Cu is deposited onto the surface of a thick Si wafer at a rate 0.85×10^{14} [atoms cm^{-2} s^{-1}]. The temperature of the wafer is T = 1000 C. Calculate the thin-film growth rate after t = 100 s from the beginning of the deposition. Ignore the re-evaporation of Cu from the wafer. Take the solid-solubility limit of Cu in Si as $N(T) = 10^{24}\exp(-18420 / T)$ [1/cm^3] and diffusion coefficient of Cu in Si as $D(T) = 1.61\times10^{-2}\exp(-10900 / T)$ [cm^2/s]. Density of Cu = 8.96 [g/cm^3]; molar weight = 63.55 [g/mol]. Hints: assume the constant-source diffusion solution $C(x,t) = C_0\left(1 - \int_0^{\frac{x}{\sqrt{4Dt}}} e^{-z^2}dz\right)$ and compare the rate at which material builds up at the surface with the rate at which it disappears through the diffusion.

If there were no diffusion of Cu into Si, the thin film would grow at the constant rate

$$G = \frac{0.85\cdot10^{14}}{6.022\cdot10^{23}}\frac{mol}{cm^2 s}\times 63.55\frac{g}{mol}\times\frac{1}{8.96}\frac{cm^3}{g} \approx 0.10\frac{Å}{s} \qquad (0)$$

(6.022×10^{23} mol^{-1} is the Avogadro's number).

However, Cu readily diffuses into Si at such a high temperature.

We use the constant-source regime to describe the diffusion.

$$C(x,t) = C_0\left(1 - \int_0^{\frac{x}{\sqrt{4Dt}}} e^{-z^2}dz\right) \qquad (1) \quad ****$$

C_0 in our case is the constant solid-solubility limit of Cu in Si, $C_0 = N(T)$.
$N(1273\text{K}) = 5.21\times10^{17}$ cm^{-3}.

The total amount of Cu atoms that has diffused into Si at the time t is:

$$S(t) = \int_0^\infty C(x,t)dx \qquad (2)$$

The rate at which Cu atoms disappear in Si is then:

$$\frac{dS(t)}{dt} = \int_0^\infty \frac{\partial C(x,t)}{\partial t}dx \qquad (3)$$

From (1) it is easy to obtain the derivative under the integral:

$$\frac{\partial C(x,t)}{\partial t} = -C_0\frac{\partial}{\partial t}\int_0^{\frac{x}{\sqrt{4Dt}}} e^{-z^2}dz = \frac{C_0}{2t}\cdot\frac{x}{\sqrt{4Dt}}\cdot\exp\left(-\frac{x^2}{4Dt}\right) \qquad (4)$$

Substituting (4) into (3) and doing the integration, we get:

$$\frac{dS(t)}{dt} = -C_0\sqrt{\frac{D}{4t}} \qquad (5)$$

Taking $D(1273\text{K}) = 3.08\times10^{-6}$ [cm^2 s^{-1}] and t = 100 s, we get $dS(t)/dt = 4.56\times10^{13}$ cm^{-2} s^{-1} that corresponds to about 0.054 Å s^{-1} of the rate at which the Cu thin film disappears in Si.

Finally, subtracting the last obtained value from G above, we get the answer: <u>0.046 Å s^{-1}</u>

**** Strictly speaking, the formula should have had a pre-factor of $2/\sqrt{\pi} = 1.13$. This changes the numerical answer to 0.039 Å/s.