The Quest for M-theory

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[1] The thesis is available at `http://fy.chalmers.se/~gran/`.

To my father and the memory of my mother,
with love and gratitude

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Abstract

In ten dimensions, there exist five consistent string theories and in eleven dimensions there is a unique supergravity theory. When trying to find *the* fundamental theory of nature this is clearly an "embarrassment of riches". In the mid 90s, however, it was discovered that these theories are all related via a kind of transformation called duality. The six theories are therefore only facets of a (largely unknown) underlying theory referred to as M-theory. This theory is intrinsically non-perturbative and therefore very hard to study. Witten has suggested that until we know more about M-theory, M can stand for 'magic', 'mystery' or 'membrane', according to taste. In this thesis, comprising an introductory text and eight appended research papers, we are going to describe some of the methods used to study M-theory. Central to this analysis are non-perturbative, solitonic objects collectively referred to as p-branes, whose properties are studied in Papers I-IV and VI. In Paper II we generalize the Goldstone mechanism to the case of tensor fields of arbitrary rank, providing an understanding of the emergence of vector and tensor fields on branes in terms of broken symmetries. In Papers III and IV we find brane solutions with finite field strengths on the brane. Lately, noncommutative theories decoupled from closed strings have been discovered. These theories are defined on branes with critical field strengths and are studied and extended in Papers VI and VII.

In Paper V we generalize eleven dimensional supergravity to obtain the most general geometrical structure in eleven dimensional superspace. The motivation for this is to examine what constraints supersymmetry imposes on possible correction terms arising from M-theory. To facilitate this study the Mathematica package GAMMA, which is capable of performing Γ-matrix algebra and Fierz transformations, were developed and is presented in Paper VIII.

This thesis consists of an introductory text and the following eight appended research papers, henceforth referred to as Paper I-VIII:

I. T. Adawi, M. Cederwall, U. Gran, M. Holm and B.E.W. Nilsson,
Superembeddings, non-linear supersymmetry and 5-branes,
Int. J. Mod. Phys. **A13** (1998) 4691-4715 [`hep-th/9711203`].

II. T. Adawi, M. Cederwall, U. Gran, B.E.W. Nilsson and B. Razaznejad,
Goldstone tensor modes,
JHEP **02** (1999) 001 [`hep-th/9811145`].

III. M. Cederwall, U. Gran, M. Holm and B.E.W. Nilsson,
Finite tensor deformations of supergravity solitons,
JHEP **02** (1999) 003 [`hep-th/9812144`].

IV. M. Cederwall, U. Gran, M. Nielsen, B.E.W. Nilsson,
(p, q) *5-branes in non-zero B-field,*
JHEP **01** (2000) 037 [`hep-th/9912106`].

V. M. Cederwall, U. Gran, M. Nielsen, B.E.W. Nilsson,
Manifestly supersymmetric M-theory,
JHEP **10** (2000) 041 [`hep-th/0007035`].

VI. D.S. Berman, V.L. Campos, M. Cederwall, U. Gran, H. Larsson,
M. Nielsen, B.E.W. Nilsson and P. Sundell,
Holographic noncommutativity,
JHEP **05** (2001) 002 [`hep-th/0011282`].

VII. U. Gran and M. Nielsen,
Non-commutative open (p, q)*-string theories,*
`hep-th/0104168`, submitted to *JHEP*.

VIII. U. Gran,
GAMMA: A Mathematica package for performing Γ*-matrix algebra and Fierz transformations in arbitrary dimensions,*
`hep-th/0105086`.

Acknowledgments

Contents

1

Introduction

String theory was originally formulated in the late 1960s as an attempt to explain the spectrum of hadrons and their interactions. It was however discarded as a theory of the strong interaction for two main reasons. Firstly, there exists a critical dimension, 26 for the bosonic string and 10 for the fermionic string, and our world has just four dimensions. Secondly, the spectrum contains a massless spin 2 particle not present in the hadronic world. These problems and the rapid success of QCD made people abandon string theory.

String theory was revived in 1974 when Scherk and Schwarz turned the existence of the massless spin 2 particle into an advantage by interpreting it as the graviton, the field quantum of gravitation. It was also discovered that at low energies string theory reduced to einsteinian general relativity. String theory was in this way elevated to be a potential "theory of everything", *i.e.,* a theory that unifies all four forces of nature.

The extension of the bosonic string to the fermionic string, thereby including fermions in the spectrum, was achieved by enforcing supersymmetry [1, 2]. This is a concept of great importance in high-energy physics. Supersymmetry can be described as an extension of special relativity taking into account that fermions exist. In the same sense supergravity can be described as an extension of general relativity.

The *first superstring revolution* (1984-85) consisted of three important discoveries. The first was an anomaly cancellation mechanism, which enabled the construction of consistent gauge theories in ten dimensions. The key result was that the gauge group has to be SO(32) or $E_8 \times E_8$. The second discovery was two new superstring theories, the *heterotic* string theories, with exactly these gauge groups. But perhaps it was the third discovery that made people set their hopes on superstring theory. By compactifying the $E_8 \times E_8$ heterotic string theory on a particular Calabi-Yau manifold one obtained a 4d effective theory with many qualitatively realistic features. There are however a great variety of possible choices of this Calabi-Yau manifold and no one stands out as particularly special. After this revolution there

were five distinct ten dimensional superstring theories with consistent weak coupling perturbation expansions and the understanding of these theories was developed in the ensuing years.

A great deal of effort has been devoted to the investigation of the non-perturbative structure of superstring theory. From field theory we know that there are many interesting non-perturbative phenomena like quark confinement, the Higgs mechanism and dynamical symmetry breaking. Since string theory contains quantum field theory one expects all of these phenomena to occur also in string theory, in addition to new stringy phenomena. Generalizing ideas from QED, the electrically charged string was found to have a magnetically charged dual partner, the solitonic five-brane, analogous to the magnetic monopole (or more precisely analogous to the 't Hooft-Polyakov monopole in the Georgi-Glashow model, which is obtained as a solitonic solution). This led to the discovery of various extended objects, collectively referred to as p-branes of dimension $d = p + 1$. The p-branes can be classified according to their world-volume field content. Papers I-IV deals with these kinds of objects and especially with branes which have vector and tensor modes living on them. Superstring theory is thus nowadays quite a misnomer since it contains so much more than just strings.

What laid the foundation to the *second superstring revolution* (mid 1990s), which has to do with the non-perturbative structure of superstring theory, was the concept of duality. By duality we mean a way of relating different superstring theories, or different "regions" of a particular superstring theory. One kind of duality, S-duality, relates weakly and strongly coupled theories and using this duality we can do calculations in the weakly coupled theory and then translate the results into the strongly coupled theory. This is of great value since we can only do calculations for weak coupling. By using this kind of duality we can obtain non-perturbative information which would be almost impossible to obtain by direct calculation. Since S-duality is a non-perturbative duality we need non-perturbative objects in order to check various conjectured S-dualities and here the above mentioned p-branes play a crucial rôle.

It was later discovered that all five superstring theories are related to each other via duality and are thus only facets of a (largely unknown) underlying fundamental theory. This all-encompassing fundamental theory is called M-theory. Witten has proposed that until we get a better understanding of what M-theory really is, M can stand for Magic, Mystery or Membrane, according to taste. However, the sense in which Witten introduced the term M-theory was to designate the eleven-dimensional quantum theory which has eleven-dimensional supergravity as its low-energy effective description. Nowadays, the term M-theory is used to designate both the all-encompassing theory and the quantum theory in eleven dimensions.

A few years ago, an enormous attention was given to the so-called AdS/CFT correspondence, which is a kind of duality between superstring theory or M-theory on certain anti-de Sitter spacetime backgrounds and gauge theory. AdS space is analogous to a sphere with negative curvature. People hope that this will help us prove quark confinement, which would be one of the greatest achievements of superstring theory to this date.

Superstring theory can also be used to calculate black hole entropy. Using D-branes (one kind of p-branes) Strominger and Vafa [3] were in 1996 able to give a statistical mechanics derivation of the Bekenstein-Hawking entropy relation $S = A/4G\hbar$. Previously this relation was only understood from a thermodynamic perspective but now the picture is complete. There are however indications that this result is universal [4], *i.e.,* that *any* quantum theory of gravity will lead to the standard result. It should be stressed that the quantitative results mentioned above consists of agreement with calculations made in more accepted theoretical models, like QCD, and does not consist of actual experiments.

Recently, also noncommutative geometry has become an important ingredient in string theory. It first appeared in the context of bound states of D-branes [5] and then reappeared in matrix theory, which attracted a lot of attention when it was understood that it provides a well defined quantum theory which reduces to a supersymmetric theory of eleven dimensional gravity at low energies [6]. However, due to the level of difficulty of performing calculations within matrix theory its use has been limited. Noncommutative geometry once again appeared in string theory with the discovery of numerous noncommutative theories decoupled from closed strings. Since the graviton is a closed string state these theories will not contain gravity. Recall that one of the reasons why people discarded string theory as a theory for the strong interaction was the presence of a massless spin 2 particle, *i.e.,* the graviton. These new theories do not suffer from this problem and could potentially be used to describe QCD. The absence of gravity also makes these theories easier to study than the full string theory itself and therefore, in any case, they provide interesting "toy" models. To be a bit more precise, the theories in question live on branes and have open strings, open Dp-branes, or open M2-branes as their light degrees of freedom. These theories will be discussed in detail in Chapter 8. Recent advances has also been made concerning the understanding of tachyon condensation, which will be briefly discussed in the outlook in Chapter 9.

This briefly recapitulates the major line of development in string theory since its birth and we will now examine some areas in more detail. In Chapter 2 we will start by presenting bosonic string theory and then move on to its supersymmetric generalization, the fermionic string, and discuss its spectrum. The various supergravity theories, which provide the setting for the work done in Papers I-VII, are reviewed in Chapter 3. It is explained how the ten dimensional supergravity theories arise as the low-energy limit of the corresponding superstring theories. Emphasis is put on the treatment of eleven dimensional supergravity in order to introduce the work in Paper V. There exist a multitude of supergravity solutions describing extended objects called p-branes. They are studied in Papers I-IV and VI, and are also important to the work done in Paper VII. In Chapter 4 the basic properties of p-branes are reviewed and a classification based on their world-volume field content is presented. Chapter 5 deals with the concept of duality and describes how the five superstring theories and M-theory are related through the *web of dualities.* Chapter 6 gives a brief introduction to the AdS/CFT correspondence, focusing on the main ideas. In Chapter 7 we extend our study of branes to the case when they have finitely excited field strengths on their world-volumes, important for the construction of decoupled,

noncommutative theories, studied in Papers VI and VII and discussed in Chapter 8. Finally, we conclude with an outlook in Chapter 9.

2

Perturbative string theory

Before the discovery of the various duality relations only perturbative aspects of string theory were accessible and the only known object in the theory was the string. This picture changed dramatically when multifarious higher-dimensional objects were discovered and found to be an integral part of string theory. Perturbative string theory is nevertheless still very important since it is for weakly coupled strings that calculations are most manageable.

In this chapter we will present the elementary concepts of bosonic string theory and superstring theory and describe how the latter is related to supergravity. For a more complete treatment of string theory see, *e.g.,* Refs. [7, 8, 9].

2.1 Bosonic string theory

The fundamental idea behind string theory is actually very simple. Consider first an ordinary point particle. The action is in this case given by the length of the path the particle traces out as it propagates, which is called the *world-line*. If we instead consider the propagation of a closed string, its orbit will be a two-dimensional tube instead of a line. In analogy with the point-particle case we take the action to be the area of this tube, the *world-sheet*. An action of this type was first written down by Nambu and Goto:

$$S_{NG}[X^\mu] = -T \int_\Sigma d^2\sigma \sqrt{-det(\partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu})}. \tag{2.1}$$

Here $T = (2\pi\alpha')^{-1}$ is the string tension, a constant of dimension $(\text{length})^{-2}$, and α' is known as the Regge slope. We can view this as an embedding of the world-sheet, Σ, in M, the target space. The target space is often, for simplicity, taken to be flat, D-dimensional Minkowski space. Nothing prevents us however from considering a general target space by just replacing $\eta_{\mu\nu}$ in (2.1) with $g_{\mu\nu}$. The fields X^μ, $\mu = 1, 2, \ldots, D$ represent the position of the string in target space. The world-sheet parameterized up by $\sigma^0 = \tau$, representing the time and $\sigma^1 = \sigma$, $0 \leq \sigma <$

π, representing the angle around the string. The embedding X induces a metric, $(X^*\eta)_{ij} = \partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu}$, and as seen in (2.1) it is with this metric the area of the world-sheet is measured. If the world-sheet has boundaries in the σ-direction, we have an open string, otherwise it is closed.

A problem with the Nambu-Goto action is that it can not be quantized preserving manifest Lorentz covariance due to the square root. A classically equivalent action without the square root can however be constructed using an auxiliary, intrinsic world-sheet metric γ_{ij}:

$$S_{BDH}[X^\mu, \gamma_{ij}] = -\frac{T}{2} \int_\Sigma d^2\sigma \sqrt{-\gamma} \gamma^{ij} \partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu}. \qquad (2.2)$$

This action was first written down by Brink, Di Vecchia and Howe [10] and by Deser and Zumino [11], but is most commonly known as the Polyakov action since Polyakov used it to construct the path integral formulation of string theory [12, 13]. The difference compared to the Nambu-Goto formulation is that the world-sheet is given an intrinsic geometry, given by γ_{ij}, but one also has an algebraic equation of motion for γ_{ij}. By using the solution to this equation the action (2.2) reduces to the one in (2.1). The property of having an intrinsic world-sheet geometry will also be important when trying to quantize the string.

The action (2.2) has three local invariances, two coming from the reparameterization invariance of the world-sheet and one from the Weyl invariance, $\gamma_{ij}(\sigma) \to e^{\Lambda(\sigma)}\gamma_{ij}(\sigma)$. In addition, we have rigid Poincaré invariance in target space, but from the world-sheet point of view this is an internal symmetry.

By using the reparameterization invariance we can locally go to the conformal gauge, $\gamma_{ij} = e^{\Lambda(\sigma)}\eta_{ij}$, where η=diag(-1,1),

$$S_{cg}[X^\mu] = -\frac{T}{2} \int_\Sigma d^2\sigma \eta^{ij} \partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu}. \qquad (2.3)$$

This is a free, conformally invariant action (which is of course also true for the action in (2.2)) and we thus have a conformal field theory [14] living on the world-sheet.

By varying the action we get the equation of motion, $\Box X = \partial_i \partial^i X^\mu = 0$. This is an ordinary wave equation and we can split the general solution into one left- and one right-moving part. Whether these parts are related or not depends on the boundary conditions. An ordinary closed string has periodic boundary conditions, while an open string can have either Dirichlet or Neumann boundary conditions (or combinations of them). The Dirichlet condition was for many years considered unphysical since it breaks Poincaré invariance. This changed with the discovery of D-branes, objects on which open strings can end, where D stands for Dirichlet. We will discuss this type of brane in Chapter 4. In the following we will concentrate on the closed string and show how quantization is achieved and how the spectrum is derived.

For the closed string, the general solution to the equation of motion is

$$X^\mu(z, \bar{z}) = q^\mu - \frac{i}{4}\alpha' p^\mu \ln(z\bar{z}) + i\sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} (\frac{\alpha_n^\mu}{n} z^{-n} + \frac{\tilde{\alpha}_n^\mu}{n} \bar{z}^{-n}), \qquad (2.4)$$

where we have performed a Wick rotation, $\sigma^2 = i\sigma^0$, and introduced complex coordinates $z = e^{2(\sigma^2 + i\sigma^1)}$. This maps the, now euclidean, world-sheet to the compactified complex plane, which is topologically S^2.

Quantization is most easily achieved by quantizing the embedding fields X^μ canonically, which leads to the commutation relations

$$[q^\mu, p^\nu] = i\eta^{\mu\nu}, \qquad [\alpha_m^\mu, \alpha_n^\nu] = m\delta_{m+n,0}\eta^{\mu\nu}, \qquad (2.5)$$

with an analogous expression for the $\tilde{\alpha}_m^\mu$ oscillators. We will from now on concentrate on the left-moving part described by the α_m^μ oscillators. Let us introduce a (momentum) vacuum, $|0\rangle$, defined by

$$p^\mu|0\rangle = \alpha_m^\mu|0\rangle = 0, m > 0. \qquad (2.6)$$

Eigenstates of p^μ can now be constructed, $|k\rangle = e^{ik \cdot q}|0\rangle$, and we have a set of Fock vacua where each vacuum is labeled by its momentum[1]. The full state space is generated by applying the creation operators $(\alpha_m^\mu)^\dagger = \alpha_{-m}^\mu, m > 0$, to the Fock vacua. The physical state space is however only a subspace of the full state space since we must take into account the constraints imposed by the equation of motion for γ_{ij} to which we will now turn.

Since the action (2.3) is free the non-trivial content of the theory is contained in the equation of motion for γ_{ij}, which is equivalent to the vanishing of the energy-momentum tensor

$$T_{ij} = -\frac{1}{T}\frac{1}{\sqrt{-\gamma}}\frac{\delta S_{BDH}}{\delta \gamma^{ij}}|_{\gamma=\eta}. \qquad (2.7)$$

The energy-momentum tensor is defined to describe the response of the system to changes in the metric according to

$$\delta S = -T\int_\Sigma d^2\sigma\sqrt{-\gamma}T_{ij}\delta\gamma^{ij}. \qquad (2.8)$$

In the complex basis we have

$$T_{zz}(z) = \frac{1}{2}\partial_z X_\mu \partial_z X^\mu, \qquad (2.9)$$

$$\bar{T}_{\bar{z}\bar{z}}(\bar{z}) = \frac{1}{2}\partial_{\bar{z}} X_\mu \partial_{\bar{z}} X^\mu, \qquad (2.10)$$

while $T_{z\bar{z}}$ and $T_{\bar{z}z}$ vanish identically due to the tracelessness of T_{ij}, which is a general feature of conformal field theories [14]. This can easily be understood by considering a Weyl invariant theory, like the one defined by the action (2.2), where only the metric transform under Weyl rescalings, $\delta\gamma_{ij} = \Lambda\gamma_{ij}$. We then have

$$0 = \delta S = -T\int_\Sigma d^2\sigma\sqrt{-\gamma}\Lambda(\sigma)T_{ij}\gamma^{ij} \qquad (2.11)$$

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[1]Note that $\langle k|k'\rangle = \delta(k-k')$ and therefore $\langle p=0|p=0\rangle = \infty$. We can, however, define a left vacuum, $\langle x=0|$, satisfying $\langle 0|x=0$ and $\langle 0|\alpha_n$, for $n < 0$, which has the property that $\langle x=0|p=0\rangle = 1$.

and since $\Lambda(\sigma)$ is an arbitrary function it follows that T_{ij} must be traceless. We now make a Fourier expansion of the stress-energy tensor,

$$T(z) = T_{zz}(z) = \sum_{n \in \mathbb{Z}} L_n z^{-n-2}. \tag{2.12}$$

The Fourier coefficients satisfy the Virasoro algebra

$$[L_m, L_n] = (m - n)L_{m+n} + \frac{c}{12}m(m^2 - 1)\delta_{m+n,0}, \tag{2.13}$$

where $c = D$ is the central charge. The equations of motion for the energy-momentum tensor, $T_{ij} = 0$, require it to vanish. In order to deduce what this implies for the L_m operators we write the condition as $\langle phys|T|phys'\rangle = 0$ which gives us the Virasoro constraints

$$(L_m - a\delta_{m,0})|phys\rangle = 0, \quad m \geq 0, \tag{2.14}$$

where the constant a is introduced due to the normal ordering ambiguity in L_0. Having a ghost free spectrum requires $D \leq 26$ and $a \leq 1$. Furthermore, to have a Lorentz invariant theory we must take $D = 26$ and $a = 1$. The fact that $L_0 - \bar{L}_0$ generates rigid σ-translations implies the level-matching constraint

$$(L_0 - \bar{L}_0)|phys\rangle = 0 \tag{2.15}$$

since no point on the string is special, *i.e.,* the string is invariant under σ-translations. Using (2.9) we can write the L_m operators as normal ordered expressions in the oscillators

$$L_m = \frac{1}{2}\sum_{n \in \mathbb{Z}} : \alpha_{m-n} \cdot \alpha_n :, \qquad \alpha_0^\mu = \sqrt{\alpha'/2}p^\mu. \tag{2.16}$$

Acting with L_0 on a physical state gives the mass-shell constraint

$$\alpha' M^2 = 4(N - a), \tag{2.17}$$

where N is the eigenvalue of the level operator $N = \sum_{m>0} \alpha_{-m} \cdot \alpha_m$ and $N = \bar{N}$ due to level-matching. As is immediately seen from (2.17) the ground state of the bosonic string is tachyonic, *i.e.,* has negative mass squared. This implies that the vacuum is not stable and therefore the bosonic string was thought not to be consistent. Recent developments, however, show that the potential associated with the open string tachyon has a local minimum corresponding to a perturbatively stable closed string vacuum (see [15] and references therein). The tachyon can also be removed by introducing supersymmetry and making a particular projection as will be described in the next section.

The first excited level, $\xi_{\mu\nu}\alpha_{-1}^\mu\tilde{\alpha}_{-1}^\nu|k\rangle$, is the massless sector of the theory. Depending on the choice of polarization tensor we get a scalar ϕ, a symmetric traceless tensor $g_{\mu\nu}$ and an antisymmetric tensor $B_{\mu\nu}$. These are, respectively, the dilaton, the graviton and the abelian two-form gauge potential.

One of the most important properties of bosonic string theory is that it reduces to einsteinian general relativity in the low-energy limit. One way to see this is to consider a non-trivial background for the string, which is described by the non-linear σ-model action

$$S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \Big(\sqrt{-\gamma}\gamma^{ij}\partial_i X^\mu \partial_j X^\nu g_{\mu\nu}(X)$$
$$+ \epsilon^{ij}\partial_i X^\mu \partial_j X^\nu B_{\mu\nu}(X) - \alpha'\sqrt{-\gamma}R^{(2)}\phi(X)\Big), \quad (2.18)$$

where $R^{(2)}$ is the Ricci scalar for the world-sheet metric. From the world-sheet point of view the massless target space fields $g_{\mu\nu}$, $B_{\mu\nu}$ and ϕ are coupling constants, or rather coupling *functionals* since they depend on X. To get the Einstein equations we require conformal invariance at the quantum level, *i.e.*, that the β-functionals vanish. A non-linear σ-model calculation, to lowest non-trivial order in α' (corresponding to the low-energy limit), gives [7]

$$\begin{aligned}
\beta_{\mu\nu}^{(g)} &= R_{\mu\nu} - \frac{1}{4}H_{\mu\rho\sigma}H_\nu{}^{\rho\sigma} + 2D_\mu D_\nu\phi + \mathcal{O}(\alpha'), \\
\beta_{\mu\nu}^{(B)} &= \frac{1}{2}D^\rho H_{\rho\mu\nu} - D^\rho\phi H_{\rho\mu\nu} + \mathcal{O}(\alpha'), \\
\beta^{(\phi)} &= \frac{(26-D)}{3\alpha'} - R + \frac{1}{12}H^2 - 4D_\mu D^\mu\phi + 4(D_\mu\phi)^2 + \mathcal{O}(\alpha').
\end{aligned} \qquad (2.19)$$

Here $R_{\mu\nu}$ is the target space Ricci tensor and $H = dB$ is the three-form field strength. By requiring that all the β-functionals vanish we obtain the equations of motion for the massless background fields.

2.2 Superstring theory

There are three different formulations of the superstring, depending on where the supersymmetry is manifest. One can have manifest supersymmetry either on the world-sheet, on the target space, or both simultaneously, which is called a doubly supersymmetric formulation. The Neveu-Schwarz-Ramond formulation, which has manifest world-sheet supersymmetry, is the formulation we will use in order to discuss the spectrum of the open fermionic string. In the Green-Schwarz formulation one embeds the bosonic world-sheet of the string into a target superspace and thus has manifest target space supersymmetry. In order to get supersymmetry also on the world-sheet one must require an additional fermionic target space symmetry called κ-symmetry, which reduces the number of fermionic degrees of freedom on the world-sheet by a factor $1/2$. A recently developed approach is the "doubly supersymmetric geometrical approach" [16, 17, 18], or the "embedding formalism" [19], which is described in detail in Paper I. In this approach one has manifest supersymmetry both on the world-sheet and in target space. This is accomplished by embedding a supermanifold, in the string case the super-world-sheet, into the supermanifold which constitutes the target space. In this sense it can be viewed as

an extension of the Green-Schwarz formalism. In order to get the dynamics we have to impose an embedding condition and in some cases supplementary conditions.

The action in the NSR formulation is

$$S = \frac{1}{4\pi} \int d^2z \left(\frac{2}{\alpha'} \partial X^\mu \bar\partial X_\mu + \psi^\mu \bar\partial \psi_\mu + \tilde\psi^\mu \partial \tilde\psi_\mu \right), \tag{2.20}$$

which leads to the equations of motion $\bar\partial \psi^\mu = 0 = \partial \tilde\psi^\mu$. The two spinor components ψ^μ and $\tilde\psi^\mu$ form a world-sheet Majorana spinor

$$\begin{pmatrix} \psi^\mu \\ \tilde\psi^\mu \end{pmatrix} \tag{2.21}$$

transforming as a spacetime vector. The Majorana condition ensures that we have the same number of bosonic and fermionic degrees of freedom on shell. The action (2.20) is obtained by gauge-fixing a supersymmetric analogue of (2.2).

We will now analyze the spectrum of the open fermionic string, which in the next chapter will be used to derive the massless spectra of the five superstring theories and hence the field content of the corresponding supergravity theories. Note first that the classical action (2.20) does not contain any spacetime spinors. They will arise as a consequence of quantization and the origin of this phenomenon has to do with the boundary conditions of the world-sheet spinors. The vanishing of the surface term when varying the action (2.20) is satisfied by two different Poincaré invariant boundary conditions

$$\begin{array}{llll} \psi^\mu(0,\sigma^2) = -\tilde\psi^\mu(0,\sigma^2), & \psi^\mu(\pi,\sigma^2) = \tilde\psi^\mu(\pi,\sigma^2) & \text{(NS)} \\ \psi^\mu(0,\sigma^2) = \tilde\psi^\mu(0,\sigma^2), & \psi^\mu(\pi,\sigma^2) = \tilde\psi^\mu(\pi,\sigma^2) & \text{(R)} \end{array} \tag{2.22}$$

which defines the Neveu-Schwarz (NS) and Ramond (R) sectors, respectively. With respect to these conditions, the solutions to the equations of motion are

$$\begin{array}{ll} \psi^\mu(\sigma^1,\sigma^2) = i^{-1/2} \sum_{r\in\mathbb{Z}+1/2} \psi_r^\mu e^{-r(\sigma^2-i\sigma^1)} \\ \tilde\psi^\mu(\sigma^1,\sigma^2) = i^{1/2} \sum_{r\in\mathbb{Z}+1/2} \psi_r^\mu e^{-r(\sigma^2+i\sigma^1)} \end{array} \quad \text{(NS)} \tag{2.23}$$

for the NS sector and

$$\begin{array}{ll} \psi^\mu(\sigma^1,\sigma^2) = i^{-1/2} \sum_{n\in\mathbb{Z}} \psi_n^\mu e^{-n(\sigma^2-i\sigma^1)} \\ \tilde\psi^\mu(\sigma^1,\sigma^2) = i^{1/2} \sum_{n\in\mathbb{Z}} \psi_n^\mu e^{-n(\sigma^2+i\sigma^1)} \end{array} \quad \text{(R)} \tag{2.24}$$

for the R sector. Proceeding with the quantization as in the bosonic case gives the commutation relations

$$\begin{aligned} \{\psi_r^\mu, \psi_s^\nu\} &= \left\{ \tilde\psi_r^\mu, \tilde\psi_s^\nu \right\} = \eta^{\mu\nu}\delta_{m+n}, \\ [\alpha_m^\mu, \alpha_n^\nu] &= [\tilde\alpha_m^\mu, \tilde\alpha_n^\nu] = m\eta^{\mu\nu}\delta_{m+n}, \end{aligned} \tag{2.25}$$

which is valid both for integer and half-integer values of r and s. The states in the NS sector are now generated from the Fock vacua $|k\rangle$ by applying the negative modes

α^μ_{-m} and ψ^μ_{-r}. The Virasoro constraint corresponding to the mass-shell constraint is

$$(L_0^{\text{(NS)}} - \frac{1}{2})|phys\rangle = 0 \tag{2.26}$$

where

$$L_0^{\text{(NS)}} = \frac{1}{2}\sum_{n\in\mathbb{Z}} : \alpha_{-n}\cdot\alpha_n : + \frac{1}{2}\sum_{r\in\mathbb{Z}+1/2} r : \psi_{-r}\cdot\psi_r : \tag{2.27}$$

and $\alpha^\mu_0 = \sqrt{2\alpha'}p^\mu$ for the open string. The mass spectrum for the NS sector is thus

$$\alpha' M^2_{\text{NS}} = N_\alpha + N_\psi - \frac{1}{2}. \tag{2.28}$$

The ground state is thus unique which implies that it is a spin 0 state.

The R sector is a bit more complicated due to the fermionic zero-modes ψ^μ_0, which commutes with the mass operator. This implies that $|0\rangle$ and $\psi^\mu_0|0\rangle$ are degenerate in mass. Since the ψ^μ_0 are the generators of a Clifford algebra (cf. Eq. (2.25)) we conclude that the R ground state is a SO(9,1) spinor. Since we have Majorana-Weyl spinors in ten dimensions we are free to choose the chirality of the vacuum. The oscillators are spacetime vectors, and can not change tensors into spinors or vice versa. Thus all states in the R sector will be fermionic and all states in the NS sector will be bosonic. In this way the emergence of spacetime fermions is due to the zero-modes ψ^μ_0.

In addition to the usual Virasoro and level-matching constraints the physical states in the R and NS sectors must satisfy

$$G_r|phys\rangle = 0; \;\; r \geq 0 \tag{2.29}$$

where

$$G_r = \sum_{n\in\mathbb{Z}} \alpha_{-n}\cdot\psi_{r+n}. \tag{2.30}$$

The G_r operators are the Fourier components of the supercurrent and the constraints come from the vanishing of the supercurrent in the same way as the Virasoro constraints come from the vanishing of the stress-energy tensor. The G_0 constraint of (2.29) actually contains the mass-shell constraint, due to the super-Virasoro algebra, and we get the mass spectrum

$$\alpha' M^2_{\text{(R)}} = N_\alpha + N_\psi. \tag{2.31}$$

We still however have tachyons in the NS sector. These can be removed by invoking the GSO projection, which is also crucial in order to have modular invariance. The projection consists of removing states with odd world-sheet fermion number after which we finally arrive at the physical spectrum. A closed string can now be considered as built up by two open strings and therefore its spectrum can be written as the direct product of two open string spectra. In this way we can obtain the spectra for the five superstring theories, see, *e.g.,* Ref. [7].

3

Supergravity

In the previous chapter we derived the spectrum for the open fermionic string. Taking the tensor product of two open string spectra yields the spectra for the five closed fermionic string theories. Since the massless spectra dominate the low-energy behavior of these theories these spectra give the field content of the corresponding supergravity theories. The various supergravity theories are thus obtained from the dynamics of the corresponding massless string states in the low-energy limit. We will now briefly review the supergravity theories in ten dimensions and then discuss eleven dimensional supergravity in more detail.

3.1   Type IIA

Type IIA supergravity can be obtained by reducing eleven dimensional supergravity on a circle, as will be explained in Chapter 5. We can also derive the massless spectrum by taking the tensor product of two copies of the massless spectrum for the open fermionic string, as mentioned above. In the NS-sector, the massless state is $\psi^{\mu}_{-1/2}|0\rangle$, which is a target space vector. In the R-sector, the massless state is, as explained in the previous chapter, a target space spinor whose chirality we are free to choose. Using the little group $SO(8)$, we therefore get the massless spectra $\mathbf{8_V} \oplus \mathbf{8_S}$ or $\mathbf{8_V} \oplus \mathbf{8_C}$ depending on the choice of chirality of the R-vacuum. By taking the product of spectra with the same or opposite chirality we get the type IIA or type IIB field content, respectively

$$
\begin{aligned}
(\mathbf{8_V} \oplus \mathbf{8_S}) \otimes (\mathbf{8_V} \oplus \mathbf{8_C}) \quad (\text{IIA})\,, \\
(\mathbf{8_V} \oplus \mathbf{8_S}) \otimes (\mathbf{8_V} \oplus \mathbf{8_S}) \quad (\text{IIB})\,.
\end{aligned}
\tag{3.1}
$$

We immediately see that type IIA is a non-chiral theory and that type IIB is chiral. The NS-NS sector

$$
\mathbf{8_V} \otimes \mathbf{8_V} = \mathbf{8} \oplus \mathbf{28} \oplus \mathbf{35_V} = \phi \oplus B_{\mu\nu} \oplus g_{\mu\nu}
\tag{3.2}
$$

is the same for type IIA and type IIB but the RR sectors differ

$$
\begin{aligned}
\mathbf{8_s} \otimes \mathbf{8_c} &= \mathbf{8_v} \oplus \mathbf{56_t} = C_{(1)} \oplus C_{(3)} & \text{(IIA)}, \\
\mathbf{8_s} \otimes \mathbf{8_s} &= \mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35_s} = C_{(0)} \oplus C_{(2)} \oplus C_{(4)}^+ & \text{(IIB)}.
\end{aligned}
\tag{3.3}
$$

Here $C_{(n)}$ is a RR n-form potential and $C_{(4)}$ has a self-dual field strength, more will be said about type IIB in the next section. The content of the products in (3.3) can be understood by considering the index structure of the Γ-matrices in eight dimensions. The fields of the NS-NS and R-R sectors are bosonic and we now turn to the fermionic NS-R and R-NS sectors. The field contents are

$$
\begin{aligned}
\mathbf{8_v} \otimes \mathbf{8_c} &= \mathbf{8_s} \oplus \mathbf{56_s} = \lambda_{\dot\alpha} \oplus \psi_\alpha^\mu, \\
\mathbf{8_v} \otimes \mathbf{8_s} &= \mathbf{8_c} \oplus \mathbf{56_c} = \lambda_\alpha \oplus \psi_{\dot\alpha}^\mu,
\end{aligned}
\tag{3.4}
$$

where we take one copy of each set for type IIA but two copies of the lower set for type IIB. The action for type IIA supergravity is

$$
\begin{aligned}
S_{\text{IIA}} = \frac{1}{2} \int d^{10}x \sqrt{-g} \bigg(& e^{-2\phi}[R + 4(\partial\phi)^2 - \frac{1}{2\cdot 3!}H_{(3)}^2] \\
& -2[\frac{1}{2\cdot 2!}R_{(2)}^2 + \frac{1}{2\cdot 4!}R_{(4)}^2] \bigg) - \frac{1}{4}\int dC_{(3)} \wedge dC_{(3)} \wedge B_{(2)}, \quad
\end{aligned}
\tag{3.5}
$$

where $H_{(3)} = dB_{(2)}$, $R_{(2)} = dC_{(1)}$ and $R_{(4)} = dC_{(3)} + H_{(3)} \wedge C_{(1)}$.

3.2   Type IIB

The type IIB supergravity in ten dimensions has an SL(2,$\mathbb{R}$) invariance, which is broken to SL(2,$\mathbb{Z}$) at the string level by charge quantization. The field content is two scalars in the coset space SL(2,$\mathbb{R}$)/U(1) (the dilaton, ϕ, from the NS-NS sector and the axion, χ, from the R-R sector), a self-dual R-R 5-form field strength $H_{(5)}$, which is an SL(2,$\mathbb{R}$) singlet, and a real SL(2,$\mathbb{R}$) doublet of 3-form field strengths, $H_{(3)r} = dC_{(2)r}$ ($r = 1, 2$), corresponding to the NS-NS and R-R field strengths respectively (r is an SL(2,$\mathbb{R}$) index), and finally the metric. We will use a formulation of the theory where the SL(2,$\mathbb{R}$) covariance is manifest [20, 21] in the notation of [22, 23]. Due to the self-dual five-form field strength it is complicated to construct a covariant action for this theory [24, 25, 26].

The scalars are described by the complex doublet $\mathcal{U}^r$ obeying the SL(2,$\mathbb{R}$) invariant constraint

$$
\frac{i}{2}\epsilon_{rs}\mathcal{U}^r\bar{\mathcal{U}}^s = 1,
\tag{3.6}
$$

where we use the convention that $\epsilon^{12} = 1$ ($\epsilon_{12} = -1$). If we gauge the U(1) we are left with the two physical scalars, which can be obtained through $\tau = \mathcal{U}^1/\mathcal{U}^2 = \chi + ie^{-\phi}$. The left-invariant SL(2,$\mathbb{R}$) Maurer-Cartan forms are

$$
Q = \frac{1}{2}\epsilon_{rs}d\mathcal{U}^r\bar{\mathcal{U}}^s, \qquad P = \frac{1}{2}\epsilon_{rs}d\mathcal{U}^r\mathcal{U}^s.
\tag{3.7}
$$

We normalize the U(1) charge to 1 for the scalar doublet and therefore $\mathcal{U}^r \to \mathcal{U}^r e^{i\theta}$ under local U(1) transformations. From the expressions for Q and P above we see that they transform as $Q \to Q + d\theta$ and $P \to P e^{2i\theta}$ under local U(1) transformations, *i.e.*, Q is a U(1) gauge field and P has U(1) charge 2. The Maurer-Cartan equations are

$$DP = 0, \qquad dQ - iP \wedge \bar{P} = 0, \tag{3.8}$$

where the covariant derivative $D = d - ieQ$ acts from the right and e is the U(1) charge. It is important to note that the scalar doublet transforms in a simple way under SL(2,$\mathbb{R}$), *i.e.*, contravariantly, compared to the physical scalars, which transforms in a complicated way via τ. The main advantage of using this formalism is that we can combine the contravariant scalar doublet with a covariant doublet, *e.g.*, the doublet of 3-form field strengths, in order to get an SL(2,$\mathbb{R}$) invariant object. The object constructed in this way is in general complex, since the scalar doublet is complex, and will be denoted by calligraphic letters. The real doublet can be retrieved by using the scalar doublet, *e.g.*,

$$\mathcal{H}_{(3)} \equiv \mathcal{U}^r H_{(3)r}, \qquad H_{(3)r} = \epsilon_{rs} \mathrm{Im}(\mathcal{U}^s \bar{\mathcal{H}}_{(3)}). \tag{3.9}$$

The equations of motion[1] can now be written as

$$D*P + \frac{i}{4}\mathcal{H}_{(3)} \wedge *\mathcal{H}_{(3)} = 0 \tag{3.10}$$

$$D*\mathcal{H}_{(3)} - i * \bar{\mathcal{H}}_{(3)} \wedge P - iH_{(5)} \wedge \mathcal{H}_{(3)} = 0 \tag{3.11}$$

and the Bianchi identities are

$$D\mathcal{H}_{(3)} + i\bar{\mathcal{H}}_{(3)} \wedge P = 0 \tag{3.12}$$

$$dH_{(5)} - \frac{i}{2}\mathcal{H}_{(3)} \wedge \bar{\mathcal{H}}_{(3)} = 0. \tag{3.13}$$

Finally, we also have the Einstein equations

$$R_{MN} = 2\bar{P}_{(M}P_{N)} + \frac{1}{4}\bar{\mathcal{H}}_{(M}{}^{RS}\mathcal{H}_{N)RS} - \frac{1}{48}g_{MN}\bar{\mathcal{H}}_{RST}\mathcal{H}^{RST} + \frac{1}{96}H_{(M}{}^{RSTU}H_{N)RSTU}. \tag{3.14}$$

3.3 Type I

The type I superstring spectrum is obtained from the type IIB spectrum by applying an orientifold projection which only keeps the left-right symmetric states. The resulting type I string theory is therefore unoriented and chiral. In the NS-NS sector (3.2) we see that the two-form $B_{\mu\nu}$ is projected out. In the RR sector (3.3)

---

[1]We have rescaled the fields as $H_{(3)r} \to \frac{1}{2}H_{(3)r}$ and $H_{(5)} \to \frac{1}{4}H_{(5)}$ compared to Paper IV in order to conform to the most commonly used conventions.

only the two-form $C_{(2)}$ survives since it is $\Gamma^{(1)}$, $\Gamma^{(2)}$ and $\Gamma^{(5)}$ that are the symmetric Γ-matrices[2]. In the fermionic sector (3.4), the orientifold projection picks out the linear combination (NS-R)+(R-NS), which is the reason why only one of each representation survives. For type I supergravity we thus get the field content

$$[\mathbf{8_V} \otimes \mathbf{8_V} \oplus \mathbf{8_S} \otimes \mathbf{8_S}]_{\text{Symmetric}} = \mathbf{1} \oplus \mathbf{28} \oplus \mathbf{35_V} = \phi \oplus C_{(2)} \oplus g_{\mu\nu} \,,$$
$$[\mathbf{8_C} \otimes \mathbf{56_C}]_{\text{Symmetric}} = \lambda_\alpha \oplus \psi^\mu_{\dot\alpha} \,. \tag{3.15}$$

The string theory obtained in this way is however anomalous and in order to cancel the anomaly we have to add an open string sector with SO(32) Chan-Paton factors. For the low-energy theory this corresponds to adding a super-Yang-Mills theory with gauge group SO(32). The low-energy effective action for the type I theory is then

$$S_{\text{I}} = \int d^{10}x\sqrt{-g}\left(e^{-2\phi}[R + 4(\partial\phi)^2] - \frac{1}{12}H_{(3)}^2 - \frac{1}{4}e^{-\phi}\operatorname{tr}F^2\right)\,, \tag{3.16}$$

where $H_{(3)} = dC_{(2)}$ and $F = dA + A \wedge A$ is the SO(32) Chan-Paton gauge field strength.

3.4   Heterotic

Since the left- and right-moving sectors of the closed string can be chosen independently, we can choose to have the degrees of freedom of a bosonic string in the left-moving sector and the degrees of freedom of a fermionic string in the right-moving sector. In order to get the spacetime dimensions to match, we must compactify 16 of the 26 dimensions of the bosonic sector. To be specific, the 16 dimensions form a torus $\mathbb{R}^{16}/\Lambda$ where Λ must, due to modular invariance, be a euclidean, even, self-dual lattice. There are actually only two lattices of this kind, the root lattice of $\text{E}_8 \times \text{E}_8$ and the lattice of $\text{Spin}(32)/\mathbb{Z}_2$. When viewed as a theory in ten dimensions, these lattices will give rise to the space-time gauge symmetry of the heterotic string, which will be $\text{E}_8 \times \text{E}_8$ or SO(32). This theory is tachyon free since the potential tachyon from the bosonic sector is removed from the spectrum due to the level-matching constraint. The massless spectrum is seen to be the same as for the type I string (without Chan-Paton factors) together with an $N = 1$ super-Yang-Mills multiplet with the gauge field A_μ in the adjoint representation of $\text{E}_8 \times \text{E}_8$ or SO(32). In the latter case the massless spectrum is identical to the massless type I spectrum, including Chan-Paton factors, hinting at a possible duality between these theories, as we will see in Chapter 5. The action for heterotic supergravity is

$$S_{\text{het}} = \int d^{10}x\sqrt{-g}e^{-2\phi}\left(R + 4(\partial\phi)^2 - \frac{1}{12}H_{(3)}^2 - \frac{1}{4}\operatorname{tr}F^2\right)\,, \tag{3.17}$$

where $H_{(3)} = dB_{(2)}$ and the gauge field strength $F = dA + A \wedge A$, where A transforms in the adjoint of either SO(32) or $\text{E}_8 \times \text{E}_8$.

---

[2]We do not have to go higher than $\Gamma^{(5)}$ since the higher Γ-matrices can be obtained by dualizing the lower ones. In the same way, we only consider up to $\Gamma^{(5)}$-terms in eleven dimensions.

3.5 Eleven-dimensional supergravity

Supergravity takes its simplest form when formulated in eleven dimensions [27]. The bosonic part of the action is simply

$$-2\kappa^2 S_{11} = \int d^{11}x \sqrt{-g} \left(R + \frac{1}{2 \cdot 4!} H^{mnpq} H_{mnpq} \right) + \frac{1}{6} \int C \wedge H \wedge H \,, \qquad (3.18)$$

where $H_{(4)} = dC_{(3)}$ is the four-form field strength. Eleven turns out to be the maximal dimension if we require that there are no particles with spin higher than two. If we do not want to make assumptions regarding higher spins we would still get into trouble by going beyond eleven dimensions since, as we shall see in the next chapter, eleven is the maximal dimension admitting supersymmetric extended objects [28]. At first, eleven dimensional supergravity was just regarded as a useful device for deriving supergravities in four dimensions. It was, however, well known that type IIA supergravity could be obtained by reducing eleven dimensional supergravity on an S^1, but it was not until the discovery that the string in ten dimensions could be obtained by wrapping the membrane in eleven dimensions around the circle [29, 30] that a fundamental theory in eleven dimensions was seriously considered.

Eleven dimensional supergravity has also been formulated in superspace [31, 32] and it is this formulation we will use. The advantage of the superspace formulation is that supersymmetry is manifest throughout all calculations. In this section we will derive eleven dimensional supergravity[3] using superspace techniques and in the next section we will generalize this treatment to obtain the most general geometrical structure in eleven dimensional superspace. The aim of this generalization is to investigate possible M-theory corrections to ordinary supergravity.

The coordinates are denoted $z^M = (x^m, \theta^\mu)$, where m enumerates the 11 bosonic and μ the 32 real fermionic coordinates, respectively. The tangent space has as structure group the Lorentz group (not the superversion of it), and hence one introduces a supervielbein and a superconnection

$$E_M{}^A(z) \,, \qquad \omega_{MA}{}^B(z) \,, \qquad (3.19)$$

where $\omega_{MA}{}^B = (\omega_{Ma}{}^b, -\frac{1}{4}(\Gamma^a{}_b)_\alpha{}^\beta \omega_{Ma}{}^b)$. A flat superindex $A = (a, \alpha)$, obtained from a curved one using the vielbein, contains an SO(10,1) vector index a and a (Majorana) spinor index α. Note that $\omega_{Ma}{}^\beta = \omega_{M\alpha}{}^b = 0$ since the connection is Lie algebra valued and therefore do not mix vector and spinor indices. The two-form field strengths corresponding to the potentials in (3.19) are

$$T^A = DE^A = dE^A + E^B \wedge \omega_B{}^A, \qquad (3.20)$$

$$R_A{}^B = d\omega_A{}^B + \omega_A{}^C \wedge \omega_C{}^B, \qquad (3.21)$$

where the covariant derivative $D = d + \omega$ is acting from the right. The associated Bianchi identities are

$$DT^A = E^B \wedge R_B{}^A, \qquad (3.22)$$

$$DR_A{}^B = 0 \,. \qquad (3.23)$$

---

[3]For details see [33].

By imposing constraints on the torsion the Bianchi identities cease to be identities and have to be solved. Since the second Bianchi identity is automatically satisfied if the first one is [34], we will only have to analyze the first Bianchi identity. For simplicity, we also introduce the four-form field strength $H_{(4)} = dC_{(3)}$, satisfying the Bianchi identity

$$dH_{(4)} = 0 \ . \tag{3.24}$$

As was shown by Candiello and Lechner [35], it is not necessary to introduce the four-form by hand; it emerges from the analysis. In the next section where we study generalized supergravity, we will perform a complete analysis not introducing the four-form by hand.

The supergravity multiplet consists of the field strengths

Field	Dimension (mass)
R_{abcd}	2
$\partial_{[a}\psi_{b]}{}^\gamma$	3/2
H_{abcd}	1

corresponding to the potentials

Field	Degrees of freedom
$e_m{}^a$	44 bosonic
$\psi_m{}^\gamma$	128 fermionic
B_{mnp}	84 bosonic

We note that we have the same number of bosonic and fermionic degrees of freedom and the theory is supersymmetric as it should be since it is formulated in superspace[4]. In order to facilitate the analysis, we can determine which components of the torsion and the four-form that can potentially be non-vanishing given the field content above. Since we are working with Bianchi identities the only objects that can appear are the field strengths. At dimension 1, the torsion components $T_{\alpha\beta}{}^\gamma$ contain two four-form representations that can be proportional to H_{abcd}, motivating the Ansatz

$$T_{\alpha\beta}{}^\gamma = xH_{ac_1c_2c_3}(\Gamma^{c_1c_2c_3})_\beta{}^\gamma + yH_{c_1\ldots c_4}(\Gamma_a{}^{c_1\ldots c_4})_\beta{}^\gamma. \tag{3.25}$$

At dimension $\frac{3}{2}$ the torsion component $T_{ab}{}^\gamma$ can be proportional to the gravitino field strength. We can make a general Ansatz for $T_{ab}{}^\gamma$, decomposed into a sum of irreducible representations, as

$$T_{ab}{}^\gamma = G_{ab}{}^\gamma + 2G_{[a}{}^\beta(\Gamma_{b]})_\beta{}^\gamma + G^\beta(\Gamma_{ab})_\beta{}^\gamma \ , \tag{3.26}$$

where $G_{ab}{}^\gamma$ and $G_a{}^\gamma$ are Γ-traceless, i.e., $G_{ab}{}^\gamma(\Gamma^b)_\gamma{}^\alpha = G_a{}^\gamma(\Gamma^a)_\gamma{}^\alpha = 0$. Other non-vanishing components can only be proportional to Γ-matrices and can therefore only occur at dimension 0. Considering the index structure, we get two possible terms,

$$T_{\alpha\beta}{}^c = 2(\Gamma^c)_{\alpha\beta} \tag{3.27}$$

---

[4]It is not enough to have an equal numbers of bosons and fermions in order to have a supersymmetric theory, the multiplet must also close under supersymmetry transformations. This, however, is manifest in a superspace formulation.

and

$$H_{ab\gamma\delta} = 2(\Gamma_{ab})_{\gamma\delta} \,, \tag{3.28}$$

where we have chosen a convenient normalization. Both these expressions turn out to be required. All other components of $T_{AB}{}^C$ and H_{ABCD}, not related to those above by, *e.g.*, derivatives, must be zero. Using this information, we can now start solving the Bianchi identities.

In order to analyze the Bianchi identities (3.22) and (3.24) it is convenient to rewrite them as tensor equations,

$$D_{[B}T_{CD)}{}^A + T_{[BC}{}^E T_{|E|D)}{}^A - R_{[BCD)}{}^A = 0 \tag{3.29}$$

and

$$D_{[A}H_{BCDE)} + 2T_{[AB}{}^F H_{|F|CDE)} = 0, \tag{3.30}$$

where $[\cdots)$ denotes graded symmetrization. We start by examining the Bianchi identity for $H_{(4)}$. The equations at dimension $-\frac{1}{2}$ and $\frac{1}{2}$ are empty. Using the Fierz identity

$$(\Gamma_a)_{(\alpha\beta}(\Gamma^{ab})_{\gamma\delta)} = 0 \,, \tag{3.31}$$

we see that the dimension 0 equation is satisfied too. At dimension 1 we are able to relate the two four-forms in $T_{\alpha\beta}{}^\gamma$ to H_{abcd}, yielding $x = -\frac{1}{36}$ and $y = -\frac{1}{288}$ in the Ansatz (3.25). The equation at dimension $\frac{3}{2}$ will be used below, together with the two equations of this dimension from the torsion Bianchi identity, to obtain the equation of motion for the spin $\frac{3}{2}$ field. Finally at dimension 2 we see that H_{abcd} is closed, enabling it (at least locally) to be written as the derivative of a three-form potential. This is crucial for the supersymmetric matching of bosonic and fermionic degrees of freedom mentioned above.

We now turn to the torsion Bianchi identity. The equation at dimension $\frac{1}{2}$ is empty. We get two equations at dimension 1, which only provide a consistency check. The two equations at dimension $\frac{3}{2}$, together with the one from the four-form Bianchi identity, imply that $G_a{}^\gamma = G^\gamma = 0$ and also give the equation of motion for the spin $\frac{3}{2}$ field

$$\partial_b \psi_c{}^\gamma (\Gamma^{abc})_{\gamma\delta} = 0 \,, \tag{3.32}$$

where the gravitino field strength sits in the torsion like

$$T_{ab}{}^\gamma = G_{ab}{}^\gamma \sim \partial_{[a}\psi_{b]}{}^\gamma. \tag{3.33}$$

At dimension 2 we get Einstein's equations

$$R_{ab} - \frac{1}{2}\eta_{ab}R = \frac{1}{96}\eta_{ab}H^2 - \frac{1}{12}H_{a(3)}H_b{}^{(3)} \tag{3.34}$$

and the equation of motion for H_{abcd}

$$D^d H_{dabc} = \frac{1}{1152} \epsilon_{abc}{}^{(4)(\tilde{4})} H_{(4)} H_{(\tilde{4})} \,.$$ (3.35)

Finally, the last equation at dimension $\frac{5}{2}$,

$$D_{[b} T_{cd]}{}^\alpha + T_{[bc}{}^\epsilon T_{|\epsilon|d]}{}^\alpha = 0 \,,$$ (3.36)

contains the Bianchi identity for the spin $\frac{3}{2}$ field.

By imposing constraints on the torsion and then solving the Bianchi identities for the torsion and the four-form, we have obtained the equations of motion for eleven dimensional supergravity. One way of motivating the torsion constraints is to study the differential geometry of certain function superspaces [36, 37]. Howe has shown that eleven dimensional supergravity follows from imposing only the single constraint $T_{\alpha\beta}{}^c \sim (\Gamma^c)_{\alpha\beta}$ on the dimension zero component of the torsion [38]. As in the analysis by Candiello and Lechner [35], the four-form is not introduced by hand. In the next section we will relax this constraint in order to study the most general geometrical structure in eleven dimensions compatible with supersymmetry. In this way we can get information regarding the allowed structure of corrections from M-theory and we also avoid the problem of having to enforce *ad hoc* torsion constraints.

3.6   Generalized eleven-dimensional supergravity

Ordinary eleven-dimensional supergravity is the low energy effective theory of M-theory. When compactified on a circle, eleven-dimensional supergravity yields type IIA supergravity, as will be explained in more detail in Chapter 5. Type IIA supergravity is the low energy effective theory of type IIA string theory, and by studying the string theory we can derive higher order corrections to the type IIA supergravity action. From the non-linear σ-model action for the string in (2.18), we see that there are two parameters we can expand in, giving quantum corrections to the supergravity action. The first parameter is α', the σ-model loop-counting parameter and the second parameter is g_s, the string loop-counting parameter, whose power is determined by the genus of the world-sheet of the string. For each power of g_s there will be a tower of α' corrections. Since $\alpha' \equiv \ell_s^2$ is a dimensionful parameter and the action is dimensionless we must group α' together with some other dimensionful object, *e.g.,* the curvature, in the expansion. Corrections to the tree-level type IIA action is shown to enter first at order ℓ_s^6 [39, 40, 41, 42]. These kinds of higher curvature corrections to the effective action is also expected to appear in eleven dimensions since we can lift the result from ten dimensions. In eleven dimensions there is, however, no simple way of deriving these terms and one must rely on anomaly cancellation arguments [43, 44] or superparticle loop calculations [45, 39, 46, 47, 41] together with the connection to string theory via dimensional reduction. Supersymmetry now puts severe restrictions on the structure of permitted corrections [40] and it would be very interesting to investigate what kinds of higher derivative terms

that are permitted by supersymmetry in eleven dimensions. This is the motivation for the work in Paper V. We are interested in the constraints imposed by supersymmetry. It is convenient to use the superspace formulation where supersymmetry is manifest. The correction terms can be viewed as *possible* M-theory corrections to ordinary supergravity, but we can not say which correction terms that will actually be used by M-theory. Related work, based on lifting results from ten dimensions, have been presented in [48, 49]. The strategy of relaxing the ordinary constraints to get information about possible higher order corrections has also been applied to maximally supersymmetric Yang-Mills theory in ten dimensions [50, 51].

We generalize the torsion constraint[5] $T_{\alpha\beta}{}^c = 2(\Gamma^c)_{\alpha\beta}$, leading to ordinary supergravity, to [53, 54]

$$T_{\alpha\beta}{}^c = 2\left((\Gamma^d)_{\alpha\beta}X_d{}^c + \frac{1}{2}(\Gamma^{d_1 d_2})_{\alpha\beta}X_{d_1 d_2}{}^c + \frac{1}{5!}(\Gamma^{d_1\dots d_5})_{\alpha\beta}X_{d_1\dots d_5}{}^c\right), \qquad (3.37)$$

where we have included the other two symmetric Γ-matrices. The X's can be decomposed into irreducible representations and the representational content of the dimension 0 torsion component, as well as the other torsion components, are shown in Tab. 3.1. We also follow Howe and use Weyl superspace, which means adding a Weyl part $K_{MA}{}^B$ to the connection used in the previous section,

$$\Omega_{MA}{}^B = \omega_{MA}{}^B + K_{MA}{}^B, \qquad (3.38)$$

where $K_{MA}{}^B = (2K_M\delta_a{}^b, K_M\delta_\alpha{}^\beta)$.

We can now enforce conventional constraints [55, 56] to eliminate some of the representations in the torsion. This amounts to using the arbitrariness in the distinction between spin connection and torsion as well as the freedom to re-define the vielbeins. To make this explicit, we can write down the expression for how the torsion transforms under a variation of the vielbein and the spin connection

$$\delta T_{AB}{}^C = 2D_{[A}H_{B)}{}^C - 2H_{[A}{}^F T_{|F|B)}{}^C + T_{AB}{}^F H_F{}^C + 2\delta\Omega_{[AB)}{}^C \qquad (3.39)$$

where $H_A{}^B = E_A{}^M \delta E_M{}^B$. By writing out the various index combinations, to linear order in the fields, we get

$$\delta T_{\alpha\beta}{}^c = 2D_{(\alpha}H_{\beta)}{}^c - 4H_{(\alpha}{}^\delta(\Gamma^c)_{|\delta|\beta)} + 2(\Gamma^e)_{\alpha\beta}H_e{}^c, \qquad (3.40)$$

$$\delta T_{\alpha b}{}^c = D_\alpha H_b{}^c - D_b H_\alpha{}^c + 2H_b{}^\gamma(\Gamma^c)_{\gamma\alpha} + \delta\Omega_{\alpha b}{}^c, \qquad (3.41)$$

$$\delta T_{\alpha\beta}{}^\gamma = 2D_{(\alpha}H_{\beta)}{}^\gamma + 2(\Gamma^c)_{\alpha\beta}H_c{}^\gamma + 2\delta\Omega_{(\alpha\beta)}{}^\gamma, \qquad (3.42)$$

$$\delta T_{ab}{}^c = 2D_{[a}H_{b]}{}^c + 2\delta\Omega_{[ab]}{}^c \qquad (3.43)$$

and

$$\delta T_{a\beta}{}^\gamma = D_a H_\beta{}^\gamma - D_\beta H_a{}^\gamma + \delta\Omega_{a\beta}{}^\gamma. \qquad (3.44)$$

---

[5] The constraint analysis presented here is an explicit account of that performed in Paper V and will be part of a forthcoming publication [52].

dim 0:	$T_{\alpha\beta}{}^c$	(00000)	(00002)	(00010)	(00100)
		(01000)	(10000)	(10002)	(11000)
		(20000)			
dim $\frac{1}{2}$:	$T_{\alpha b}{}^c$	2(00001)	(01001)	2(10001)	(20001)
	$T_{\alpha\beta}{}^\gamma$	3(00001)	(00003)	(00011)	(00101)
		2(01001)	3(10001)		
dim 1:	$T_{ab}{}^c$	(00100)	(10000)	(11000)	
	$T_{a\beta}{}^\gamma$	(00000)	2(00002)	2(00010)	2(00100)
		2(01000)	2(10000)	(10002)	(10010)
		(10100)	(11000)	(20000)	
dim $\frac{3}{2}$:	$T_{ab}{}^\gamma$	(00001)	(01001)	(10001)	

Table 3.1: Representations in $T_{AB}{}^C$, in Dynkin notation, before enforcing the conventional constraints.

We can now directly use the spin connection to eliminate representations in the torsion in Eqs. (3.40)-(3.44). It is important to keep in mind, however, that due to the Lorentz condition $\Omega_{A\alpha}{}^\beta$ and $\Omega_{Aa}{}^b$ are related and we can therefore only use one of these to remove parts of the torsion. In those cases that we can solve *algebraically* for a representation in $H_A{}^B$ in terms of a representation in the torsion, we can use that part of $H_A{}^B$ to remove the torsion part in question. We can therefore not use the terms in which there is a derivative acting on $H_A{}^B$. Since all the components of the torsion have dimensions[6] ≥ 0, we can not use the dimension $-\frac{1}{2}$ part $H_\alpha{}^b$ to cancel parts of the torsion. In Tab. 3.2 we have collected the results of the analysis of the conventional constraints. Note that there is not a unique choice of which part of $H_A{}^B$ or the spin connection that is used to cancel a specific part of the torsion and hence alternatives to the choices in Tab. 3.2 exist. We can now write down all the representations that are left in the torsion after imposing the conventional constraints

$$\text{dim } 0: \qquad T_{\alpha\beta}{}^c = 2(\Gamma^c)_{\alpha\beta} + (\Gamma^{d_1 d_2})_{\alpha\beta} X_{d_1 d_2,}{}^c \qquad\qquad (11000)$$

$$+ \frac{2}{5!}(\Gamma^{d_1 \ldots d_5})_{\alpha\beta} X_{d_1 \ldots d_5,}{}^c \qquad\qquad (10002)$$

---

[6]Each vector (spinor) index contributes with mass dimension 1 ($\frac{1}{2}$) if it sits downstairs and contributes with the opposite sign if it sits upstairs, as can be seen from, *e.g.,* Eq. (3.20).

$H_\alpha{}^b$ (dim $-\frac{1}{2}$)	(00001) (10001)	Not used	
$H_a{}^b$ (dim 0)	(00000) (01000) (20000)	$T_{\alpha\beta}{}^c$	(00000) (01000) (20000)
$H_\alpha{}^\beta$ (dim 0)	(00000) (01000)	Not used	
	(10000) (00100) (00010) (00002)	$T_{\alpha\beta}{}^c$	(10000) (00100) (00010) (00002)
$H_a{}^\beta$ (dim $\frac{1}{2}$)	(00001) (10001)	$T_{\alpha\beta}{}^\gamma$	(00001) (10001)
$\Omega_{\alpha b}{}^c$ (dim $\frac{1}{2}$)	(00001) (00001) (10001) (01001)	$T_{\alpha b}{}^c$	(00001) (00001) (10001) (01001)
$\Omega_{ab}{}^c$ (dim 1)	(10000)	$T_{a\beta}{}^\gamma$	(10000)
	(10000) (11000) (00100)	$T_{ab}{}^c$	(10000) (11000) (00100)

Table 3.2: Summary of which parts of $H_A{}^B$ that is used to remove specific parts of the torsion. Note that the connection $\Omega_{Ab}{}^c$ contains both Lorentz and Weyl parts.

$$\text{dim } \frac{1}{2}: \qquad T_{\alpha b}{}^{c} \;=\; S_b{}^c{}_\alpha \tag{20001}$$

$$+\, 2(\Gamma_{(b}S_{d)})_\alpha \eta^{cd} \tag{10001}$$

$$T_{\alpha\beta}{}^{\gamma} \;=\; \frac{1}{120}\Gamma_{\alpha\beta}^{d_1\ldots d_5} Z_{d_1\ldots d_5}{}^{\gamma} \tag{00003}$$

$$+\,\frac{1}{24}\Gamma_{\alpha\beta}^{d_1\ldots d_5}(\Gamma_{d_1}Z_{d_2\ldots d_5})^{\gamma} \tag{00011}$$

$$+\,\frac{1}{12}\Gamma_{\alpha\beta}^{d_1\ldots d_5}(\Gamma_{d_1 d_2}Z_{d_3 d_4 d_5})^{\gamma} \tag{00101}$$

$$+\,\frac{1}{12}\Gamma_{\alpha\beta}^{d_1\ldots d_5}(\Gamma_{d_1 d_2 d_3}Z_{d_4 d_5})^{\gamma} + \frac{1}{2}\Gamma_{\alpha\beta}^{d_1 d_2} Y_{d_1 d_2}{}^{c} \qquad 2(01001)$$

$$+\,\frac{1}{24}\Gamma_{\alpha\beta}^{d_1\ldots d_5}(\Gamma_{d_1\ldots d_4}Z_{d_5})^{\gamma} + \Gamma_{\alpha\beta}^{d_1 d_2}(\Gamma_{d_1} Y_{d_2})^{\gamma} \qquad 2(10001)$$

$$+\,\frac{1}{120}\Gamma_{\alpha\beta}^{d_1\ldots d_5}(\Gamma_{d_1\ldots d_5}Z)^{\gamma} + \frac{1}{2}\Gamma_{\alpha\beta}^{d_1 d_2}(\Gamma_{d_1 d_2} Y)^{\gamma} \qquad 2(00001)$$

$$\text{dim } 1: \qquad T_{ab}{}^{c} \;=\; 0$$

$$T_{a\beta}{}^{\gamma} \;=\; \frac{1}{24}(\Gamma^{d_1\ldots d_4})_\beta{}^{\gamma} A_{d_1\ldots d_4 a} + \frac{1}{120}(\Gamma_a{}^{d_1\ldots d_5})_\beta{}^{\gamma} A'_{d_1\ldots d_5} \quad 2(00002)$$

$$+\,\frac{1}{6}(\Gamma^{d_1 d_2 d_3})_\beta{}^{\gamma} A_{d_1 d_2 d_3 a} + \frac{1}{24}(\Gamma_a{}^{d_1\ldots d_4})_\beta{}^{\gamma} A'_{d_1\ldots d_4} \quad 2(00010)$$

$$+\,\frac{1}{2}(\Gamma^{d_1 d_2})_\beta{}^{\gamma} A_{d_1 d_2 a} + \frac{1}{6}(\Gamma_a{}^{d_1 d_2 d_3})_\beta{}^{\gamma} A'_{d_1 d_2 d_3} \quad 2(00100)$$

$$+\,(\Gamma^{d})_\beta{}^{\gamma} A_{da} + \frac{1}{2}(\Gamma_a{}^{d_1 d_2})_\beta{}^{\gamma} A'_{d_1 d_2} \quad 2(01000)$$

$$+\,(\Gamma_a{}^{d})_\beta{}^{\gamma} A'_{d} \tag{10000}$$

$$+\,(\Gamma_a)_\beta{}^{\gamma} A' \tag{00000}$$

$$+\,\frac{1}{120}(\Gamma^{d_1\ldots d_5})_\beta{}^{\gamma} B_{d_1\ldots d_5, a} \tag{10002}$$

$$+\,\frac{1}{24}(\Gamma^{d_1\ldots d_4})_\beta{}^{\gamma} B_{d_1\ldots d_4, a} \tag{10010}$$

$$+\,\frac{1}{6}(\Gamma^{d_1 d_2 d_3})_\beta{}^{\gamma} B_{d_1 d_2 d_3, a} \tag{10100}$$

$$+\,\frac{1}{2}(\Gamma^{d_1 d_2})_\beta{}^{\gamma} B_{d_1 d_2, a} \tag{11000}$$

$$+\,(\Gamma^{d})_\beta{}^{\gamma} B_{d, a} \tag{20000}$$

$$\text{dim } \frac{3}{2}: \qquad T_{ab}{}^{\gamma} \;=\; t_{ab}{}^{\gamma} \tag{01001}$$

$$+\, 2(\Gamma_{[a}t_{b]})^{\gamma} \tag{10001}$$

$$+\,(\Gamma_{ab}t)^{\gamma} \tag{00001}$$

Using the torsion components above we start solving the Bianchi identity (3.22) in Paper V. We start at dimension $\frac{1}{2}$ and then work our way through the equations with progressively higher dimensions. The analysis is very technical For each dimension of the Bianchi identity we have to decompose the equation into irreducible representations and study each representation separately. This requires a lot of Γ-matrix algebra and Fierz transformations to be performed, motivating the development of the Mathematica package GAMMA in paper VIII, which is capable of performing both Γ-matrix algebra and Fierz transformations. In order to simplify the analysis we have so far only taken into account the (10002) part $T_{\alpha\beta}{}^c$; in the complete analysis we of course also have to consider the (11000) part. We have also restricted ourselves to a linear analysis where ordinary supergravity fields and the auxiliary ones are treated on equal footing. We also drop vector derivatives on the auxiliary superfield, which can be thought of as a kind of low-energy expansion. The analysis is not yet complete, but in Paper V we have obtained a correction to the spin $\frac{3}{2}$ field equation showing that the prescription we use indeed takes us off-shell.

4

p-branes

p-branes are solitonic solutions to the low-energy effective supergravity theories. An important property is that they interpolate between different vacua. This means that they are topological in nature and therefore their stability is guaranteed. Since they are topological objects they are not included in the perturbative spectrum and are thus intrinsically non-perturbative. By saturating a Bogomol'nyi bound, *i.e.,* having the charge equal to the mass, in appropriate units, we get BPS states, *i.e.,* states preserving some fraction of the supersymmetry. They therefore belong to short supersymmetry multiplets and are thus protected from quantum corrections. This means that BPS states can give us vital information about the exact theory even at strong coupling. This feature makes BPS p-branes the best candidate to use in exploring the non-perturbative structure of string theory and they play a central rôle in verifying the duality conjectures in the previous chapter.

In this chapter we are first going to derive the *brane-scan* in Fig. 4.1, which shows the p-branes allowed by supersymmetry. We are then going to review some of the salient features of p-branes and explain how the M2 and M5 brane in $D = 11$ are related to various branes in type IIA string theory. Finally, we are going say a few words about D-branes, *e.g.,* explain how they arise when T-dualizing an open string and why they are dynamical objects.

4.1   The brane-scan

Unlike bosonic p-branes, which can be formulated in an arbitrary spacetime dimension D, supersymmetric p-branes can only be formulated for certain combinations of $d = p + 1$ and D. This restriction, enforced by supersymmetry, gives rise to the brane-scan in Fig. 4.1. It is important to note that the brane-scan only tells us which branes are *not forbidden* by supersymmetry. If these branes actually *exist* as solutions to any supersymmetric field theory is another question.

Let us now derive the brane-scan[1]. We start by considering the world volume

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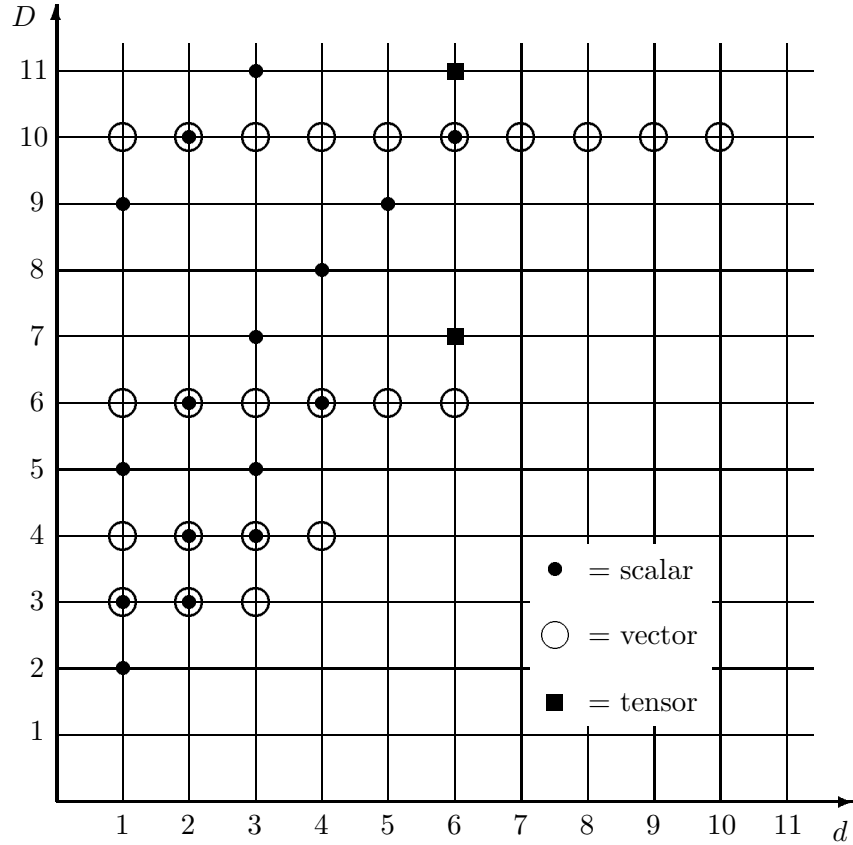[1]We are not considering Kaluza-Klein monopoles in this analysis.

Figure 4.1: The brane-scan.

scalar multiplets. This analysis can be done using two different methods. The first method is to list all scalar supermultiplets and interpret the space-time dimension as $D = d +$ the number of scalars. We are only considering space-times having Minkowski signature. The second method, which we will use, is to require world-volume supersymmetry by matching the numbers of bosonic and fermionic on-shell degrees of freedom in the superspace embeddings $X^a(\xi)$ and $\theta^\alpha(\xi)$. The bosonic degrees of freedom are

$$N_B = D - d, \tag{4.1}$$

where we have taken into account the reparameterization invariance of the world-volume. The bosonic degrees of freedom correspond in this case to the directions transverse to the brane. Motion in these directions will give rise to the simplest example of Goldstone modes, *i.e., scalar Goldstone modes*. The concept of Goldstone modes is extended to the case of Goldstone *tensor* modes of arbitrary rank in Paper II and will be reviewed in Chapter 7.

By taking into account that kappa symmetry halves the fermionic degrees of

freedom and going on-shell halves them again, we obtain

$$N_F = \frac{1}{2}mn = \frac{1}{4}MN, \tag{4.2}$$

where m (M) is the number of real components of an irreducible spinor in d (D) dimensions and n (N) is the numbers of supersymmetries. By matching bosonic and fermionic degrees of freedom we get

$$D - d = \frac{1}{2}mn = \frac{1}{4}MN, \tag{4.3}$$

which must be fulfilled in order to allow the existence of a scalar multiplet (for $d > 2$). By consulting Tab. 4.1 we find that Eq. (4.3) has eight solutions, represented by dots in Fig. 4.1. We also see that $D_{\text{MAX}} = 11$ since $M \geq 64$ for $D \geq 12$ and therefore (4.3) can not be satisfied for $D \geq 12$. This means that there exist no supersymmetric extended objects for $D \geq 12$ [28], as was mentioned in Section 3.5. The case $d = 2$, *i.e.,* the string, is special since the left- and right-handed modes can be treated independently. By having fermions in both sectors, *i.e.,* having a type II theory, we get the same condition as in Eq. (4.3) resulting in strings in $D = 3$, 4, 6 and 10 with $N = 2$. By having fermions in only one sector, *i.e.,* having a heterotic theory, we get the condition

$$D - 2 = n = \frac{1}{2}MN \tag{4.4}$$

resulting in strings in $D = 3$, 4, 6 and 10 with $N = 1$. For completeness we have also included the superparticles $(p = 0)$ in $D = 2$, 3, 5 and 9.

We must also consider higher spin multiplets, a possibility that was originally overlooked. In the case of a vector multiplet we get $d - 2$ additional bosonic degrees of freedom from the vector gauge field, giving

$$D - 2 = \frac{1}{4}MN, \tag{4.5}$$

which can be satisfied in $D = 3$, 4, 6 and 10 for arbitrary d, giving the circles in Fig. 4.1. The branes with vector multiplets living on them are called D-branes.

Finally, branes with tensor multiplets are allowed in $D = 7$ and $D = 11$. The first case is considered in Paper I and the second in Papers II and III.

4.2 *p*-branes

We are now going to review some of the general properties of *p*-branes. We can describe bosonic *p*-branes by a generalization of the Nambu-Goto action (2.1)

$$S_{DNG}[X^\mu] = -T_p \int d^{p+1}\xi \sqrt{-\det(\partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu})}, \tag{4.6}$$

Dimension	Spinor	Type of spinor	Number of susy
11	32	Majorana	1
10	16	Maj & Weyl	1,2
9	16	Majorana	1,2
8	16	Weyl	1,2
7	16	Dirac	1,2
6	8	Weyl	$1, \dots, 4$
5	8	Dirac	$1, \dots, 4$
4	4	Maj or Weyl	$1, \dots, 8$
3	2	Majorana	$1, \dots, 16$
2	1	Maj & Weyl	$1, \dots, 32$

Table 4.1: Irreducible spinor representations in various dimensions.

which is called the Dirac-Nambu-Goto action. Since the p-branes we are interested in are charged with respect to the gauge fields in the low-energy supergravity theories we must add a Wess-Zumino term[2]

$$S_{WZ} = T_p \int X^* A_{(d)}, \qquad (4.7)$$

where $A_{(d)}$ is a d-form gauge potential which is pulled back to the world-volume. This generalizes the coupling of $B_{\mu\nu}$ to the string world-sheet in Eq. (2.18). The d-form gauge potential $A_{(d)}$ couples naturally to a p-brane, where $d = p + 1$. Since the p-brane can be surrounded by a space-like surface S^{D-d-1} we can define an "electric" charge

$$q_e = \int_{S^{D-d-1}} *F_{(d+1)}, \qquad (4.8)$$

where $*F_{(d+1)}$ is the Hodge dual of the field strength $F_{(d+1)} = dA_{(d)}$. This is a direct generalization of Gauss law. By using the field equations for F it follows that the electric charge is conserved. We can also define a "magnetic" charge by

$$q_m = \int_{S^{d+1}} F_{(d+1)}, \qquad (4.9)$$

which is a topological charge, *i.e.*, it is conserved by virtue of the Bianchi identity for F. The electric and magnetic charges defined above now satisfy the generalized Dirac quantization rule [57, 58]

$$q_e q_m = 2\pi n, \ n \in \mathbb{Z}. \qquad (4.10)$$

Since the potential corresponding to the dual field strength is a $(D-d-2)$-form it couples naturally to a $(D-d-3)$-brane. We thus have an electric-magnetic dual pair consisting of a p-brane and a $\tilde{p}$-brane satisfying $p + \tilde{p} = D - 4$.

$D = 11$			M2			M5	
Type IIA	D0	F1	D2		D4	S5	D6
Type IIB		F1+D1		D3$^+$		S5+D5	
Type I		D1				D5	
Heterotic		F1				S5	

Table 4.2: The M-theory brane-scan.

Branes carrying electric charge are called *fundamental* or *elementary*, in analogy with the electron, while branes carrying magnetic charge are called *solitonic*, in analogy with the magnetic monopole. It is now interesting to note how the brane tensions depend on the string coupling constant $g_s = \langle e^\phi \rangle$. For the fundamental *p*-branes the tension do not depend on the string coupling constant and we have $T_{Fp} \sim (m_s)^{p+1}$, where we have used the *string mass* $m_s = 1/\sqrt{\alpha'}$ as the dimensionful parameter. Such *p*-branes only occur for $p = 1$ (cf. Tab. 4.2) and are thus fundamental strings. For the solitonic *p*-branes we have $T_{Sp} \sim (m_s)^{p+1}/g_s^2$ which indicates that they will dominate the dynamics at strong coupling. Finally, for the D-branes, which lies outside the electric-magnetic considerations above, we have $T_{Dp} \sim (m_s)^{p+1}/g_s$ which is an intermediate behavior compared to the fundamental and solitonic cases.

If we now consider eleven dimensional supergravity we have a 3-form potential that couple to an electric M2-brane, which has the M5-brane as its magnetic dual. Since M-theory compactified on a circle is conjectured to be equivalent to the strong coupling limit of type IIA string theory [59, 60] it should be possible to obtain the branes of type IIA theory by suitable wrappings of the M2 and M5 branes around the compact direction [30, 61, 62, 59]. The branes we are considering are, among others, collected in Tab. 4.2. We start by noting that in type IIA theory there are two parameters, the string coupling constant g_s and the string mass m_s. In M-theory there are only one parameter, the Plank mass m_p, but since we consider compactified M-theory we also have the radius R of the compact direction. The M2 brane tension is $T_{M2} = m_p^3$ and by wrapping it around the compact direction we get

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[2]The Wess-Zumino term turns out to be required in order to cancel the κ-variation of the Dirac-Nambu-Goto term.

the F1 string tension

$$T_{F1} = m_s^2 = RT_{M2} = Rm_p^3. \tag{4.11}$$

This implies the identification $m_s^2 = Rm_p^3$. By instead wrapping one of the transverse directions we obtain the D2 brane tension

$$T_{D2} = \frac{m_s^3}{g_s} = T_{M2} = m_p^3 \tag{4.12}$$

implying the identification $g_s = Rm_s$. Let us now check if these identifications work for the M5 brane. By wrapping it around the compact direction we get the D4 brane tension

$$RT_{M5} = Rm_p^6 = \frac{m_s^5}{g_s} = T_{D4} \tag{4.13}$$

and by wrapping a transverse direction we get

$$T_{M5} = m_p^6 = \frac{m_s^6}{g_s^2} = T_{S5}, \tag{4.14}$$

which is the correct tension for the solitonic 5-brane. By this simple procedure we have thus related the M2 and M5 branes to various branes in type IIA theory.

   We end this section by indicating how the supersymmetry algebra can be used to deduce which kinds of branes that exist in a given dimension [63]. Consider $D = 11$ where we have the *M-theory algebra*

$$\{Q_\alpha, Q_\beta\} = (C\Gamma^M)_{\alpha\beta}P_M + \frac{1}{2!}(C\Gamma^{MN})_{\alpha\beta}Z_{MN} + \frac{1}{5!}(C\Gamma^{MNPQR})_{\alpha\beta}Z_{MNPQR}, \tag{4.15}$$

where the supercharge Q_α is a 32-component Majorana spinor. By counting the degrees of freedom in the RHS we get

$$11 + 55 + 462 = 528 \tag{4.16}$$

which equals the degrees of freedom in the LHS. It can be shown that the spatial components of Z_{MN} correspond to the electric charge of the M2-brane while the spatial components of Z_{MNPQR} correspond to the magnetic charge of the M5-brane. We can thus get a good hint of what branes there exist in a particular theory by just looking at the supersymmetry algebra.

4.3   Dp-branes

The p-branes which have vector multiplets living on their world-volumes are called Dp-branes, or simply D-branes. They are dynamical objects on which open strings can end. This implies that the D-branes have an exact description at the string

level in contrast to the solitonic branes. The low-energy dynamics is given by the Dirac-Born-Infeld action

$$S_{DBI}[X^\mu, A_i] = -T_p \int d^{p+1}\xi e^{-\phi}\sqrt{-\det(g + \mathcal{F})}, \qquad (4.17)$$

where T_p is the world-volume tension and $\mathcal{F}_{ij} = 2\pi\alpha' F_{ij} - B_{ij}$ where $F_{ij} = \partial_i A_j - \partial_j A_i$ is the field strength of A_i. The target-space fields ϕ, g and B are understood to be pulled back to the world-volume by the embedding X. These fields represent the closed string background in which the D-brane is embedded. The scaling of the tension with the dilaton as described in the previous section can directly be seen to agree with that given by the action (4.17). The reason for the dilaton dependence $e^{-\phi} = g_s^{-1}$ is that (4.17) is an open string tree level action and we thus get a factor $e^{-\phi}$ from the disc. From the brane-scan, D-branes are seen to be allowed in $D = 3, 4, 6$ and 10. In type II string theory no states in the perturbative spectrum are charged under the RR gauge fields. This is because only the gauge-invariant field strengths appear in the vertex operators creating the RR vacuum out of the NS-NS ground state. The D-branes now restore the balance since they are RR-charged objects which in addition satisfy charge quantization conditions of the form $q_e q_m = 2\pi n$.

Let us now go back and explain how D-branes can be seen to arise when applying T-duality to open strings. Consider an open bosonic string and take one of the target space directions to be compact, *i.e.*, $X^{25} \sim X^{25} + 2\pi R$. The corresponding component of the embedding field decomposes as $X^{25}(\sigma^1, \sigma^2) = X^{25}(z) + \tilde{X}^{25}(\bar{z})$, where

$$\begin{aligned}
X^{25}(z) &= \frac{x^{25}}{2} + \frac{c^{25}}{2} - i\alpha' p^{25}\ln z + i\sqrt{\frac{\alpha'}{2}}\sum_{m \neq 0}\frac{\alpha_m^{25}}{mz^m}, \\
\tilde{X}^{25}(\bar{z}) &= \frac{x^{25}}{2} - \frac{c^{25}}{2} - i\alpha' p^{25}\ln\bar{z} + i\sqrt{\frac{\alpha'}{2}}\sum_{m \neq 0}\frac{\alpha_m^{25}}{m\bar{z}^m}.
\end{aligned} \qquad (4.18)$$

We have here added and subtracted the constant $c^{25}/2$ and used that $z = e^{\sigma^2 + i\sigma^1}$. By now looking at the T-dual field[3]

$$X'^{25}(z, \bar{z}) = X^{25}(z) - \tilde{X}^{25}(\bar{z}) = c^{25} + 2\alpha' p^{25}\sigma^1 + \text{osc.} \qquad (4.19)$$

and noting that the oscillator terms vanish at the endpoints $\sigma^1 = 0, \pi$ we see that the endpoints are fixed at the hyperplane $x^{25} = c^{25}$

$$X'^{25}(\sigma^1 = 0) = c^{25}, \qquad X'^{25}(\sigma^1 = \pi) = c^{25} + 2\pi n R' \sim c^{25}. \qquad (4.20)$$

We have here used that the momentum is quantized in the compact direction, $p^{25} = n/R$, and that $R' = \alpha'/R$. The T-dual field thus lives on a circle of radius R' and we also note that the string is wound n times around the compact direction. The

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[3]For an explanation of why this is the T-dual field see Section 5.1.

Neumann boundary condition has thus been converted into a Dirichlet boundary condition by T-duality.

To see why the D-branes are dynamical objects and why there is a vector field living on them we examine the massless states of the T-dualized open string. The massless states correspond to non-winding states at oscillator level $N = 1$. In the original open string theory this corresponds to the massless U(1) gauge field $\alpha^{\mu}_{-1}|k\rangle$. In the T-dual theory (where we now have dualized $25 - p$ directions in order to get a Dp-brane) this field decomposes into a longitudinal part, giving a U(1) gauge field A_i on the world-volume, and a transverse part, giving $25 - p$ scalars ϕ^m representing the transverse oscillations of the world-volume.

4.4   Branes ending on branes

When examining, *e.g.,* the theory induced on a probe brane configuration, as we will do in Chapter 8, it is important to consider whether the brane used to describe the theory on the probe is actually allowed to end on all the branes in the probe configuration, or if it sees only some of the branes. In Paper VII we get an example where this actually happens. We are therefore sometimes able to examine just a subset of the branes in a configuration by choosing a special kind of brane to study the configuration. One, however, has to be careful when trying to find an S-dual theory in these cases since the S-dual brane[4] might not see the same branes in the configuration and we can therefore not regard the theories to be S-dual. It is therefore important to determine the rules for when one brane is allowed to end on another brane [64, 65, 66, 67].

What will constrain the possible intersections is charge conservation. Consider, *e.g.,* an M2-brane ending on an M5-brane. As was discussed in Section 4.2 we can measure the (electric) charge of the membrane by computing

$$q_{\text{M2}} = \int_{S^7} *H_{(4)}, \tag{4.21}$$

where the seven sphere surrounds the M2-brane. If we were able to neglect the M5-brane in calculating the M2-brane charge, we would be able to take the S^7 through the M5-brane, thereby sliding it off the M2-brane, after which it would be contractible and we would not get any charge. This scenario would therefore violate charge conservation. We thus have to require that the membrane charge can be accounted for on the M5-brane, *i.e.,* generating a flux on the M5-brane, preventing us from sliding off the S^7. In the case we are considering it turns out that the boundary of the M2-brane gives rise to a self-dual string soliton on the M5-brane. In order for the intersection to be permitted by charge conservation the boundary of the lower dimensional brane must have an interpretation as a charged object living on the higher dimensional brane.

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[4]We keep the background fixed and S-dualize the brane used to study the configuration, as will be explained in detail in Chapter 8.

Continuing this reasoning in eleven dimensions we can also get intersection rules concerning M-waves and Kaluza-Klein monopoles [67]. By denoting a k-dimensional intersection between an A-type and a B-type object $A||B(k)$ or $A \perp B(k)$ depending on whether A and B are parallel or perpendicular to each other, we get [68]

$$\begin{gathered} \text{M2} \perp \text{M2}(0), \qquad \text{M2} \perp \text{M5}(1), \qquad \text{M5} \perp \text{M5}(1) \text{ or } \text{M5} \perp \text{M5}(3), \\ \text{W} \, || \, \text{M2}, \qquad \text{W} \, || \, \text{M5}, \qquad \text{M2} \, || \, \text{KK or M2} \perp \text{KK}(0), \\ \text{M5} \, || \, \text{KK or M5} \perp \text{KK}(1) \text{ or } \text{M5} \perp \text{KK}(3), \\ \text{W} \, || \, \text{KK}, \qquad \text{KK} \perp \text{KK}(4, 2). \end{gathered} \tag{4.22}$$

In ten dimensions the same reasoning of course applies, but we can also start from a simple system, *e.g.,* an F-string ending on a Dp-brane, and then use T- and S-duality to generate other allowed intersections. Either way we get

$$\begin{gathered} \text{D}m \, || \, \text{D}m + 4(m), \;\; m = 0, 1, 2 \;, \qquad \text{D}p \perp \text{D}q(m), \; p + q = 4 + 2m, \\ \text{F1} \, || \, \text{NS5}, \qquad \text{NS5} \perp \text{NS5}(3), \qquad Dp \perp \text{NS5}(p - 1), \end{gathered} \tag{4.23}$$

where we for simplicity only consider Dp-branes with $p \leq 6$ whose asymptotic geometries are flat. For multi-brane intersections, these rules must be obeyed by each pair of branes.

5

Duality

The discovery of various duality relations led to a dramatic advance in the understanding of the non-perturbative structure of string theory. Each string theory corresponds to a point in the moduli space of vacua, *i.e.,* a particular choice of vacuum. The dualities take us between different points in this moduli space, relating all the string theories. This indicates the existence of one all-embracing theory, M-theory. The big question is whether there exists a deeper formulation of M-theory or if the best definition we can get is in terms of perturbation expansions and various non-perturbative dualities. As Vafa pointed out [69] this latter alternative is much like how one defines a manifold in terms of charts, being the perturbative string theories, and transition functions, being the dualities. Here we will describe how different string theories and M-theory are related through the basic T- and S-dualities, which have been shown to be subgroups of a larger class of duality called U-duality [70]. In this way we will obtain the *web of dualities* in which all string theories and M-theory are related.

5.1 T-duality

T-duality is an equivalence between two weakly coupled string theories compactified on manifolds of different volume. A generic feature is that when one volume is large the other one is small and vice versa. More concretely, look at the zero-modes of a closed bosonic string

$$X^\mu(z, \bar{z}) = -i\sqrt{\frac{\alpha'}{2}}(\alpha_0^\mu + \tilde{\alpha}_0^\mu)\sigma^2 + \sqrt{\frac{\alpha'}{2}}(\alpha_0^\mu - \tilde{\alpha}_0^\mu)\sigma^1 + \cdots, \qquad (5.1)$$

where, as in Chapter 2, σ^1 is the coordinate along the string and σ^2 is the world-sheet time coordinate. The spacetime momentum of a string is

$$p^\mu = \frac{1}{\sqrt{2\alpha'}}(\alpha_0^\mu + \tilde{\alpha}_0^\mu). \qquad (5.2)$$

When going around the string, *i.e.*, taking $\sigma^1 \rightarrow \sigma^1 + 2\pi$, $X^\mu(z,\bar{z})$ changes by $2\pi\sqrt{(\alpha'/2)}(\alpha_0^\mu - \tilde{\alpha}_0^\mu)$. In the non-compact (spatial) directions, however, $X^\mu(z,\bar{z})$ must be periodic, leading to

$$\alpha_0^\mu = \tilde{\alpha}_0^\mu = \sqrt{\frac{\alpha'}{2}} p^\mu\,. \tag{5.3}$$

For a compact direction, say X^{25}, of radius R we get a weaker condition since $X^{25}(z,\bar{z})$ is now allowed to change by $2\pi\omega R$, where ω is the winding number. Using also that the compact momentum is quantized, $p^{25} = n/R$, where n is the Kaluza-Klein level, we get

$$\alpha_0^{25} = \left(\frac{n}{R} + \frac{\omega R}{\alpha'}\right)\sqrt{\frac{\alpha'}{2}}\,,$$
$$\tilde{\alpha}_0^{25} = \left(\frac{n}{R} - \frac{\omega R}{\alpha'}\right)\sqrt{\frac{\alpha'}{2}}\,. \tag{5.4}$$

We thus get a modification to the mass spectrum (2.17)

$$M^2 = -p^\mu p_\mu = \frac{4}{\alpha'}(N-1) + \frac{2}{\alpha'}(\alpha_0^{25})^2$$
$$= \frac{4}{\alpha'}(\tilde{N}-1) + \frac{2}{\alpha'}(\tilde{\alpha}_0^{25})^2\,, \tag{5.5}$$

where μ runs over all non-compact dimensions and N and $\tilde{N}$ are the oscillator levels for the left- and right-moving excitations, respectively. Note that if we let $R \rightarrow R' = \alpha'/R$ and also interchange n and ω the mass spectrum is unchanged. The effect on the oscillators under this transformation is

$$\alpha_0^{25} \rightarrow \alpha_0^{25}\,,$$
$$\tilde{\alpha}_0^{25} \rightarrow -\tilde{\alpha}_0^{25} \tag{5.6}$$

and we can therefore write the T-dual field as

$$X'^{25}(z,\bar{z}) = X^{25}(z) - \tilde{X}^{25}(\bar{z})\,, \tag{5.7}$$

where $X^{25}(z)$ and $\tilde{X}^{25}(\bar{z})$ are defined in Eq. (4.18). This explains the form of the T-dual field used in Section 4.3 where we studied the effect of T-duality on open bosonic strings. Since $X^{25}(z,\bar{z})$ and $X'^{25}(z,\bar{z})$ have the same energy momentum tensor and OPEs, and therefore also the same correlation functions, we conclude that T-duality is an exact symmetry of perturbative closed string theory.

The invariance under T-duality also implies the existence of a smallest scale in string theory since we can always go from the radius which is smaller than the self-dual radius, $R_{sd} = \sqrt{\alpha'}$, to a radius larger than R_{sd} by using the duality. It is as if an extra term is present in the Heisenberg uncertainty relation[1]

$$\Delta x \geq \frac{1}{2}\frac{\hbar}{\Delta p} + \frac{1}{2}\frac{\Delta p}{\hbar}\alpha' \tag{5.8}$$

---

[1] See, *e.g.*, Ref. [71] and references therein.

giving a minimum length of order[2] $\sqrt{\alpha'} \approx 10^{-33}$cm. Note that the construction of this extra term is possible due to the presence of α'. Since T-duality involves a compactification it will be a duality in nine dimensions[3]. There are two examples, type II and heterotic duality, to which we will now turn.

By compactifying type IIA and type IIB theory on circles with different radii we can identify a T-duality map between the two theories. More generally, dualizing in an odd number of coordinates relates IIA to IIB and dualizing in an even number of coordinates relates IIA to IIA or IIB to IIB. A very efficient way of performing T-duality transformations is to use the *Buscher rules* [72, 73, 74, 75] which we will use in Section 7.3 where we discuss solution generating transformations for brane bound states.

The two heterotic theories are also T-dual when compactified as above. This can be understood by noting that the existence of the two heterotic theories in ten dimensions is due to the existence of two 16-dimensional euclidean self-dual even lattices. By further compactification on T^d, the lattice must now be lorentzian (still being even and self-dual). It is however known that for each $d \geq 1$ there exists a unique lattice with these properties and therefore this duality should come as no surprise [76].

5.2   S-duality

S-duality is a strong-weak coupling duality and therefore of great interest since it enables us to obtain information about the non-perturbative structure of string theory. This, however, also means that conjectured S-dualities can not be investigated using perturbation theory, as in the case of T-dualities. Instead, the p-branes will now play a central rôle, especially those whose properties remain unaltered when going to the strong coupling region, the so called BPS branes. In this way we can devise some non-trivial tests which the conjectured S-dualities must pass. We will now turn to four important S-dualities.

To start with we will consider the duality between M-theory on $\mathbb{R}^{10} \times S^1$ and type IIA string theory. A strong indication of this duality is that by wrapping the M2-brane around the compact circle we get the string in type IIA [30], as explained in the previous chapter. This compactification does not break any supersymmetry and we can hope to obtain one of the two $N = 2$ type II theories. It turns out that we obtain the type IIA supergravity action, *i.e.,* the low-energy limit of type IIA string theory. This is reasonable since this theory is non-chiral. As was shown in Section 4.2, we get the relations [60]

$$\ell_p^3 = R\ell_s^2 \quad \text{and} \quad g_s = \frac{R}{\ell_s} \tag{5.9}$$

---

[2]There are two different conventions for calculating the Planck length. By using $\hbar$ we get $\ell_P \approx 10^{-33}$cm and by using h we get $\ell_P \approx 10^{-32}$cm.

[3]By further compactification we can of course obtain T-dualities in less than nine dimensions but the primary ones, which we will examine, will be in nine dimensions.

by requiring that the branes in M-theory reduce to the branes of type IIA. We can now solve for the string coupling g_s in terms of M-theory parameters

$$g_s = \left(\frac{R}{\ell_p}\right)^{3/2} , \tag{5.10}$$

saying that strong coupling corresponds to large compactification radius. In type IIA theory the strong coupling limit thus effectively corresponds to a decompactification and we obtain an eleven dimensional theory. When doing perturbation theory in the type IIA theory, *i.e.,* expanding around $g_s = 0$, this extra dimension is invisible explaining why it passed unnoticed for so long. We can use this duality to relate brane solution in the two theories. In that context the following formulas are very useful

$$\frac{ds_{11}^2}{\ell_p^2} = e^{-2\phi/3}\frac{ds_{\text{IIA}}^2}{\alpha'} + e^{4\phi/3}\left(\frac{dx^{11}}{R} - \frac{R_{(1)}}{\sqrt{\alpha'}}\right)^2 , \tag{5.11}$$

$$\frac{C_{(3)}}{\ell_p^3} = \frac{R_{(3)}}{(\alpha')^{3/2}} + \frac{dx^{11}}{R} \wedge \frac{B_{(2)}}{\alpha'} , \tag{5.12}$$

where x^{11} is the compact direction, $C_{(3)}$ is the three-form potential in eleven dimensions, $R_{(3)}$, $R_{(1)}$ and $B_{(2)}$ are the RR three- and one-form potentials and the NS-NS two-form potential of type IIA, respectively, and the type IIA metric is the string frame metric.

If we instead compactify M-theory on $S^1/\mathbb{Z}_2$ we get a theory dual to the $\text{E}_8 \times \text{E}_8$ heterotic string theory [77, 78, 79]. The orbifolding breaks half the supersymmetry so we must obtain an $N = 1$ theory. The orbifold $S^1/\mathbb{Z}_2$ is effectively an interval and one E_8 factor is associated with each endpoint. These endpoints are ten dimensional hyperplanes, or "end-of-the-world 9-branes" as they are metaphorically called, and are separated by a distance determined by the coupling constant.

Type IIB supergravity has an $\text{SL}(2,\mathbb{R})$ symmetry, as explained in Section 3.2. In type IIB string theory this symmetry is broken to $\text{SL}(2,\mathbb{Z})$ by charge quantization [80, 70, 60]. Since we are only allowed to have integer multiples of the fundamental F- and D-string charges, the transformation matrix must now only contain integers. The $\text{SL}(2;\mathbb{Z})$ transformations contain in particular a transformation taking $\phi \to -\phi$. Since the string coupling constant is given by $g_s = \langle e^\phi \rangle$ we have that type IIB theory is S-selfdual. The $\text{SL}(2,\mathbb{Z})$ symmetry of type IIB are used in Papers II-IV to obtain $\text{SL}(2,\mathbb{Z})$ covariant brane solution and in Paper VII to obtain a $\text{SL}(2,\mathbb{Z})$ covariant generalization of open string theories. In Chapter 8 we will see some explicit examples of how this symmetry is utilized.

The last duality we are going to mention is that between type I and $\text{SO}(32)$ heterotic string theory. Both theories have $\text{SO}(32)$ as gauge group and this indicates that the two theories may be related by duality. By comparing the low-energy limits of the two theories one sees that they only differ by a field redefinition, *i.e.,*

$$
\begin{aligned}
(g_{\mu\nu})_{\text{I}} &= e^{-\phi_{\text{het}}}(g_{\mu\nu})_{\text{het}} , & \phi_{\text{I}} &= -\phi_{\text{het}} \\
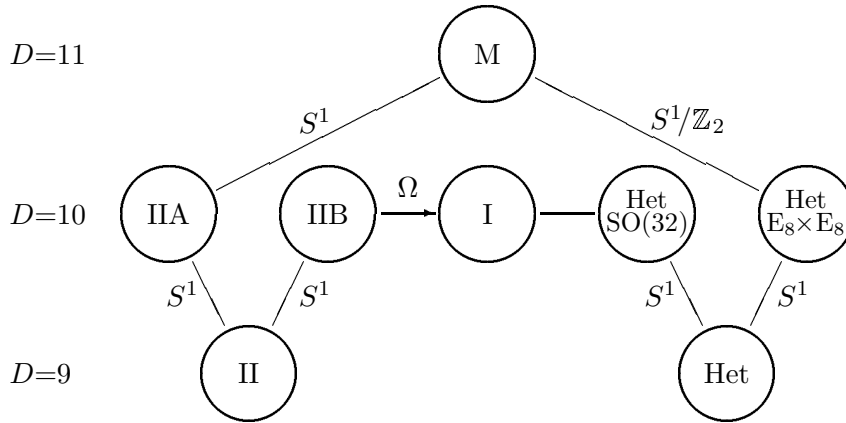C_{(2)} &= B_{(2)} , & A_{\text{I}} &= A_{\text{het}} .
\end{aligned}
\tag{5.13}
$$

Figure 5.1: The web of dualities.

Note in particular that the mapping for the dilaton is $\phi_{\mathrm{I}} = -\phi_{het}$. Based on this Witten [60] conjectured that these theories are related by S-duality. Since then further evidence has strengthened this conjecture.

5.3 The web of dualities

Collecting the results above gives us the *web of dualities*, as illustrated in Fig. 5.1. The fact that the five string theories and M-theory are related in this way indicates that there is really only one fundamental theory.

In Fig. 5.1 we have also included the orientifold projection by which Type I theory can be obtained from Type IIB theory, as explained in Section 3.3. This projection, $P_+^{(\Omega)} = \frac{1}{2}(\mathbb{1} + \Omega)$, is constructed from the orientifold operation Ω which reverses the roles of the left- and right-moving sectors. The resulting theory is left-right symmetric and therefore unoriented. Thus, by keeping only the left-right symmetric states of Type IIB theory we end up with Type I theory.

6

The AdS/CFT correspondence

In November 1997 Juan Maldacena published a paper [81] which was to be the main influence on the string community in the years to come. What Maldacena did was to identify a duality between string theory and gauge theory. It is important to note that string theory inherently includes gravity which gauge theory does not. The person first to propose the existence of such a duality was 't Hooft in 1974 [82]. He was trying to expand the equations for QCD in the variable $1/N$, where N is the number of colours, taking N to be large. This idea of looking at the large N limit will be central in the AdS/CFT correspondence as described below. However, 't Hooft's approach fell short of solving the problems of interest in QCD, but he proposed that one should be able to find a string theory describing QCD where $1/N$ played the role of some coupling constant. Despite the considerable interest aroused by this proposal no one was able to find a relation between string theory and gauge theory until now.

More precisely, the proposed duality is between type IIB string theory on $AdS_5 \times S^5$ and a conformal field theory on the 4-dimensional Minkowski space which is the boundary of AdS_5. The important property is that weakly coupled string theory is dual to strongly coupled gauge theory, where calculations are intractable. As described in Chapter 3, the low-energy limit of weakly coupled string theory is given by supergravity. Thus, to lowest order we have a correspondence between supergravity and gauge theory.

Since Maldacena's first paper, a more precise version of the correspondence has been developed by Gubser, Klebanov and Polyakov [83] and independently by Witten [84], which made it possible to relate quantities in the interior of AdS_5 to quantities in the gauge theory living on the boundary[1]. The correspondence was still limited to maximally supersymmetric and conformal gauge theories, but progress has been made in order to rid it from these two restrictions. In order to describe gauge theories with reduced supersymmetry we must take string theory on $AdS_5 \times X_5$, where X_5 is a positively curved Einstein manifold, *i.e.,* one for which $R_{\mu\nu} = \Lambda g_{\mu\nu}$

---

[1]See [85] for an extensive review of the AdS/CFT correspondence.

with $\Lambda > 0$. The number of supersymmetries in the dual gauge theory is determined by the number of Killing spinors on X_5. Some progress has also been made towards non-conformal gauge theories. This development is very important in order to make contact with QCD, the prime application, since it is neither supersymmetric nor conformal.

It is amusing to note that string theory has come full circle; it was invented to provide a description of QCD and now, after thirty years of development, it might just do that.

6.1   The large N limit

What Maldacena did in order to discover the duality between supergravity and gauge theory was to relate both theories to D-branes. In order to get a four-dimensional gauge theory one must use D3-branes, which can be embedded in ten-dimensional spacetime. Each D-brane carries a $U(1)$ charge and by stacking N D-branes one obtains $\mathcal{N} = 4$ $U(N)$ super-Yang-Mills in the low-energy limit. To be more precise, we take the low energy (in units of α') limit by keeping the energy (not in units of α'), and dimensionless quantities like g_s and N, fixed and sending $\alpha' \to 0$. Since the Newton constant is proportional to α', $\kappa \sim g_s(\alpha')^2$, the gauge theory on the D-branes will decouple from gravity in the low-energy limit. Also, higher derivative corrections to the super-Yang-Mills action depend on positive power of κ and will therefore be suppressed.

On the supergravity side, the stacked D3-branes make up a black hole solution to the supergravity equations [86]

$$
\begin{aligned}
ds^2 &= f^{-1/2}\left(-(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2\right) + f^{1/2}(dr^2 + r^2 d\Omega_5^2)\,, \\
H_{(5)} &= (1 + *)dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge df^{-1}\,, \\
f &= 1 + \frac{R^4}{r^4}\,, \qquad R^4 \equiv 4\pi g_s(\alpha')^2 N\,,
\end{aligned}
\tag{6.1}
$$

where $x^0 \equiv t$. Far away from the D-branes space is flat, but near the D-branes there is an infinite throat leading down to the horizon of the black hole as depicted in Fig. 6.1. In order to understand what happens in the low-energy limit we consider particle waves in the asymptotically flat region. Their wave lengths will increase as the energy is lowered and in the low-energy limit their wave lengths will be much larger than the typical gravitational size of the D-brane system (which is of order R). Since g_{00} is non-constant, the energy E_r measured by an observer at radius r and the energy E measured at infinity are related by the redshift factor given by $f^{-1/4}$ in (6.1)

$$
E = f^{-1/4} E_r\,.
\tag{6.2}
$$

An object that is brought closer to $r = 0$ will therefore appear to have lower and lower energy for an observer at infinity. In this way, excitations inside the throat region will lie closer and closer to the event horizon as the energy is lowered and it will be
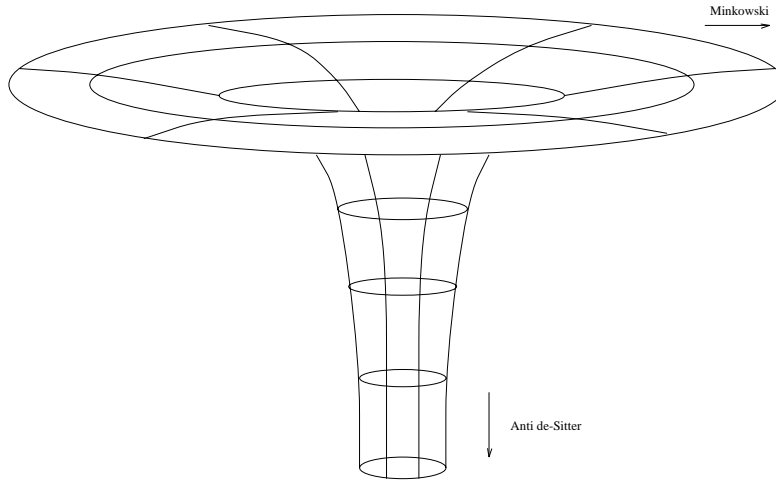
Minkowski

Anti de-Sitter

Figure 6.1: The geometry near the stacked D-branes.

increasingly harder for them to climb the gravitational potential and escape to the asymptotically flat region. We therefore conclude that the bulk physics decouples from the boundary physics near the horizon in the low energy limit. Since it is the low-energy dynamics of the D-branes that we are interested in it is the low-energy region near the horizon we should be focusing at. By taking the near horizon limit, *i.e.*, $r \ll R$, in the the stacked D3-brane solution (6.1) we get the near horizon geometry

$$ds^2 = \frac{r^2}{R^2} \left(-(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2 \right) + R^2 \frac{dr^2}{r^2} + R^2 d\Omega_5^2, \qquad (6.3)$$

where we have used that $f \sim R^4/r^4$ for $r \ll R$. This metric describes the geometry of $AdS_5 \times S^5$ and since the radius of curvature of this AdS space is proportional to $N^{1/4}$, the supergravity solution is most reliable when N is large. On the supergravity side the low energy limit of the stacked D3-branes are therefore described by supergravity in an $AdS_5 \times S^5$ background. Maldacena's conclusion was that the two descriptions are dual to each other.

On the gauge theory side, N is the number of colours and we are interested in how the *effective* coupling scales with N. To see this, consider the exchange of a gluon between two quarks. Since the emitting quark can turn into N different colours, the effective coupling will be $\lambda = g_{eff}^2 = g_{YM}^2 N$, where λ is called the 't Hooft parameter. As we will see below, the duality takes its simplest form when λ is large and since λ corresponds to the effective coupling we have a duality between strongly coupled gauge theory and supergravity.

When trying to relate quantities in the two pictures we have to be a bit more careful when taking the near horizon limit. We take $\alpha' \to 0$ but want to be able to study arbitrary string states in the near horizon region, requiring that we keep

the energy in string units, *i.e.*, $\sqrt{\alpha'}E_r$, fixed. In the dual field theory, the energy is measured at infinity. In the near horizon region it follows from (6.2) that $E \sim \frac{r}{\sqrt{\alpha'}}E_r$. In order to keep also this energy fixed in the low energy limit we get the condition[2] that r/α' has to be fixed as $\alpha' \to 0$. It is now convenient to rewrite the near horizon metric (6.3) in terms of the new variable $U \equiv r/\alpha'$, which is fixed in the low energy limit, giving

$$ds^2 = \alpha' \left[\frac{U^2}{\sqrt{4\pi g_s N}}(-(dx^0)^2 + (dx^1)^2 + (dx^2)^2 + (dx^3)^2) \right.$$
$$\left. + \sqrt{4\pi g_s N}\frac{dU^2}{U^2} + \sqrt{4\pi g_s N}d\Omega_5^2 \right] . \quad (6.4)$$

Note that U has the dimension of energy. An alternative way of seeing that r/α' must be fixed is to consider a D3-brane that has been pulled out from the stack at $\vec{r} = 0$ to a location[3] $\vec{r}$. In the Yang-Mills theory this corresponds to giving a vacuum expectation value to one of the scalars, which gives rise to massive W-bosons described by strings stretching between the brane at $\vec{r}$ and the branes in the stack. The mass of a W-boson is given simply by the energy of the corresponding stretched string as $E_W \sim |\vec{r}|/\alpha' = U$, where we use the euclidean distance since we are considering D-branes in flat space. In order to keep this energy fixed we arrive at the same conclusion as before that r/α' must be fixed in the $\alpha' \to 0$ limit. Note that we get the same result by using the supergravity solution to compute the energy of a stretched string since the factor $f(|\vec{r}|)^{1/4}$ from the transverse metric is cancelled by the redshift factor $f(|\vec{r}|)^{-1/4}$ when converting the string tension into energy.

Let us finally examine the validity of the supergravity-gauge theory correspondence a bit more carefully. The argument above that string theory reduces to supergravity in the low-energy limit near the horizon is a bit too weak. We must also demand that the radius of AdS_5 and S^5 (they have the same radius) is large compared to the string scale in order to be able to neglect stringy effects. The radius, expressed in the 't Hooft parameter, is

$$R = \lambda^{1/4}\ell_s. \quad (6.5)$$

This implies that we must have $\lambda \gg 1$. In order to be able to neglect quantum effect, the string scale must be large compared to the Planck scale[4], given by

$$(\ell_p)_{10} \sim g_s^{1/4}\ell_s. \quad (6.6)$$

---

[2]The same chain of reasoning can be used to motivate the electric near-horizon limit used in Paper VI. In that case there exist a finite length scale, corresponding to the open strings, in the limit $\alpha' \to 0$. Energies should therefore be measured in this fixed unit and consequently E_r is fixed in the limit $\alpha' \to 0$ using the same reasoning as above. To also keep E in (6.2) fixed we get the condition that $r/\sqrt{\alpha'}$ must be fixed as $\alpha' \to 0$, which is the radial scaling used in the electric near-horizon limit.

[3]We are here viewing the D-branes as hyper surfaces in flat space.

[4]The Planck length in Eq. (6.6) is the *ten* dimensional Planck length, defined from the Newton constant $\kappa \sim g_s(\alpha')^2$. For relations between Newton constants in various dimensions see, *e.g.,* [87].

Using that $g_s = g_{YM}^2$ we see that we must have $g_{YM} \ll 1$, which implies $N \gg 1$ since λ is large. To conclude, we have the following relations between gauge theory and supergravity quantities

$$\lambda = \frac{R^4}{\ell_s^4}\,, \qquad\qquad N = \frac{R^4}{g_s \ell_s^4}\,, \qquad\qquad (6.7)$$

and in order for the correspondence to be valid the large N limit is not enough, we must also take λ to be large.

6.2 Holography

The supergravity-gauge theory duality also gives new insights into an intriguing concept which 't Hooft has named "holography". By holography we mean a relation between the information carried on a surface and that within the volume it encloses. To be more precise, 't Hooft [88] and Susskind [89] proposed that the degrees of freedom in the bulk of a region matches the degrees of freedom on the surface of that region with an upper bound on the amount of information per unit area.

Before the discovery of the supergravity-gauge theory duality, the best candidate for realizing holography was black holes. As Bekenstein and Hawking showed, the entropy of a black hole is proportional to its surface area. But if we consider the creation of a black hole, in order not to lose any information, all the information carried by what forms the inside of the black hole must be carried by its surface. The holographic principle then means that the hologram captures all the information but in a non-transparent way. Considering this, it is not surprising that Maldacena got his idea while studying black holes.

Not everyone believes in the idea of holography, but the supergravity-gauge theory duality may provide a new realization of holography and can thus bring some new insights. The important difference to the black hole realization mentioned above is that the supergravity-gauge theory realization would provide a microscopic understanding of the physics behind holography. Witten and Susskind [90] have used this new realization to get an order of magnitude estimate of the degrees of freedom of a black hole [3]. They also found that infrared, *i.e.,* long distance, effects in the bulk are related to ultraviolet, *i.e.,* short distance, effects on the boundary.

7

Branes in background fields

In Chapter 4, we studied the simplest cases of brane solutions, *i.e.,* brane solutions with only electric or magnetic charge. A more general situation would be to allow for non-zero background fields, which from the brane point of view is equivalent to excite the brane by including a non-zero field strength on the brane world volume. After this excitation the brane will carry both electric and magnetic charge, the new charge being interpreted as coming from new branes within the brane, and will therefore sometimes be referred to as a dyonic brane.

There are two alternative approaches to the problem of obtaining these more general solutions. We will start by explaining the method of analyzing the zero-modes, used in Papers III and IV to obtain M5, D3 and (p, q) 5-brane solutions in non-zero background fields. We also review the more commonly used method of solution generating transformations, used in Paper VI, and in the last section we explain the relation between solutions obtained by using these two methods.

7.1 Goldstone tensor modes

It is well known that massless degrees of freedom, so called Goldstone modes, arise when a continuous symmetry is broken. Put, *e.g.,* an M2-brane into an eleven dimensional space. This breaks half of the supersymmetry. Another half is broken by the Dirac equation when going on-shell, resulting in eight fermionic zero-modes. The translational symmetry in the transverse directions are also broken, generating eight bosonic zero-modes. Since we get the same number of bosonic and fermionic degrees of freedom, there is a supersymmetric theory living on the M2-brane. The M2-brane case, however, contains only scalar modes, which have been understood for quite some time. The situation is a bit different if we instead look at the M5-brane. Here we get eight fermionic degrees of freedom as above, but now we only get five bosonic degrees of freedom from the breaking of translational symmetries. The three extra bosonic degrees of freedom needed to get a supersymmetric theory come from an anti-self-dual three-form field strength on the brane. In Paper II, we generalize

the Goldstone mechanism to tensor fields of arbitrary rank, providing the same level of understanding in terms of broken symmetries as for the scalar modes. We will end this section by explaining how the anti-self-dual three-form field strength on the M5-brane arise from broken gauge symmetries of the background three-form potential. The presentation in Paper II is very thorough and we will here only recapitulate the main ideas.

In order to understand how to make the proper generalization to tensor modes, it is important to understand in detail how the scalar modes arise. Since we are studying a theory with gravity, *i.e.,* a theory having local diffeomorphism invariance, we have to be more careful when we say that introducing a brane breaks translational symmetry in the transverse directions. We can always make a *small* diffeomorphism, *i.e.,* a diffeomorphism taking the same value in the two asymptotic regions of the brane solution, $r \to 0$ and $r \to \infty$, without changing any conserved quantities like, *e.g.,* the momentum. If we instead make a *large* diffeomorphism, taking different values in the asymptotic regions, we change conserved quantities and these symmetries are therefore broken in the presence of a brane. Since diffeomorphisms are the gauge symmetry associated with gravity, we come to the conclusion that Goldstone modes are associated with broken large gauge symmetries. In the case of the fermionic modes it is the large supersymmetry transformations that are broken and, *e.g.,* the tensor modes on the M5-brane come from broken large gauge transformations of the background three-form potential.

Now that we know where the Goldstone modes come from, we can deduce exactly how they appear in the target space fields. Let us again look at the scalar modes. Under an infinitesimal diffeomorphism we have

$$\delta g_{MN} = \mathcal{L}_\epsilon g_{MN} = 2D_{(M}\epsilon_{N)}, \tag{7.1}$$

where $M = (\mu, m)$ corresponds to the split into longitudinal and transverse directions. For a diffeomorphism in the transverse directions we now write $\epsilon^m = \Delta^s \bar{\phi}^m$, where Δ is the harmonic function in the brane solution, s is a parameter that will be determined by imposing the supergravity field equations and $\bar{\phi}^m$ are *constant* moduli corresponding to a rigid transverse displacement of the brane in the usual zero-mode picture. Inserting this form of ϵ^m into (7.1) gives us how the scalar zero-modes sit in the metric. So far, all we have done is a gauge transformation and this can not give rise to any non-trivial dynamics. Now, however, we make $\bar{\phi}^m$ x-dependent, where x is a coordinate on the brane, corresponding to allowing for transverse wiggles of the brane. Since we turned on the x-dependence after calculating δg_{MN}, which includes a derivative, we have no longer just performed a gauge transformation. By using the supergravity field equations we can now get the equations of motion for the zero-modes and we are also able to determine the parameter s, which turns out to be equal to -1 for *all* brane solutions. Thus, by doing a "rigid" transformation we get information on how to introduce the zero-modes in the relevant field and by turning on the x-dependence we can then get the equations of motion for the zero-modes using the supergravity field equations. In the same way the fermionic modes will be introduced in the gravitino field and the tensor modes in the corresponding gauge potential. Since we want to have a theory living on the brane, we must require that

all the zero-modes are normalizable when integrating out the transverse directions, which, as we will soon see, will be very important for the M5-brane.

We now return to the analysis of the M5-brane. Using the notation of Paper II, we make a gauge transformation of the background three-form potential $\delta C = d\Lambda$, where $\Lambda = \Delta^k A$ and A is a *constant* two-form which lies in the transverse directions, since we want a theory on the brane, and k is a constant that will be determined using the supergravity equations. Following the prescription above we first calculate $\delta C = d\Delta^k \wedge A$ and then turn on the x-dependence of A. We can now compute the variation of the four-form field strength $H = dC$,

$$h = \delta H = d\Delta^k \wedge F \,, \tag{7.2}$$

where $F = dA$. The field equation for H is, to linear order in h,

$$d * h - H \wedge h = 0 \tag{7.3}$$

yielding

$$\Delta d *_x F \wedge *_y d\Delta - (k *_x F - F) \wedge d\Delta \wedge *_y d\Delta = 0 \,, \tag{7.4}$$

where $*_x$ and $*_y$ denotes dualization in the longitudinal and transverse directions, respectively, and we have used $H = *_y d\Delta$. By considering the two duality components of F separately (fulfilling $*_x F = \pm F$) we get that $k = -1$ for the anti-self-dual part and $k = 1$ for the self-dual part. We also get the equation of motion $d *_x F = 0$. Since each duality component of F contributes with three bosonic degrees of freedom, we seem to have twice the number of extra degrees of freedom that we needed in order to get supersymmetry. By considering normalizability, however, we see that the self-dual part of F has non-normalizable zero-modes, and must therefore be discarded. We have thus seen how the tensor modes on the M5-brane can be understood as arising from broken large gauge transformation of the background three-form potential. In Paper II the same procedure is also applied to the vector modes on a D3-brane and in Paper III to the modes on a (p, q) 5-brane.

7.2 Finite tensor deformations

We can start from any "host" brane, whose normalizable zero-modes we identify using the prescription described above. This gives us exact knowledge of how the zero-modes appear in all target space fields and enables us to make an Ansatz for the full solution. The non-linear supergravity equations are then solved for the unknown functions in the Ansatz. The reason for the designation "host" brane is that the full solution with finite field strength on the brane can also be viewed as a non-threshold bound state where various smaller branes, depending on the rank of the field strength, have dissolved into the "host" brane and become smeared in some directions. In this way the original brane acts as a host for all kinds of lower-dimensional branes representing the excitation. Using this method, we were able

to obtain the most general[1] bound state solutions, with *constant* fields on the host brane, for the D3- and M5-branes[2] as host branes in Paper III and for the (p, q)-5 brane as host brane in Paper IV. Apart from the general M5-brane solution, only special cases of these solutions were previously obtained.

7.3   Solution generating transformations

The simplest way to derive bound state solutions is to use solution generating transformations, consisting of T-duality, Lorentz and gauge transformations, on existing solutions. See, *e.g.,* [68] for a nice review on the subject. The advantage of using this method compared to the method described in the previous section is that it is purely algebraic and it is therefore not necessary to solve any non-linear differential equations. It is also possible to combine the action of diagonal T-dualities, constant NS gauge transformations and $SO(p, 1)$ transformations to get an arbitrary element of the T-duality group $O(p + 1, p + 1)$. Using the full T-duality group, it is possible to generate a general Dp-brane bound state in a very efficient way [95]. This method was used in Paper VI to generate electrically and magnetically deformed supergravity duals.

T-duality transformations are most easily performed using the *Buscher rules* [72, 73, 74, 75]. Let y be the direction in which we T-dualize and let all the other indices be different from y, then the Buscher rules for the NS fields are

$$
\hat{g}_{yy} = \frac{1}{g_{yy}}, \qquad \hat{g}_{mn} = g_{mn} - \frac{g_{my}g_{ny} - B_{my}B_{ny}}{g_{yy}},
$$

$$
\hat{B}_{my} = \frac{g_{my}}{g_{yy}}, \qquad \hat{B}_{mn} = B_{mn} - \frac{B_{my}g_{ny} - g_{my}B_{ny}}{g_{yy}}, \qquad (7.5)
$$

$$
\hat{g}_{my} = \frac{B_{my}}{g_{yy}}, \qquad e^{2\hat{\phi}} = \frac{e^{2\phi}}{g_{yy}}
$$

and the RR-fields transform as

$$
\hat{C}_{(p)m\cdots nqy} = C_{(p-1)m\cdots nq} - (p - 1)\frac{C_{(p-1)[m\cdots n|y|}g_{q]y}}{g_{yy}}, \qquad (7.6)
$$

$$
\hat{C}_{(p)m\cdots nqr} = C_{(p+1)m\cdots nqry} + p\, C_{(p-1)[m\cdots nq}B_{r]y} + p(p + 1)\frac{C_{(p-1)[m\cdots n|y|}B_{q|y|}g_{r]y}}{g_{yy}}.
$$

Note that the off-diagonal parts of the metric and the B-field get interchanged under T-duality. When doing a T-duality transformation in a direction longitudinal to a Dp-brane (except for the time direction) we get a D$(p - 1)$-brane and a transverse

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[1]The motivation for this statement is that we start from the zero-modes and then find a *unique* solution with finite field strength. This method do not generate any waves in the solutions, but solutions including waves [91] has been shown to be related to solutions without waves via finite boosts [92]. Solutions with light-like fields have been shown to be obtainable by taking a limit involving an infinite boost [93]

[2]As will be explained in Section 7.4, our M5-brane solution in Paper III is equivalent to the solution already obtained in [94].

T-duality gives a D(p+1)-brane. It is important to note, however, that the Buscher rules apply only when dualizing in a direction that is an isometry of the solution. A longitudinal direction is automatically an isometry, but in order to be able to T-dualize in a direction transverse to the brane we must first "smear" it in that direction to create an isometry. We will now derive the (F,Dp) bound state using two different methods.

7.3.1 T-duality and Lorentz transformations

We start from the Dp brane solution, for $p \leq 6$, in the string frame

$$ds^2 = H^{-\frac{1}{2}}\left(-(dx^0)^2 + (dx^1)^2 + \cdots + (dx^p)^2\right) + H^{\frac{1}{2}}dy^2\,,$$
$$C_{(p+1)} = \frac{1}{g_s H}dx^0 \wedge \cdots \wedge dx^p + (7-p)N(\alpha')^{\frac{7-p}{2}}\epsilon_{7-p}\,, \tag{7.7}$$
$$e^{2\phi} = g_s^2 H^{\frac{3-p}{2}}\,,$$

where $d\epsilon_{7-p}$ is the volume element on the transverse $(8-p)$-sphere and the harmonic function is

$$H = 1 + \frac{g_s N(\alpha')^{\frac{7-p}{2}}}{r^{7-p}}\,. \tag{7.8}$$

In order to turn on an electric B-field, B_{01}, we can T-dualize in the x^1-direction and then boost in the same direction, creating an off-diagonal term in the metric, and finally T-dualize back, transforming the off-diagonal metric component into a B-field. If we instead want to generate a magnetic field on the brane we have to T-dualize and rotate in the magnetic directions where we want to generate the B-field, yielding a bound state of a Dp-brane and a D$(p-2)$-brane.

T-dualizing in the x^1-direction gives

$$\hat{g}_{11} = H^{\frac{1}{2}}\,, \qquad e^{2\hat{\phi}} = g_s^2 H^{\frac{4-p}{2}}\,,$$
$$\hat{C}_{01\cdots p} = \frac{1}{g_s H}\,. \tag{7.9}$$

Boosting in the x^1-direction,

$$dx^0 = \cosh\gamma\, d\tilde{x}^0 + \sinh\gamma\, d\tilde{x}^1\,,$$
$$dx^1 = \sinh\gamma\, d\tilde{x}^0 + \cosh\gamma\, d\tilde{x}^1\,, \tag{7.10}$$

leads to transformed metric and p-form components

$$\tilde{g}_{00} = H^{\frac{1}{2}}\left(\sinh^2\gamma - \cosh^2\gamma H^{-1}\right)\,, \qquad \tilde{C}_{02\cdots p} = \cosh\gamma\frac{(-1)^{p-1}}{g_s H}\,,$$
$$\tilde{g}_{01} = H^{\frac{1}{2}}(1 - H^{-1})\cosh\gamma\sinh\gamma\,, \tag{7.11}$$
$$\tilde{g}_{11} = H^{\frac{1}{2}}\left(\cosh^2\gamma - \sinh^2\gamma H^{-1}\right)\,, \qquad \tilde{C}_{12\cdots p} = \sinh\gamma\frac{(-1)^{p-1}}{g_s H}\,.$$

By T-dualizing back in the x^1-direction the off-diagonal component of the metric gets transformed into a B-field, giving[3]

$$ds^2 = H^{-\frac{1}{2}} \left(h \left(-(dx^0)^2 + (dx^1)^2 \right) + (dx^2)^2 + \cdots + (dx^p)^2 \right) + H^{\frac{1}{2}} dy^2 \,,$$

$$e^{2\phi} = g_s^2 H^{\frac{3-p}{2}} h \,, \qquad B_{01} = -\tanh\gamma H^{-1} h \,, \tag{7.12}$$

$$C_{01\cdots p} = \frac{h\cosh\gamma}{g_s H} \,, \qquad C_{23\cdots p} = \frac{\sinh\gamma}{g_s H} \,,$$

where

$$h^{-1} = \cosh^2\gamma - \sinh^2\gamma H^{-1}. \tag{7.13}$$

We have also performed a gauge transformation, $\delta B = -\tanh\gamma dx^0 \wedge dx^1$, in order to get the solution on standard form with a B-field that is asymptotically non-vanishing and that goes to zero as $r \to 0$. That fact that we now have a non-zero B_{01}, in addition to a non-zero $C_{01\cdots p}$, means that we have a (F,Dp) bound state. In general, all q-form potentials having a component along the time direction corresponds to a $(q-1)$-brane lying in the same directions as the potential.

7.3.2   T-duality and gauge transformations

We can also generate the same bound state using (double) T-duality and gauge transformations. In the case of the (F,Dp) bound state, we T-dualize first in the x^0-direction and then in the x^1-direction. Since the x^0 and x^1 directions after the T-dualities are transverse to the brane we can turn on a B-field in these directions using a gauge transformation. By then T-dualizing back, first in the x^1-direction and then in the x^0-direction, the B-field gets interchanged with the off-diagonal part of the metric twice and therefore ends up as a B-field on the brane.

T-dualizing in the x^0 and x^1 directions and performing a gauge transformation give

$$\hat{g}_{00} = -H^{\frac{1}{2}} \,, \qquad \hat{g}_{11} = H^{\frac{1}{2}} \,,$$

$$e^{2\hat{\phi}} = -g_s^2 H^{\frac{5-p}{2}} \,, \qquad \hat{C}_{2\cdots p} = -\frac{1}{g_s H} \,, \tag{7.14}$$

$$\hat{B}_{01} = b \,.$$

By T-dualizing back in the x^1 and x^0 directions we get the bound state solution

$$ds^2 = H^{-\frac{1}{2}} \left(\hat{h} \left(-(dx^0)^2 + (dx^1)^2 \right) + (dx^2)^2 + \cdots + (dx^p)^2 \right) + H^{\frac{1}{2}} dy^2 \,,$$

$$e^{2\phi} = g_s^2 H^{\frac{3-p}{2}} \hat{h} \,, \qquad B_{01} = b H^{-1} \hat{h} \,, \tag{7.15}$$

$$C_{01\cdots p} = \frac{\hat{h}}{g_s H} \,, \qquad C_{23\cdots p} = -\frac{b}{g_s H} \,,$$

---

[3]We have used the same notation as in (7.7), but keep in mind that the fields have been T-dualized twice and the coordinates have been boosted.

where

$$\hat{h}^{-1} = 1 - b^2 H^{-1} \,. \tag{7.16}$$

By making the replacements

$$
\begin{aligned}
b &\to -\tanh\gamma \,, \\
g_s &\to \frac{g_s}{\cosh\gamma} \,, \\
x^{0,1} &\to \frac{x^{0,1}}{\cosh\gamma}
\end{aligned}
\tag{7.17}
$$

we get the solution in the previous section. We have thus explicitly checked that these two methods yield equivalent solutions. In the next section we will explicitly verify the equivalence of various bound state solutions in eleven dimensions.

7.4   Relations between different forms of solutions

In the two previous sections we have discussed two different approaches to obtaining brane solutions with non-zero background field strengths. While the method of analyzing the zero-modes generally gives a unique solution, starting from an un-excited host brane, parameterized by the field strength on the host brane, the method of using solution generating transformations generally gives a multitude of solutions for the same host brane. An important question is whether these two approaches are equivalent or in what way they differ. We argue that the method of analyzing the zero-modes always gives the most general half supersymmetric solution by construction, since there is a unique way of fitting the half supersymmetric zero-modes into the target space fields and since the subsequent full solution is uniquely obtained from the zero-mode solution. Therefore, all the solutions obtained using solution generating transformations are related to solutions obtained by analyzing the zero-modes and thus do not generate a larger family of solutions despite the multitude of different looking solutions[4]. As a consequence of this, the most general bound state in type IIB having a (p, q) 5-brane as host brane was first obtained in Paper IV. The most general bound state having a D3-brane as a host brane was first obtained in Paper III.

In order to explicitly prove this assertion we have chosen to look at bound states in eleven dimensions and want to show that all the obtained bound state solutions can be related to the one we obtained in Paper III using the zero-mode approach. One reason for this choice is that in eleven dimensions we will not have to deal with the additional complication of SL(2,$\mathbb{Z}$) covariance, which we have to in the case of bound states in type IIB theory. More importantly, due to the various dualities described in Chapter 5, all bound state solutions in lower dimensions can be obtained from the *one* in eleven dimensions.

We will start from the solution obtained in Paper III, representing an M5-M2 bound state, and derive an explicit mapping to the M5-M2 solution of Izquierdo *et*

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[4]The results in this section are part of a forthcoming publication [96].

al. [94], who were the first to obtain the M5-M2 solution. We will then boost the solution of Izquierdo *et al.* leading to the M5-M2-M2′-MW solution of Bergshoeff *et al.* [91], essentially following [92] but using a slightly generalized form of the mapping.

The M5-M2 solution obtained in Paper III is[5]

$$ds^2 = (\Delta^2 - \nu^2)^{1/3} \left[\frac{1}{(\Delta - \nu)} \left(-dt^2 + (dx^1)^2 + (dx^2)^2 \right) \right.$$
$$\left. + \frac{1}{(\Delta + \nu)} \left((dx^3)^2 + (dx^4)^2 + (dx^5)^2 \right) + dr^2 + r^2 d\Omega_4^2 \right],$$
$$\ell_p^3 C_3 = \sqrt{2\nu} \left[\frac{1}{(\Delta - \nu)} dt \wedge dx^1 \wedge dx^2 - \frac{1}{(\Delta + \nu)} dx^3 \wedge dx^4 \wedge dx^5 \right], \tag{7.18}$$

$$H_4 = dC_3 + 3\pi N \epsilon_4,$$

where ℓ_p is the eleven dimensional Planck length, N is the number of M5-branes in the bound state, ϵ_4 is the volume element on the unit 4-sphere and

$$\Delta = k + \left(\frac{R}{r} \right)^3, \qquad R \equiv \pi N^{1/3} \ell_p \tag{7.19}$$

is the harmonic function and ν is proportional to the square of the field strength on the brane, see Paper III for details. We must have $\nu \leq k$ in order to avoid naked singularities and it is for the critical case, $\nu = k$, that we can take a decoupling limit and obtain OM theory.

If we make the following substitutions [97]

$$\frac{(\Delta - \nu)}{2\nu} = \frac{H}{\tan^2 \alpha},$$
$$\frac{(\Delta + \nu)}{2\nu} = \frac{H}{h \sin^2 \alpha}, \tag{7.20}$$
$$\frac{(k - \nu)}{2\nu} = \frac{A}{\tan^2 \alpha},$$

keeping R unchanged and rescaling the coordinates according to

$$r \to \left(\frac{\tan^2 \alpha \cos \alpha}{2\nu} \right)^{1/3} r,$$
$$x^{0,1,2} \to \left(\frac{2\nu \cos^2 \alpha}{\tan^2 \alpha} \right)^{1/6} x^{0,1,2}, \tag{7.21}$$
$$x^{3,4,5} \to \left(\frac{2\nu}{\tan^2 \alpha \cos^4 \alpha} \right)^{1/6} x^{3,4,5},$$

---

[5]Note that we have chosen a sign such that $h_{+++} = -h_{---} = -\sqrt{2\nu}$ in the formulation of Paper III.

where $x^0 \equiv t$, we get the M5-M2 solution of Izquierdo *et al.* [94],

$$ds^2 = H^{-1/3}h^{-1/3}\left[-dt^2 + (dx^1)^2 + (dx^2)^2 + h\left((dx^3)^2 + (dx^4)^2 + (dx^5)^2\right)\right.$$

$$\left. + H(dr^2 + r^2 d\Omega_4^2)\right],$$

$$\ell_p^3 C = H^{-1}\sin\alpha\, dt \wedge dx^1 \wedge dx^2 - H^{-1}h\tan\alpha\, dx^3 \wedge dx^4 \wedge dx^5, \tag{7.22}$$

$$H_4 = dC_3 + 3\pi N\epsilon_4,$$

where the function h and the harmonic function H are defined as

$$H = A + \frac{R^3}{\cos\alpha\, r^3}, \qquad h^{-1} = H^{-1}\sin^2\alpha + \cos^2\alpha, \tag{7.23}$$

where we have allowed for an arbitrary constant A in the harmonic function.

Since we at this point can not solve for A and α in terms of k and ν, we are free to impose further constraints on the mapping. One alternative is to set, *e.g.*, $A = 1$. This, however, prevents us from being able to map our critical solution in a non-singular manner, which will be important later on. The constraint we are going to impose instead is that we obtain exactly the solution of Bergshoeff *et al.* after a Lorentz boost as explained below. We can then map a solution with arbitrary k and ν to a solution of Bergshoeff *et al.* with arbitrary a and θ_1, as shown in Eqs. (7.36) and (7.37), except in the critical case.

We now boost the solution of Izquierdo *et al.* (7.22), for details see [92], letting

$$t \to t\cosh\gamma - x^5\sinh\gamma, \qquad x^5 \to x^5\cosh\gamma - t\sinh\gamma. \tag{7.24}$$

We introduce the angles θ_1 and θ_2, where $\theta_1 \leq \theta_2$, through the definitions

$$\cos\alpha = \frac{\cos\theta_2}{\cos\theta_1}, \quad \cosh\gamma = \frac{\sin\theta_2}{\cos\theta_1}\frac{1}{\sin\alpha}, \quad \sinh\gamma = \frac{\sin\theta_1}{\cos\theta_2}\frac{1}{\tan\alpha}. \tag{7.25}$$

We also introduce

$$H' = B + \frac{R^3}{\cos\theta_1\cos\theta_2\, r^3}, \qquad h_i^{-1} = H'^{-1}\sin^2\theta_i + \cos^2\theta_i, \tag{7.26}$$

satisfying

$$H = H'h_1^{-1}, \qquad h^{-1} = h_1 h_2^{-1}, \tag{7.27}$$

where

$$B = \frac{A - \sin^2\theta_1}{\cos^2\theta_1}. \tag{7.28}$$

In [92] they use $A = B = 1$ which is consisted with (7.28). Using the relations above we now get

$$ds^2 = (H' h_1 h_2)^{-1/3} \Bigg[-dt^2 + h_1 \left((dx^1)^2 + (dx^2)^2\right) + h_2 \left((dx^3)^2 + (dx^4)^2\right)$$

$$+ h_1 h_2 \left(dx^5 + \sin\theta_1 \sin\theta_2 (H'^{-1} - 1) dt \right)^2 + H'(dr^2 + r^2 d\Omega_4^2) \Bigg],$$

$$\ell_p^3 C_3 = H'^{-1} \left(\frac{\sin\theta_2}{\cos\theta_1} h_1 dt \wedge dx^1 \wedge dx^2 + \frac{\sin\theta_1}{\cos\theta_2} h_2 dt \wedge dx^3 \wedge dx^4 \right. \tag{7.29}$$

$$\left. - h_1 \tan\theta_1 dx^1 \wedge dx^2 \wedge dx^5 - h_2 \tan\theta_2 dx^3 \wedge dx^4 \wedge dx^5 \right),$$

$$H_4 = dC_3 + 3\pi N \epsilon_4 \,.$$

We now want to match this to the solution of Bergshoeff *et al.* [91]

$$ds^2 = (E_1 E_2)^{1/3} \Bigg[-\tilde{H}^{-1} \left[1 - (1 - \tilde{H})^2 \frac{s_1^2 s_2^2}{E_1 E_2} \right] dt^2$$

$$+ \frac{2}{E_1 E_2} (1 - \tilde{H}) s_1 s_2 dt dx^5 + \frac{\tilde{H}}{E_1 E_2} (dx^5)^2 + \frac{1}{E_1} \left((dx^1)^2 + (dx^2)^2\right)$$

$$+ \frac{1}{E_2} \left((dx^3)^2 + (dx^4)^2\right) + dr^2 + r^2 d\Omega_4^2 \Bigg], \tag{7.30}$$

$$dC_3 = d\left(\frac{1 - \tilde{H}}{E_1}\right) c_1 s_2 \wedge dt \wedge dx^1 \wedge dx^2 + d\left(\frac{1 - \tilde{H}}{E_2}\right) c_2 s_1 \wedge dt \wedge dx^3 \wedge dx^4$$

$$- d\left(\frac{1 - \tilde{H}}{E_1}\right) c_1 s_1 \wedge dx^1 \wedge dx^2 \wedge dx^5 - d\left(\frac{1 - \tilde{H}}{E_2}\right) c_2 s_2 \wedge dx^3 \wedge dx^4 \wedge dx^5$$

$$- c_1 c_2 \star d\tilde{H},$$

where $s_i = \sin\theta_i$, $c_i = \cos\theta_i$ and

$$E_i = s_i^2 + \tilde{H} c_i^2 \tag{7.31}$$

with the harmonic function

$$\tilde{H} = a + \left(\frac{\tilde{R}}{r}\right)^3 . \tag{7.32}$$

We have allowed for a constant $\tilde{R}$ in order not to have to rescale the radial coordinate in the mapping. The mapping is obtained by setting

$$H' = \tilde{H} \,, \qquad H' h_i^{-1} = E_i \tag{7.33}$$

without any coordinate rescalings. The first requirement implies that

$$\tilde{R}^3 = \frac{R^3}{\cos\theta_1 \cos\theta_2} \tag{7.34}$$

and also that the constants in the harmonic functions must match, *i.e.,*

$$a = \frac{A - \sin^2 \theta_1}{\cos^2 \theta_1}, \tag{7.35}$$

which, if we view k, ν, a and θ_1 as independent parameters, impose an additional constraint on A. We can now solve for A and α,

$$A = (a - 1)\cos^2 \theta_1 + 1, \tag{7.36}$$

$$\tan^2 \alpha = A\left(\frac{2\nu}{k - \nu}\right). \tag{7.37}$$

We have thus obtained a complete mapping between our solution in Paper III, the solution of Izquierdo *et al.* and the solution of Bergshoeff *et al.*

It is also interesting to examine how the critical case $k = \nu$ is mapped into the parameters of solution (7.30). This analysis is only relevant if one wants to be able to boost the critical solution or go to the critical case $k = \nu$ without having to take any limits like $\tan \alpha \to \infty$. From (7.37) and (7.21) we have

$$x^{0,1,2} \rightarrow \left(\frac{(k-\nu)}{A}\left[2\nu\left(\frac{A}{k-\nu}\right) + 1\right]^{-1}\right)^{1/6} x^{0,1,2},$$

$$x^{3,4,5} \rightarrow \left(\frac{(k-\nu)}{A}\left[2\nu\left(\frac{A}{k-\nu}\right) + 1\right]^2\right)^{1/6} x^{3,4,5}, \tag{7.38}$$

$$r \rightarrow \left(\frac{A}{(k-\nu)}\left[2\nu\left(\frac{A}{k-\nu}\right) + 1\right]^{-1/2}\right)^{1/3} r.$$

and we must require that the mapping is non-singular. Therefore we let $A \to 0$ as $\nu \to k$ in such a way that the quotient $A/(k - \nu)$ is kept fixed. We also note that $A = 0$ corresponds to

$$a = -\tan^2 \theta_1 \tag{7.39}$$

and therefore, as soon as we have boosted the solution (see (7.25)), the value of a which gives the critical solution is negative. We can obviously not regard k, ν, a and θ_1 as independent parameters, as we did above, when we consider the critical case. Since we now have a relation between k and ν, which gives us the critical solution, we will also get a relation between a and θ_1, *i.e.,* relation (7.39).

It is interesting to note that unlike the ordinary DLCQ compactification of M-theory in flat space, where one compactifies on a light-like circle, we can compactify on a (finitely boosted) space-like circle, and still get a system with critical D0-branes in ten dimensions, provided that we have an M-wave in eleven dimensions. To see this take the M5-M2-M2'-MW solution (7.30) and reduce it to ten dimensions along the M-wave. By setting the constant in the harmonic function to zero, we get smeared D0-branes at infinity, *i.e.,* the D0-branes are critical. It is interesting to note that lowering the constant in the harmonic function below zero yields a singular

solution in ten dimension, while the solution in eleven dimensions is regular [98] all the way down to $a = -\tan^2 \theta_1$, where the M2-branes become critical, and the metric at infinity describes membranes smeared in three dimensions. Since the ten dimensional theory does not have to be in the non-relativistic "infinite momentum frame" in order to have critical D0-branes, it might be possible to extend the analysis of how the D0-branes describe M-theory in more general backgrounds.

We end this section with some comments regarding the recently proposed V-duality [99, 100, 92]. In [92] it is argued that it is only possible to obtain a decoupled OM theory from an infinitesimally boosted (M5,M2) bound state, but not from a finitely boosted one. Important for this conclusion is that the decoupling limit is assumed to be the same after the boost as before. We see that the restriction to infinitesimal boosts, *i.e.,* galilean transformations, follows directly from this assumption regarding the decoupling limit. The decoupling limit is obtained by scaling $t \sim \epsilon^0$ and $x^5 \sim \epsilon^{3/2}$ and demanding this scaling both before and after the boost gives, when inserted into the Lorentz transformation (7.24), that $\sinh \gamma \sim \epsilon^{3/2}$, *i.e.,* $\gamma \sim \epsilon^{3/2}$. The restriction to infinitesimal Lorentz transformations can therefore be seen to arise due to the different scalings of the coordinates when one tries to keep the decoupling limit fixed. If we instead use the decoupling limit reviewed in Section 8.3 [97], where all the coordinates on the M5-brane scale in the same way, we do not get any restriction to infinitesimal boosts. We get exactly the same supergravity dual, and therefore the same decoupled theory, for all values of n in the scaling limit (8.46). Since the coordinates scale in the same way, boosting before or after taking the decoupling limit commute. The scaling used in [92] corresponds to $n = 1$, but if we instead map the scaling limit with $n = 0$ to their coordinates it gives a decoupling limit where all coordinates on the M5-brane scale in the same way, thereby avoiding the restriction to infinitesimal boosts. It is therefore unclear to us in what sense V-duality is a non-trivial concept.

8

Decoupled theories

An obvious generalization of the AdS/CFT correspondence described above, is to try to take the same decoupling limit for a brane in a background field. By doing this in a magnetic background, one obtains a noncommutative Yang-Mills (NCYM) theory [101] with space-space noncommutativity instead of an ordinary commutative one. The case with an electric background, which leads to space-time noncommutativity, is a bit harder to make sense of since we are used to viewing time as just a parameter which labels the time evolution of a system. In this sense time is not an operator and it is unclear how it could fail to commute. It turns out that taking the decoupling limit in a critical electric field yields a noncommutative open string (NCOS) theory [102, 103], instead of a field theory, on the brane. The effect of the critical electric field is to make the effective string tension finite in the decoupling limit, in contrast to the Yang-Mills case where it diverges, keeping the full open string spectrum in the theory. After this was done, these ideas were generalized to eleven dimensions where one can take an analogous decoupling limit for an M5-brane in a critical background field, obtaining an open membrane (OM) theory [101, 104, 105, 106, 107] on the M5-brane world volume. By reducing OM-theory to ten-dimensions and using various dualities, decoupled theories on NS5 branes containing light open Dp-branes (ODp) [106, 108, 109], can be defined. It is also possible to define open Dp-brane theories on other host branes than NS5-branes [110, 111]. In Paper VII we generalize NCYM and NCOS theory, which can be defined as (0,1) or (1,0)-string theory, respectively, on a D3-brane probe in an (F,D3) background. By instead studying a (p, q)-string theory on the probe brane we get new noncommutative theories with interesting S-duality properties. We will now examine some of these noncommutative theories in more detail.

8.1   NCYM and NCOS

In their very influential paper [101], Seiberg and Witten study open string theory on a Dp-brane in a background B-field. The B-field affects the boundary conditions

for open strings ending on the Dp-brane according to[1][2]

$$(g_{\mu\nu}\partial_n x^\nu + iB_{\mu\nu}\partial_t x^\nu)\,\big|_{\partial\Sigma} = 0, \tag{8.1}$$

where μ is along the Dp-brane and ∂_n (∂_t) is a derivative normal (tangential) to the string world-sheet. If $B_{\mu\nu} = 0$ we get the ordinary Neumann boundary conditions, but as $B_{\mu\nu}$ becomes large compared to $g_{\mu\nu}$ the boundary conditions turn into Dirichlet ones. With these boundary conditions, the two-point function is

$$\langle X^\mu(0)X^\nu(\tau)\rangle = -\alpha' G^{\mu\nu}\log\tau + i\pi\Theta^{\mu\nu}\epsilon(\tau), \tag{8.2}$$

$$\langle X^i(0)X^j(\tau)\rangle = -\alpha' g^{ij}\log\tau, \tag{8.3}$$

where

$$\alpha' G^{\mu\nu} + \Theta^{\mu\nu} = \alpha'\left(\frac{1}{g + 2\pi\alpha'\langle\mathcal{F}\rangle}\right)^{\mu\nu}. \tag{8.4}$$

Here $G_{\mu\nu}$ is the effective metric seen by open strings, $\Theta^{\mu\nu}$ measures the noncommutativity and $\langle\mathcal{F}\rangle$ is the background value of the gauge invariant field strength on the Dp-brane[3],

$$\mathcal{F} = dA + \frac{1}{2\pi\alpha'}B, \tag{8.5}$$

i.e.,

$$\langle\mathcal{F}\rangle_{\mu\nu} = \frac{1}{2\pi\alpha'}B_{\mu\nu}. \tag{8.6}$$

The effective open string coupling is

$$G_o = e^\phi\sqrt{\frac{\det(g + B)}{\det g}} = e^\phi\left(\frac{\det G}{\det g}\right)^{\frac{1}{4}} \tag{8.7}$$

and from (8.4) we have the following expressions for $G^{\mu\nu}$, $\Theta^{\mu\nu}$ and $G_{\mu\nu}$:

$$G^{\mu\nu} = \left(\frac{1}{g+B}\right)_S^{\mu\nu} = \left(\frac{1}{g+B}g\frac{1}{g-B}\right)^{\mu\nu}, \tag{8.8}$$

$$\frac{\Theta^{\mu\nu}}{\alpha'} = \left(\frac{1}{g+B}\right)_A^{\mu\nu} = -\left(\frac{1}{g+B}B\frac{1}{g-B}\right)^{\mu\nu} \tag{8.9}$$

and

$$G_{\mu\nu} = g_{\mu\nu} - B_{\mu\rho}g^{\rho\sigma}B_{\sigma\nu}, \tag{8.10}$$

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[1] As in Paper VI, our convention for B_{MN} differs by a factor $2\pi\alpha'$ from that in [101].

[2] We have not yet added any gauge fields on the world-sheet.

[3] All background fields that appear in the formulas regarding open string data are implicitly assumed to be pulled back to the brane.

where S and A denote the symmetric and antisymmetric part, respectively, and $G_{\mu\nu}$ is the inverse of $G^{\mu\nu}$. The metric g_{MN} will always refer to the string metric and will sometimes be denoted g_{MN}^{str} to avoid confusion.

As was shown by Seiberg and Witten [101], depending on what type of regularization we choose we get seemingly different theories. With Pauli-Villars regularization we get an ordinary commutative theory, while with point-splitting regularization we get a noncommutative theory. However, since it is known that the difference between different regularizations is always in a choice of contact terms these theories must be related via coupling constant redefinitions. Since for a world-sheet lagrangian, the coupling constants are the space-time fields, the two theories must be related via a field redefinition. Noncommutative Yang-Mills is therefore shown to be equivalent to ordinary Yang-Mills and the equivalence is realized by a field redefinition.

Originally, the noncommutative theories were obtained in a certain flat space scaling limit [101, 102, 103]. This limit is obtained by demanding that the massive closed string states, having a rest mass proportional to $\sqrt{|g_{00}|/\alpha'}$, become infinitely heavy, while at the same time the mass of open string states are kept finite in the NCOS case and are sent to infinity in the NCYM case. In order to decouple also the massless closed string sector we must take $\alpha' \to 0$, as is in the AdS/CFT case. A more careful analysis [112] based on calculations of absorption cross-sections shows, however, that the massless closed string sector only decouples for $p \leq 5$. There is also compelling evidence [109] that for $p = 5$ the resulting theory contains a (closed) little string sector. Since the rest-mass of an open string state with oscillator number $N \geq 1$ is proportional to $\sqrt{|G_{00}|(N-1)/\alpha'}$, we can read off the effective tension for an F-string from the open string metric as[4]

$$\frac{1}{\alpha'_{eff}} = \frac{|G_{00}|}{\alpha'}.$$ (8.11)

By requiring, as was mentioned above, that the effective open string tension is fixed for NCOS we keep the full open string spectrum in the theory, while for NCYM, by sending the effective open string tension to infinity, we only keep the massless level, *i.e.*, we get a field theory. An alternative way of computing the effective open string tension is to view an open string as a dipole situated in an electric field. The ordinary string tension, striving to contract the string, is then counteracted by the pull on the open string endpoints due to the electric field. We therefore get[5]

$$\frac{1}{\alpha'_{eff}} = \frac{|g_{00}^{str}|}{\alpha'} - \frac{\epsilon^{01}B_{01}}{\alpha'}.$$ (8.12)

What makes these two definitions equivalent (up to a constant factor), despite $(\alpha'_{eff})^{-1}$ being quadratic in B in the first definition and linear in the second, is

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[4]In this argument, we use the mass formula for an open *bosonic* string. For a *fermionic* open string the tension will only differ by a constant factor.

[5]The first term in the RHS can also be written as $\frac{\sqrt{-\det g}}{\alpha'}$, where $\sqrt{-\det g}\, dx^0 dx^1$ is the invariant volume element in the string directions. It is this form of the effective tension that generalizes to higher dimensions.

that in both cases the B term manages to cancel the divergent term in g_{00}^{str}/α'. The remaining finite terms then give the effective open string tension.

Instead of defining the decoupling limit as a flat space scaling limit [102, 103], we can derive the decoupling limit from the supergravity dual as in Paper VI and [104, 107, 97]. This provides a more physical interpretation of the decoupling limit as a certain limit of a probe brane in the background generated by a stack of branes, as will be explained below. We choose the canonical dimensions of the fields such that if ds^2/α' and $C_{(p)}/(\alpha')^{p/2}$, where $C_{(p)}$ is a p-form potential, are held fixed in the limit $\alpha' \to 0$, then the supergravity action is finite. We start by going to critical field strength, which is possible for the brane solutions found in Papers III and IV without having to take any limits. Requiring the quantities above to be fixed in the $\alpha' \to 0$ limit tells us how the coordinates and the parameters of the supergravity dual, *i.e.*, the field strengths, must scale with α'. We can then write the supergravity dual in terms of quantities that are fixed in the $\alpha' \to 0$ limit. By putting in a probe brane, of the same type as the host branes, at some radius, we can now study the induced theory on the probe. The radial coordinate can be viewed as the energy scale, in appropriate units, above which higgsed W-bosons, *i.e.*, interactions between the probe and the stack, are no longer negligible. By pushing the probe brane to infinity, and taking $\alpha' \to 0$, we can obtain a theory decoupled from both gravity and the stack. Taking the probe brane to infinity, which will be called the UV limit [113, 99, 97], is the only way to obtain a theory that is *potentially* UV complete. For critical field strength, the metric in the UV region generally approaches some configuration of smeared branes, as observed in Papers III and IV, indicating the presence of light open branes in the theory (of the same type as those being smeared), implying that the decoupled theory might not be a field theory but rather a theory of the light open branes in question. Two cases where we get open brane theories are OM theory and NCOS theory, in which cases the metric at infinity approaches that of smeared membranes and strings, respectively. For NCYM, however, despite that the metric is the same as for NCOS, we in this case get a field theory. Note that there are three separate limits involved here, the critical limit, the $\alpha' \to 0$ limit and the UV limit. These three limits together will be referred to as the decoupling limit. When using the flat space scalings, the three limits are combined into one ϵ scaling limit.

As a concrete example, we will now use this method to derive the NCYM/NCOS decoupling limit. As we will see, using the supergravity solution describing the most general ((F,D1),D3) bound state, obtained in Paper III, the limits giving NCYM and NCOS are actually identical. This also serves as an example of how to choose the scalar doublet $\mathcal{U}^r$ in our SL(2,$\mathbb{R}$) invariant solutions in order to get a specific bound state, *e.g.*, a (F,D3) or a (D1,D3) bound state. The scalars can then be obtained form the projective invariant according to $\mathcal{U}^1/\mathcal{U}^2 = \chi + ie^{-\phi}$. We will first compute the open string metric for NCYM and NCOS and then derive the decoupling limit.

The scalar doublet satisfies (see Section 3.2 for details)

$$1 = \frac{i}{2}\epsilon_{rs}\mathcal{U}^r\bar{\mathcal{U}}^s\,, \qquad P = \frac{1}{2}\epsilon_{rs}d\mathcal{U}^r\mathcal{U}^s \tag{8.13}$$

and

$$Q = \frac{1}{2}\epsilon_{rs}d\mathcal{U}^r\bar{\mathcal{U}}^s, \tag{8.14}$$

where $\epsilon_{12} = -1$.[6] From the D3 brane solution in Paper III we have

$$\begin{aligned} P &= i\mu \left(\Delta^2 - \nu^2\right)^{-1} d\Delta, \\ Q &= 0, \end{aligned} \tag{8.15}$$

where

$$\Delta = k + \left(\frac{R}{r}\right)^4, \qquad R^4 = g_s N(\alpha')^2, \tag{8.16}$$

is the harmonic function, μ is a measure of the (complex) field strength and $\nu = 2|\mu|$ (see Paper III for details). Solving for $\mathcal{U}^r$ gives

$$\begin{aligned} \mathcal{U}^1 &= c\eta(\Delta_+)^{\mp\frac{1}{4}}(\Delta_-)^{\pm\frac{1}{4}}, \\ \mathcal{U}^2 &= -\frac{i}{c\bar{\eta}}(\Delta_+)^{\pm\frac{1}{4}}(\Delta_-)^{\mp\frac{1}{4}}, \end{aligned} \tag{8.17}$$

where c is a real constant, $\eta^2 = \pm\frac{\mu}{|\mu|}$ and the lower (upper) sign corresponds to the NCYM (NCOS) background as we will see shortly. We have also introduced the notation $\Delta_\pm = \Delta \pm \nu$, which is convenient since the harmonic function will only be deformed in these two ways when we turn on a background field. We can obtain the field strength doublet according to

$$H_{(3)r} = \epsilon_{rs}\mathrm{Im}\left(\mathcal{U}^s\bar{\mathcal{H}}_{(3)}\right), \tag{8.18}$$

where

$$\mathcal{H}_{(3)} = \mathcal{U}^r dB_{(2)r} \tag{8.19}$$

and[7]

$$\mathcal{H}_{p\mu\nu} = 2\partial_p\Delta\left(\Delta^2 - \nu^2\right)^{-\frac{1}{4}}\left(\Delta_-^{-\frac{3}{2}}\Pi_+F + \Delta_+^{-\frac{3}{2}}\Pi_-F\right)_{\mu\nu} \tag{8.20}$$

are given by the solution[8]. To express the field strength we have used the projectors

$$\Pi_\pm = \frac{1}{2}\left(1 \pm \frac{2}{\nu}F\bar{F}\right), \tag{8.21}$$

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[6]The factor i in P is due to the different conventions used in Papers III and IV.

[7]We have rescaled the fields as $H_{(3)r} \to \frac{1}{2}H_{(3)r}$ and $H_{(5)} \to \frac{1}{4}H_{(5)}$ compared to Paper IV in order to conform to the most commonly used conventions.

[8]We use the notation that Greek indices denote longitudinal directions and Latin indices denote transverse directions.

where time is in the plus sector. Integrating $H_{(3)r}$ gives the potentials

$$B_1 = -\frac{2}{c\eta}\Delta_+^{-1}(\Pi_- F),$$
$$B_2 = -2ic\bar{\eta}\Delta_-^{-1}(\Pi_+ F). \tag{8.22}$$

The fact that B_1 is proportional to $\Pi_- F$, *i.e.*, B_1 has no components along the time direction, means that this choice of the scalar doublet implies a magnetic field on the brane and we therefore get the NCYM case. Using

$$ds_E^2 = (\Delta^2 - \nu^2)^{-\frac{1}{4}}\left[\left(\frac{\Delta_+}{\Delta_-}\right)^{\frac{1}{2}} dx_+^2 + \left(\frac{\Delta_-}{\Delta_+}\right)^{\frac{1}{2}} dx_-^2\right] + (\Delta^2 - \nu^2)^{1/4} dy^2, \tag{8.23}$$

$$e^\phi = c^{-2}\left(\frac{\Delta_-}{\Delta_+}\right)^{\frac{1}{2}} \tag{8.24}$$

and the following useful properties of the projectors,

$$(\Pi_\pm F)(\Pi_\pm F) = \mu\Pi_\pm,$$
$$(\Pi_\pm F)(\Pi_\mp F) = 0, \tag{8.25}$$

we can now compute the open string metric for NCYM

$$G_{\mu\nu}^{NCYM} \equiv g_{\mu\nu}^s - (B_1 g_{str}^{-1} B_1)_{\mu\nu} = \frac{1}{c}\Delta_-^{-\frac{1}{2}}\eta_{\mu\nu}, \tag{8.26}$$

where $g_{MN}^{str} = e^{\frac{\phi}{2}} g_{MN}^E$ is the string metric. From equation (8.24) we note that c is related to the value of the undeformed dilaton.

To obtain the open string metric for NCOS we use the upper sign in the solution for the scalar doublet. The background potentials are

$$B_1 = -\frac{2c}{\eta}\Delta_-^{-1}(\Pi_+ F),$$
$$B_2 = i\frac{2\bar{\eta}}{c}\Delta_+^{-1}(\Pi_- F) \tag{8.27}$$

and the dilaton is

$$e^\phi = c^2\left(\frac{\Delta_+}{\Delta_-}\right)^{\frac{1}{2}}. \tag{8.28}$$

The fact that B_1 is proportional to $\Pi_+ F$ means that this choice of the scalar doublet implies an electric field on the brane, *i.e.*, we get the NCOS case, and the open string metric is

$$G_{\mu\nu}^{NCOS} \equiv g_{\mu\nu}^s - (B_1 g_{str}^{-1} B_1)_{\mu\nu} = c\Delta_+^{-\frac{1}{2}}\eta_{\mu\nu}. \tag{8.29}$$

In order to derive the decoupling limit, we first go to critical field strength, *i.e.,* $\nu = k$, and then demand that ds^2/α' is fixed in the limit when $\alpha' \to 0$. We then get the following scalings

$$\nu = k = \frac{1}{2}\left(\frac{\ell}{\ell_s}\right)^{4n}, \qquad \tilde{x}_\pm = \left(\frac{\ell_s}{\ell}\right)^{n-1} x_\pm, \qquad \tilde{r} = \frac{\ell^{n+1}r}{\ell_s^{n+1}(g_s N)^{1/4}}, \qquad (8.30)$$

where the tilded coordinates and the length scale ℓ are fixed in the limit. Rewriting the supergravity dual in terms of these fixed coordinates gives[9]

$$\frac{ds^2}{\alpha'} = \frac{1}{\ell^2}\left(f^{1/4}\tilde{r}^3\ell^{-3}d\tilde{x}_+^2 + f^{-3/4}\tilde{r}^{-1}\ell d\tilde{x}_-^2 + f^{1/4}\tilde{r}^{-1}\ell(g_s N)^{1/2}(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2)\right)$$
$$(8.32)$$

and

$$\frac{(B_1)_{23}}{\alpha'} = -\frac{2}{c\ell^2}f^{-1}, \qquad \frac{(B_2)_{01}}{\alpha'} = -\frac{2c}{\ell^6}\tilde{r}^4, \qquad e^\phi = c^{-2}\tilde{r}^{-2}\ell^2 f^{-1/2} \qquad (8.33)$$

for NCYM and

$$\frac{(B_1)_{01}}{\alpha'} = -\frac{2c}{\ell^6}\tilde{r}^4, \qquad \frac{(B_2)_{23}}{\alpha'} = -\frac{2}{c\ell^2}f^{-1}, \qquad e^\phi = c^2\tilde{r}^2\ell^{-2}f^{1/2} \qquad (8.34)$$

for NCOS where

$$f = 1 + \left(\frac{\ell}{\tilde{r}}\right)^4. \qquad (8.35)$$

For sub-critical field strength, both g_{00}/α' and e^ϕ are finite in the UV limit (since the free $\tilde{r}$ coming from Δ_- would be replaced with a constant in the UV limit) and therefore closed strings are not decoupled.

Now, in order to obtain an NCOS theory we must demand that the effective tension of the F-strings are finite in the UV limit $\tilde{r} \to \infty$. Using the critical decoupling limit above we get

$$\frac{G_{\mu\nu}}{\alpha'} = \frac{c}{\ell^2}f^{-1/2}\eta_{\mu\nu}, \qquad (8.36)$$

which is finite in the UV. By applying the same limit to NCYM we see that

$$\frac{G_{\mu\nu}}{\alpha'} = \frac{\tilde{r}^2}{c\ell^4}\eta_{\mu\nu}, \qquad (8.37)$$

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[9]In order to get explicit expressions for the two-form potentials we have used the following parameterization of F_{ij},

$$F_{ij} = \begin{pmatrix} 0 & \sqrt{\mu} & 0 & 0 \\ -\sqrt{\mu} & 0 & 0 & 0 \\ 0 & 0 & 0 & i\sqrt{\mu} \\ 0 & 0 & -i\sqrt{\mu} & 0 \end{pmatrix}, \qquad (8.31)$$

which satisfies the algebra used in Paper III.

which diverges in the UV as it should. Note that we do not get any constraint on how α' should scale with $\tilde{r}$. We have now obtained the decoupling limit, given in the supergravity dual by $\tilde{r} \to \infty$, for NCYM and NCOS. Using translations between different forms of the deformed D3-brane solution, analogous to the ones for the deformed M5-brane solutions in Section 7.4, we see that the decoupling limit used in Paper VI corresponds to taking $n = 0$ in the scaling limit (8.30).

8.2   Open (p, q)-string theories

We can now unify and generalize the treatment in the previous section by considering open (p, q)-strings theories, on the probe brane, instead of only $(1, 0)$-string theory, *i.e.,* ordinary string theory. This treatment can be done both for the D3 and the (p, q) 5-branes of type IIB, as is done in Paper VII, but here we will concentrate on the D3-brane case for simplicity. We use the (F,D3) bound state solution from the previous section, which is given in the Einstein frame. In our approach, we hold the background fixed, while SL(2,$\mathbb{Z}$) rotating the string we use to study the theory on the probe in order to get different theories. If we were to apply the same SL(2,$\mathbb{Z}$) transformation to both the background and the string we would get a theory equivalent to the one we started with. Since we have chosen the background to be the (F,D3) bound state solution, we get NCOS if we use a (1,0)-string to study the probe. Since S-dualizing the background is equivalent to S-dualizing the string in our picture, we get NCYM if we use a (0,1)-string instead. It is only the angle between the SL(2) charges of the probe (*i.e.,* the background), given by the doublet of three forms, and the string used to study it that matters. Let $(p^1, p^2) = (p, q)$ be the string charges. The charges with indices downstairs are then obtained as $p_r = \epsilon_{rs} p^s$ yielding $(p_1, p_2) = (-q, p)$, since we use $\epsilon_{01} = -1$. In the gauge $\text{Im}(\mathcal{U}^2) = 0$, the string tension, in units of $1/\alpha'$, is given by [22]

$$|\mathcal{U}^r p_r| = \sqrt{e^{\phi}(p - q\chi)^2 + q^2 e^{-\phi}}. \tag{8.38}$$

In order to get the SL(2,$\mathbb{Z}$)-covariant open string data we therefore have to replace $e^{\phi/2}$ by $|\mathcal{U}^r p_r|$ giving

$$G^{\mu\nu} = \tfrac{1}{|\mathcal{U}^r p_r|} \left(\left(g^{\text{E}} + \tfrac{p^s C_s}{|\mathcal{U}^t p_t|} \right)^{-1}_{\text{S}} \right)^{\mu\nu} = \tfrac{1}{|\mathcal{U}^r p_r|} \left(\left(g^{\text{E}} + \tfrac{p^s C_s}{|\mathcal{U}^t p_t|} \right)^{-1} g^{\text{E}} \left(g^{\text{E}} - \tfrac{p^m C_m}{|\mathcal{U}^n p_n|} \right)^{-1} \right)^{\mu\nu},$$

$$\tfrac{\Theta^{\mu\nu}}{\alpha'} = \tfrac{1}{|\mathcal{U}^r p_r|} \left(\left(g^{\text{E}} + \tfrac{p^s C_s}{|\mathcal{U}^t p_t|} \right)^{-1}_{\text{A}} \right)^{\mu\nu} = -\tfrac{1}{|\mathcal{U}^r p_r|} \left(\left(g^{\text{E}} + \tfrac{p^s C_s}{|\mathcal{U}^t p_t|} \right)^{-1} \tfrac{p^k C_k}{|\mathcal{U}^l p_l|} \left(g^{\text{E}} - \tfrac{p^m C_m}{|\mathcal{U}^n p_n|} \right)^{-1} \right)^{\mu\nu},$$

$$G_{\mu\nu} = |\mathcal{U}^r p_r| \left(g^{\text{E}}_{\mu\nu} - \tfrac{(p^s C_s)^2_{\mu\nu}}{|\mathcal{U}^t p_t|^2} \right), \quad G_{\text{O}} = |\mathcal{U}^r p_r| \left(\tfrac{\det G}{\det g^{\text{E}}} \right)^{\frac{1}{4}}. \tag{8.39}$$

The effective open string tension can now be obtained from the open string metric as before, or equivalently from the dipole analogy as

$$\frac{1}{\alpha'_{eff}} = |\mathcal{U}^r p_r| \frac{|g^E_{00}|}{\alpha'} - \frac{(C_{(2)r} p^r)_{01}}{\alpha'}. \tag{8.40}$$

A possible source of confusion in this context is that we keep α' fixed when rotating the string charges. Since we do not change the background, α' will always be the F-string tension. Most authors instead perform the S-duality transformation on the background, and since an F-string should turn into a D-string they have to transform α'.

In order to look at the decoupled theory we choose to use $n = 0$ in (8.30) giving

$$k = \nu = \frac{1}{2}, \quad \hat{x}_\pm = \frac{\ell}{\ell_s}x_\pm, \quad u = \frac{\ell}{\ell_s}r, \qquad c \quad \text{fixed.} \tag{8.41}$$

We can now calculate the open string data for arbitrary string charges

$$\begin{aligned}
\frac{G_{\mu\nu}}{\alpha'} &= \frac{1}{\ell^2}c^{-1}(q^2 + c^4(p - q\chi)^2)(q^2\tilde{r}^{-4}\ell^4 + c^4(p - q\chi)^2 f)^{-\frac{1}{2}}\eta_{\mu\nu} \\
\Theta^{01} &= -\ell^2\sqrt{2\nu}\,c^3(p - q\chi)(q^2 + c^4(p - q\chi)^2)^{-1} \\
\Theta^{23} &= -\ell^2\sqrt{2\nu}\,c\,q(q^2 + c^4(p - q\chi)^2)^{-1} \\
G_{\text{O}} &= c^{-2}(q^2 + c^4(p - q\chi)^2)
\end{aligned} \tag{8.42}$$

We can now look at the two special cases with $(1,0)$ and $(0,1)$-strings and $\chi = 0$, giving NCOS and NCYM respectively. The open string data for the $(1,0)$-probe are

$$\begin{aligned}
\frac{G_{\mu\nu}}{\alpha'} &= \frac{c}{\sqrt{2}\ell^2}\eta_{\mu\nu}, \qquad \Theta^{01} = -\sqrt{2}\ell^2 c^{-1}, \\
G_{\text{O}} &= c^2, \qquad\qquad \Theta^{23} = 0
\end{aligned} \tag{8.43}$$

and for the $(0,1)$-probe we have

$$\begin{aligned}
\frac{G_{\mu\nu}}{\alpha'} &= \frac{1}{\ell^4}c^{-1}\tilde{r}^2\eta_{\mu\nu}, \qquad \Theta^{23} = -\sqrt{2}\ell^2 c, \\
G_{\text{O}} &= c^{-2}, \qquad\qquad \Theta^{01} = 0.
\end{aligned} \tag{8.44}$$

We thus see that this result agrees with (8.36) and (8.37) and that the open string coupling for NCOS is the inverse of that of NCYM implying a strong/weak coupling duality.

An important result of Paper VII is that every strongly coupled open (p, q)-string theory has a weakly coupled SL(2,$\mathbb{Z}$)-dual theory. Therefore, most of the moduli space of noncommutative theories on the D3-brane are accessible for perturbative calculations.

8.3 OM theory

The ideas leading to decoupled theories containing light open strings can be generalized to eleven dimensions giving a theory containing light open membranes, OM theory [101, 104, 105, 106, 107]. However, since we can not quantize the membrane we do not have the same fundamental understanding of this theory as of, *e.g.,* NCOS. For example, the open membrane metric can only be argued for up to a

conformal factor. We will now derive the decoupling limit for OM theory using the same technique as in the NCYM/NCOS case [97].

We start from the metric for the deformed M5-brane solution in Paper III,

$$
g_{\mu\nu} = \left(\Delta^2 - \nu^2\right)^{-\frac{1}{6}} \left[\left(\frac{\Delta_+}{\Delta_-}\right)^{\frac{1}{2}} dx_-^2 + \left(\frac{\Delta_-}{\Delta_+}\right)^{\frac{1}{2}} dx_+^2 \right] + \left(\Delta^2 - \nu^2\right)^{1/3} dy^2, \quad (8.45)
$$

where time now is in the minus directions. By requiring that ds^2/ℓ_p^2 is fixed in the $\ell_p \to 0$ limit, we are led to the following scaling limit

$$
k = \nu = \frac{1}{2}\left(\frac{\ell}{\ell_p}\right)^{3n}, \qquad \tilde{x}_\pm = \left(\frac{\ell_p}{\ell}\right)^{\frac{n}{2}-1} x_\pm\,, \qquad \tilde{r} = \frac{\ell^{n+1} r}{\ell_p^{n+1} N^{1/3}}, \qquad (8.46)
$$

where again the tilded coordinates and the length scale ℓ are fixed in the limit. Rewriting the supergravity dual in terms of these fixed coordinates gives[10]

$$
\frac{ds^2}{\alpha'} = \frac{1}{\ell^2}\left(f^{1/4}\tilde{r}^3\ell^{-3}d\tilde{x}_+^2 + f^{-3/4}\tilde{r}^{-1}\ell d\tilde{x}_-^2 + f^{1/4}\tilde{r}^{-1}\ell(g_s N)^{1/2}(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2)\right), \tag{8.47}
$$

$$
\frac{H_{(4)}}{\ell_p^3} = \frac{N}{\ell^3}\epsilon_4(S^4) + \frac{1}{\ell^3}d(\tilde{r}^3\ell^{-3}d\tilde{x}_-^3 - f^{-1}d\tilde{x}_+^3)
$$

where now

$$
f = 1 + \left(\frac{\ell}{\tilde{r}}\right)^3. \tag{8.48}
$$

The effective open membrane tension can be computed by generalizing the dipole argument for the open string. We then get

$$
\frac{1}{\ell_{eff}^3} = \frac{\sqrt{-\det g}}{\ell_p^3} - \frac{\epsilon^{012}C_{012}}{\ell_p^3}, \tag{8.49}
$$

where $\sqrt{-\det g}\, dx^0 dx^1 dx^2$ is the invariant volume element in the membrane directions. This expression for the open membrane effective tension reduces to the expression for the effective string tension (8.12) under dimensional reduction. The effective tension must be finite in the UV limit in order to have light open membranes in the spectrum. By using the solution above in (8.49), we get

$$
\frac{1}{\ell_{eff}^3} = f^{1/2}\frac{\tilde{r}^3}{\ell^6} - \frac{\tilde{r}^3}{\ell^6} \tag{8.50}
$$

and since $f \to 1$ in the UV limit we see that the divergent parts cancel. This, however, does not mean that the effective tension has to go to zero since we can always generate a constant finite term in the effective tension using a gauge transformation of $C_{(3)}$. Note that as in the D3-brane case we get no constraint on how ℓ_p should scale with $\tilde{r}$. By choosing $\ell_p \sim \tilde{r}^{-1}$ we get the limit used in [106, 107].

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[10]In order to get explicit expressions for the three-form potential we have used $h_{012} = -h_{345} = \sqrt{2}\nu$ which satisfy the algebra used in Paper III.

8.4 OD*p* theories

It is natural to generalize the ideas above to theories containing light open branes other than strings and membranes. This has been done for open D*p*-branes on NS5-branes in type IIA/B [106, 109], where there also exist a limit giving light open D*p*-branes, *i.e.,* yielding OD*p* theories. Very little is known about these theories due to the lack of any fundamental understanding of them, *i.e.,* it is only the open string that we can quantize. We would, however, like to make a comment regarding the OD5 theory. This theory is defined on an NS5-brane in type IIB theory and contains light open D5-branes [106, 109]. When studying the metrics obtained in Papers III and IV, for critical field strength, the asymptotic metric generally approaches some smeared brane configuration. For a D3-brane we get a smeared string, for an M5-brane we get a smeared membrane and for a (p, q) 5-brane we generically get a smeared string, but for the special case of a magnetic rank 2 field strength we get a smeared 3-brane. In all the cases we have studied, the object appearing smeared at infinity is the light object in the theory. For an NS5-brane in type IIB we therefore see that we can get OD1 and OD3 theories, depending on the choice of field strength. However, at first it seems difficult to obtain an SO(5,1) invariant solution for critical field strength, which would yield an OD5 theory. It turns out that the OD5 theory is obtained by taking $k = \nu = 0$ in the (p, q) 5-brane solution in Paper IV, which amounts to taking zero background field strength and setting the constant in the harmonic function to zero. This is still a critical limit in the sense that $k = \nu$, even though the field strength in this case is zero. This corresponds to embedding the (p, q) 5-brane in an AdS background instead of a Minkowski one. Using the analogue of (8.49) for the effective open D5-brane tension, where the metric used should be the closed D5-brane metric defined as

$$g_{\mu\nu}^{Dp} \equiv e^{-\frac{2\phi}{p+1}} g_{\mu\nu}^{str}, \qquad (8.51)$$

we see that the effective tension is finite in the UV limit.

9

Outlook

The last dramatic advance in our understanding of string theory was the discovery of the AdS/CFT correspondence, reviewed in Chapter 6. The correspondence has been scrutinized in a large number of situations and has passed all tests so far. Matrix theory has also attracted a lot of attention and has been shown to be a well defined quantum theory which reduces to a supersymmetric theory of gravity at low energies [6]. The formulation is, however, not background independent. There has also been the discovery of a multitude of decoupled, noncommutative theories, reviewed in Chapter 8. It will be interesting to see how much can be learnt from these theories and what rôle they will play in the future developments of string theory. Lately, there has been significant progress in the understanding of tachyon condensation (see [15] and references therein), which is a very active area at the moment.

It will also be interesting to follow the developments concerning higher derivative corrections to the lowest order effective supergravity actions, especially in eleven dimensions. Note that in ten dimensions we have a microscopic understanding in terms of string theory and can derive the correction terms. In eleven dimensions, however, it has not yet been possible to compute any correction terms using the current microscopic understanding through matrix theory. The terms we are able to deduce by, *e.g.,* relating to results in ten dimensions, can instead be viewed as providing information about the microscopic eleven dimensional theory. The higher derivative terms are also interesting since they can be used to check various conjectured dualities [45, 39]. It is also believed that many of the space-time singularities appearing in supergravity solutions will be resolved when taking into account the higher derivative terms [68].

There are, however, still some great challenges for string theory. One fundamental problem is that the formulation of string theory is not background independent. Each background has its own description of string theory, *e.g.,* string theory is described by a CFT when formulated in AdS space and by matrix theory when formulated in flat eleven-dimensional space. In light of the recent discovery that

the expansion of the universe is accelerating[1] we are ultimately forced to formulate string theory in a background like this. There are two ways of modeling an accelerating universe, either by including a positive cosmological constant, giving deSitter space, or by using the theory of quintessence. The reason for the accelerated expansion in the case of a positive cosmological constant is that the cosmological constant can be interpreted as a vacuum energy density counter-acting gravity. If correct, this leads to an ever expanding universe and also implies that most of the energy in the universe is associated with the vacuum. In the theory of quintessence, on the other hand, the dark energy of the universe is dominated by the potential of a scalar field ϕ, which is still rolling towards the minimum of the potential, typically at $\phi = \infty$. Quintessence models that accommodate the current acceleration seem to accelerate eternally [114, 115]. There are, however, profound problems concerning the formulation of string theory in both these approaches since it seems impossible to define an S-matrix in the respective backgrounds [116, 114, 115]. This might require a fundamental revision of our ideas about string theory. There is a recent proposal how to obtain de Sitter space from modified (or massive) M-theory [117].

Finally, the experimental verification of supersymmetry seems to be in reach in the accelerators under construction. If traces of supersymmetry can be detected it will no doubt be one of the greatest experimental discoveries. Hopefully the "stock price" for strings [118] will continue to rise!

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[1] See, *e.g.,* http://www.eso.org/outreach/press-rel/pr-1998/pr-21-98.html and http://-www.sciencemag.org/cgi/content/summary/282/5397/2156a.

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