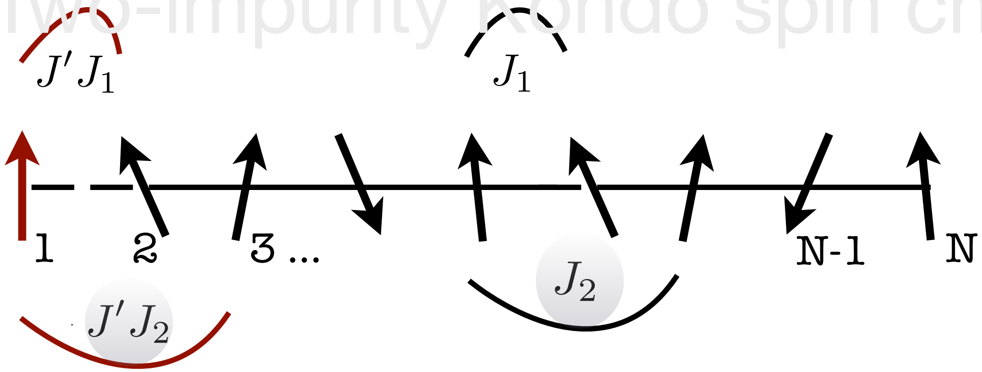
$$+ \; \textit{marginally irrelevant} \; + \; \textit{irrelevant} \text{ terms}$$

Two-impurity Kondo spin chain



$$H = J_1 \sum_{i=2}^{N-1} \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+1} + J' J_1 \boldsymbol{S}_1 \cdot \boldsymbol{S}_2 + J_2 \sum_{i=2}^{N-2} \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+2} + J' J_2 \boldsymbol{S}_1 \cdot \boldsymbol{S}_3$$

Non-Abelian bosonization:

$$\boldsymbol{S}_i = \boldsymbol{J}_L(ai) + \boldsymbol{J}_R(ai) + (-1)^i \text{constant} \cdot \boldsymbol{n}(ai)$$

+ continuum limit

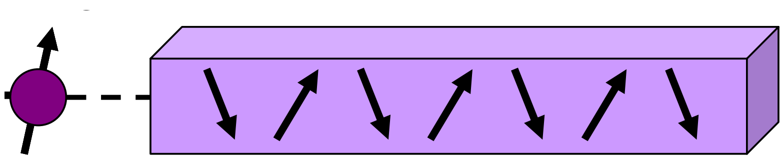
$$J_2 = 0.2412\, J_1$$

$$H_{\text{spin}} = \frac{v_F}{6\pi} \int_0^\infty \mathrm{d}x (\boldsymbol{J}_L \cdot \boldsymbol{J}_L + \boldsymbol{J}_R \cdot \boldsymbol{J}_R) + v_F \lambda \boldsymbol{J}_L(0) \cdot \boldsymbol{S}_{\text{imp}}$$

$$+ \; \textit{marginally irrelevant} \; + \; \textit{irrelevant} \text{ terms}$$

S. Eggert, PRB **54**, 9616(R) (1996)

Single-impurity Kondo spin chain

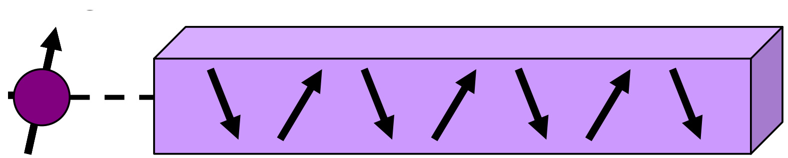


$$H = \sum_{j=1}^{2} J_j \left(J' \boldsymbol{S}_1 \cdot \boldsymbol{S}_{j+1} + \sum_{i=2}^{N-j} \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+j} \right) \qquad J_2 = 0.2412\, J_1$$

same low-energy physics as the (spin sector) of the single-impurity Kondo model

S. Rommer and S. Eggert, PRB **62**, 4370 (2000)
N. Laflorencie *et al.*, J. Stat. Mech. P02007 (2008)

Single-impurity Kondo spin chain



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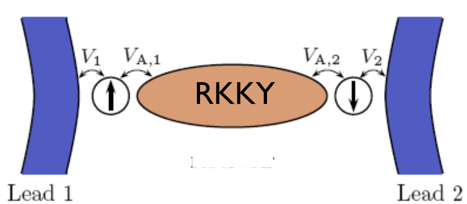
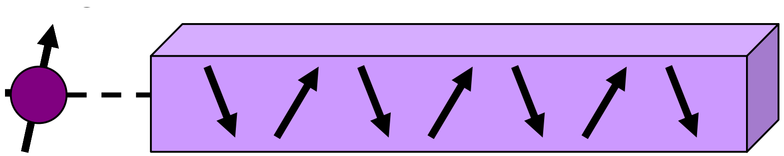
Implication for two-impurity Kondo model?



Single-impurity Kondo spin chain



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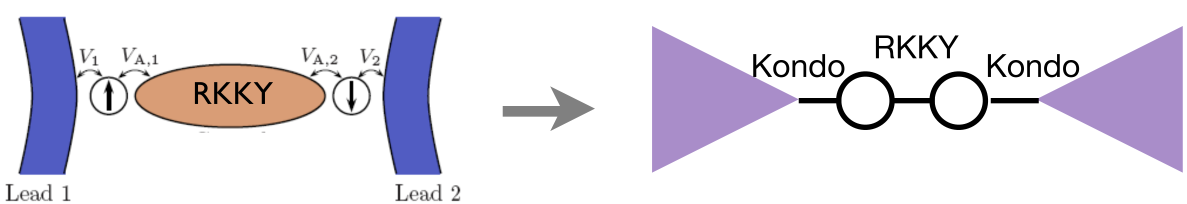
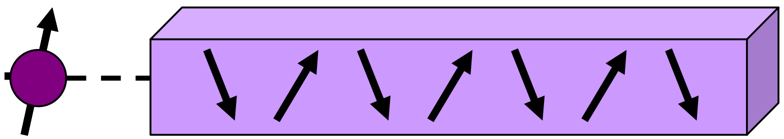


Single-impurity Kondo spin chain



$$H = \sum_{j=1}^{2} J_j \left(J' \boldsymbol{S}_1 \cdot \boldsymbol{S}_{j+1} + \sum_{i=2}^{N-j} \boldsymbol{S}_i \cdot \boldsymbol{S}_{i+j} \right)$$

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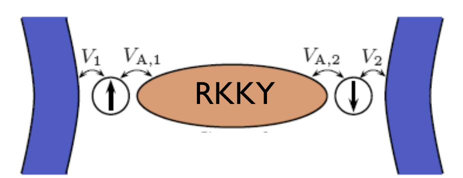
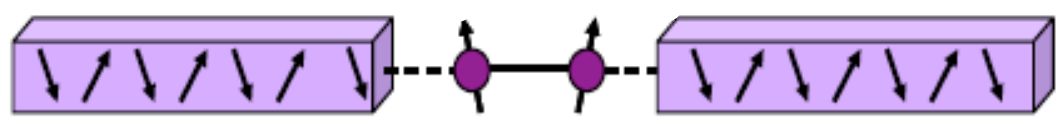
Implication for two-impurity Kondo model?

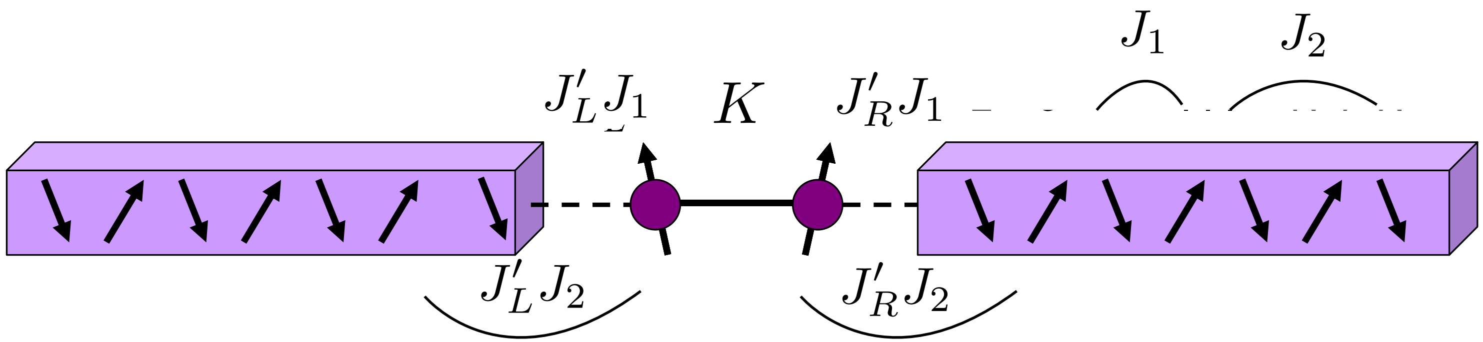


→



two RKKY-coupled 'Kondo spin chains'
(a.k.a. *'two-impurity Kondo chain'*)

Two-impurity Kondo spin chain



$$H = \sum_{k=L,R} H_k + H_I$$

$$H_k = \sum_{j=1}^{2} J_j \left(J'_k \boldsymbol{S}_1^k \cdot \boldsymbol{S}_{j+1}^k + \sum_{i=2}^{N-j} \boldsymbol{S}_i^k \cdot \boldsymbol{S}_{i+j}^k \right)$$

$$J'_L = J'_R \equiv J' < 1$$

Kondo

$$H_I = K \boldsymbol{S}_1^L \cdot \boldsymbol{S}_1^R$$

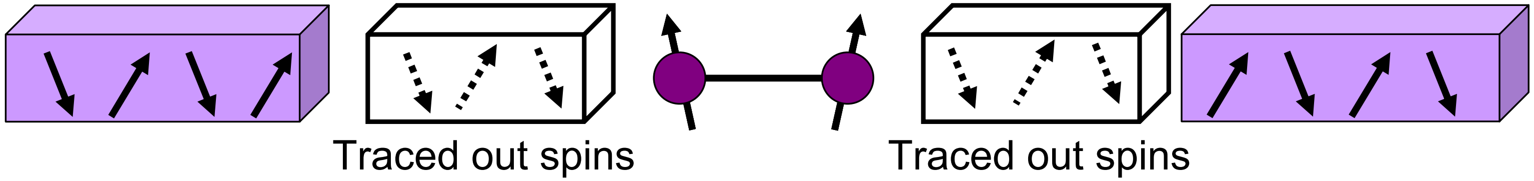
$$J_2 = 0.2412\, J_1$$

RKKY

Well adapted for DMRG... and entanglement probes!

Kondo screening cloud

Entanglement probe:



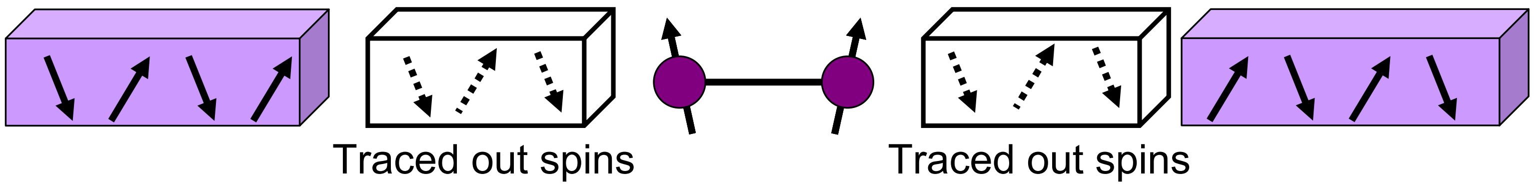
- Trace out, compute the negativity between the impurities and the rest of the system

- Define $\xi_E \equiv$ length beyond which the negativity is smaller than some cutoff (here, = 0.01)

entanglement length

Kondo screening cloud

Entanglement probe:



- Trace out, compute the **negativity** between the impurities and the rest of the system

$$\mathcal{N}(\rho_{AB}) = \sum_i \frac{|a_i| - a_i}{2}$$

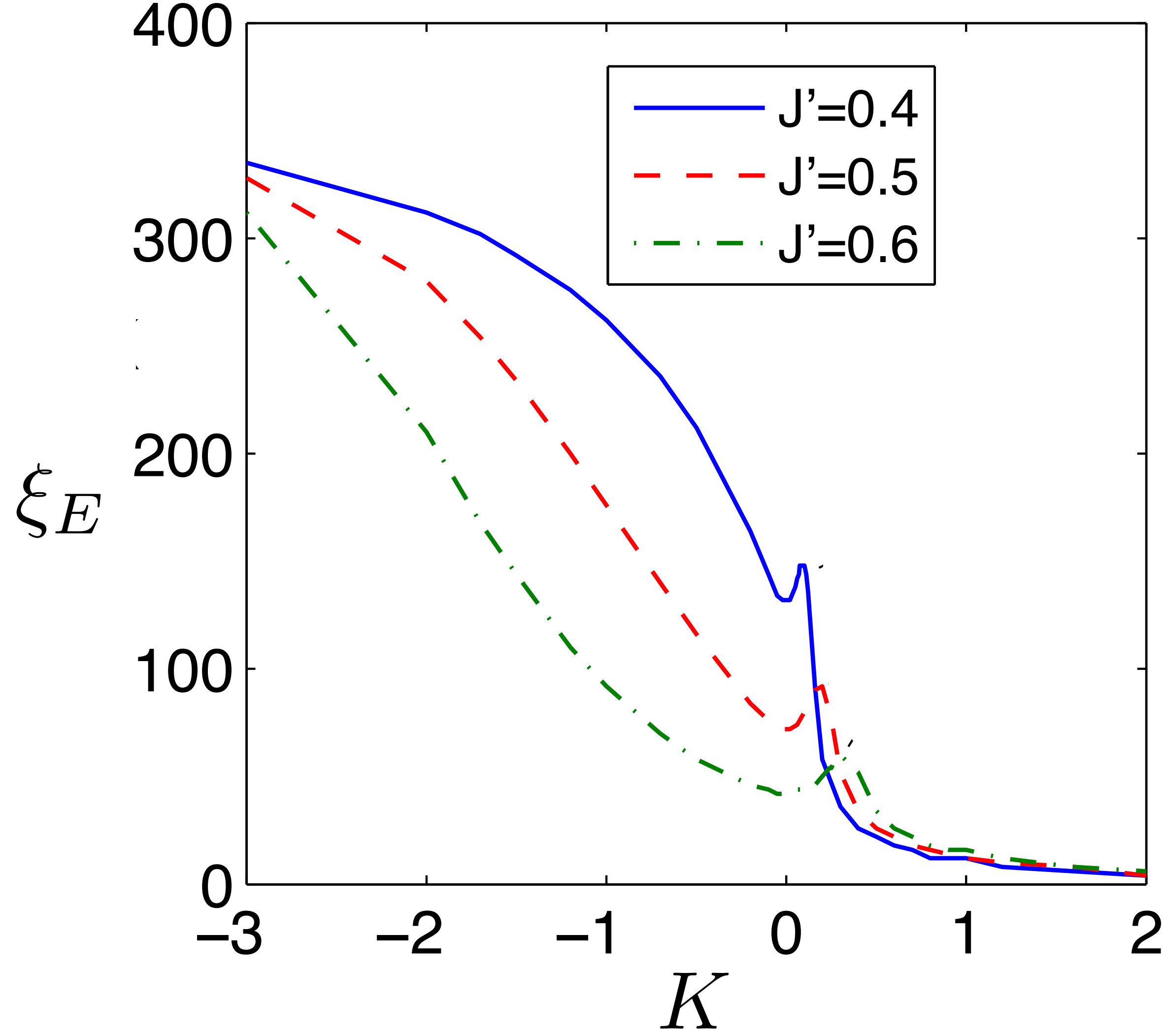
a_i eigenvalues of $\rho_{AB}^{T^A}$

$\rho_{AB} = \mathrm{Tr}_{(AB)^c} |\Psi\rangle\langle\Psi|$

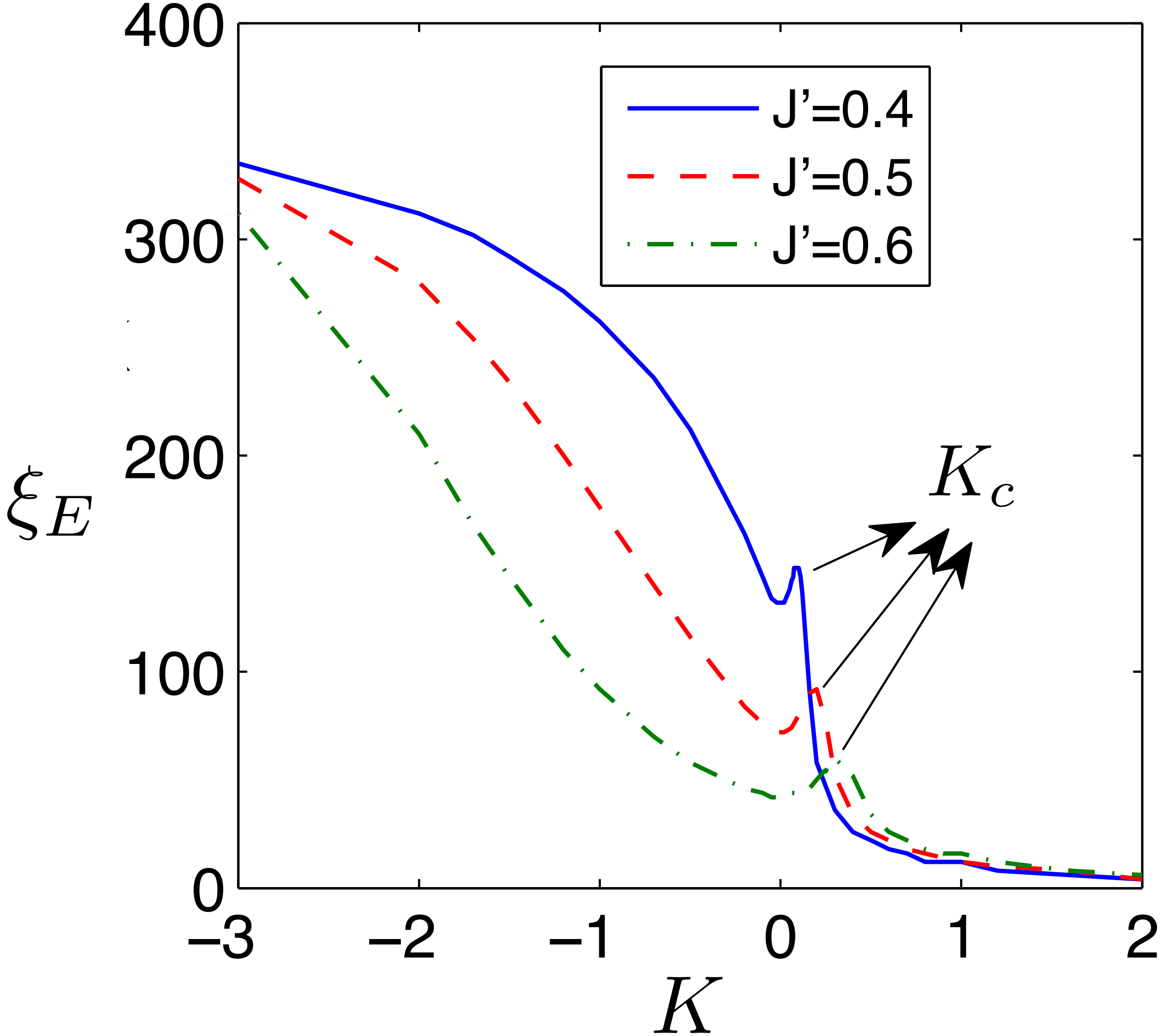
- Define $\xi_E \equiv$ length beyond which the negativity is smaller than some cutoff (here, = 0.01)

G. Vidal and R. F. Werner, PRA **65**, 032314 (2002)

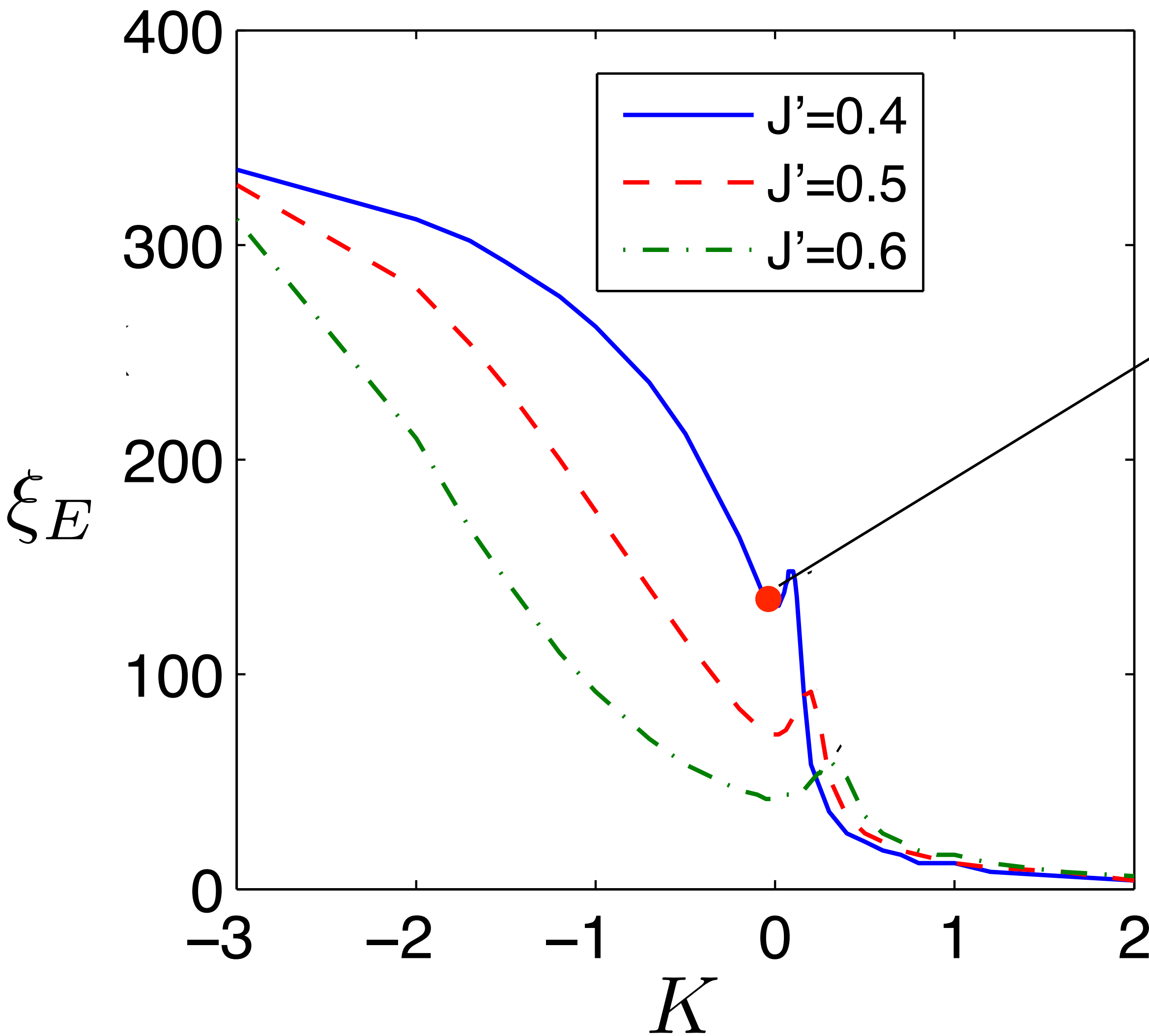
Kondo screening cloud



Kondo screening cloud



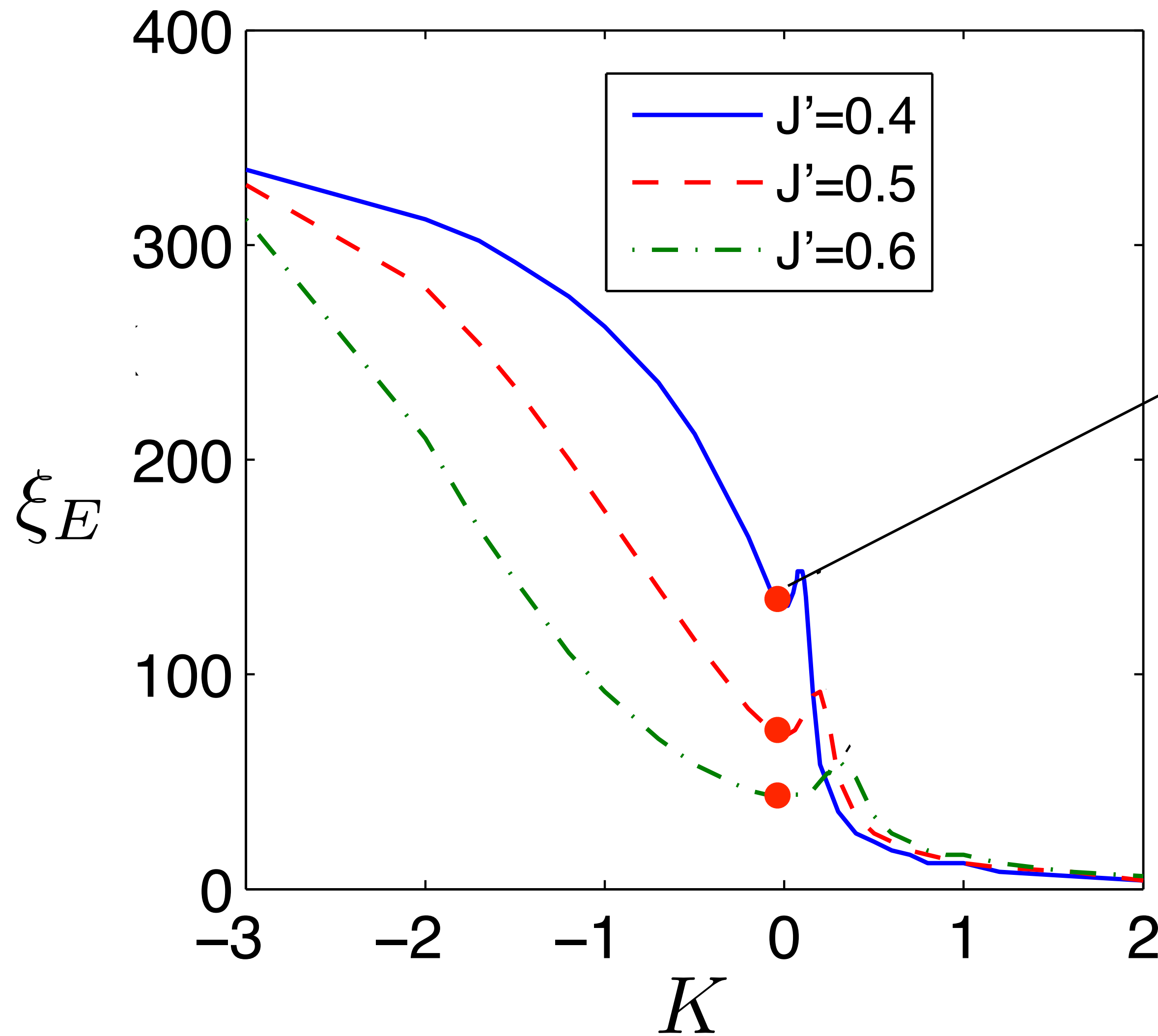
Kondo screening cloud



fit to single-impurity Kondo length ξ_K from perturbative RG

Nevidomskyy and Coleman, PRL 103, 147205 (2009)

Kondo screening cloud



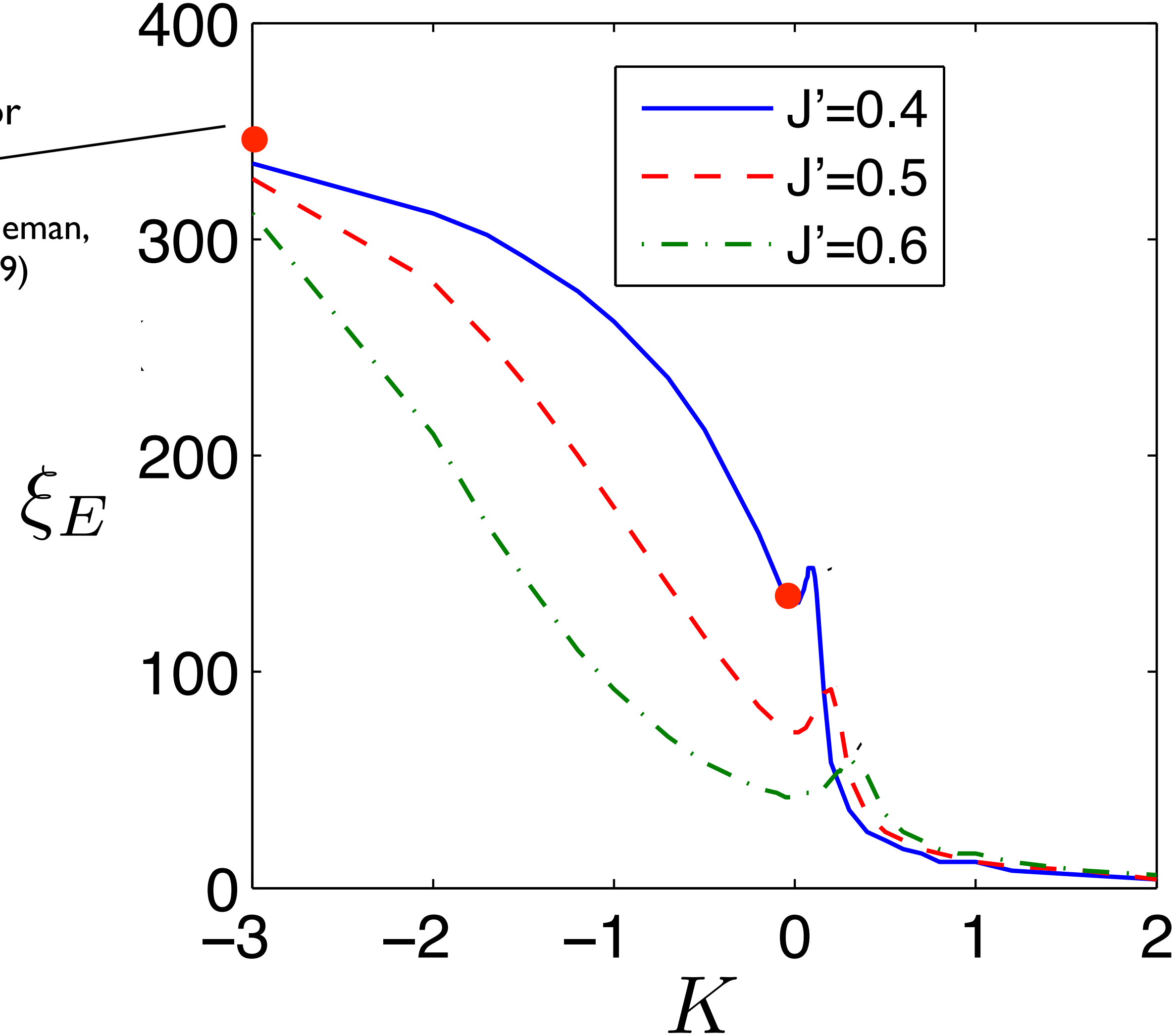
fit to single-impurity Kondo length ξ_K from perturbative RG

Nevidomskyy and Coleman, PRL 103, 147205 (2009)

Kondo screening cloud

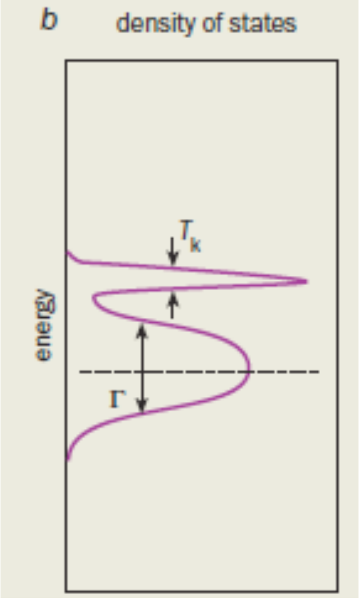
spin-1 two-channel Kondo length ξ_K for $K \to -\infty$

Nevidomskyy and Coleman, PRL 103, 147205 (2009)



Kondo screening cloud

Kondo resonance narrowing



Experiments indicate that the width of the Kondo resonance narrows with larger impurity spin (at complete screening)

M.D. Daybell and W.A. Steyert, RMP **40**, 380 (1968)

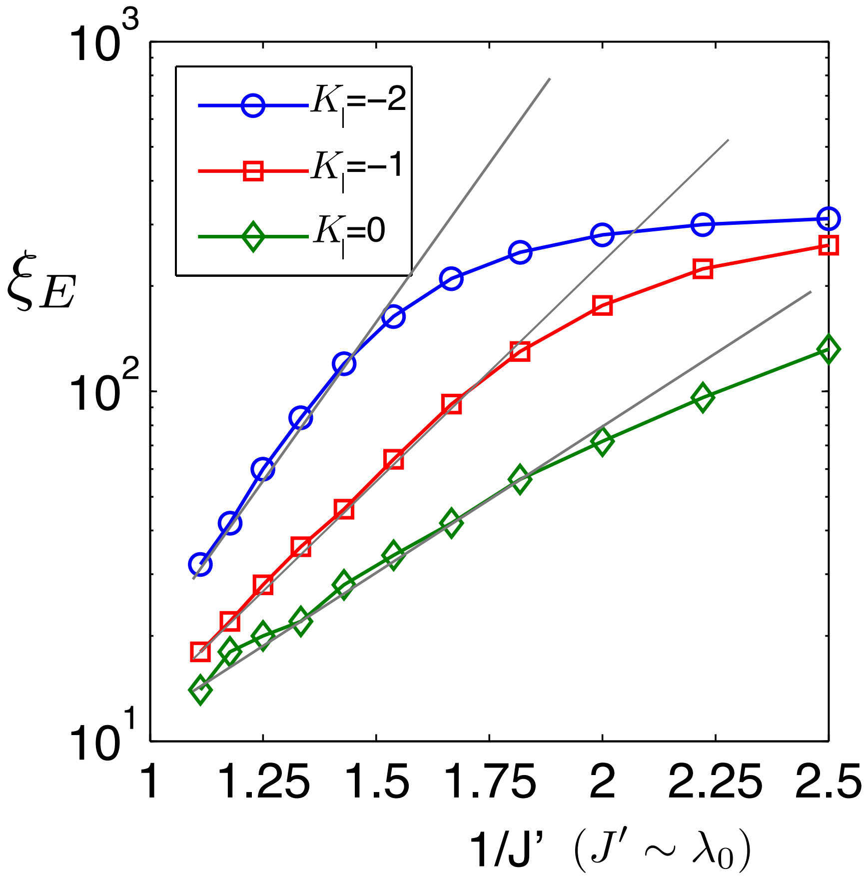
Perturbative scaling theory suggests

$$T_K = D_0 \exp(-2S \times \text{const.}/\lambda_0)$$

A. H. Nevidomskyy and P. Coleman, PRL 103, 147205 (2009)

Kondo screening cloud

Kondo resonance narrowing



ξ_E scales exponentially with 1/J', like

$\xi_K \sim \exp(\alpha(\bar{K})/J')$

J_I	-3.00	-2.50	-2.00	-1.50	-1.00	-0.50	0.00
$\alpha(J_I)$	3.2175	3.1407	2.9403	2.6838	2.3106	2.0718	1.7686

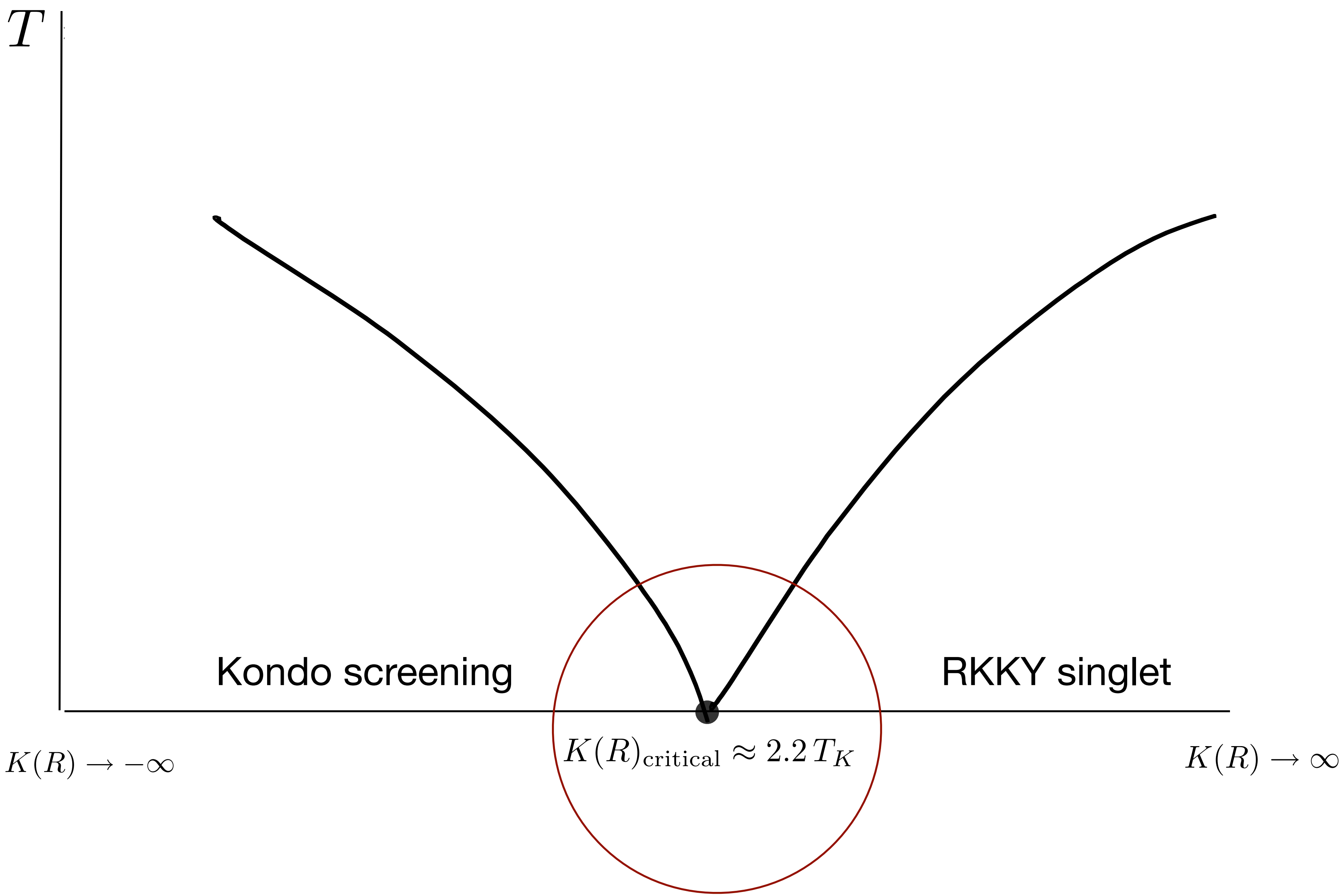
→ $\alpha(-\infty) \approx 2.5\alpha(0)$

Nevidomskyy-Coleman predicts

$\alpha(-\infty) = 2\alpha(0)$

Order parameter at the Kondo-RKKY quantum phase transition?

Order parameter at the quantum phase transition?



Order parameter at the quantum phase transition?

Kondo screening

RKKY singlet

$K(R) \to -\infty$

$K(R)_{\text{critical}} \approx 2.2\, T_K$

$K(R) \to \infty$

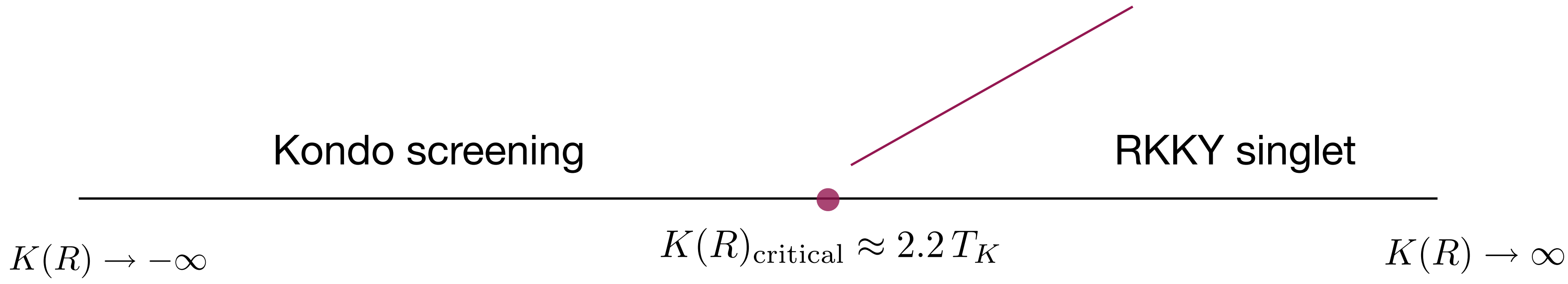
impurity

Second-order quantum phase transition

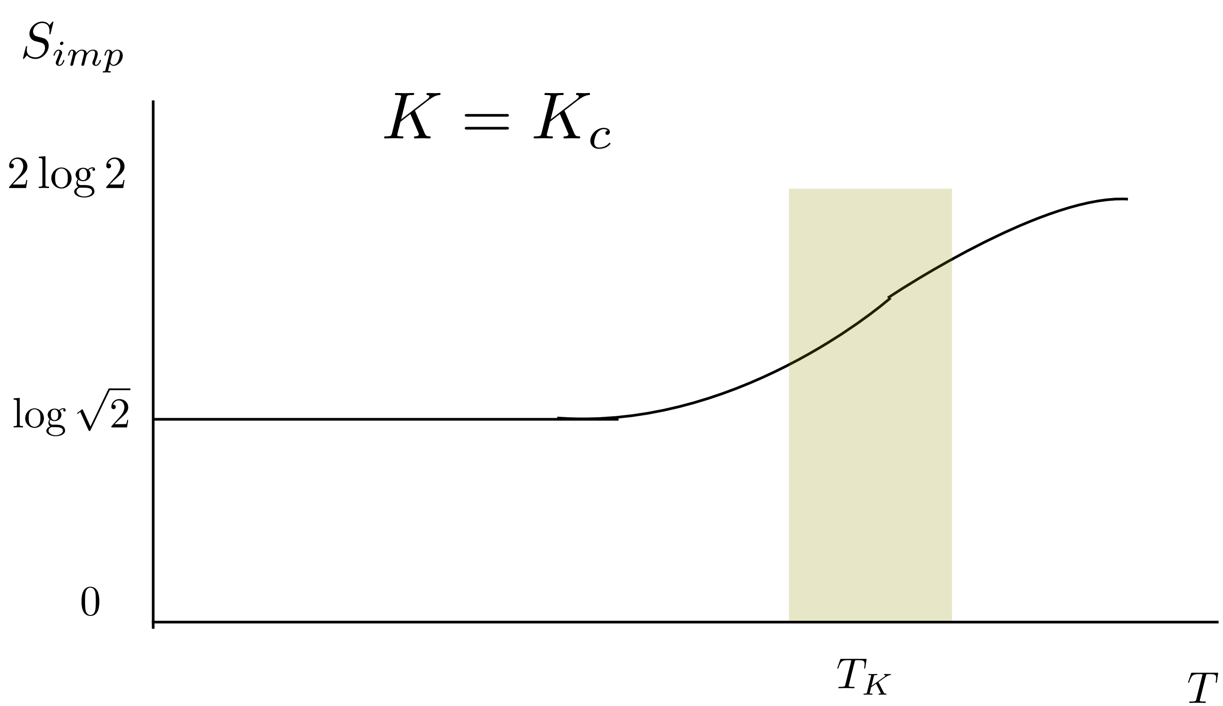
$$\frac{C_{\text{imp}}}{T} = \gamma \longrightarrow \frac{T_K}{(K - K_c)^2}$$

critical exponent

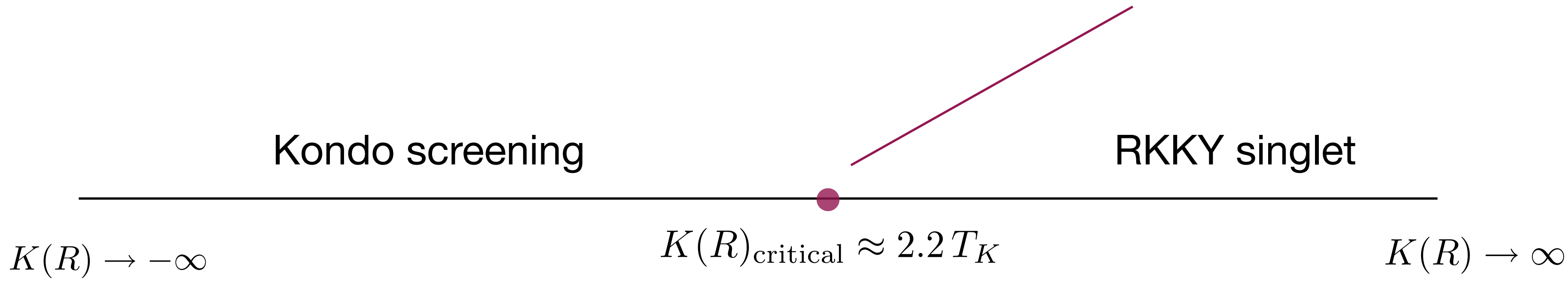
Order parameter at the quantum phase transition?



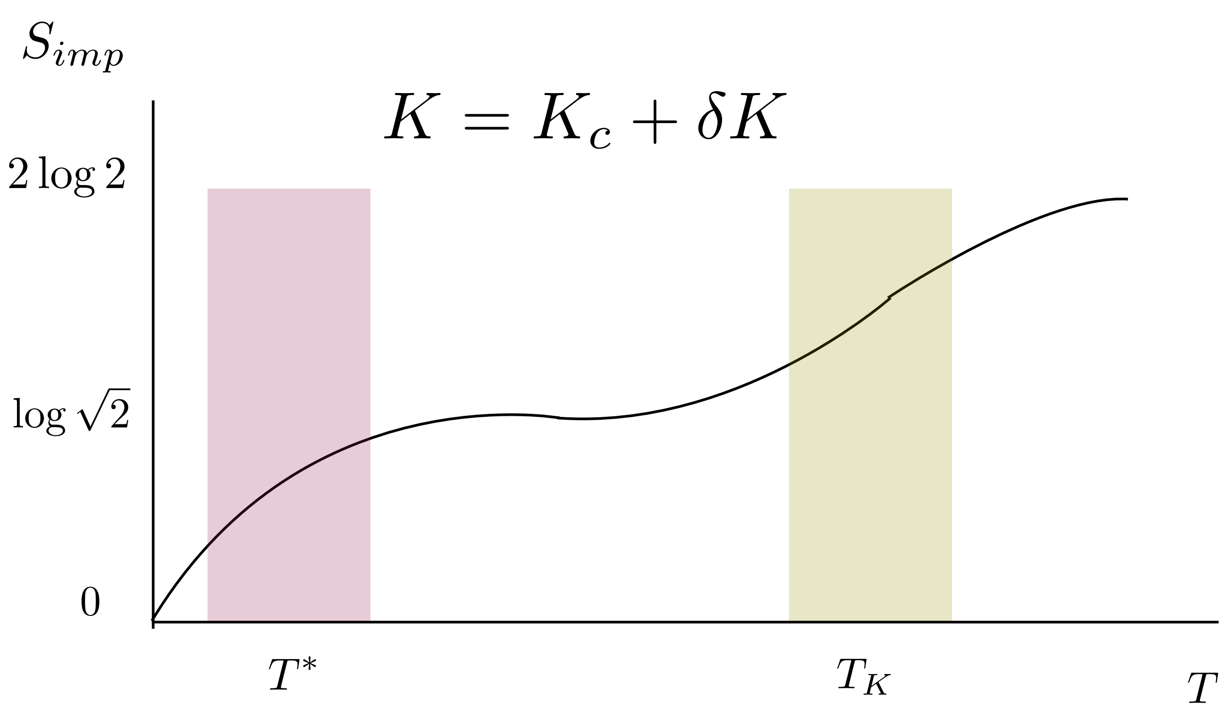
Second-order (impurity) quantum phase transition



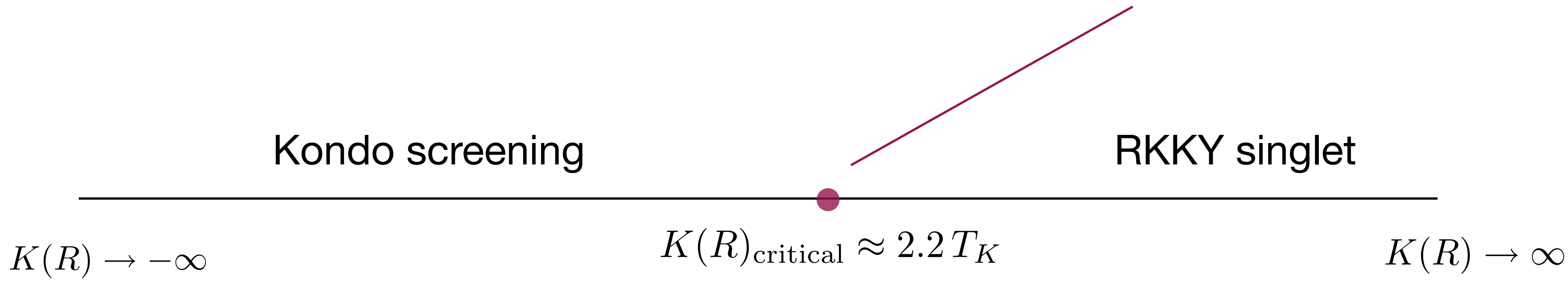
Order parameter at the quantum phase transition?



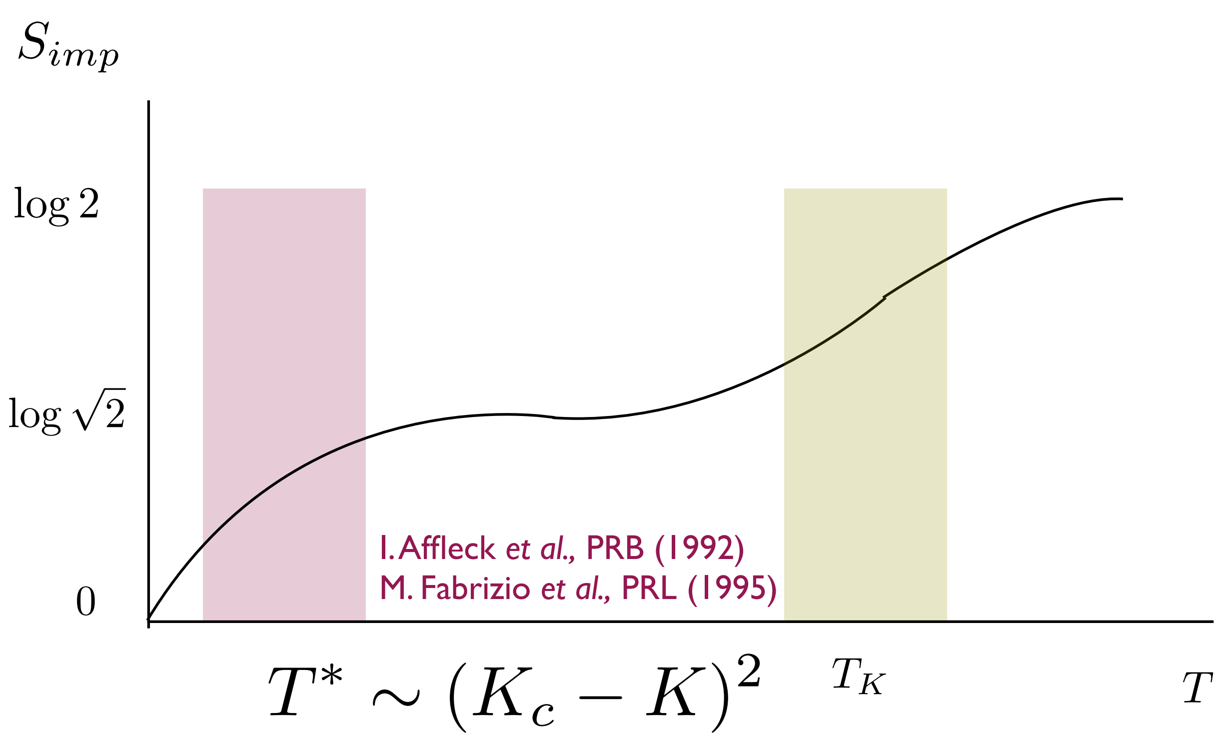
Second-order (impurity) quantum phase transition



Order parameter at the **quantum phase transition?**



Second-order impurity quantum phase transition



Order parameter at the quantum phase transition?

Kondo screening | RKKY singlet

$K(R) \to -\infty$ | $K(R)_{\text{critical}} \approx 2.2\, T_K$ | $K(R) \to \infty$

Second-order (impurity) quantum phase transition

$$T^* \sim (K_c - K)^2$$

$$\xi^* \sim v/T^* \sim (K_c - K)^{-2}$$

Order parameter at the quantum phase transition?

Kondo screening

RKKY singlet

$K(R) \to -\infty$

$K(R)_{\text{critical}} \approx 2.2\, T_K$

$K(R) \to \infty$

Second-order quantum phase transition (impurity)

$$T^* \sim (K_c - K)^2$$

$$\xi^* \sim v/T^* \sim (K_c - K)^{-2}$$

diverging length scale at criticality

Order parameter at the quantum phase transition?

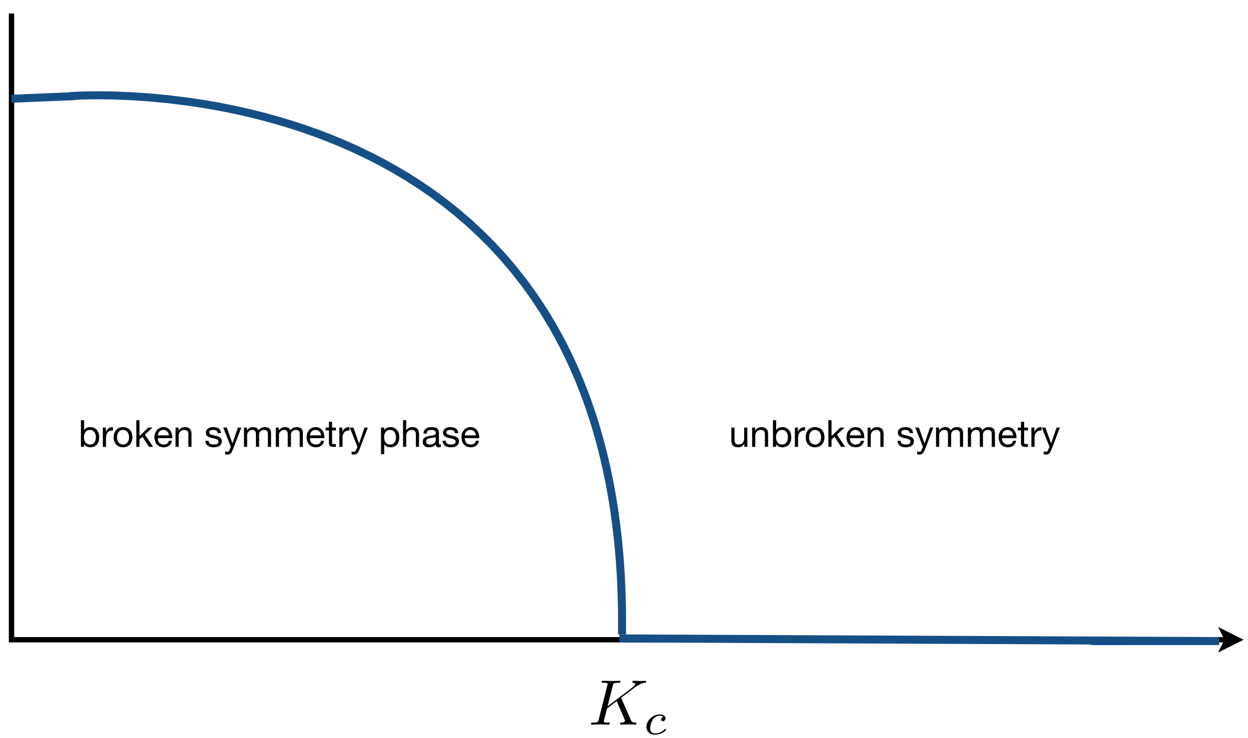
Kondo screening — RKKY singlet

$K(R) \to -\infty$ — $K(R)_{\text{critical}} \approx 2.2\, T_K$ — $K(R) \to \infty$

impurity

Second-order quantum phase transition

Expectation from "Landau-Ginzburg-Wilson-Herz":



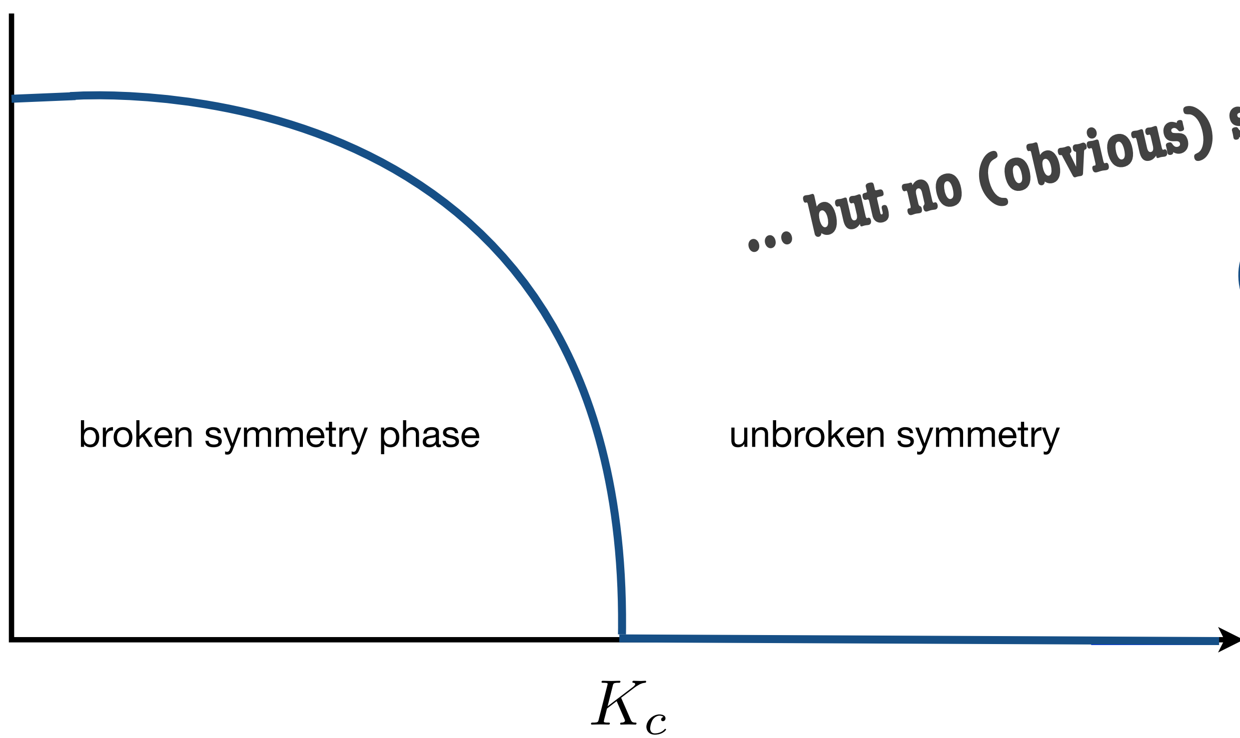
Order parameter at the quantum phase transition?

Kondo screening | RKKY singlet

$K(R) \to -\infty$ | $K(R)_{\text{critical}} \approx 2.2\, T_K$ | $K(R) \to \infty$

Second-order (impurity) quantum phase transition

Expectation from "Landau-Ginzburg-Wilson-Herz":



... but no (obvious) symmetry breaking!

?

Order parameter at the quantum phase transition

PRL 109, 237208 (2012) PHYSICAL REVIEW LETTERS week ending 7 DECEMBER 2012

Entanglement Spectrum, Critical Exponents, and Order Parameters in Quantum Spin Chains

G. De Chiara,[1,2] L. Lepori,[1] M. Lewenstein,[3,4] and A. Sanpera[3,1]

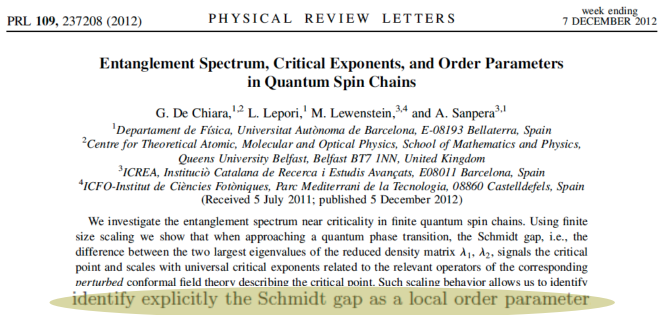
[1]*Departament de Física, Universitat Autònoma de Barcelona, E-08193 Bellaterra, Spain*
[2]*Centre for Theoretical Atomic, Molecular and Optical Physics, School of Mathematics and Physics, Queens University Belfast, Belfast BT7 1NN, United Kingdom*
[3]*ICREA, Instituciò Catalana de Recerca i Estudis Avançats, E08011 Barcelona, Spain*
[4]*ICFO-Institut de Ciències Fotòniques, Parc Mediterrani de la Tecnologia, 08860 Castelldefels, Spain*

(Received 5 July 2011; published 5 December 2012)

We investigate the entanglement spectrum near criticality in finite quantum spin chains. Using finite size scaling we show that when approaching a quantum phase transition, the Schmidt gap, i.e., the difference between the two largest eigenvalues of the reduced density matrix λ_1, λ_2, signals the critical point and scales with universal critical exponents related to the relevant operators of the corresponding *perturbed* conformal field theory describing the critical point. Such scaling behavior allows us to

identify explicitly the Schmidt gap as a local order parameter

Order parameter at the quantum phase transition


PRL 109, 237208 (2012) PHYSICAL REVIEW LETTERS week ending 7 DECEMBER 2012

Entanglement Spectrum, Critical Exponents, and Order Parameters in Quantum Spin Chains

G. De Chiara,[1,2] L. Lepori,[1] M. Lewenstein,[3,4] and A. Sanpera[3,1]
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[3]*ICREA, Institució Catalana de Recerca i Estudis Avançats, E08011 Barcelona, Spain*
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We investigate the entanglement spectrum near criticality in finite quantum spin chains. Using finite size scaling we show that when approaching a quantum phase transition, the Schmidt gap, i.e., the difference between the two largest eigenvalues of the reduced density matrix λ_1, λ_2, signals the critical point and scales with universal critical exponents related to the relevant operators of the corresponding *perturbed* conformal field theory describing the critical point. Such scaling behavior allows us to identify

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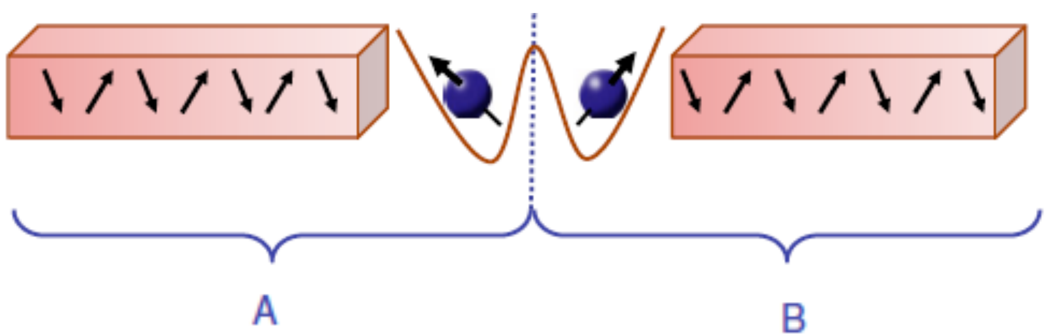
adapted to the two-impurity Kondo spin chain



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$$\rho_\alpha = \sum_k \lambda_k\,|\alpha_k\rangle\langle\alpha_k|, \quad \alpha = A, B.$$

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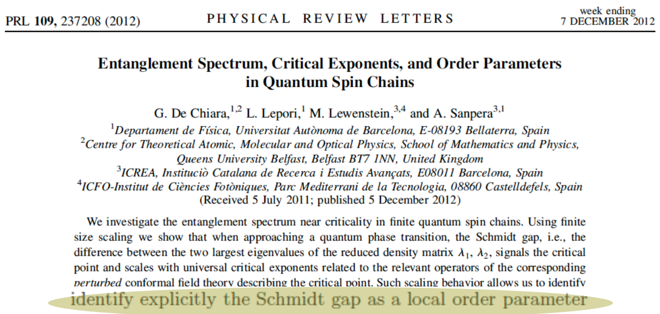
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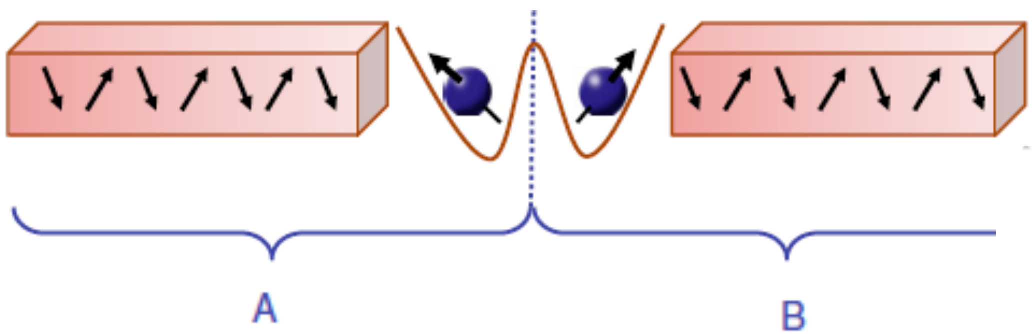

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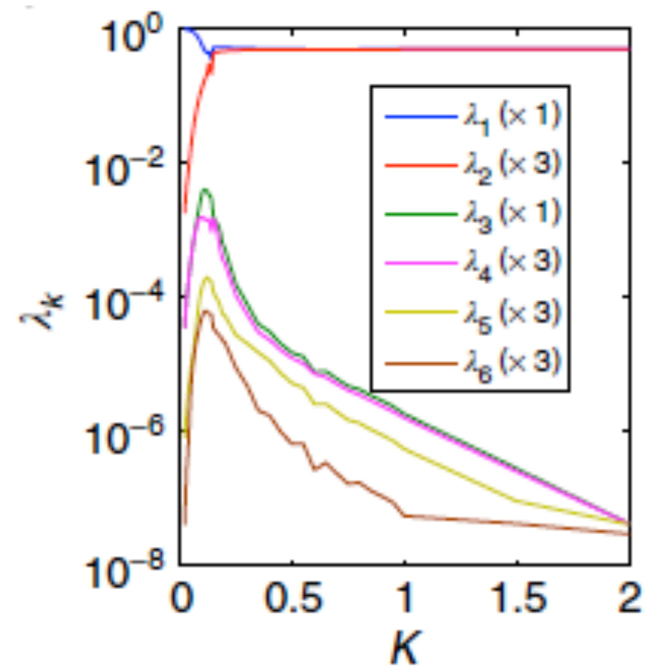
N=800



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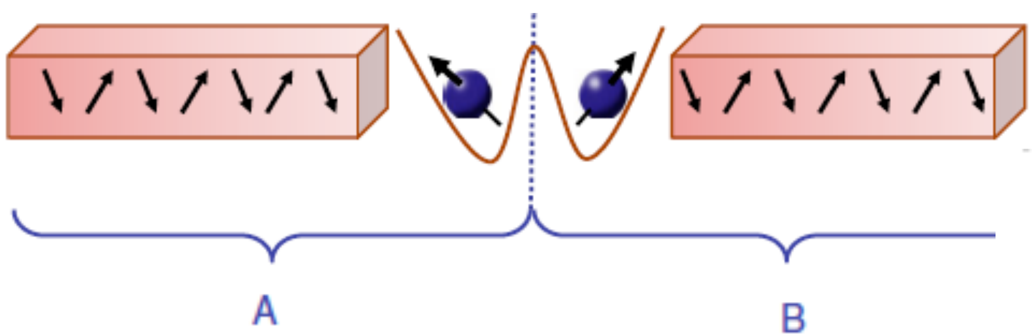

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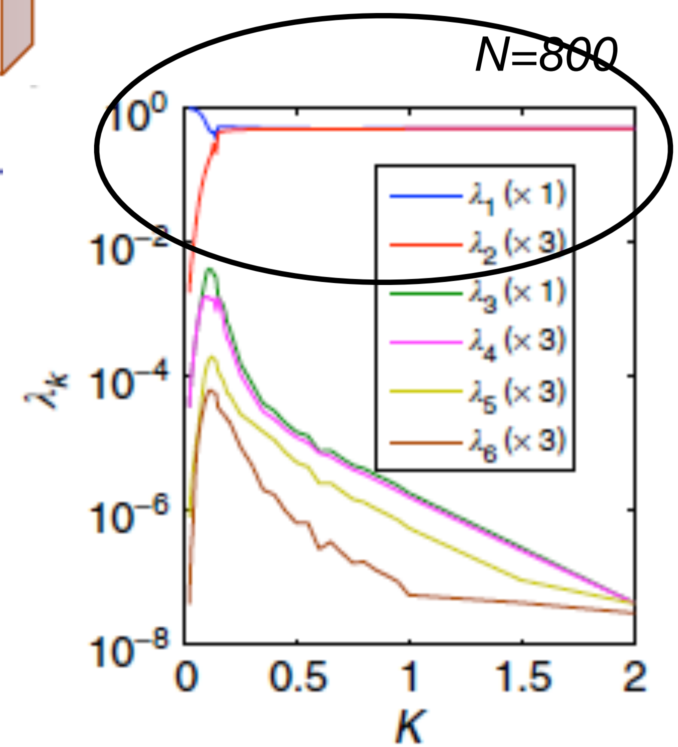
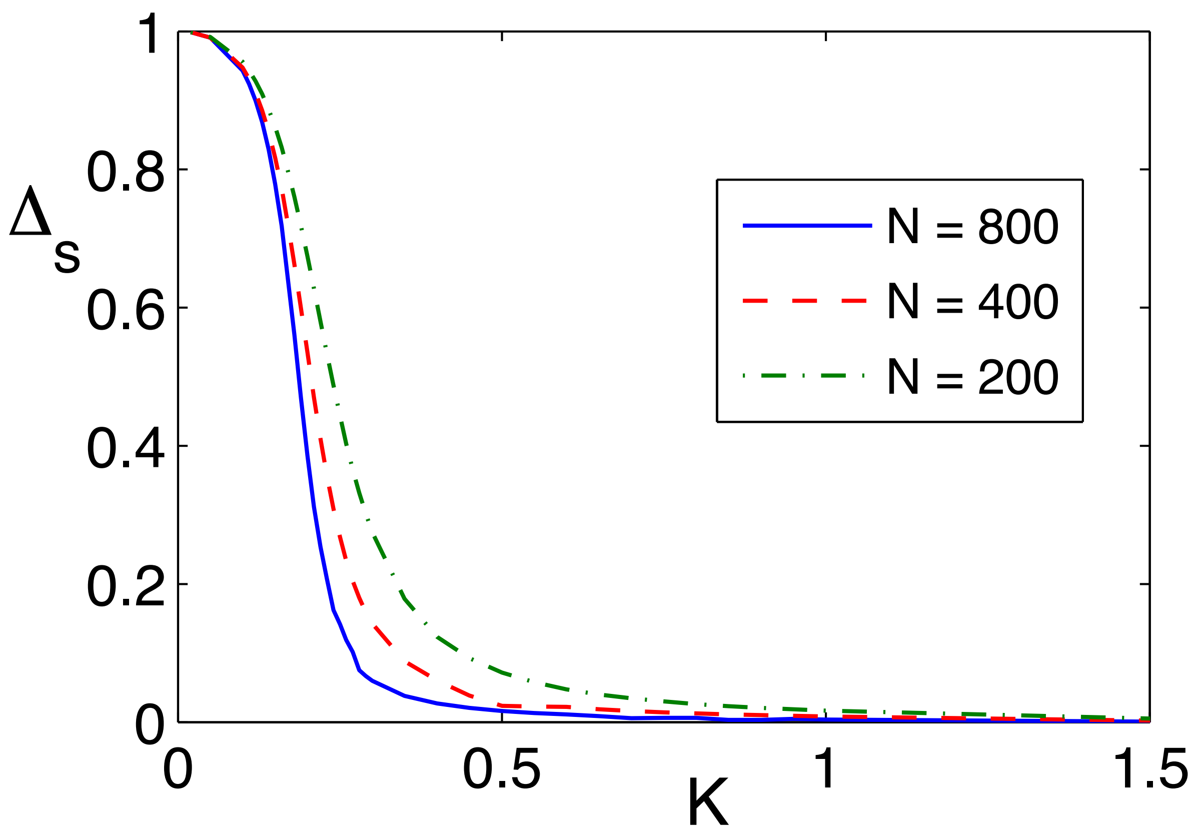


Order parameter at the quantum phase transition

Schmidt gap Δ_S



Order parameter at the quantum phase transition

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finite-size scaling

$$\Delta_S = N^{-\beta/\nu} f_{\Delta_S}(|K - K_c| N^{1/\nu})$$

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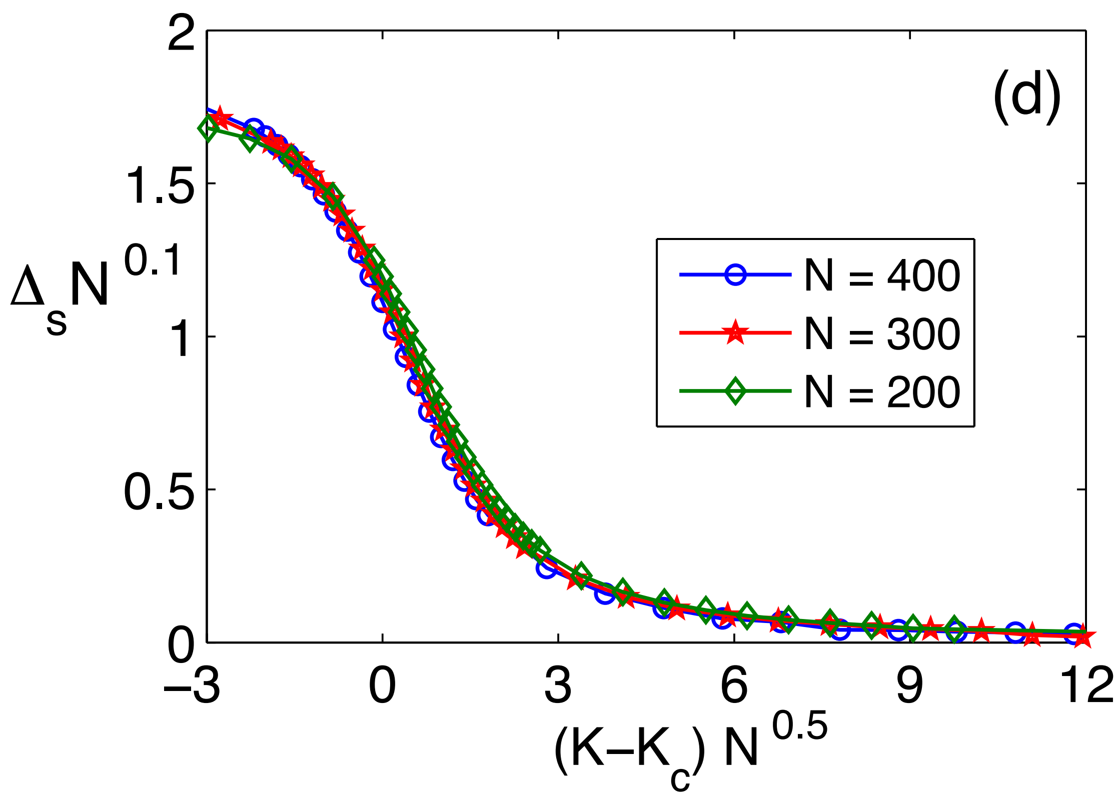

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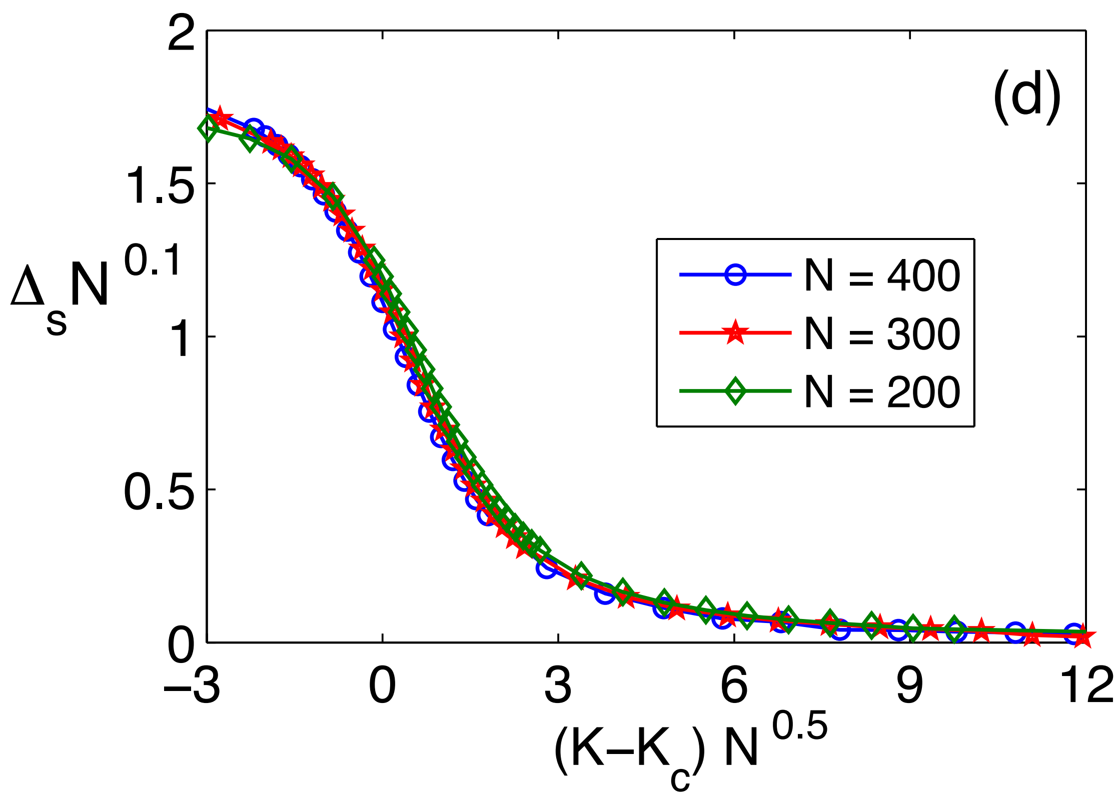

$$\beta = 0.2 \pm 0.05$$

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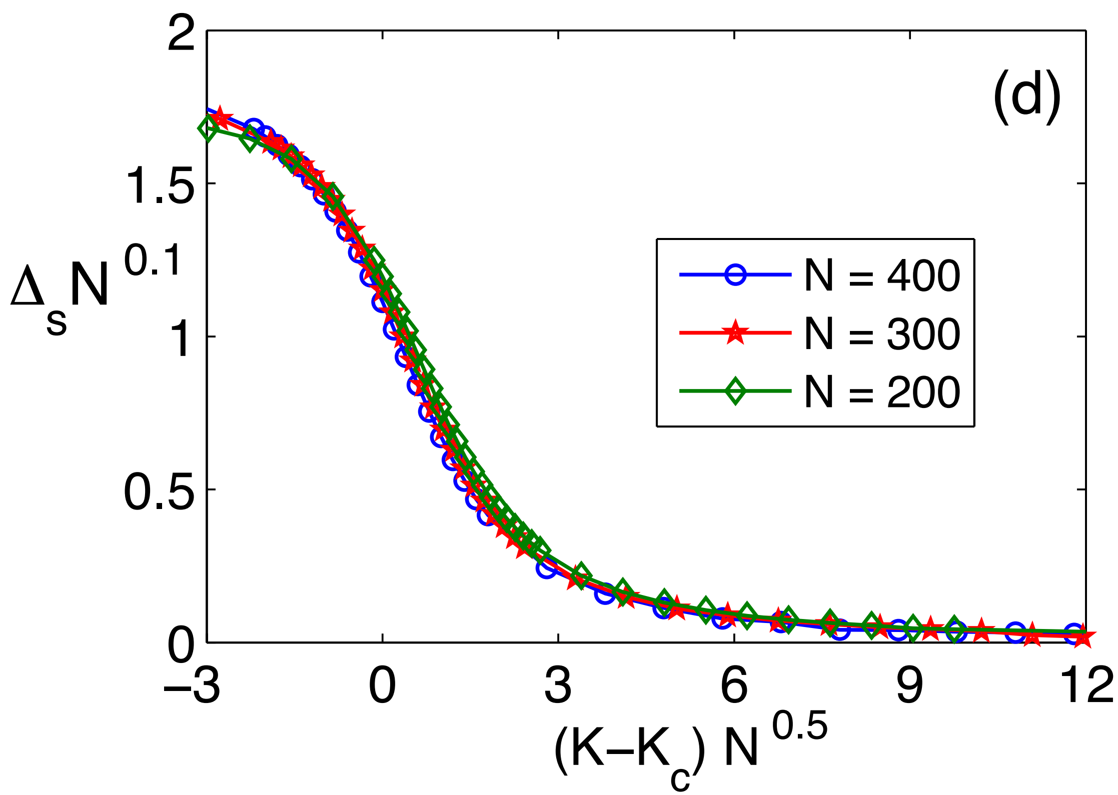

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I. Affleck *et al.*, PRB (1992)

M. Fabrizio *et al.*, PRL (1995)

Summary & outlook

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- Kondo cloud reconstruction for *any* (subcritical) RKKY and Kondo couplings
- Prediction of *Kondo resonance narrowing*
- Nonperturbative diagnostic of Kondo-RKKY quantum phase transition
- Schmidt gap as order parameter for an impurity quantum phase transition

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- Universal single-frequency oscillations in quantum impurity systems after a quantum quench
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- Entanglement structure of the two-channel Kondo model
  B. Alkurtass, A. Bayat, I. Affleck, S. Bose, H. J., P. Sodano, E. Sorensen, K. Le Hur, Phys. Rev. B 93, 081106(R) (2016)