Vacuum

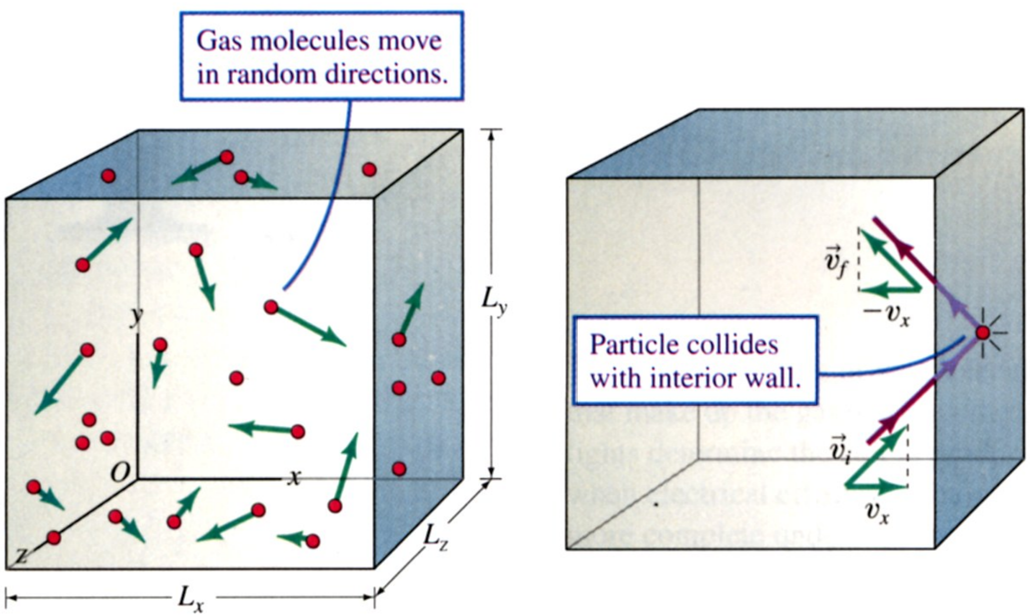
Gas kinetics, pumping, gauges

Kinetic Theory of Gases

- Microscopic View of Gases
- Molecular Velocities

$$\langle v^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle$$

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle$$ equivalence of directions



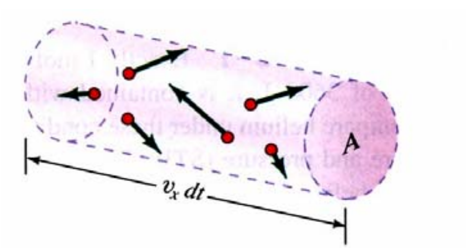
The internal energy of a gas :

$$U = N\langle K \rangle = N \cdot \frac{m\langle v^2 \rangle}{2}$$

$$\langle v_x^2 \rangle = \frac{1}{3}\langle v^2 \rangle = \frac{2}{3}\frac{U}{m\,N}$$

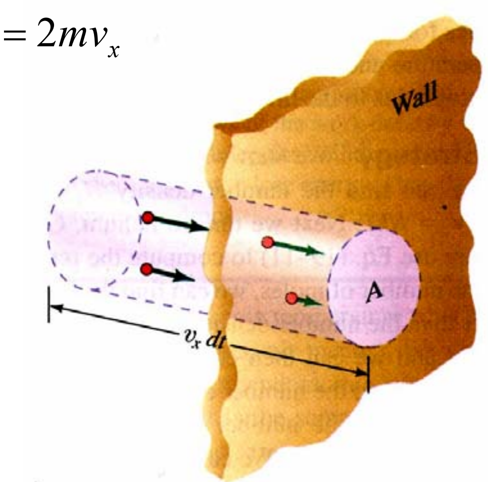
The Origin of Pressure

- Pressure results from multiple collisions between molecules and walls of a pot containing gas.



The momentum transfer to the wall:

$$\delta M = 2mv_x$$



The No. of collisions with the wall during the time dt:

$$N_{coll} = \frac{1}{2}\frac{N}{V}(v_x dt)A$$

The total momentum transferred:

$$dM = N_{coll}\cdot\delta M = (2mv_x)\frac{1}{2}\frac{N}{V}(v_x dt)A = \frac{N}{V}mv_x^2\,A\,dt$$

The pressure:

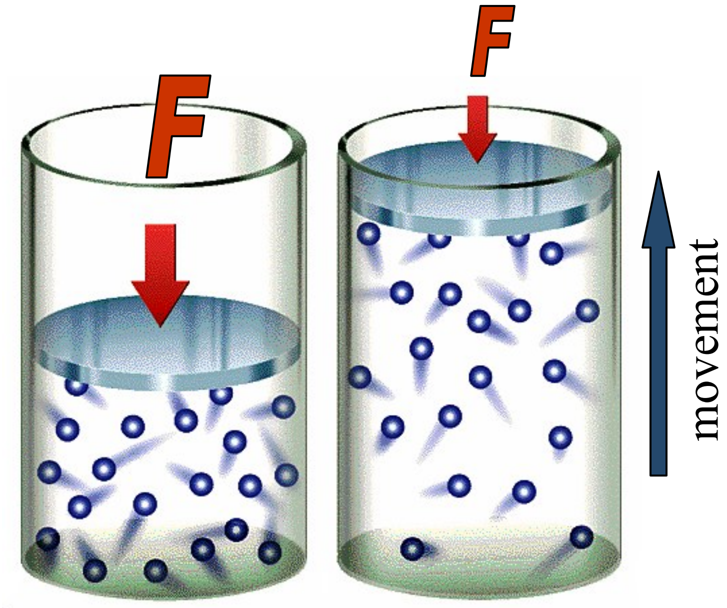
$$P = \frac{F_x}{A} = \frac{1}{A}\frac{dM}{dt} = \frac{N}{V}mv_x^2$$

$$P = m\langle v_x^2\rangle\frac{N}{V} = m\left(\frac{2}{3}\frac{U}{m\,N}\right)\frac{N}{V} = \frac{2}{3}\frac{U}{V}$$

Units:
1 atm = $1.013\cdot10^5$ N/m^2 ≡ $1.013\cdot10^5$ Pa
1 Torr ≡ 1 mm Hg = 133.3 Pa
1 bar = 0.987 atm = 750 Torr
1 psi = 51.7 Torr = 68.9 mbar

The Meaning of Temperature

- T is a measure of the kinetic energy of gas molecules

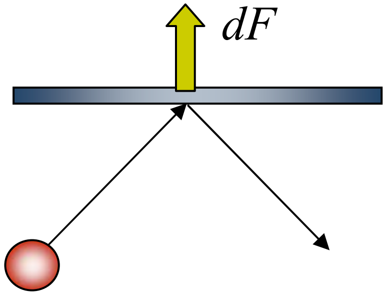


Molecules transfer energy to piston to move it ; <v> drops and the temperature decreases

$$U = \frac{1}{2} Nm\langle v^2 \rangle; \quad U = \frac{3}{2} PV$$

$$PV = \frac{3}{2} N\,kT$$

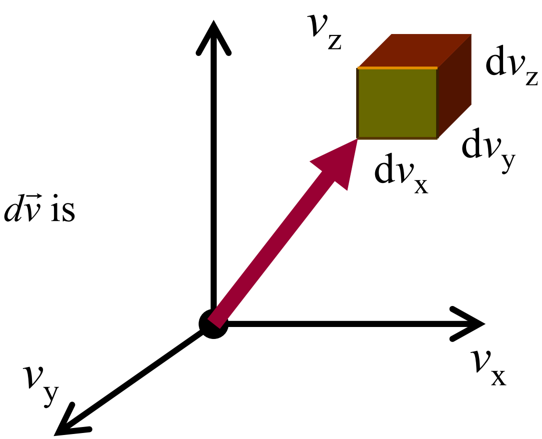
$$kT = \frac{2}{3}\frac{U}{N} = \frac{2}{3}\langle K \rangle$$



The Velocity Distribution of Gases

The number distribution $N(\vec{v})$:
so that the number of molecules with a velocity between $\vec{v}$ and $\vec{v}+d\vec{v}$ is $N(\vec{v})dv_x dv_y dv_z$, and

$$\int N(\vec{v})dv_x dv_y dv_z = N$$



The probability distribution $\varphi(\vec{v}) = \frac{N(\vec{v})}{N}$:

$\varphi(\vec{v})\,d^3v$ is the probability that a molecule velocity is between $\vec{v}$ and $\vec{v}+d\vec{v}$.

$\langle v^2 \rangle = \int v^2 \varphi(\vec{v})\,d^3v$, and

$$\int \varphi(\vec{v})d^3v = 1$$

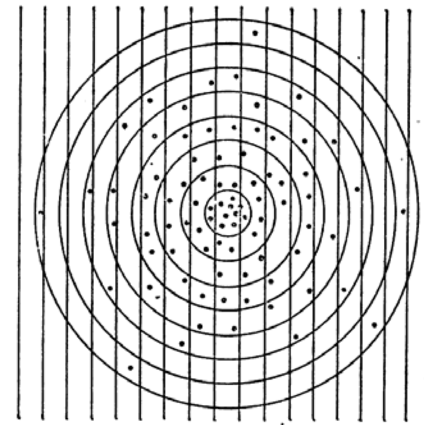
Postulate: Any way in which the total energy and momentum can be shared among the molecules is equally probable

$\varphi(\vec{v}) = \varphi(v_x)\,\varphi(v_y)\,\varphi(v_z)$, independent events

$$\varphi(v_x) = \sqrt{\frac{m}{2\pi kT}}\exp\left(-\frac{mv_x^2}{2kT}\right) \qquad \varphi(\vec{v}) = \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}}\exp\left(-\frac{mv^2}{2kT}\right)$$

derivation is beyond the scope of the course

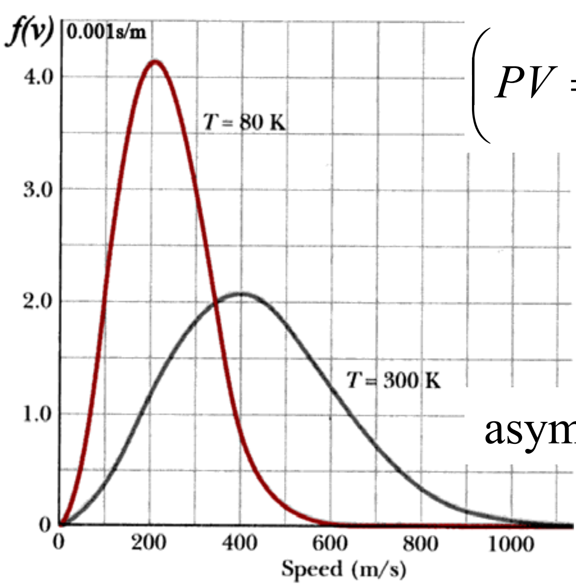
Maxwell Distribution



- Distribution of the *absolute* values of velocities

$$f(|v|) = 4\pi v^2 \varphi(v) = 4\pi v^2 \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \exp\left(-\frac{mv^2}{2kT}\right)$$

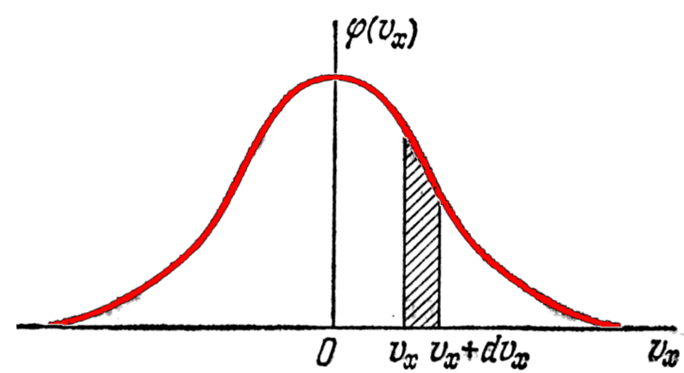
$$f(|v|) = \frac{4v^2}{\sqrt{\pi}}\left(\frac{M}{2RT}\right)^{\frac{3}{2}} \exp\left(-\frac{Mv^2}{2RT}\right)$$ (Eq. 2.1 in Ohring)



$$\left(PV = \frac{Nm}{M}RT = NkT\right)$$

asymmetrical

$$\varphi(\vec{v}) = \left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \exp\left(-\frac{mv^2}{2kT}\right)$$



symmetrical

Characteristic Velocities

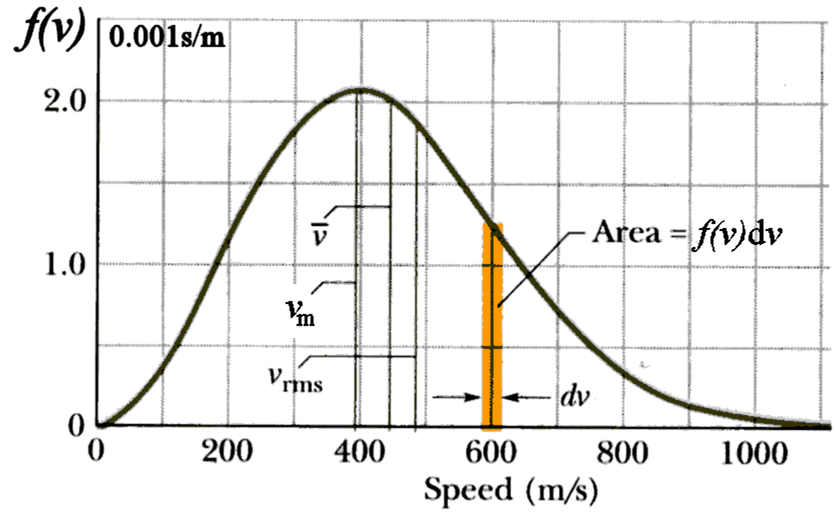
$v \equiv |v|$ now on

$$v_m = \sqrt{\frac{2RT}{M}}$$

$$\bar{v} \equiv \langle v \rangle = \frac{\int_0^\infty v\, f(v) dv}{\int_0^\infty f(v) dv} = \sqrt{\frac{8RT}{\pi M}}$$

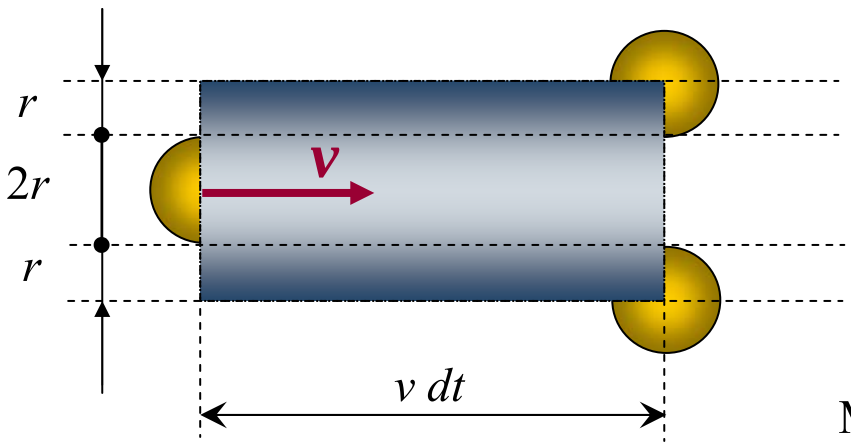
$$\overline{v^2} \equiv \langle v^2 \rangle = \frac{\int_0^\infty v^2\, f(v) dv}{\int_0^\infty f(v) dv} = \frac{3RT}{M}$$

$$v_{rms} \equiv \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3RT}{M}}$$



N.B. it is $\bar{v}^2 \equiv \langle v \rangle^2 \neq \langle v^2 \rangle$ in Ohring's book

Collisions in Gas



$\sigma = \pi(2r)^2$ Collision cross section

$$\sigma\lambda = \pi(2r)^2 \cdot v\,dt \approx \frac{V}{N}$$

Mean-free path: $\lambda \approx \dfrac{V}{\sqrt{2}\,\sigma\, N} \approx \dfrac{kT}{\sqrt{2}\,\sigma\, P}$

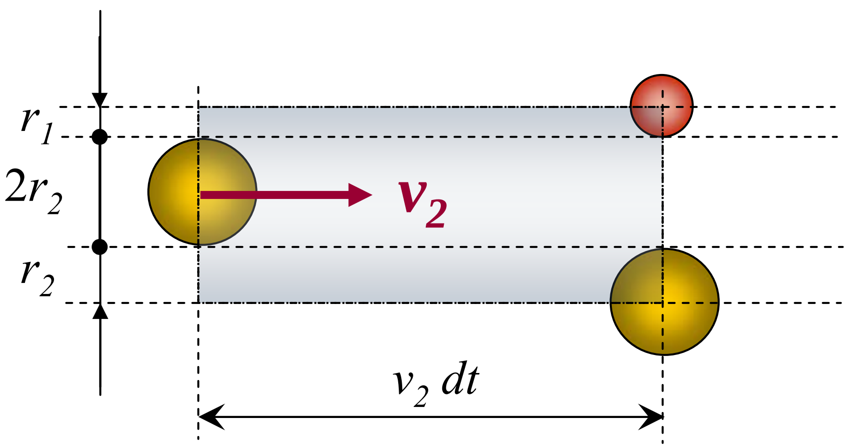
Type of vacuum	P(Torr)	λ (air at 300K)
rough	760 – 1	70 nm – 50 μm
medium	1 – 0.001	50 μm – 5 cm
high	$10^{-3} – 10^{-7}$	5 cm – 500 m
ultra-high	$< 10^{-7}$	> 0.5 km

$\sqrt{2}$ comes from the fact that the target molecules move as well.

$$\lambda \sim \frac{0.005}{P_{\,[\mathrm{Torr}]}}\ [\mathrm{cm}] \quad \text{for air}$$

Collisions in Gas

- Different Molecules



$$\sigma_{11} = \pi (2r_1)^2 = 4\pi r_1^2$$

$$\sigma_{12} = \sigma_{21} = \pi (r_1 + r_2)^2$$

$$\sigma_{22} = \pi (2r_2)^2 = 4\pi r_2^2$$

$$n_1 \equiv N_1/V \qquad n_2 \equiv N_2/V$$

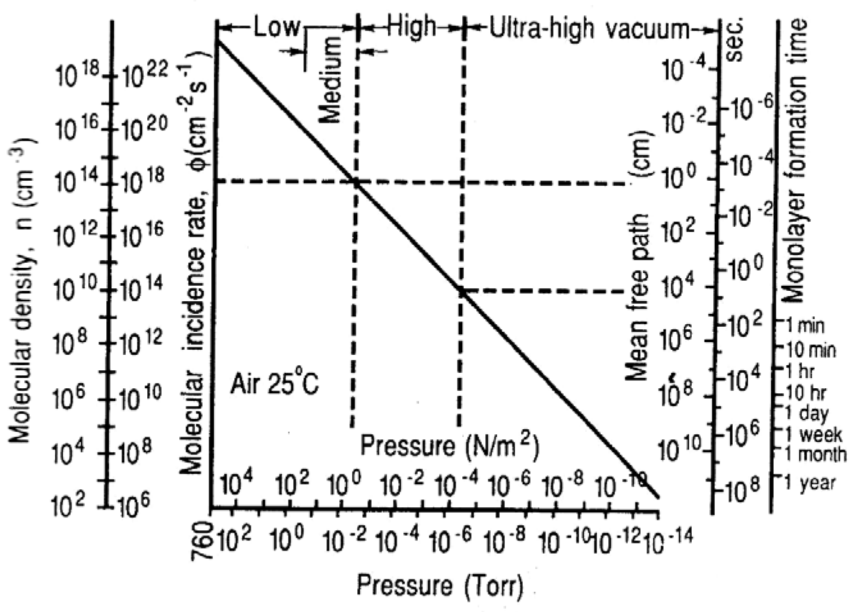
$$\lambda_1 = \frac{1}{n_1\sigma_{11} + n_2\sigma_{12}} \qquad \lambda_2 = \frac{1}{n_1\sigma_{21} + n_2\sigma_{22}}$$

After collision: elastic scattering, absorption (nuclear fusion), dissociation, ionization. One can introduce an *effective cross-section* for each of such processes.

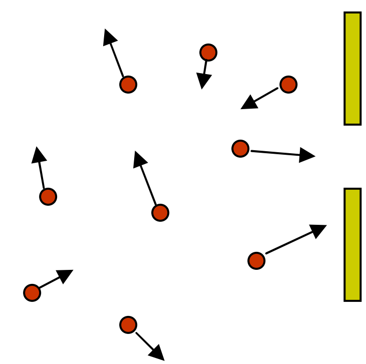
Gas Impingement on Surfaces

$$\Phi = \int_0^\infty v_x \, dn_x \qquad \Phi = n\sqrt{\frac{M}{2\pi RT}}\int_0^\infty v_x \exp\left(-\frac{Mv_x^2}{2RT}\right) dv_x = n\sqrt{\frac{RT}{2\pi M}}$$

$$\Phi = 3.5\times10^{22}\,\frac{P_{\,[\mathrm{Torr}]}}{\sqrt{MT}} \quad \left[\frac{1}{\mathrm{cm}^2\,\mathrm{s}}\right]$$



Pumping:



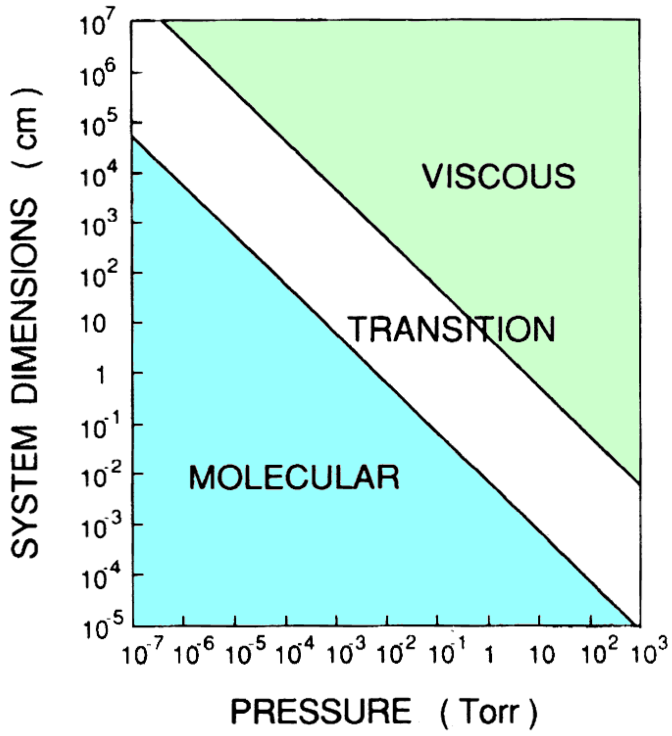
$$\dot{V} = 3.6\times10^3 \cdot A[\mathrm{cm}^2]\sqrt{\frac{T}{M}} \quad \left[\frac{\mathrm{cm}^3}{s}\right]$$

$$\dot{V}(\text{air at 300K}) = 11.7\times \text{A liter/s}$$

Gas Flow Regimes

D – characteristic dimension of the system (chamber or pipe diameter, for instance)

- Molecular flow: $\lambda/D > 1$
- Intermediate flow: $0.01 < \lambda/D < 1$
- Viscous flow: $\lambda/D < 0.01$



Conductance

Is only a function of geometry at a given temperature (not for the viscous flow which depends on pressure)

$$Q = C(P_1 - P_2) \quad \text{[Torr - liters / s]}$$

For a viscous flow: $184(P_1 + P_2)D^4/2L$

$D = 2.5\text{ cm};\ L = 100\text{ cm};\ (P_1+P_2)/2 = 380\text{ Torr}$

$C_{mol} = 1.9\text{ liters/s};\ C_{visc} = 27300\text{ liters/s}$

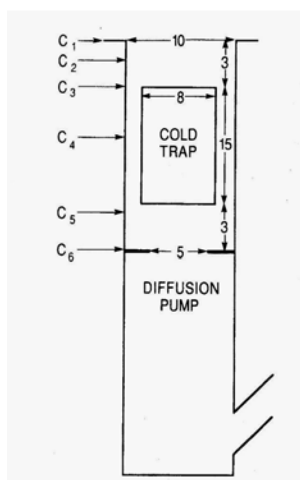
Electrical analogy:

Connection in series:

$$\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$$

Connection in parallel:

$$C_{tot} = C_1 + C_2 + C_3 + \ldots$$



Molecular flow:

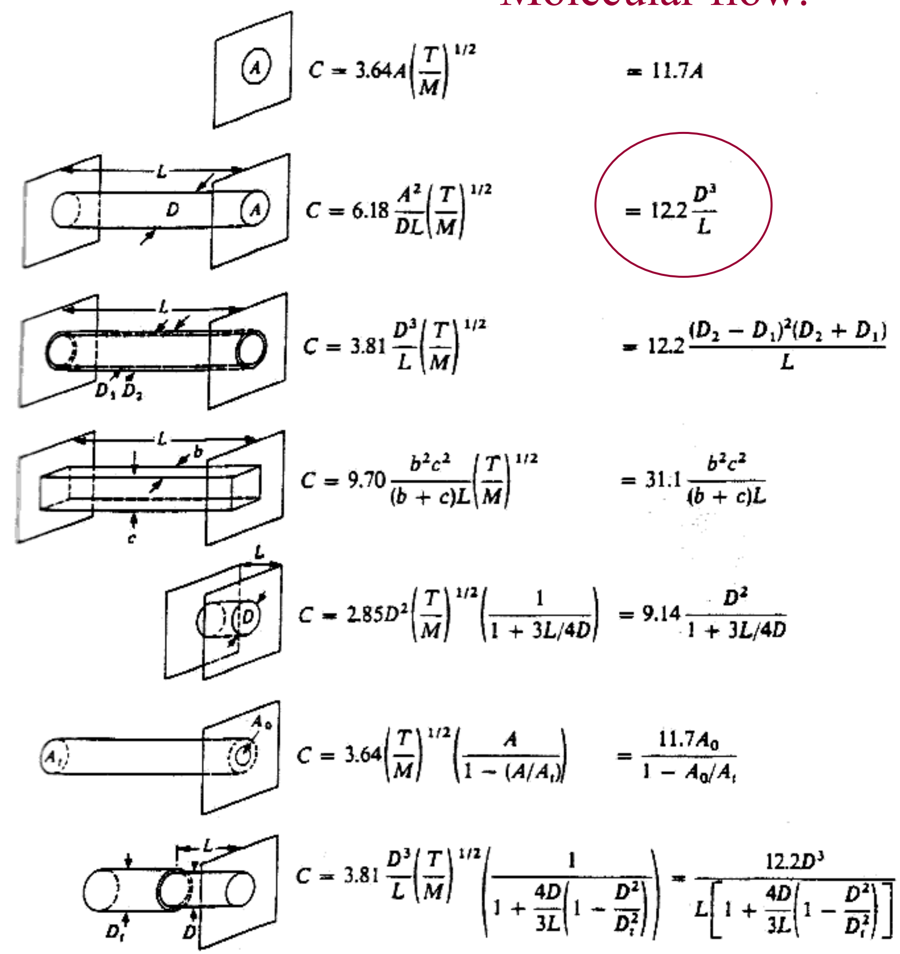


Figure 2-4 Conductances of various geometric shapes for molecular flow of air at 25°C. Units of C are liters/s. (Reprinted with permission from Ref. 7.)

Pumping Speed

The volume of gas passing the plane of the inlet port per unit time at a given pressure at the inlet. ***S* = *Q* / *P***

- Depends on a pump but also on a pipe line: $Q_{visc} \sim D^4 \Delta P$
- It is more efficient to increase D than ΔP!
- Refers to a given plane, while conductance refers to an element across which a given pressure difference is set.

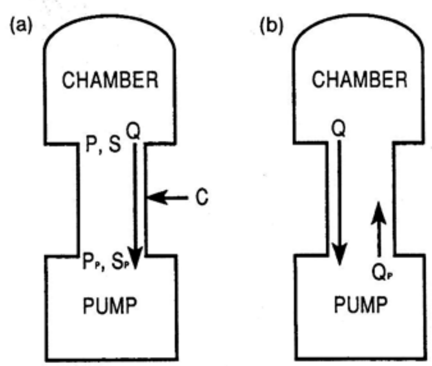


Figure 2-6 Chamber-pipe-pump assembly: (a) no outgassing, (b) with outgassing.

$$Q = C(P - P_p); \quad S = Q/P; \quad S_p = Q/P_p$$

$$S = \frac{S_p}{1 + S_p/C}$$ an effective pumping speed

With outgassing: $Q = S_p P - Q_p$

When $Q = 0$, the minimum pressure P_0 that a pump can give is reached and $Q_p = S_p P_0$.

The effective pumping speed is then $S = Q/P = S_p(1 - P_0/P) \rightarrow 0$ when $P \rightarrow P_0$;

Pump-Down Time

$$Q = -V\frac{dP(t)}{dt} = S_p P - Q_p^*$$; Q^* includes both the pump and chamber outgassing

$$-V\frac{dP(t)}{(S_p P - Q_p^*)} = dt$$

$$-\frac{V}{S_p}\frac{d(S_p P(t) - Q_p^*)}{(S_p P(t) - Q_p^*)} = dt$$

$$-\frac{V}{S_p} d\ln(S_p P(t) - Q_p^*) = dt$$

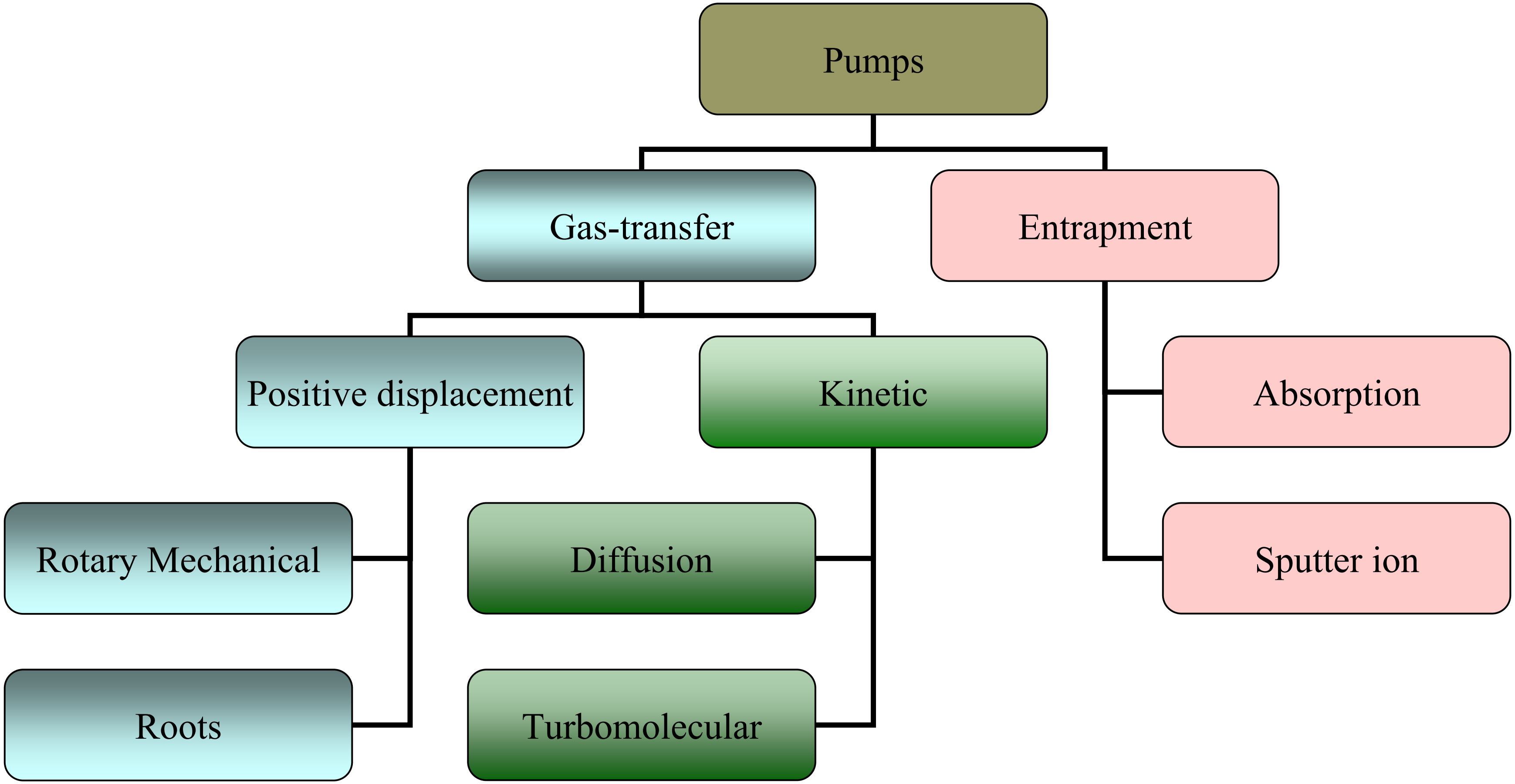
$$S_p P_{init} - Q_p^* = A; \quad Q_p^* = S_p P_0; \quad P_{init} = P(0)$$

$$\ln(S_p P(t) - Q_p^*) = -\frac{S_p}{V}t + B$$

$$S_p P(t) - Q_p^* = \exp\left(-\frac{S_p}{V}t + B\right) = A\exp\left(-\frac{S_p}{V}t\right)$$

$$\frac{P(t) - P_0}{P_{init} - P_0} = \exp\left(-\frac{S_p}{V}t\right)$$

Pumps

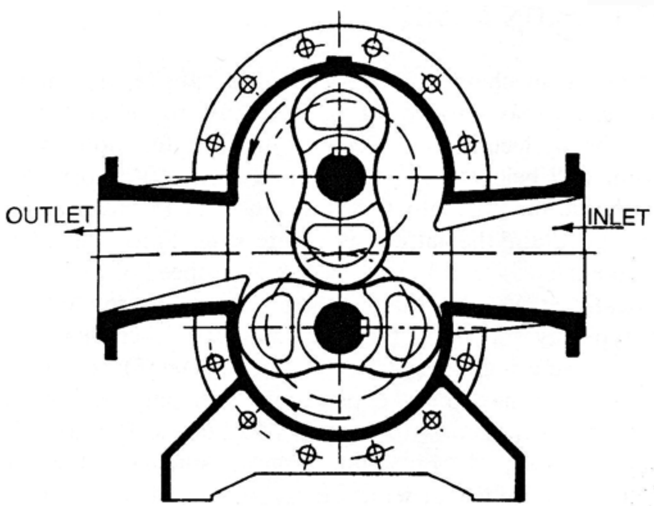


Pumps
Gas-transfer
Entrapment
Positive displacement
Kinetic
Absorption
Rotary Mechanical
Diffusion
Sputter ion
Roots
Turbomolecular

Rotary Mechanical

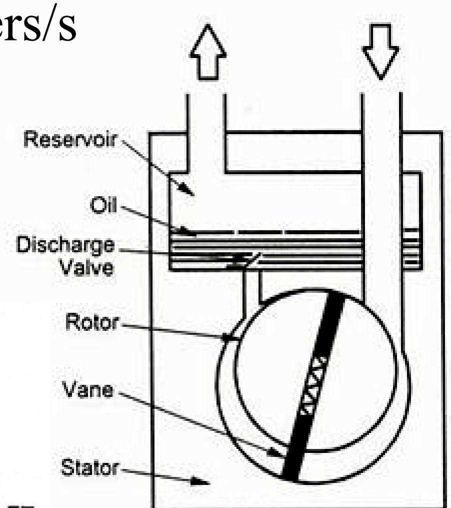
- Rotary-piston: ~10−400 liters/s
- Rotary-vane: ~3−50 liters/s
- Roots: ~1000 liters/s

$10^{-2} - 10^{-4}$ Torr

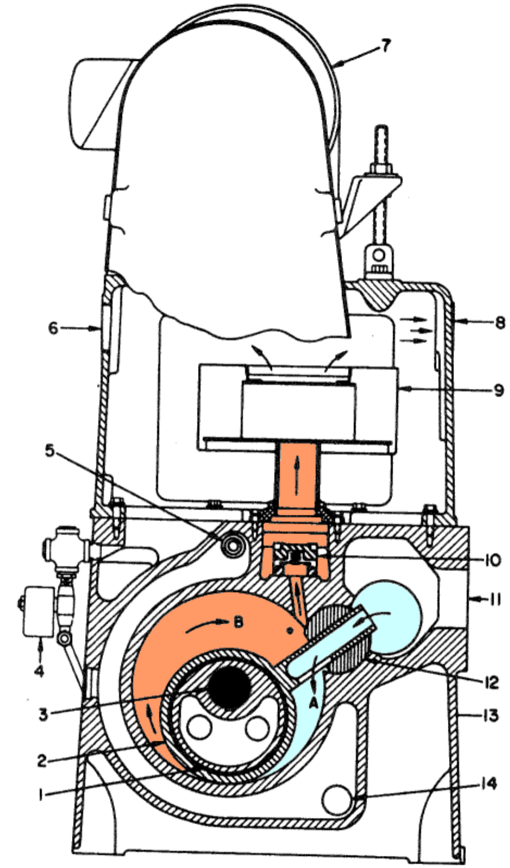


(b)

Figure 2-7 (b) Schematic of a Roots pump.



Rotary vane

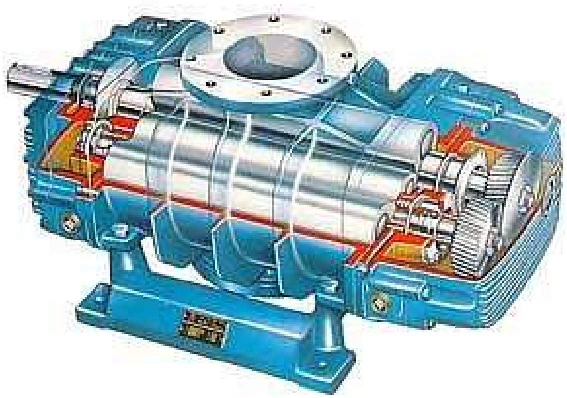


(a)

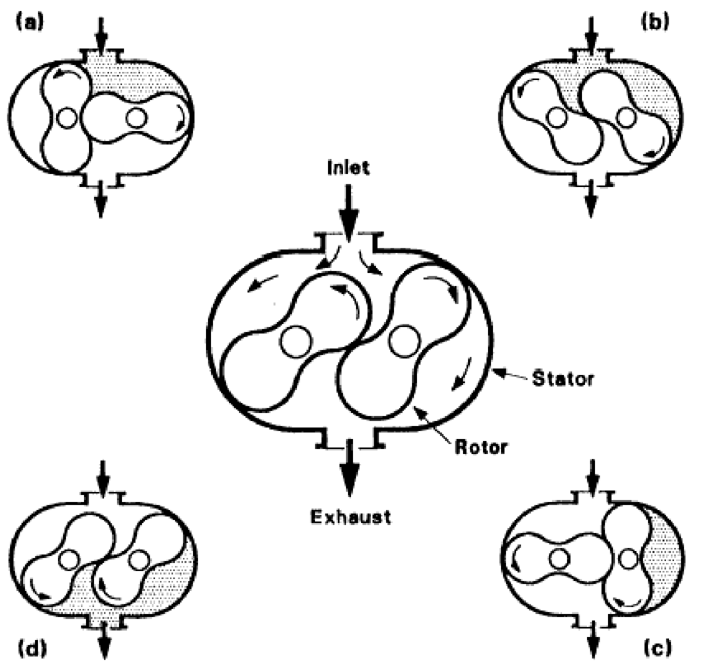
Figure 2-7 (a) Schematic of a rotary piston pump: 1, eccentric; 2, piston; 3, shaft; 4, gas ballast; 5, cooling water inlet; 6, optional exhaust; 7, motor; 8, exhaust; 9, oil mist separator; 10, poppet valve; 11, inlet; 12, hinge bar; 13, casing; 14, cooling water outlet. (Courtesy of Stokes Vacuum Inc.)

Roots Pump

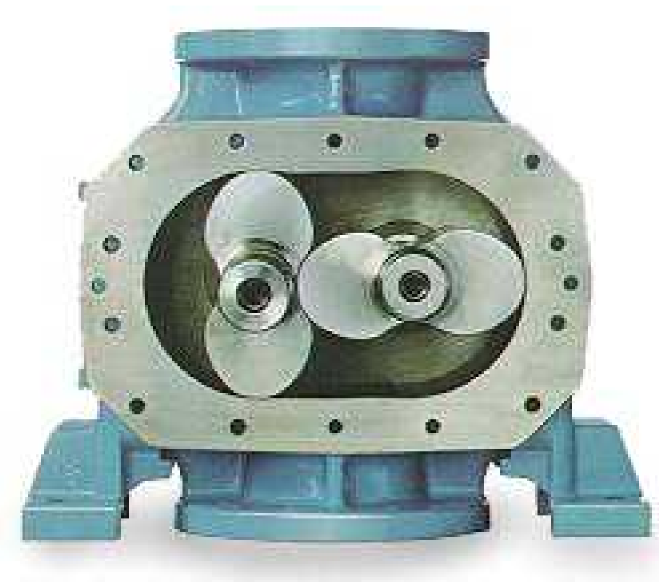
- Dry pump, minimum contamination
- Requires forepumping, susceptible to overheating



POSITIVE DISPLACEMENT VACUUM PUMPS

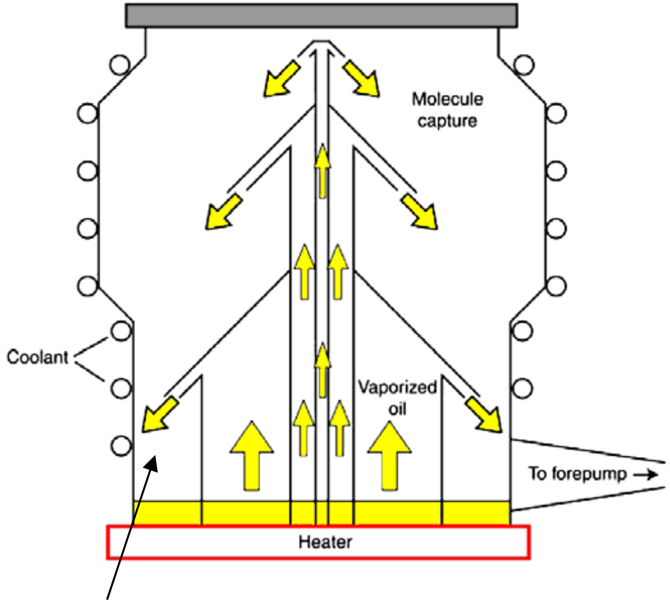


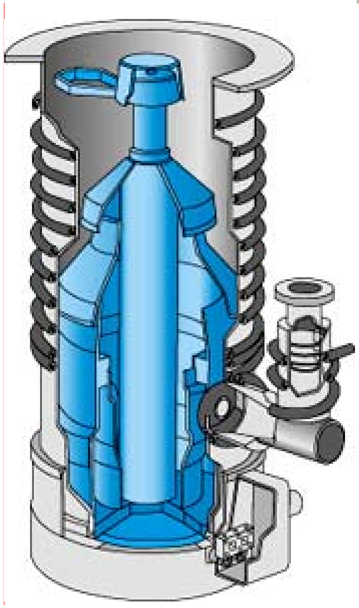
Cross section through a Roots (mechanical booster) pump and its operating cycle.

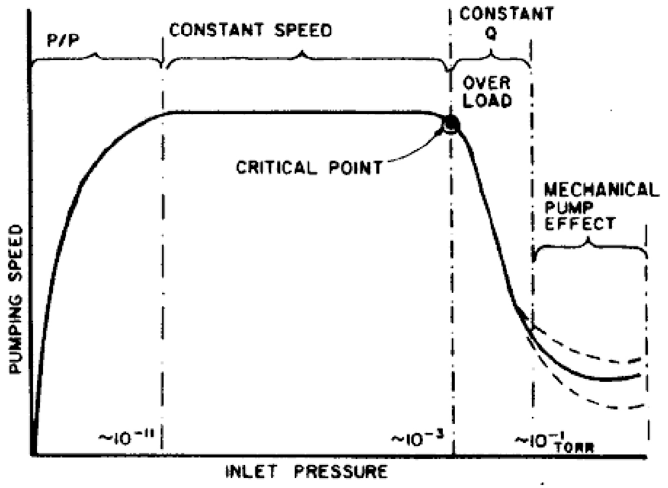


Diffusion Pump

- Rough pump required
- $5 \times 10^{-2} - 10^{-10}$ Torr
- 1 – 20 000 liters/s



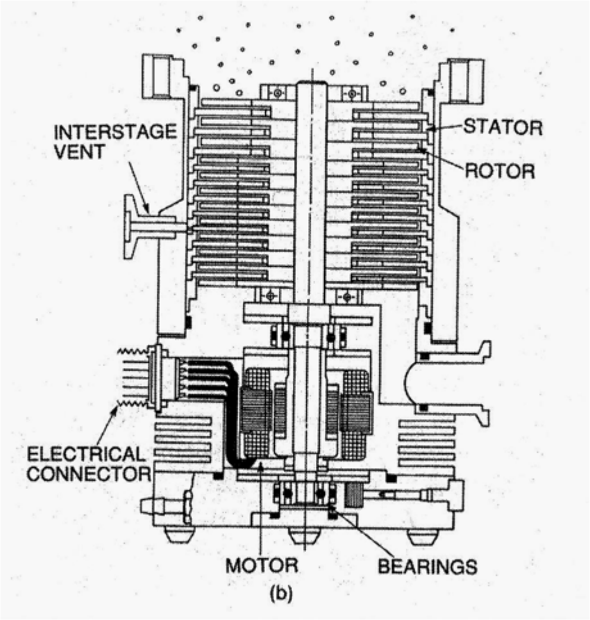


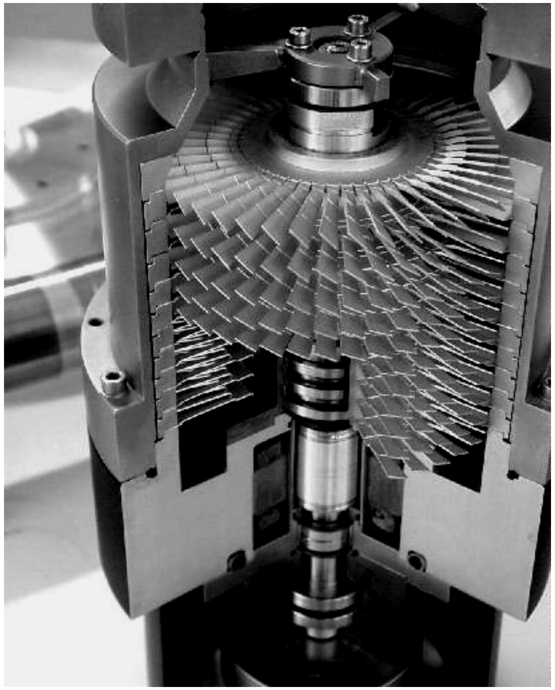


oil vapors impart a preferred direction to molecular motion

Turbomolecular Pump

- $>10^{-10}$ Torr; $\sim 10^3$ liters/s
- 20-30 thousand revolutions per min
- very clean (no oil contamination)

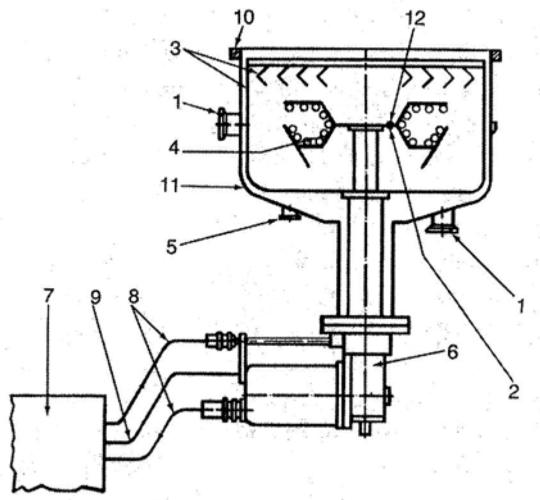




collision with turbine blades impart a preferred direction to molecular motion

Cryo-pump

- Very clean (no oil contamination)
- Different absorbing surfaces: bare metal, covered with pre-condensed gases, micro-porous charcoal or zeolite)
- Fore-pumps required (10^{-3} Torr)
- Pumping speed is only limited by impingement rate: $P = P_s(T)(300/T)^{0.5}$
  (P_s is the saturation pressure of the pumped gas P_s(20K) for LN_2 is 10^{-11} Torr)
- He, H_2 is problematic to pump out



CTI cryopumps

Cryo-pump

Vapor Pressure of Common Gases as a Function of Temperature in K

	Vapor Pressure (Pa)								
	10^{-11}	10^{-9}	10^{-7}	10^{-5}	10^{-3}	10^{-1}	10	10^{3}	10^{5}
Helium							1.0	1.7	4.5
Hydrogen	2.9	3.0	3.5	4.0	4.8	6.1	8.0	12	21
Neon	5.5	6.1	6.9	7.9	9.2	11	14	18	28
Nitrogen	18	20	22	25	29	34	42	54	80
Argon	20	23	25	29	33	39	48	63	90
Carbon monoxide	21	23	25	28	33	38	46	58	84
Oxygen	22	24	27	30	34	40	48	63	93
Krypton	28	31	35	39	46	54	66	86	124
Xenon	39	43	48	54	63	74	92	119	170
Carbon dioxide	60	65	72	81	92	106	125	154	198
Water	113	124	137	153	173	199	233	284	381

Sputter Ion Pump

- Gases are ionized and accelerated by high voltage
- Ions sputter Ti cathode
- Ti is deposited elsewhere and getters the gas molecules
- Fresh layer of Ti buries those inside the metal film

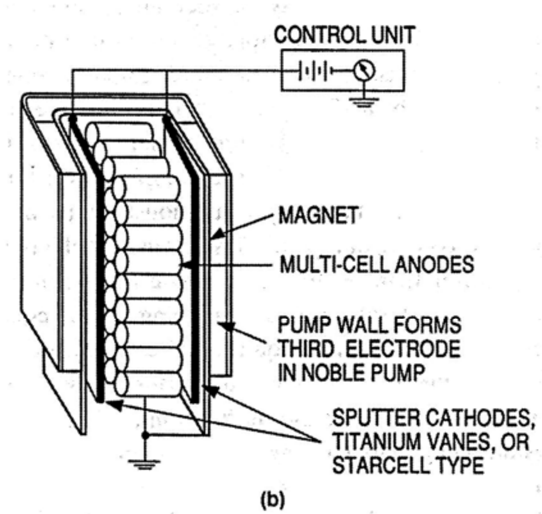
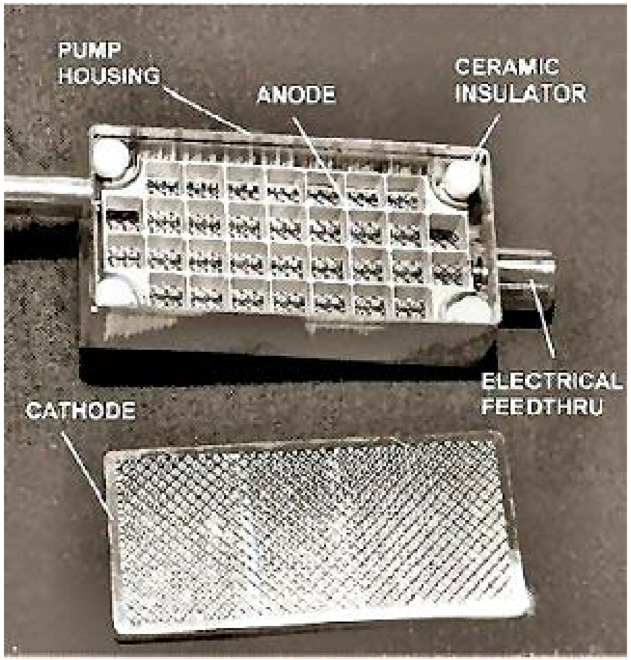
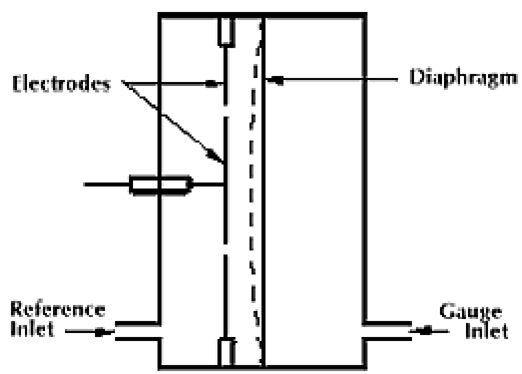


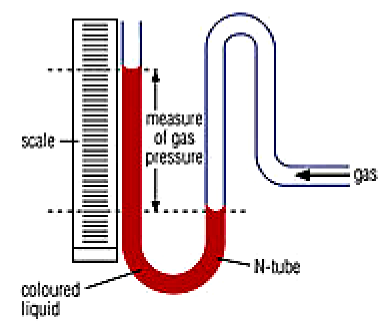
Figure 2-11 Sputter ion pump: (a) photograph; (b) schematic of pump interior. (Cc Varian Associates, Vacuum Products Division.)



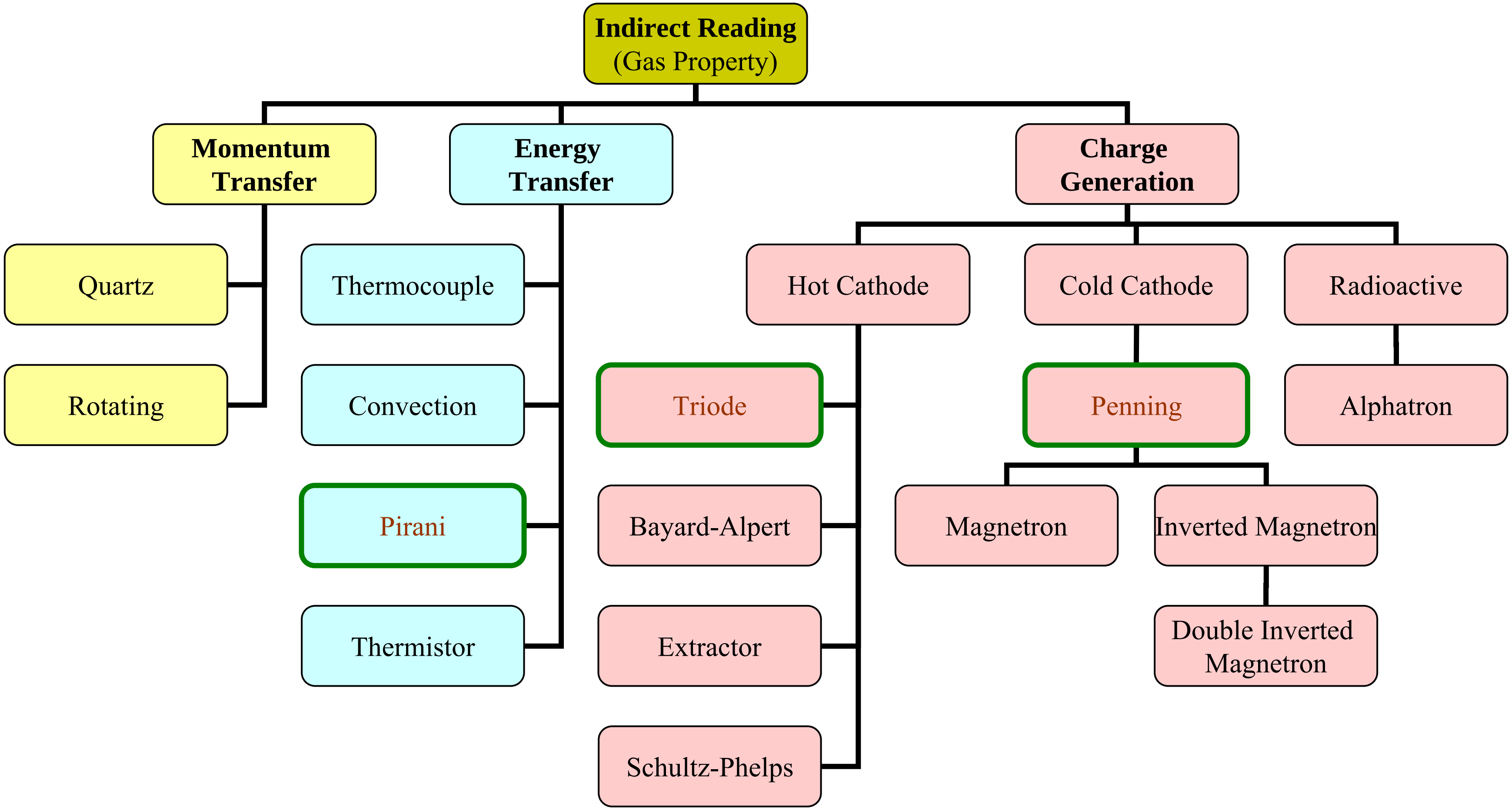
Pressure Gauges: DIRECT READING

- **Direct Reading** (Displacement of a wall)
  - **Solid Wall**
    - Diaphragm
      - Capacitance Diaphragm
      - Strain-gauge Diaphragm
    - Bourdon
    - Radiometer
  - **Liquid Wall**
    - Liquid Manometer
      - U-tube Manometer
    - McLeod Compression

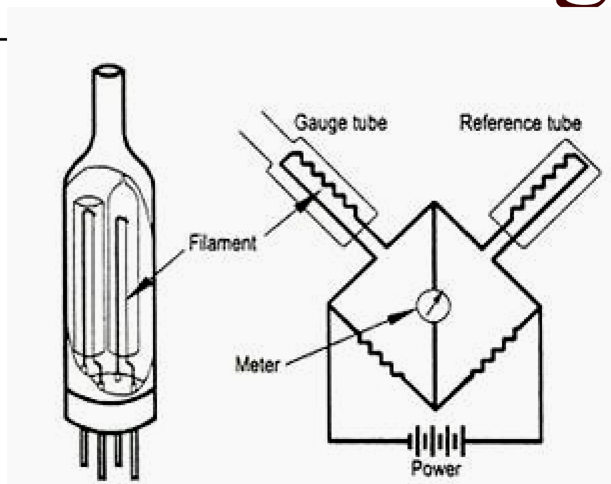




Pressure Gauges: INDIRECT READING



Pressure Gauges: Pirani



http://www.lesker.com/

In a Pirani gauge, the reference filament is immersed in a fixed-gas pressure, while the measurement filament is exposed to the system gas. A current through the bridge heats both filaments. Gas molecules hit the heated filaments and conduct away some of the heat. If the gas pressures (or composition) around the filaments is not identical, the bridge is unbalanced and the degree of unbalance is a measure of the pressure..

Pressure Gauges: Penning

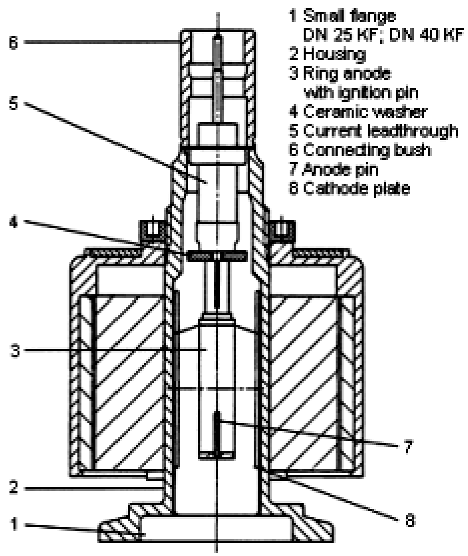


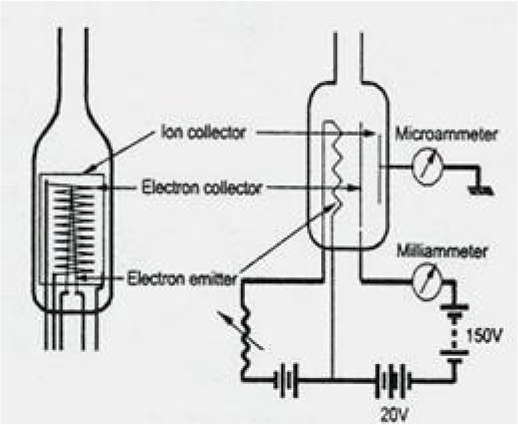
Figure 1 Cross-section of PENNINGVAC PR 35 gauge

Range Dependence
Measuring
10^{-2}-10^{-8} mbar
specific gas dependent

Advantages
- robust, no filament, operation guaranteed
- simple construction, supply and measurement
- Penningcel stays on room temperature; degassing generally not necessary

Disadvantages
- discharge power not linear with pressure
- sensitivity specific gas dependent
- Penning cel and magnet are heavy and extensive
- with magnet not suitable for build in
- magnetic disturbed field
- at low pressure difficult to start
- bad reproducibility in consequence of the getter ion pump principal;

Pressure Gauges: Bayard-Alpert



In a Bayard-Alpert gauge, a tungsten filament is heated to give off electrons which are accelerated then to ~70 eV.

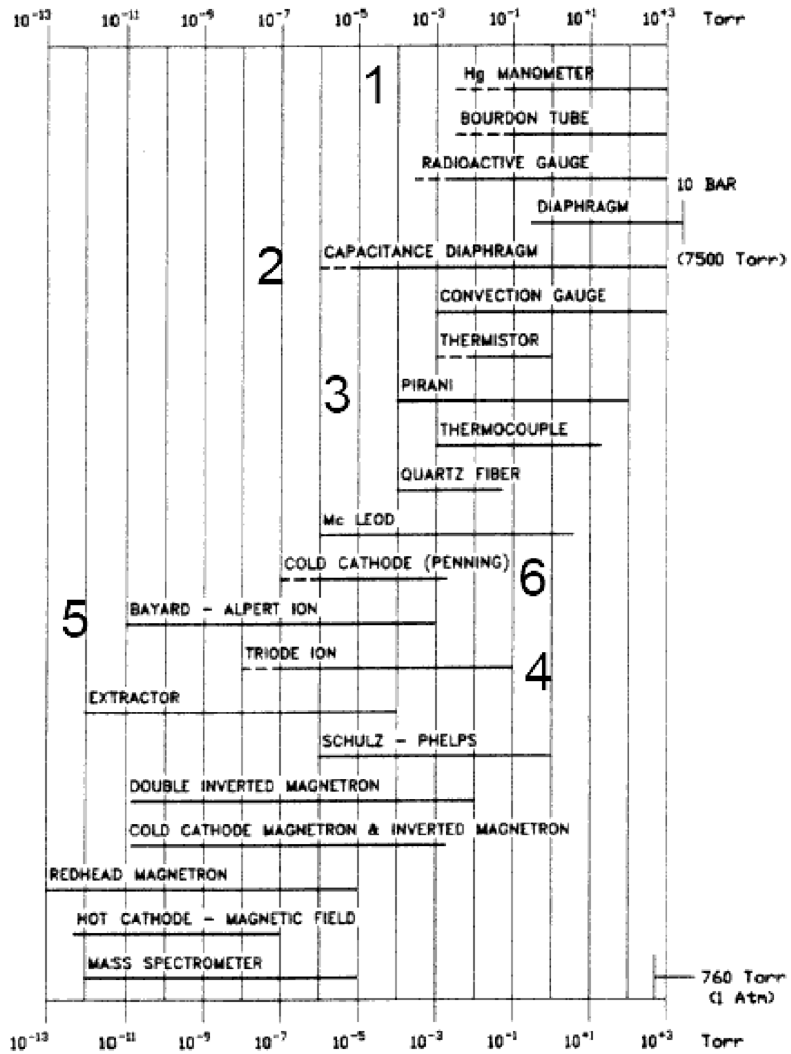
These energetic electrons ionize the residual gas molecules which they collide with.

The positive ions formed are then collected by a wire collector held at about -150V.

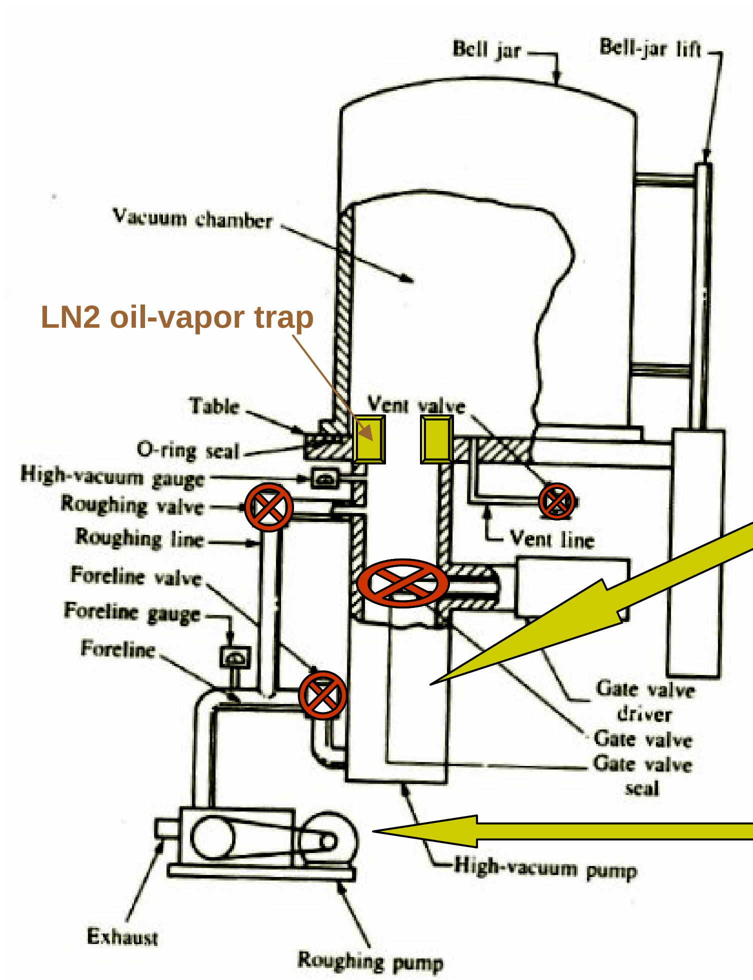
The ion current at the collector wire varies with the gas density which is a direct measure of gas pressure.

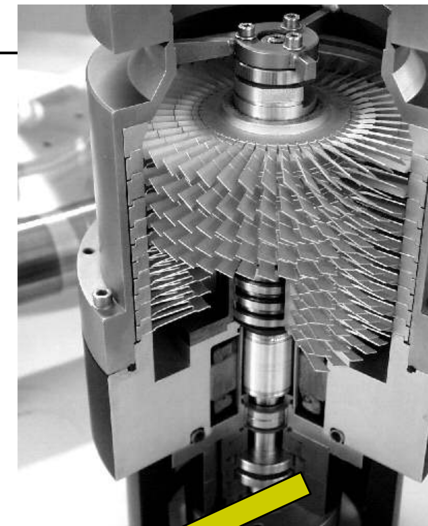
Ionization gauges are dependent on gas composition
Cannot give accurate absolute pressure measurements.
The practical working range of the Bayard-Alpert gauge is in the 10^{-4} Torr to 10^{-10} Torr.

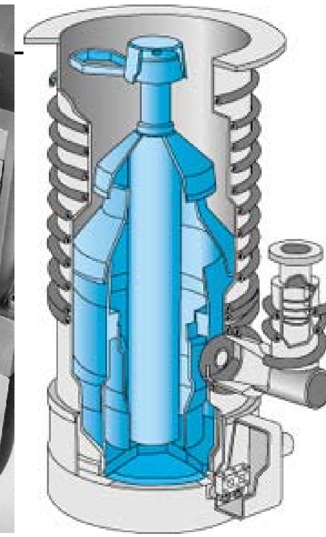
Pressure Gauges: A COMPARISON

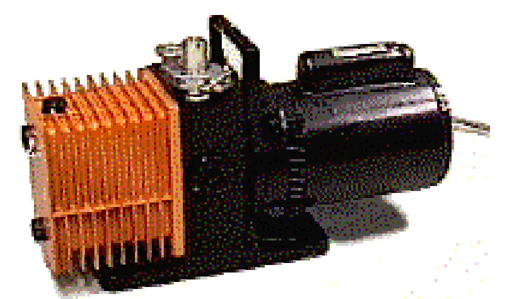


Vacuum Systems









Vacuum Leaks

- Very leak tight $<10^{-6}$ Torr·l/s
- Adeqautely leak tight ~ 10^{-5} Torr·l/s
- Not leak tight $>10^{-4}$ Torr·l/s

Technical Data		L 200 dry
Smallest detectable helium leak rate (Vacuum mode)	mbar x l x s^{-1}	$< 3 \times 10^{-10}$
Smallest detectable helium leak rate (Sniffer mode)	mbar x l x s^{-1}	$< 1 \times 10^{-7}$
Max. detectable helium leak rate (Vacuum mode)	mbar x l x s^{-1}	1×10^{-1}
Pumping speed for helium at the inlet	l/s	0.6
Leak rate measurement range	mbar x l x s^{-1}	1×10^{-11} to 1×10^{-1}
Time until ready for operation	minutes	< 3
Mass spectrometer		180° magnetic sector field
Detectable masses	amu	2, 3 and 4

Leybold Inficon UL 200 Leak Detector

= Mass Spectrometer

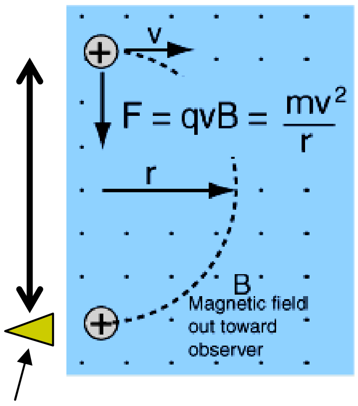
$$r = \frac{mv^2}{qvB} = \frac{mv}{qB}$$ Radius of path produced by magnetic field

If the velocity v is produced by an accelerating voltage V:

$$\frac{1}{2}mv^2 = qV\ ;\quad v = \sqrt{\frac{2qV}{m}}$$

Substitution gives:

$$r = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$



detector

Residual Gas Analysis

- Mass Spectra = species' "fingerprint"