Branestorming

On Various Aspects of p-branes

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Licentiate Thesis

Institute for Theoretical Physics
Chalmers University of Technology
and
Göteborg University

Göteborg 1999

Thesis for the degree of Licentiate of Enginering[1]

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Abstract

In recent years dramatic progress has been made in the understanding of the non-perturbative structure of superstring theory and M-theory. Central to this progress are non-perturbative, solitonic objects collectively referred to as p-branes. In this thesis, comprising an introductory text and three appended research papers, we are going to briefly review superstring theory and M-theory. Emphasis will be given to the dynamics of p-branes, which is the subject of Papers I-III.

In the past year enormous attention has been given to the so-called AdS/CFT correspondence, which is a kind of duality between superstring theory or M-theory on certain anti de Sitter spacetime backgrounds and gauge theory. For completeness this subject will also be briefly reviewed.

This thesis consists of an introductory text and the following three appended research papers, henceforth referred to as Papers I-III:

I. T. Adawi, M. Cederwall, U. Gran, M. Holm and B.E.W. Nilsson,
Superembeddings, non-linear supersymmetry and 5-branes,
Int. J. Mod. Phys. **A13** (1998) 4691-4715 (`hep-th/9711203`).

II. T. Adawi, M. Cederwall, U. Gran, B.E.W. Nilsson and B. Razaznejad,
Goldstone tensor modes,
JHEP **02** (1999) 001 (`hep-th/9811145`).

III. M. Cederwall, U. Gran, M. Holm and B.E.W. Nilsson,
Finite tensor deformations of supergravity solitons,
JHEP **02** (1999) 003 (`hep-th/9812144`).

Acknowledgments

First, I would like to thank my supervisor professor Bengt E.W. Nilsson for all his encouragement and support during my three years at the Institute and especially for always taking his time to answer my questions. I am also greatly indebted to Martin Cederwall for his support and for the vital contributions he has made to our joint publications. Furthermore, I would like to thank my other collaborators, Tom Adawi, Magnus Holm and Behrooz Razaznejad for a stimulating time together. In addition, I would like to thank all the members of the Institute for stimulating discussions and for creating a nice atmosphere. Last, but certainly not least, I would like to thank my fiancée Jenny Josefsson for her kind support and for helping me with the text.

Göteborg, February 1999
Ulf Gran

Contents

1

Introduction

String theory was originally formulated in the late 1960's as an attempt to explain the spectrum of hadrons and their interactions. It was however discarded as a theory of the strong interaction for two main reasons. Firstly, there exists a critical dimension, 26 for the bosonic string and 10 for the fermionic string, and our world has just four dimensions. Secondly, the spectrum contains a massless spin 2 particle not present in the hadronic world. These problems and the rapid success of QCD made people abandon string theory.

String theory was revived in 1974 when Scherk and Schwarz turned the existence of the massless spin 2 particle into an advantage by interpreting it as the graviton, the field quantum of gravitation. It was also discovered that at low energies string theory reduced to general relativity. String theory was in this way elevated to be a potential "theory of everything", i.e. a theory that unifies all four forces of nature.

The extension of the bosonic string to the fermionic string, thereby including fermions in the spectrum, was achieved by enforcing supersymmetry [1, 2]. This is a concept of great importance in high-energy physics. Supersymmetry can be described as an updating of special relativity to take into account that fermions exist. In the same sense supergravity can be described as an updating of general relativity.

The *first superstring revolution* (1984-85) consisted of three important discoveries. The first was an anomaly cancellation mechanism, which enabled the construction of consistent gauge theories in ten dimensions. The key was that the gauge group had to be SO(32) or $E_8 \times E_8$. The second discovery was two new superstring theories, the *heterotic* string theories, with exactly these gauge groups. But perhaps it was the third discovery that made people set their hopes on superstring theory. By compactifying the $E_8 \times E_8$ heterotic string theory on a particular Calabi-Yau manifold one obtained a 4d effective theory with many qualitatively realistic features. There are however a great variety of possible choices of this Calabi-Yau manifold and no one stands out as particularly special. After this revolution there were five distinct ten dimensional superstring theories with consistent weak coupling perturbation expansions and the understanding of these theories was developed in the ensuing years.

A great deal of effort was put into the investigation of the non-perturbative structure of superstring theory. This led to the discovery of various extended objects, collectively referred to as p-branes. The p-branes can be classified according to their world-volume field content. Papers I-III deals with these kinds of objects and especially with branes which have vector and tensor modes living on them. Superstring theory is thus nowadays quite a misnomer since it contains so much more that just strings.

What laid the foundation to the *second superstring revolution* (mid 1990s), which has to do with the non-perturbative structure of superstring theory, was the concept of duality. By duality we mean a way of relating different superstring theories, or different "regions" of a particular superstring theory. Using this relation we can do calculations in the theory where it is most conveniently done and then just translate the results to the other theory. One kind of duality, S-duality, relates weak- and strong-coupling regions. This is of great value since we can only do calculations in the weak-coupling regime. By using this duality we can obtain non-perturbative information which would be almost impossible to obtain by direct calculation. Since S-duality is a non-perturbative duality we need non-perturbative objects in order to check various conjectured S-dualities and here the above mentioned p-branes play a crucial rôle.

It was later discovered that all five superstring theories are related to each other via duality and are thus only facets of a (largely unknown) underlying fundamental theory. Sen has suggested that this fundamental theory should be called *U-theory* [3], where U can stand for either "unknown" or "unified". Sometimes this fundamental theory is designated M-theory but Sen proposes that this term should only be used in the sense Witten introduced it, i.e. for the 11-dimensional quantum theory which has 11-dimensional supergravity as its low-energy effective description. M-theory is also related to the superstring theories via duality and thus U-theory encompasses both superstring theory and M-theory.

In the past year enormous attention has been given to the so-called AdS/CFT correspondence, which is a kind of duality between superstring theory or M-theory on certain anti-de Sitter spacetime backgrounds and gauge theory. AdS space is analogous to a sphere with negative curvature. People hope that this will provide the theory of quark confinement, which would be one of the greatest achievements of superstring theory to this date. Actually the mass spectrum of a glueball, a bound state of gluons, has been calculated using this correspondence [4] and is in perfect agreement with lattice QCD calculations.

Superstring theory has also been used to calculate black hole entropy. Using D-branes (one kind of p-branes) Strominger and Vafa [5] were able to give a statistical mechanical derivation of the Bekenstein-Hawking entropy relation $S = A/4G\hbar$. Previously this relation was only understood from a thermodynamic perspective but now the picture was complete. There are however indications that this result is universal [6], i.e. that *any* quantum theory gives the standard result. It should be stressed that the quantitative results mentioned above consists of agreement with calculations made in more accepted theoretical models, like QCD, and does not consist of actual experiments.

This briefly recapitulates the major line of development in string theory since its birth and we will now examine some areas in more detail. In chapter 2 we will start by presenting bosonic string theory and then move on to its supersymmetric generalization, the fermionic string, and discuss its spectrum. We also discuss the relation between the low-energy limit of the superstring theories and supergravity. Chapter 3 deals with the concept of duality and describes how the five superstring theories and M-theory are related through the *web of dualities*. In chapter 4 we introduce the various branes and present a classification based on their world-volume field content. Chapter 5 finally gives a brief introduction to the AdS/CFT correspondence, focusing on the main ideas. For a more complete presentation of superstring theory and M-theory see e.g. Ref. [7] and for an introduction to the AdS/CFT correspondence see e.g. Ref. [8, 9].

Papers I-III are appended and since they are fairly self-contained, the introductory text should provide enough background to be able to understand them.

2

Perturbative string theory

Until the discovery of the various duality relations only perturbative aspects of string theory were accessible and the only known object in the theory was the string. This picture changed dramatically when multifarious higher-dimensional objects were discovered and found to be an integral part of string theory. Perturbative string theory is nevertheless still very important since it is within this part of the theory the calculations are most manageable.

In this chapter we will present the elementary concepts of bosonic string theory and superstring theory and describe how the latter is related to supergravity. For a more complete treatment of string theory see e.g. Ref. [10, 11, 12].

2.1 Bosonic strings

The fundamental idea behind string theory is actually very simple. Consider first an ordinary point particle. The action is in this case given by the length of the path which the particle traces out as it propagates, which is called the *world-line*. If we instead consider the propagation of a string, its orbit will be a two-dimensional tube instead of a line. In analogy with the point-particle case we take the action to be the area of this tube, the *world-sheet*. An action of this type was first written down by Nambu and Goto:

$$S_{NG}[X^\mu] = -T \int_\Sigma d^2\sigma \sqrt{-det(\partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu})}. \tag{2.1}$$

Here $T = (2\pi\alpha')^{-1}$ is the string tension, a constant of dimension $(\text{length})^{-2}$, and α' is known as the Regge slope. We can view this as an embedding of the world-sheet, Σ, in M, the target space. The target space is often, for simplicity, taken to be flat, D-dimensional Minkowski space, explaining the designation M. Nothing prevents us however from considering a general target space by just replacing $\eta_{\mu\nu}$ in (2.1) with $g_{\mu\nu}$. The fields X^μ, $\mu = 1, 2, \ldots, D$ represent the position of the string in target space. The world-sheet, Σ, is only a collection of parameters used to parameterize the string as it propagates and has no intrinsic geometry of its own. The world-sheet is made up by $\sigma^0 = \tau$, representing the time and $\sigma^1 = \sigma$, $0 \leq \sigma < \pi$, representing the angle around the string. The embedding X induces a metric, $(X^*\eta)_{ij} = \partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu}$, and as seen in (2.1) it is with this metric the area of the world-sheet is measured. If the world-sheet has boundaries, we have an open string, otherwise it is closed.

A problem with the Nambu-Goto action is that it can not be quantized preserving manifest Lorentz covariance due to the square root. A classically equivalent action without the square root can however be constructed using an auxiliary, intrinsic world-sheet metric γ_{ij}:

$$S_{BDH}[X^\mu, \gamma_{ij}] = -\frac{T}{2}\int_\Sigma d^2\sigma \sqrt{-\gamma}\gamma^{ij}\partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu}. \tag{2.2}$$

This action was first written down by Brink, Di Vecchia and Howe and by Deser and Zumino, but is actually most commonly known as the Polyakov action. The difference compared to the Nambu-Goto formulation is that the world-sheet is given an intrinsic geometry, given by γ_{ij}, but one also has an algebraic equation of motion for γ_{ij}. By using the solution to this equation the action (2.2) reduces to the action (2.1). The property of having an intrinsic world-sheet geometry will also be important when trying to quantize the string. In this way, the spacetime defines a two-dimensional field theory on the world-sheet.

The action (2.2) has three local invariances, two coming from the reparameterization invariance of the world-sheet and one from the Weyl invariance, $\gamma_{ij}(\sigma) \to e^{\Lambda(\sigma)}\gamma_{ij}(\sigma)$. In addition, we have rigid Poincaré invariance in target space, but from the world-sheet point of view this is an internal symmetry.

By using the reparameterization invariance we can locally go to the conformal gauge, $\gamma_{ij} = e^{\Lambda(\sigma)}\eta_{ij}$, where η=diag(-1,1),

$$S_{cg}[X^\mu] = -\frac{T}{2}\int_\Sigma d^2\sigma \eta^{ij}\partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu}. \tag{2.3}$$

The conformal factor cancels and we are left with a free, conformally invariant action. We thus have a conformal field theory [13] living on the world-sheet.

By varying the action we get the equation of motion, $\Box X = \partial_i\partial^i X^\mu = 0$. This is an ordinary wave equation and we can split the general solution into one left- and one right-moving part If these parts are related or not depends on the boundary conditions. An ordinary closed string has periodic boundary conditions, while an open string can have either Dirichlet or Neumann boundary conditions (or combinations of them). The Dirichlet condition was for many years considered un-physical since it breaks Poincaré invariance. This changed with the discovery of D-branes, objects on which open strings can end. This amounts to having just Dirichlet boundary conditions, therefore D-branes. We will discuss this type of brane in chapter 4. In the following we will concentrate on the closed string and show how quantization is achieved and how the spectrum is derived.

For the closed string, the general solution to the equation of motion is

$$X^\mu(z,\bar{z}) = q^\mu - \frac{i}{4}\alpha' p^\mu \ln(z\bar{z}) + i\sqrt{\frac{\alpha'}{2}}\sum_{n\neq 0}(\frac{\alpha_n^\mu}{n}z^{-n} + \frac{\tilde{\alpha}_n^\mu}{n}\bar{z}^{-n}), \tag{2.4}$$

where we have performed a Wick rotation, $\sigma^2 = i\sigma^0$, and introduced complex coordinates $z = e^{2(\sigma^2 + i\sigma^1)}$. This maps the, now euclidean, world-sheet to the compactified complex plane, which is topologically S^2.

Quantization is most easily achieved by quantizing the embedding fields X^μ canonically, which leads to the commutation relations

$$[q^\mu, p^\nu] = i\eta^{\mu\nu}, \qquad [\alpha_m^\mu, \alpha_n^\nu] = m\delta_{m+n,0}\eta^{\mu\nu}, \tag{2.5}$$

with an analogous expression for the $\tilde{\alpha}_m^\mu$ oscillators. We will from now on concentrate on the left-moving part described by the α_m^μ oscillators. Let us introduce a vacuum, $|0\rangle$, defined by

$$\langle 0|0\rangle = 1,\ \langle 0| = (|0\rangle)^\dagger,\ p^\mu|0\rangle = \alpha_m^\mu|0\rangle = 0, m > 0. \tag{2.6}$$

Eigenstates of p^μ can now be constructed, $|k\rangle = e^{ik\cdot q}|0\rangle$, and we have a set of Fock vacua. The full state space is generated by applying the creation operators $(\alpha_m^\mu)^\dagger = \alpha_{-m}^\mu, m > 0$, to the Fock vacua. The physical state space is however only a subspace of the full state space since we must take into account the constraints imposed by the equation of motion for γ_{ij} to which we will now turn.

Since the action (2.3) is free the non-trivial content of the theory is contained in the equation of motion for γ_{ij}, which is equivalent to the vanishing of the energy-momentum tensor

$$T_{ij} = -\frac{1}{T}\frac{1}{\sqrt{-\gamma}}\frac{\delta S_{BDH}}{\delta\gamma^{ij}}|_{\gamma=\eta}. \tag{2.7}$$

The energy-momentum tensor is defined to describe the response of the system to changes in the metric according to

$$\delta S = -T\int_\Sigma d^2\sigma\sqrt{-\gamma}T_{ij}\delta\gamma^{ij}. \tag{2.8}$$

In the complex basis we have

$$T_{zz}(z) = \frac{1}{2}\partial_z X_\mu \partial_z X^\mu, \tag{2.9}$$

$$\bar{T}_{\bar{z}\bar{z}}(\bar{z}) = \frac{1}{2}\partial_{\bar{z}} X_\mu \partial_{\bar{z}} X^\mu, \tag{2.10}$$

while $T_{z\bar{z}}$ and $T_{\bar{z}z}$ vanish identically due to the tracelessness of T_{ij}, which is a general feature of conformal field theories [13]. This can easily be understood by considering a Weyl invariant theory, like the one defined by the action (2.2), where only the metric transform under Weyl rescalings, $\delta\gamma_{ij} = \Lambda\gamma_{ij}$. We then have

$$0 = \delta S = -T\int_\Sigma d^2\sigma\sqrt{-\gamma}\Lambda(\sigma)T_{ij}\gamma^{ij} \tag{2.11}$$

and since $\Lambda(\sigma)$ is an arbitrary function it follows that T_{ij} must be traceless. We now make a Fourier expansion of the stress-energy tensor,

$$T(z) = T_{zz}(z) = \sum_{n\in\mathbb{Z}} L_n z^{-n-2}. \tag{2.12}$$

The Fourier coefficients satisfy the Virasoro algebra

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{c}{12}m(m^2-1)\delta_{m+n,0}, \tag{2.13}$$

where $c = D$ is the central charge. The equations of motion for the energy-momentum tensor, $T_{ij} = 0$, require it to vanish. In order to deduce what this implies for the L_m operators we write the condition as $\langle phys|T|phys'\rangle = 0$ which gives us the Virasoro constraints

$$(L_m - a\delta_{m,0})|phys\rangle = 0, \quad m \geq 0, \tag{2.14}$$

where the constant a is introduced due to the normal ordering ambiguity in L_0. Unitarity requires that $D = 26$ and $a = 1$. The fact that $L_0 - \bar{L}_0$ generates σ-translations implies the level-matching constraint

$$(L_0 - \bar{L}_0)|phys\rangle = 0 \tag{2.15}$$

since no point on the string is special, i.e. the string is invariant under σ-translations. Using (2.9) we can write the L_m operators as normal ordered expressions in the oscillators

$$L_m = \frac{1}{2}\sum_{n\in\mathbb{Z}} :\alpha_{m-n}\cdot\alpha_n:, \qquad \alpha_0^\mu = \sqrt{\alpha'/2}\,p^\mu. \tag{2.16}$$

Acting with L_0 on a physical state gives the mass-shell constraint

$$\alpha' M^2 = 4(N-a), \tag{2.17}$$

where N is the eigenvalue of the level operator $N = \sum_{m>0}\alpha_{-m}\cdot\alpha_m$ and $N = \bar{N}$ due to level-matching. As is immediately seen from (2.17) the ground state of the bosonic string is tachyonic, i.e. has negative mass squared. This implies that the vacuum is not stable and therefore the bosonic string is not consistent. This problem is taken care of by introducing supersymmetry and making a particular projection as will be seen in the next section.

The first excited level, $\xi_{\mu\nu}\alpha_{-1}^\mu\alpha_{-1}^\nu|k\rangle$, is the massless sector of the theory. Depending on the choice of polarization tensor we get a scalar ϕ, a symmetric traceless tensor $g_{\mu\nu}$ and an antisymmetric tensor $B_{\mu\nu}$. These are, respectively, the dilaton, the graviton and the abelian two-form gauge potential which are all SO(24) fields since this is the little group in the massless case.

One of the most important properties of bosonic string theory is that it reduces to general relativity in the low-energy limit. In order to see this we must consider non-trivial backgrounds for the string, which is described by the non-linear σ-model action

$$S = \frac{1}{4\pi\alpha'} \int d^2\sigma \Big(\sqrt{-\gamma}\gamma^{ij}\partial_i X^\mu \partial_j X^\nu g_{\mu\nu}(X) \tag{2.18}$$

$$+\epsilon^{ij}\partial_i X^\mu \partial_j X^\nu B_{\mu\nu}(X) - \alpha'\sqrt{-\gamma}R^{(2)}\phi(X)\Big), \tag{2.19}$$

where $R^{(2)}$ is the Ricci scalar for the world-sheet metric. From the world-sheet point of view the massless target space fields $g_{\mu\nu}$, $B_{\mu\nu}$ and ϕ are coupling constants, or rather coupling *functionals* since they depend on X. To get the Einstein equations we require Weyl invariance at the quantum level, i.e. that the β-functionals vanish. Perturbation theory gives to lowest non-trivial order in α' (corresponding to the low-energy limit)

$$\beta^{(g)}_{\mu\nu} = R_{\mu\nu} - \frac{1}{4}H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma} + 2D_\mu D_\nu\phi + \mathcal{O}(\alpha'), \tag{2.20}$$

$$\beta^{(B)}_{\mu\nu} = \frac{1}{2}D^\rho H_{\rho\mu\nu} - D^\rho\phi H_{\rho\mu\nu} + \mathcal{O}(\alpha'), \tag{2.21}$$

$$\beta^{(\phi)} = -R + \frac{1}{12}H^2 - 4D_\mu D^\mu\phi + 4(D_\mu\phi)^2 + \mathcal{O}(\alpha'). \tag{2.22}$$

Here $R_{\mu\nu}$ is the target space Ricci tensor and $H = dB$ is the three-form field strength. By requiring that all the β-functionals vanish we obtain the equations of motion for the massless background fields.

It is important to note that the picture generalized here is that of a classical point particle whose path is given by the shortest path in curved spacetime. The more fundamental description of this particle is by some kind of wave which should then be quantized in order to give a particle. This wave description would involve analogs to the Einstein, Maxwell or Yang-Mills equations and is certainly much closer to the fundamental concepts of physics. These equations are closely related to the principle of gauge invariance and we thus have no analogous understanding in the string picture. This is one of the fundamental problems in string theory.

2.2 Superstrings

There are three different formulations of the superstring, depending on where the supersymmetry is manifest. One can either have manifest world-sheet or target space supersymmetry or have manifest supersymmetry in both places, which is called a doubly supersymmetric formulation. In the end, one wants to have both world-sheet and target space supersymmetry but one has to derive the non-manifest supersymmetry. The Neveu-Schwarz-Ramond formulation, which in its modern form is covariant and has manifest world-sheet supersymmetry, is the formulation we will use in order to discuss the spectrum of the open fermionic string. In the Green-Schwarz formulation one embeds the bosonic world-sheet of the string into a target superspace and thus has manifest target space supersymmetry. World-sheet supersymmetry then arises as a consequence of κ-symmetry. A recently developed approach is the "doubly supersymmetric geometrical approach" [14, 15, 16], or the "embedding formalism" [17], which is described in detail in Paper I. In this approach one has manifest supersymmetry both on the world-sheet and in target space. This is accomplished by embedding a supermanifold, in the string case the super-world-sheet, into the supermanifold which constitutes the target space. In this sense it can be viewed as an extension of the Green-Schwarz formalism. In order to get the dynamics we have to impose an embedding condition and in some cases supplementary conditions.

The action in the NSR formulation is

$$S = \frac{1}{4\pi}\int d^2z\left(\partial X^\mu \bar{\partial} X_\mu - \psi^\mu \bar{\partial}\psi_\mu - \tilde{\psi}^\mu \partial \tilde{\psi}_\mu\right), \tag{2.23}$$

which leads to the equations of motion $\bar{\partial}\psi^\mu = 0 = \partial\tilde{\psi}^\mu$. The two spinor components ψ^μ and $\tilde{\psi}^\mu$ form a set of world-sheet Majorana spinors

$$\begin{pmatrix} \psi^\mu \\ \tilde{\psi}^\mu \end{pmatrix}$$

transforming as a spacetime vector. The Majorana condition ensures that we have the same number of bosonic and fermionic degrees of freedom on shell. The action (2.23) is obtained by gauge-fixing a supersymmetric analogue of (2.2).

We will now analyze the spectrum of the open fermionic string, which can be used to derive the spectra of the five superstring theories. Note first that the classical action (2.23) does not contain any spacetime spinors. They will arise as a consequence of quantization and the origin of the phenomena has to do with the boundary conditions of the world-sheet spinors. The vanishing of the surface term when varying the action (2.23) is satisfied by two different Poincaré invariant boundary conditions:

$$\begin{array}{lll} \psi^\mu(0,\sigma^2) = \tilde{\psi}^\mu(0,\sigma^2), & \psi^\mu(2\pi,\sigma^2) = -\tilde{\psi}^\mu(2\pi,\sigma^2) & \text{(NS)} \\ \psi^\mu(0,\sigma^2) = \tilde{\psi}^\mu(0,\sigma^2), & \psi^\mu(2\pi,\sigma^2) = \tilde{\psi}^\mu(2\pi,\sigma^2) & \text{(R)} \end{array} \tag{2.24}$$

With respect to these conditions, the solutions to the equations of motion are

$$\begin{array}{l} \psi^\mu(\sigma^1,\sigma^2) = \sum_{r\in\mathbb{Z}+1/2}\psi^\mu_r e^{-r(\sigma^2+i\sigma^1)} \\ \tilde{\psi}^\mu(\sigma^1,\sigma^2) = \sum_{r\in\mathbb{Z}+1/2}\psi^\mu_r e^{-r(\sigma^2-i\sigma^1)} \end{array} \quad \text{(NS)} \tag{2.25}$$

for the NS sector and

$$\begin{array}{l} \psi^\mu(\sigma^1,\sigma^2) = \sum_{n\in\mathbb{Z}}\psi^\mu_n e^{-n(\sigma^2+i\sigma^1)} \\ \tilde{\psi}^\mu(\sigma^1,\sigma^2) = \sum_{n\in\mathbb{Z}}\psi^\mu_n e^{-n(\sigma^2-i\sigma^1)} \end{array} \quad \text{(R)} \tag{2.26}$$

for the R sector. Proceeding with the quantization as in the bosonic case gives the commutation relation

$$\{\psi^\mu_m, \psi^\nu_n\} = \eta^{\mu\nu}\delta_{m+n}, \tag{2.27}$$

which is valid both for integer and half-integer values of m and n. The states in the NS sector are now generated from the Fock vacua by applying the negative modes α^μ_{-m} and ψ^μ_{-r}. The Virasoro constraint corresponding to the mass-shell constrain is

$$(L_0^{\text{(NS)}} - \frac{1}{2})|phys\rangle = 0 \tag{2.28}$$

where

$$L_0^{(\mathrm{NS})} = \frac{1}{2}\sum_{n\in\mathbb{Z}} :\alpha_{-n}\cdot\alpha_n: + \frac{1}{2}\sum_{r\in\mathbb{Z}+1/2} r:\psi_{-r}\cdot\psi_r: \tag{2.29}$$

and $\alpha_0^\mu = \sqrt{2\alpha'}p^\mu$ for the open string. The mass spectrum for the NS sector is thus

$$\alpha' M_{\mathrm{NS}}^2 = N_\alpha + N_\psi - \frac{1}{2}. \tag{2.30}$$

The ground state is thus unique which implies that it is a spin 0 state.

The R sector is a bit more complicated due to the fermionic zero-modes ψ_0^μ, which commutes with the mass operator. This implies that $|0\rangle$ and $\psi_0^\mu|0\rangle$ are degenerate in mass. Since the ψ_0^μ are the generators of a Clifford algebra (cf. Eq. (2.27)) we conclude that the R ground state is a SO(9,1) spinor. Since we have Majorana-Weyl spinors in ten dimensions we are free to choose the chirality of the vacuum. The oscillators are spacetime vectors, and can not change bosons into fermions or vice versa, thus all states in the R sector will be fermionic and all states in the NS sector will be bosonic. In this way the emergence of spacetime fermions is due to the zero-modes ψ_0^μ.

In addition to the usual Virasoro and level-matching constraints the physical states in the R and NS sectors must satisfy

$$G_r|phys\rangle = 0; \quad r \geq 0 \tag{2.31}$$

where

$$G_r = \sum_{n\in\mathbb{Z}} \alpha_{-n}\cdot\psi_{r+n}. \tag{2.32}$$

The G_r operators are the Fourier components of the supercurrent and the constraints comes from the vanishing of the supercurrent in the same way as the Virasoro constraints come from the vanishing of the stress-energy tensor. The G_0 constraint of (2.31) actually contains the mass-shell constraint, due to the super-Virasoro algebra, and we get the mass spectrum

$$\alpha' M_{(\mathrm{R})}^2 = N_\alpha + N_\psi. \tag{2.33}$$

We still however have tachyons in the NS sector. These can be removed by invoking the GSO projection, which is also crucial in order to have modular invariance. The projection consists of removing states with odd world-sheet fermion number after which we finally arrive at the physical spectrum.

A closed string can now be considered as built up by two open strings and therefore its spectrum can be written as the direct product of two open string spectra. In this way we can obtain the massless spectra for the five superstring theories, see e.g. Ref. [10]. The massless spectra is then seen to correspond to supergravity theories and this is the reason why we say that superstring theory reduces to supergravity in the low-energy limit.

3

Duality

The discovery of various duality relations led to a dramatic advance in the understanding of the non-perturbative structure of string theory. Each string theory corresponds to a point in the moduli space of vacua, i.e. a particular choice of vacuum. The dualities take us between different points in this moduli space, relating all the string theories. This indicates the existence of one all-embracing theory, U-theory. The big question is whether there exists a deeper formulation of U-theory or if the best definition we can get is in terms of perturbation expansions and various non-perturbative dualities. As Vafa pointed out [18] this latter alternative is much like how one defines a manifold in terms of charts, being the perturbative string theories, and transition functions, being the dualities. Here we will briefly describe how different string theories and M-theory are related through the basic T- and S-dualities. In this way we will obtain the *web of dualities* in which all string theories and M-theory are related.

3.1 T-duality

T-duality is an equivalence between two weakly coupled string theories compactified on manifolds of different volume. A generic feature is that when one volume is large the other one is small and vice versa. More concretely, we have the basic relation $R' = \alpha'/R$, where R and R' are the compactification radii. This relation implies the existence of a smallest scale in string theory since we can always go from the radius which is smaller than the self-dual radius, $R_{sd} = \sqrt{\alpha'}$, to a radius larger than R_{sd} by using the duality. Technically, what happens in this transformation is that the Kaluza-Klein modes and the winding modes get exchanged while the spectrum is unaltered. It is as if an extra term is present in the Heisenberg uncertainty relation[1]

$$\Delta x \geq \frac{1}{2}\frac{\hbar}{\Delta p} + \frac{1}{2}\frac{\Delta p}{\hbar}\alpha' \tag{3.1}$$

[1] See e.g. Ref. [19] and references therein.

giving a minimum length of order[2] $\sqrt{\alpha'} \approx 10^{-33}$cm. Note that the construction of this extra term is possible due to the presence of α'. Since T-duality involves a compactification it will be a duality in nine dimensions[3]. There are two examples, type II and heterotic duality, to which we will now turn.

By compactifying type IIA and type IIB theory on circles with different radii we can identify a T-duality map between the two theories. More generally, dualizing in an odd number of coordinates relates IIA to IIB and dualizing in an even number of coordinates relates IIA to IIA or IIB to IIB. There are strong reasons to believe that this perturbative equivalence extends to the non-perturbative level.

The two heterotic theories are also T-dual when compactified as above. This can be understood by noting that the existence of the two heterotic theories in ten dimensions is due to the existence of two 16-dimensional euclidean self-dual even lattices. By further compactification on T^d, the lattice must now be lorentzian (still being even and self-dual). It is however known that there for each $d \geq 1$ exists a unique lattice with these properties and therefore this duality should come as no surprise.

3.2 S-duality

S-duality is a strong-weak coupling duality and therefore of great interest since it enables us to obtain information about the non-perturbative structure of string theory. This however also means that conjectured S-dualities can not be investigated using perturbation theory, as in the case of T-dualities. Instead, the p-branes will now play a central rôle, especially those whose properties remain unaltered when going to the strong coupling region, the so called BPS branes. In this way we can devise some non-trivial tests which the conjectured S-dualities must pass. We will now turn to four important S-dualities.

To start with we will consider the duality between M-theory on $\mathbb{R}^{10} \times S^1$ and type IIA string theory. The first hint to this duality is obtained by compactifying the $D = 11$ supergravity action, i.e. the low-energy limit of M-theory, on a circle [20]. This compactification does not break any supersymmetry and we can hope to obtain one of the two $N = 2$ type II theories. It turns out that we obtain the type IIA supergravity action, i.e. the low-energy limit of type IIA string theory. This is reasonable since this theory is non-chiral. We can identify a relation between the compactification radius and the coupling constant in type IIA theory,

$$R \sim \lambda^{2/3}, \tag{3.2}$$

saying that strong coupling corresponds to large compactification radius. In type IIA theory the strong coupling limit thus effectively corresponds to a decompactification and we obtain an eleven dimensional theory. When doing perturbation theory in

[2] There are two different conventions for calculating the Plank length. By using $\hbar$ we get $\ell_P = 10^{-33}$cm and by using h we get $\ell_P = 10^{-32}$cm.

[3] By further compactification we can of course obtain T-dualities in less than nine dimensions but the primary ones, which we will examine, will be in nine dimensions.

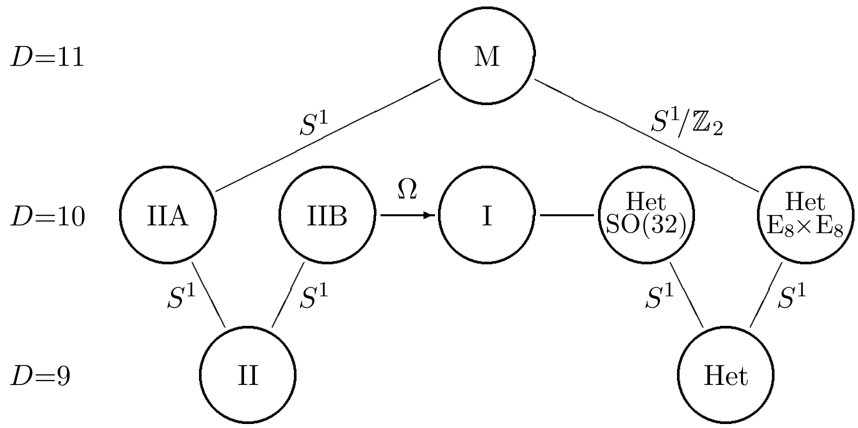

Figure 3.1: The web of dualities.

the type IIA theory, i.e. expanding around $\lambda = 0$, this extra dimension is invisible explaining why it passed unnoticed for so long. So far we have only made the duality plausible by low-energy analysis and in order to check if the duality also extends to the non-perturbative level we need to examine the spectra of the two theories. In this analysis the BPS branes play a central rôle and there are now much non-perturbative evidence for this duality.

If we instead compactify M-theory on $S^1/\mathbb{Z}_2$ we get a theory dual to the $E_8 \times E_8$ heterotic string theory. The orbifolding breaks half the supersymmetry so we must obtain a $N = 1$ theory. The orbifold $S^1/\mathbb{Z}_2$ is effectively an interval and one E_8 factor is associated with each endpoint. These endpoints are ten dimensional hyperplanes, or "end-of-the-world 9-branes" as they are metaphorically called, and are separated a distance determined by the coupling constant.

Type IIB string theory is conjectured to be invariant under SL(2;$\mathbb{Z}$) transformations which in particular transforms the dilaton as $\phi \to -\phi$. Since the string coupling constant is given by $g_s = \langle e^{\phi} \rangle$ we have that type IIB theory is S-selfdual. Further analysis of the spectrum strengthen this conjecture.

The last duality we are going to mention is that between type I and SO(32) heterotic string theory. Both theories have SO(32) as gauge group and this indicates that the two theories may be related by duality. By comparing the low-energy limits of the two theories one sees that they can be mapped into each other and particularly that the mapping for the dilaton is $\phi_I = -\phi_{het}$. Based on this Witten [21] conjectured that these theories are related by S-duality. Since then further evidence has strengthened this conjecture.

3.3 The web of dualities

Collecting the results above gives us the *web of dualities*, as illustrated in Fig. 3.1. The fact that the five string theories and M-theory are related in this way indicates that there is really only one theory, U-theory.

In Fig. 3.1 we have also included the orientifold projection by which Type I theory can be obtained from Type IIB theory. This projection, $P_+^{(\Omega)} = \frac{1}{2}(\mathbb{1} + \Omega)$, is constructed from the orientifold operation Ω which reverses the roles of the left- and right-moving sectors. The resulting theory is left-right symmetric and therefore unoriented. Thus, by keeping only the left-right symmetric states of Type IIB theory we end up with Type I theory.

4

p-branes

The p-branes are solitonic solutions to the low-energy effective supergravity theories. An important property is that they interpolate between different vacua. This means that they are topological in nature and therefore their stability is guaranteed. Since they are topological objects they are not included in the perturbative spectrum and are thus intrinsically non-perturbative. By saturating a Bogomol'nyi bound we get BPS states belonging to short supersymmetry multiplets implying that they preserve some fraction of the supersymmetry and are stable to quantum corrections in the strong coupling limit. This feature makes BPS p-branes the best candidate to use in exploring the non-perturbative structure of string theory and they play a central rôle in verifying the duality conjectures in the previous chapter.

In this chapter we are first going to derive the *brane-scan* in Fig. 4.1, which shows the p-branes allowed by supersymmetry. We are then going to review some of the salient features of p-branes and explain how the M2 and M5 brane in $D = 11$ are related to various branes in type IIA string theory. Finally, we are going say a few words about D-branes, e.g. explain how they arise when T-dualizing an open string and why they are dynamical objects.

4.1 The brane-scan

Unlike bosonic p-branes, which can be formulated in arbitrary spacetime dimension D, supersymmetric p-branes can only be formulated for certain combinations of $d = p + 1$ and D. This restriction, enforced by supersymmetry, gives rise to the brane-scan in Fig. 4.1. It is important to note that the brane-scan only tells us which branes are *not forbidden* by supersymmetry. If these branes actually *exist* as solutions to any supersymmetric field theory is another question.

Let us now derive the brane-scan. We start by considering the scalar world volume multiplets. This analysis can be done using two different methods. The first method is to list all scalar supermultiplets and interpret the space-time dimension as

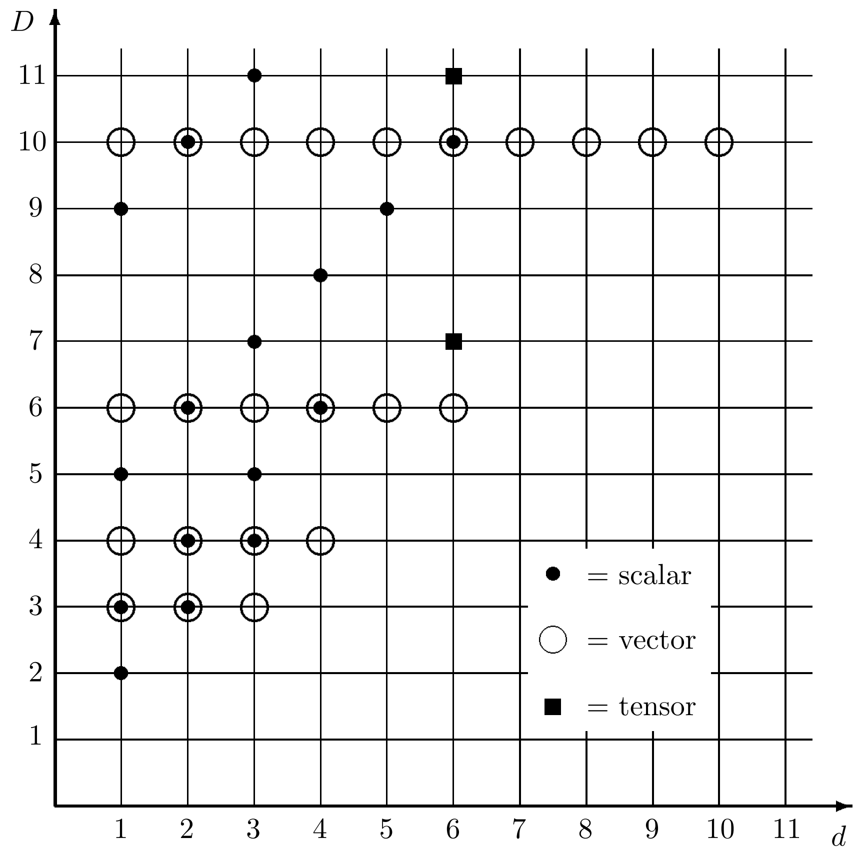

Figure 4.1: The brane-scan.

$D = d +$ the number of scalars. The second method, which we will use, is to require world-volume supersymmetry by matching the numbers of bosonic and fermionic on-shell degrees of freedom in the superspace embeddings $X^a(\xi)$ and $\theta^\alpha(\xi)$. The bosonic degrees of freedom are

$$N_B = D - d, \tag{4.1}$$

where we have taken into account the reparameterization invariance of the world-volume. The bosonic degrees of freedom correspond in this case to the directions transverse to the brane. Motion in these directions will give rise to the simplest example of Goldstone modes, i.e. *scalar Goldstone modes*. The concept of Goldstone modes is analyzed in detail in Paper II and extended to the case of Goldstone *tensor* modes of arbitrary rank.

By taking into account that kappa symmetry halves the fermionic degrees of freedom and going on-shell halves them again, we obtain

$$N_F = \frac{1}{2}mn = \frac{1}{4}MN, \tag{4.2}$$

Dimension	Spinor	Type of spinor	Number of susy
11	32	Majorana	1
10	16	Maj & Weyl	1,2
9	16	Majorana	1,2
8	16	Weyl	1,2
7	16	Dirac	1,2
6	8	Weyl	1,...,4
5	8	Dirac	1,...,4
4	4	Maj or Weyl	1,...,8
3	2	Majorana	1,...,16
2	1	Maj & Weyl	1,...,32

Table 4.1: Irreducible spinor representations in various dimensions.

where m (M) is the number of real components of an irreducible spinor in d (D) dimensions and n (N) is the numbers of supersymmetries. By matching bosonic and fermionic degrees of freedom we get

$$D - d = \frac{1}{2}mn = \frac{1}{4}MN, \tag{4.3}$$

which must be fulfilled in order to allow the existence of a scalar multiplet (for $d > 2$). By consulting Tab. 4.1 we find that Eq. (4.3) has eight solutions, represented by the dots in Fig. 4.1. The case $d = 2$, i.e. the string, is special since the left- and right-handed modes can be treated independently. By having fermions in both sectors, i.e. having a type II theory, we get the same condition as in Eq. (4.3) resulting in strings in $D = 3$, 4, 6 and 10 with $N = 2$. By having fermions in only one sector, i.e. having a heterotic theory, we get the condition

$$D - 2 = n = \frac{1}{2}MN \tag{4.4}$$

resulting in strings in $D = 3$, 4, 6 and 10 with $N = 1$. For completeness we have also included the superparticles ($p = 0$) in $D = 2$, 3, 5 and 9.

We must also consider higher spin multiplets, a possibility that was originally overlooked. In the case of a vector multiplet we get $d-2$ additional bosonic degrees of freedom from the vector gauge field, giving

$$D - 2 = \frac{1}{4}MN, \tag{4.5}$$

which can be satisfied in $D = 3$, 4, 6 and 10 for arbitrary d, giving the circles in Fig. 4.1. The branes with vector multiplets living on them are called D-branes.

Finally, branes with tensor multiplets are allowed in $D = 7$ and $D = 11$. The first case is considered in Paper I and the second in Paper II and III.

4.2 p-branes

We are now going to review some of the general properties of p-branes. We can describe bosonic p-branes by a generalization of the Nambu-Goto action (2.1)

$$S_{DNG}[X^\mu] = -T_p \int d^{p+1}\xi \sqrt{-\det(\partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu})}, \tag{4.6}$$

which is called the Dirac-Nambu-Goto action. Since the p-branes we are interested in are charged with respect to the gauge fields in the low-energy supergravity theories we must add a Wess-Zumino term[1]

$$S_{WZ} = T_p \int X^* A_{(d)}, \tag{4.7}$$

where $A_{(d)}$ is a d-form gauge potential which is pulled back to the world-volume. This generalizes the coupling of $B_{\mu\nu}$ to the string world-sheet in Eq. (2.19). The d-form gauge potential $A_{(d)}$ couples naturally to a p-brane, where $d = p+1$. Since the p-brane can be surrounded by a space-like surface S^{D-d-1} we can define an "electric" charge

$$q_e = \int_{S^{D-d-1}} *F_{(d+1)}, \tag{4.8}$$

where $*F_{(d+1)}$ is the Hodge dual of the field strength $F_{(d+1)} = dA_{(d)}$. This is a direct generalization of Gauss law. By using the field equations for F it follows that the electric charge is conserved. We can also define a "magnetic" charge by

$$q_m = \int_{S^{d+1}} F_{(d+1)}, \tag{4.9}$$

which is a topological charge, i.e. it is conserved by virtue of the Bianchi identity for F. The electric and magnetic charges defined above now satisfy the generalized Dirac quantization rule [22, 23]

$$q_e q_m = 2\pi n,\ n \in \mathbb{Z}. \tag{4.10}$$

Since the potential corresponding to the dual field strength is a $(D-d-2)$-form it couples naturally to a $(D-d-3)$-brane. We thus have an electric-magnetic dual pair consisting of a p-brane and a $\tilde{p}$-brane satisfying $p + \tilde{p} = D - 4$.

Since the branes carrying electric charge have an associated source term (the Wess-Zumino term) they are called *fundamental* or *elementary*. The branes carrying magnetic charge have no associated source term and are just solutions to the effective supergravity theory. They are therefore called *solitonic*. Note that the designation electric/magnetic reflects our choice of basic fields in the effective supergravity theory which at the string level corresponds to a particular choice of vacuum [24]. It is now

[1] The Wess-Zumino term turns out to be required in order to cancel the κ-variation of the Dirac-Nambu-Goto term.

M-theory			M2			M5	
Type IIA	D0	F1	D2		D4	S5	D6
Type IIB		F1+D1		$D3^+$		S5+D5	
Type I		D1				D5	
Heterotic		F1				S5	

Table 4.2: The U-theory brane-scan.

interesting to note how the brane tensions depend on the string coupling constant $g_s = \langle e^\phi \rangle$. For the fundamental p-branes the tension do not depend on the string coupling constant and we have $T_{Fp} \sim (m_s)^{p+1}$, where we have used the *string mass* $m_s = 1/\sqrt{\alpha'}$ to get the correct dimensions. Such p-branes only occur for $p = 1$ (cf. Tab. 4.2) and are thus fundamental strings. For the solitonic p-branes we have $T_{Sp} \sim (m_s)^{p+1}/g_s^2$ which indicates that they will dominate the dynamics at strong coupling. Finally, for the D-branes, which lies outside the electric-magnetic considerations above, we have $T_{Dp} \sim (m_s)^{p+1}/g_s$ which is an intermediate behavior compared to the fundamental and solitonic cases.

If we now consider eleven dimensional supergravity we have a 3-form potential coupling to an electric M2 brane and giving rise to a dual magnetic M5 brane. Since M-theory compactified on a circle is conjectured to be equivalent to the strong coupling limit of type IIA string theory it should be possible to obtain the branes of type IIA theory by suitable wrappings of the M2 and M5 branes around the compact direction. The branes we are considering are, among others, collected in Tab. 4.2. We start by noting that in type IIA theory there are two parameters, the string coupling constant g_s and the string mass m_s. In M-theory there are only one parameter, the Plank mass m_p, but since we consider compactified M-theory we also have the radius R of the compact direction. The M2 brane tension is $T_{M2} = m_p^3$ and by wrapping it around the compact direction we want to get the F1 string

$$T_{F1} = m_s^2 = RT_{M2} = Rm_p^3. \tag{4.11}$$

This implies the identification $m_s^2 = Rm_p^3$. By instead wrapping one of the transverse directions we want to obtain the D2 brane

$$T_{D2} = \frac{m_s^3}{g_s} = T_{M2} = m_p^3 \tag{4.12}$$

implying the identification $g_s = Rm_s$. Let us now check if these identifications work for the M5 brane. By wrapping it around the compact direction we want to get the D4 brane

$$RT_{M5} = Rm_p^6 = \frac{m_s^5}{g_s} = T_{D4}. \tag{4.13}$$

By wrapping a transverse direction we get

$$T_{M5} = m_p^6 = \frac{m_s^6}{g_s^2} = T_{S5} \tag{4.14}$$

and we see that we get exactly what we wanted. By this simple procedure we have related the M2 and M5 branes to various branes in type IIA theory.

We end this section by explaining how the supersymmetry algebra can be used to deduce which kinds of branes that exist in a given dimension. Consider $D = 11$ where we have the *M-theory algebra*

$$\{Q_\alpha, Q_\beta\} = (C\Gamma^M)_{\alpha\beta} P_M + \frac{1}{2!}(C\Gamma^{MN})_{\alpha\beta} Z_{MN} + \frac{1}{5!}(C\Gamma^{MNPQR})_{\alpha\beta} Z_{MNPQR}, \tag{4.15}$$

where the supercharge Q_α is a 32-component Majorana spinor. By counting the degrees of freedom in the RHS we get

$$11 + 55 + 462 = 528 \tag{4.16}$$

which equals the degrees of freedom in the LHS. It turns out that the spatial components of Z_{MN} correspond to the electric charge while the spatial components of Z_{MNPQR} correspond to the magnetic charge. We can thus deduce that there must exist M2 and M5 branes by just looking at the supersymmetry algebra.

4.3 Dp-branes

The p-branes which have vector multiplets living on their world-volumes are called Dp-branes, or simply D-branes. They are dynamical objects on which open strings can end. This implies that the D-branes have an exact description at the string level, i.e. an exact description in term of a conformal field theory, in contrast to the other types of branes. The low-energy dynamics are given by the Dirac-Born-Infeld action

$$S_{DBI}[X^\mu, A_i] = -T_p \int d^{p+1}\xi e^{-\phi}\sqrt{-\det(g + \mathcal{F})}, \tag{4.17}$$

where T_p is the world-volume tension and $\mathcal{F}_{ij} = 2\pi\alpha' F_{ij} - B_{ij}$ where $F_{ij} = \partial_i A_j - \partial_j A_i$ is the field strength of A_i. The target-space fields ϕ, g and B are understood to be pulled back to the world-volume by the embedding X. These fields represent the closed string background in which the D-brane is embedded. The scaling of the tension with the dilaton as described in the previous section can directly be seen to agree with that given by the action (4.17). The reason for the dilaton dependence $e^{-\phi} = g_s^{-1}$ is that (4.17) is an open string tree level action and we thus get a factor $e^{-\phi}$ from the disc. From the brane-scan, D-branes are seen to be allowed in

$D = 3, 4, 6$ and 10. In type II string theory no states in the perturbative spectrum are charged under the RR gauge fields. This is because only the gauge-invariant field strengths appear in the vertex operators creating the RR vacuum out of the NS-NS ground state. The D-branes now restore the balance since they are RR-charged objects which in addition satisfy charge quantization conditions of the form $q_e q_m = 2\pi n$.

Let us now go back and explain how D-branes can be seen to arise when applying T-duality to open strings. Consider an open bosonic string and take one of the target space directions to be compact, i.e. $x^{25} \sim x^{25} + 2\pi R$. The corresponding component of the embedding field decomposes as $X^{25}(\sigma^1, \sigma^2) = X^{25}(z) + \tilde{X}^{25}(\bar{z})$, where

$$X^{25}(z) = \frac{q^{25}}{2} + \frac{c^{25}}{2} - i\alpha' p^{25} \ln z + i\sqrt{\frac{\alpha'}{2}} \sum_{m \neq 0} \frac{z^{-m}}{m} \alpha^{25}_m, \tag{4.18}$$

$$\tilde{X}^{25}(\bar{z}) = \frac{q^{25}}{2} - \frac{c^{25}}{2} - i\alpha' p^{25} \ln \bar{z} + i\sqrt{\frac{\alpha'}{2}} \sum_{m \neq 0} \frac{\bar{z}^{-m}}{m} \alpha^{25}_m. \tag{4.19}$$

We have here added and subtracted the constant $c^{25}/2$ and used that $z = e^{\sigma^2 + i\sigma^1}$. By now looking at the T-dual field

$$X'^{25}(z, \bar{z}) = X^{25}(z) - \tilde{X}^{25}(\bar{z}) = c^{25} + 2\alpha' p^{25} \sigma^1 + \text{osc.} \tag{4.20}$$

and noting that the oscillator terms vanish at the endpoints $\sigma^1 = 0, 2\pi$ we see that the endpoints are fixed at the hyperplane $x^{25} = c^{25}$

$$X'^{25}(\sigma^1 = 0) = c^{25}, \qquad X'^{25}(\sigma^1 = 2\pi) = c^{25} + 2\pi n R' \sim c^{25}. \tag{4.21}$$

We have here used that the momentum is quantized in the compact direction, $p^{25} = n/R$, and that $R' = \alpha'/R$. We also note that the string is wound n times around the compact direction. The Neumann boundary condition has thus been converted into a Dirichlet boundary condition by T-duality.

To see why the D-branes are dynamical objects and why there is a vector field living on them we look at the massless states of the T-dualized open string. The massless states correspond to non-winding states at oscillator level $N = 1$. In the original open string theory this corresponds to the massless U(1) gauge field $\alpha^\mu_{-1}|k\rangle$. In the T-dual theory (where we now have dualized $25 - p$ directions in order to get a Dp-brane) this field decomposes into a longitudinal part, giving a U(1) gauge field A_i on the world-volume, and a transverse part, giving $25 - p$ scalars ϕ^m representing the transverse oscillations of the world-volume.

5

The AdS/CFT correspondence

In November 1997 Juan Maldacena published a paper [25] which was to be the main influence on the string community in the year to come. Only one year after appearing on the net his paper had received around 500 citations.

What Maldacena did was to identify a duality between string theory and gauge theory. It is important to note that string theory inherently includes gravity which gauge theory does not. The person first to propose the existence of such a duality was 't Hooft in 1974 [26]. He was trying to expand the equations for QCD in the variable $1/N$, where N is the number of colours, taking N to be large. This idea of looking at the large N limit will be central in the AdS/CFT correspondence as described below. However 't Hooft's approach fell short of solving the problems of interest in QCD, but he proposed that one should be able to find a string theory describing QCD where $1/N$ played the role of some coupling constant. Despite the considerable interest aroused by this proposal no one was able to find the anticipated string theory until now.

More precisely, the proposed duality is between type IIB string theory on $AdS_5 \times S^5$ and a conformal field theory on the 4-dimensional Minkowski space which is the boundary of AdS_5. The important property is that weakly coupled string theory is dual to strongly coupled gauge theory, where calculations are intractable. As described in chapter 2, the low-energy limit of weakly coupled string theory is given by supergravity. Thus, to lowest order we have a correspondence between supergravity and gauge theory.

Since Maldacena's first paper, a more precise version of the correspondence has been developed by Gubser, Klebanov and Polyakov and independently by Witten, which made it possible to relate quantities in the interior of AdS_5 to quantities in the gauge theory living on the boundary. The correspondence was still limited to maximally supersymmetric and conformal gauge theories, but progress has been made in order to rid it from these two restrictions. In order to describe gauge theories with reduced supersymmetry we must take string theory on $AdS_5 \times X_5$, where X_5 is a positively curved Einstein manifold, i.e. one for which $R_{\mu\nu} = \Lambda g_{\mu\nu}$ with $\Lambda > 0$. The number of supersymmetries in the dual gauge theory is determined by the number of Killing spinors on X_5. Some progress has also been made towards non-conformal gauge theories. This development is very important in order to make contact with QCD, the prime application, since it is neither supersymmetric nor conformal.

It is amusing to note that string theory has come full circle; it was invented to provide a description of QCD and now, after thirty years of development, it might just do that.

5.1 The large N limit

What Maldacena did in order to discover the duality between supergravity and gauge theory was to formulate both theories in terms of D-branes. In order to get a four-dimensional gauge theory one must use D3 branes, which can be embedded in ten-dimensional spacetime. Each D-brane carries a $U(1)$ charge and by stacking N D-branes one obtains a $U(N)$ Born-Infeld theory. By taking the low-energy limit the gauge fields decouple from gravity, simply because Newton's constant depends on the energy. In this limit we can also neglect higher order terms F^n in the Born-Infeld action. In this way we get that the low-energy dynamics is governed by a $U(N)$ super-Yang-Mills theory. We will thus describe the gauge theory as the low-energy limit of stacked D-branes.

On the supergravity side, the stacked D-branes make up a black hole solution to the supergravity equations. Far away from the D-branes space is flat, but near the D-branes there is an infinite throat leading down to the horizon of the black hole as depicted in Fig. 5.1. In order to understand what happens in the low-energy limit we consider particle waves in the Minkowski region. Their wave length will increase as the energy is lowered and in the low-energy limit they will not notice the D-branes. Similarly, excitations inside the throat region will lie closer and closer to the event horizon as the energy is lowered. In this way we see that the bulk physics decouples from the boundary physics near the horizon. Since it is the low-energy dynamics of the D-branes that we are interested in it is the low-energy region near the horizon we should be focusing at.

From the metric for the stacked D-brane solution it is easy to see that the near horizon geometry is $AdS_5 \times S^5$. Since the radius of curvature of this AdS space is proportional to $N^{1/4}$, the supergravity solution is most reliable when N is large. Maldacena's conclusion was that these two descriptions are dual to each other.

On the gauge theory side, N is the number of colours and we are interested in how the *effective* coupling scales with N. To see this, consider the exchange of a gluon between two quarks. Since the emitting quark can turn into the order of N colours, the effective coupling will be $\lambda = g_{eff}^2 = g_{YM}^2 N$, where λ is called the 't Hooft parameter. As we will see below, the duality takes its simplest form when λ is large and since λ corresponds to the effective coupling we have a duality between strongly coupled gauge theory and supergravity.

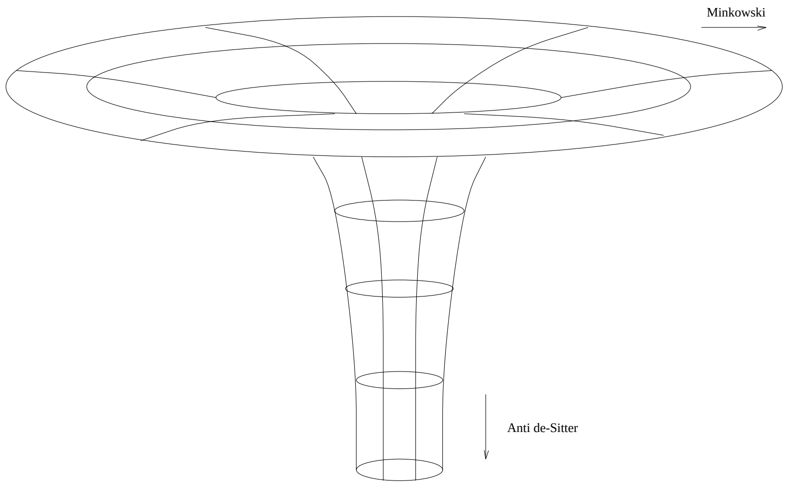

Figure 5.1: The geometry near the stacked D-branes.

Let us finally examine the validity of the supergravity-gauge theory correspondence a bit more carefully. The argument above that string theory reduces to supergravity in the low-energy limit near the horizon is a bit too weak. We must also demand that the radius of AdS_5 and S^5 (they have the same radius) is large compared to the string scale in order to be able to neglect stringy effects. The radius, expressed in the 't Hooft parameter is

$$R = \lambda^{1/4} \ell_s. \tag{5.1}$$

This implies that we must have $\lambda \gg 1$. In order to be able to neglect quantum effect, the string scale must be large compared to the Plank scale, given by

$$\ell_p = g_s^{1/4} \ell_s. \tag{5.2}$$

Using that $g_s = g_{YM}^2$ we see that we must have $g_{YM} \ll 1$, which implies $N \gg 1$ since λ is large. To conclude, in order for the correspondence to be valid between supergravity and gauge theory the large N limit is not enough, we must also take λ to be large.

5.2 Holography

The supergravity-gauge theory duality also gives new insights into an intriguing concept which 't Hooft has named "holography". By holography we mean a relation between the information carried on a surface and that within the volume it encloses. To be more precise, 't Hooft [27] and Susskind [28] proposed that the degrees of freedom in the bulk of a region matches the degrees of freedom on the surface of that region with an upper bound on the amount of information per unit area.

Before the discovery of the supergravity-gauge theory duality, the best candidate for realizing holography was black holes. As Bekenstein and Hawking showed, the entropy of a black hole is proportional to its surface area. But if we consider the creation of a black hole, in order not to lose any information, all the information carried by what forms the inside of the black hole must be carried by its surface. The holographic principle then means that the hologram captures all the information but in a non-transparent way. Considering this, it is not surprising that Maldacena got his idea while studying black holes.

Not everyone believes in the idea of holography, but the supergravity-gauge theory duality may provide a new realization of holography and can thus bring some new insights. The important difference to the black hole realization mentioned above is that the supergravity-gauge theory realization would provide a microscopic understanding of the physics behind holography. Witten and Susskind have used this new realization to get an order of magnitude estimate of the degrees of freedom of a black hole [29]. They also found that infrared, i.e. long distance, effects in the bulk are related to ultraviolet, i.e. short distance, effects on the boundary.

6

Outlook

In light of the recent years' dramatic progress we are getting closer to answering some long-standing problems in string theory. The Maldacena duality, together with matrix theory, bring us a step closer to a non-perturbative definition of U-theory. It is also possible that the Maldacena duality can be applied to QCD as described above.

There are however some problems that still seem to be out of reach, e.g. the problems related to vacuum selection, supersymmetry breaking and the cosmological constant. In the latter case there may be some new input from the recent discovery that the expansion of the universe is accelerating[1]. This implies the existence of a non-zero (positive) cosmological constant, meaning that the universal expansion is dominated by a repulsive force, counter-acting gravity. If correct, this leads to an ever expanding universe and also implies that most of the energy in the universe is associated with the vacuum (the cosmological constant is related to the vacuum energy density).

Finally, the experimental verification of supersymmetry seems to be in reach in the accelerators under construction. If traces of supersymmetry can be detected it will no doubt be one of the greatest experimental discoveries. There will no doubt be much excitement in the years to come!

[1] See e.g. `http://www.eso.org/outreach/press-rel/pr-1998/pr-21-98.html` and `http://www-.sciencemag.org/cgi/content/summary/282/5397/2156a`.

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Paper I

Paper II

Paper III