CHAPTER VI

HEAT TRANSFER

1. Introduction

A DETERMINATION of the physical properties of materials at temperatures considerably different from the ambient temperature requires some degree of thermal isolation of the material from its surroundings; the degree of isolation must be sufficient to meet the demands of temperature control, temperature measurement, and in the case of low-temperature research, the available refrigerating capacity of the coolant. In a measurement such as the determination of heat capacity by the adiabatic method, the sample must also be thermally isolated from its immediate environment—the cooling medium. These conditions imply that an important factor in the design of a successful cryostat is the ability to predict the degree of thermal isolation or in other words, to calculate the transfer of heat that will take place between the specimen and its surroundings; then materials and methods of construction must be used which will ensure this heat transfer being within or below certain allowable limits.

In general, heat may be transferred by conduction, convection, and radiation. In most low-temperature applications, thermal isolation is assisted by partial evacuation of gas from the interior of the cryostat so that convection is eliminated. Then effective heat transfer takes place by conduction through the residual low-pressure gas, conduction through the solids that interconnect the various parts of the cryostat, and by radiation. In addition such factors as Joule heating in electrical leads, eddy current heating, mechanical vibration, adsorption or desorption of gases may contribute to the heat transfer.

Of the three major processes responsible for heat conduction at low temperatures that due to conduction by solids can be estimated generally with a fair degree of accuracy; low-pressure gas conduction and radiation transfer may be estimated with rather less accuracy, the uncertainty depending on our lack of

knowledge of the accommodation coefficient and emissivity respectively. However, it is usually possible to estimate an upper bound for the heat transfer by these processes and therefore ensure that materials, degree of high vacuum, etc., used are sufficient to meet the required demands.

The remainder of this chapter deals in some detail with the methods of calculating heat transferred by these processes.

2. Conduction of heat by a gas

An elementary treatment on the basis of kinetic theory indicates that in a gas at normal pressures the thermal conductivity λ and viscosity η are given by (see, for example, Roberts, 1940, or Jeans, 1948)

$$\lambda = \tfrac{1}{3}mln\bar{v}C_v, \quad \eta = \tfrac{1}{3}mln\bar{v},$$

where
$m = $ mass of one molecule,

$l = $ mean free path,

$n = $ number of molecule per cm³,

$\bar{v} = $ mean velocity,

$C_v = $ specific heat per gm.

Thus if ρ is the density,

$$\lambda = \tfrac{1}{3}\rho l\bar{v}C_v, \quad \eta = \tfrac{1}{3}\rho l\bar{v},$$

and
$$\lambda = \eta C_v.$$

Since the mean free path $l \propto 1/p$, both λ and η are seen to be pressure independent at least to a first approximation, and to depend on the temperature through their dependence on the mean velocity $\bar{v}$.

Experiment (and a more detailed theory) indicates that $\lambda = \text{constant} . \eta C_v$ where the constant has a value of 1·5–2·5 for most common gases. As may be seen from Figures 50 and 51 (Chapter III), λ and η increase monotonically with increasing temperature as T^n where the exponent n has experimental values in the range 0·6–0·9 for hydrogen, helium, nitrogen, and oxygen.

However, as we have pointed out before, the residual gas pressure in a cryostat is nearly always reduced to a point where

the mean free path becomes comparable with the dimensions of the system; at room temperature a pressure of $\sim 10^{-4}$ mm Hg is sufficiently low that the mean free path is ~ 100 cm. At such pressures, the average molecule may travel from a hot wall to a cold wall without collision with another gas molecule, and the thermal conductivity becomes a function of the number of molecules present (and also their mean velocity), i.e. at low pressures $\lambda \propto n \propto p$.

For approximately parallel surfaces at temperatures T_1 and T_2, the heat transferred, $\dot{Q}$, by conduction through a gas at low pressure p dynes cm^{-2} is given by (see Kennard, 1938):

$$Q = \frac{a_0}{4} \frac{\gamma+1}{\gamma-1} \sqrt{\left(\frac{2R}{\pi M}\right)} p \frac{T_2-T_1}{\sqrt{T}} \text{ ergs cm}^{-2} \text{ sec}^{-1}, \qquad (40)$$

where M is the molecular weight and γ is the ratio of specific heats of the gas, R is the gas constant in ergs mole^{-1} deg^{-1}; a_0 is related to the individual accommodation coefficients a_1 and a_2 and the areas A_1 and A_2 of the two surfaces by

$$a_0 = \frac{a_1 a_2}{a_2 + (A_2/A_1)(1-a_2)a_1},$$

so that if $A_1 \simeq A_2$, then

$$a_0 = \frac{a_1 a_2}{a_1 + a_2 - a_1 a_2};$$

if also

$$a_1 = a_2 = a, \quad a_0 = \frac{a}{2-a}$$

$$\simeq \tfrac{1}{2}a \text{ for } a \to 0$$

$$\simeq a \quad \text{for } a \to 1.$$

Accommodation coefficients are normally measured for the case of a wire at temperature T_2 slightly higher than a surrounding gas at temperature T_1. In this case molecules colliding with the wire have an average energy corresponding to T_1 so that T in the denominator of (40) may be identified with T_1. The molecules leave the wire with an energy corresponding to a temperature T_2' intermediate between T_1 and T_2; then

$$(T_2'-T_1) = a(T_2-T_1)$$

defines the accommodation coefficient a.

In many low-temperature applications the molecules may travel from one wall to the other without collision and it is impossible to identify T in (40) with either T_1 or T_2. Also the value of p used in equation (40) must be considered very carefully, since at low pressures (under molecular or Knudsen conditions) the thermal transpiration or thermo-molecular pressure effect arises and the local pressure and temperature vary together as expressed by

$$p/(T)^{\frac{1}{2}} = \text{constant.}$$

R. J. Corruccini of the National Bureau of Standards (Boulder Laboratories) has examined recently this question of gaseous heat conduction at low pressures and low temperatures. He points out† that p and T are associated in the derivation of (40), and that p is observed by a vacuum gauge usually at room temperature. Under low-pressure conditions $l \gg d$, the diameter of the tube connecting the gauge to the cryostat chamber, and if the system is in equilibrium, $p/(T)^{\frac{1}{2}}$ is constant. Therefore, if a value of p obtained from the gauge is inserted in (40), the appropriate value for T is the temperature at the pressure gauge.

Equation (40) may be simplified to

$$\dot{Q} = 0.243 \frac{\gamma+1}{\gamma-1} a_0 \frac{T_2-T_1}{\sqrt{(MT)}} p_{\text{mm}} \text{ W cm}^{-2}, \qquad (41)$$

and using $T = 295°$ K,

$$\dot{Q} = 0.014 \frac{\gamma+1}{\gamma-1} a_0 \frac{T_2-T_1}{\sqrt{M}} p_{\text{mm}}$$

$$= \text{constant } a_0 \cdot p_{\text{mm}}(T_2-T_1) \text{ W cm}^{-2}, \qquad (42)$$

where the constant has approximate values of 0.028, 0.059, and 0.016 for helium, hydrogen, and air respectively. The chief uncertainty in calculating the heat conducted by a low-pressure gas lies in the accommodation coefficient a. As an upper limit $a = 1$, but experimental research has shown that with a clean metallic surface exposed to helium gas, a may be as low as 0.025.

† I am very grateful to Dr. Corruccini for his kindness in giving me a draft of this interesting paper, which was presented in part at the 1957 Cryogenic Engineering Conference (Boulder, Colorado) and which is also expected to be submitted to a suitable journal in the near future, e.g. *Vacuum*, Edwards High Vacuum Ltd., Sussex.

Keesom (1942) has given a number of values for a for helium gas obtained by measuring the heat loss from a wire stretched along the axis of a tube. Some of the values found (original references may be found in Keesom, 1942) are given in Table XIV below.

TABLE XIV

Accommodation Coefficient for Helium Gas

Metal

Platinum . . .	. 0·49 (90° K, 153° K); 0·38 (34°–264° C)
Bright platinum .	. 0·44 (50°–150° C)
Blackened platinum .	. 0·91 (50°–150° C)
Clean fresh tungsten .	. 0·025 (79° K); 0·046 (195° K); 0·057 (22° C)
Gas-filled tungsten .	. 0·19 to 0·82
Gas free nickel .	. 0·048 (90° K); 0·060 (195° K); 0·071 (273° K)
Nickel (gas layer adsorbed).	0·413 (90° K); 0·423 (195° K); 0·360 (273° K)
Glass . . .	. 0·67 (12° K) 0·38 (77° K); 0·34 (273° K)

The experimental figures suggest that for a metal surface in the usual condition encountered in a cryostat, exposed to helium gas at low pressure, a value

$$a \leqslant 0·5$$

should provide a useful upper limit for calculating heat transfer.

Available data for the accommodation coefficient of other gases on various metallic surfaces seems rather meagre. Values quoted in the *International Critical Tables* (1929) for the accommodation coefficients at about room temperature for He, H_2, air, N_2, O_2, and argon at a platinum surface range from 0·2 to 0·9.

More recent experimental work on accommodation coefficients (for example, Bremner, 1950; Eggleton, Tompkins, and Wanford, 1952; Thomas and Schofield, 1955; Schäfer, 1952) deals principally with surfaces of tungsten which are clean and gas free. The results, of fundamental rather than practical cryogenic interest, serve to confirm some of the experimental disagreements and the considerable experimental difficulties that are faced in these determinations. In contrast to Roberts (1930), Thomas and Schofield find very little change in accommodation coefficient with temperature for helium on a clean gas-free tungsten filament; from 80° to 300° K they find a has a value of 0·015–0·017 but increases to $\sim 0·2$ when the surface is con-

taminated by gas (oxygen). Eggleton, Tompkins, and Wanford report values of $a \simeq 0·056$ after flashing a tungsten wire, but find $a \simeq 0·3$ when the surface has adsorbed gas layers on it.

Generally for clean gas-free surfaces, a increases with molecular weight: for example, from Bremner (1950) for flashed tungsten at 90° K, $a = 0·041$ for He, 0·081 for Ne, 0·16 for A, 0·09 for H_2 0·20 for O_2. For heavy polyatomic molecules (e.g. organic gases) it appears that generally $0·8 < a < 0·9$.

3. Heat transfer through solids

As we mentioned in the last chapter, the heat flow $\dot{Q}$ through a solid of cross-section A cm² under a temperature gradient $\partial T/\partial x$ is given by

$$\dot{Q} = \lambda(T) A \frac{\partial T}{\partial x}. \tag{43}$$

Thus, if the ends of a solid bar of uniform cross-section and length l are at temperatures T_1 and T_2,

$$\dot{Q} = \frac{A}{l} \int_{T_1}^{T_2} \lambda(T)\, dT. \tag{44}$$

$\lambda(T)$ is the temperature-dependent thermal conductivity of the solid, of which some typical examples are shown in the graphs of Figure 96. In Chapter XI are summarized experimental data (with references to compiled tables) of the thermal conductivity of the various typical groups of solids: (i) glasses, (ii) metallic alloys, (iii) pure metallic elements, (iv) crystalline dielectric solids.

In the case of metallic alloys, e.g. brass, German silver, monel, stainless steel, available data enable us to give a fairly accurate estimate of $\lambda(T)$ at any temperature and therefore of the integrated heat conductivity $\int \lambda(T)\, dT$, provided that the alloy composition is not markedly different from that of alloys already investigated and that we know the physical state, e.g. strained or annealed. In the case of pure metallic elements, however, the conductivity at low temperature is very sensitive to small traces of chemical impurities and physical defects; but as discussed in Chapter XI, a comparatively simple measurement of the
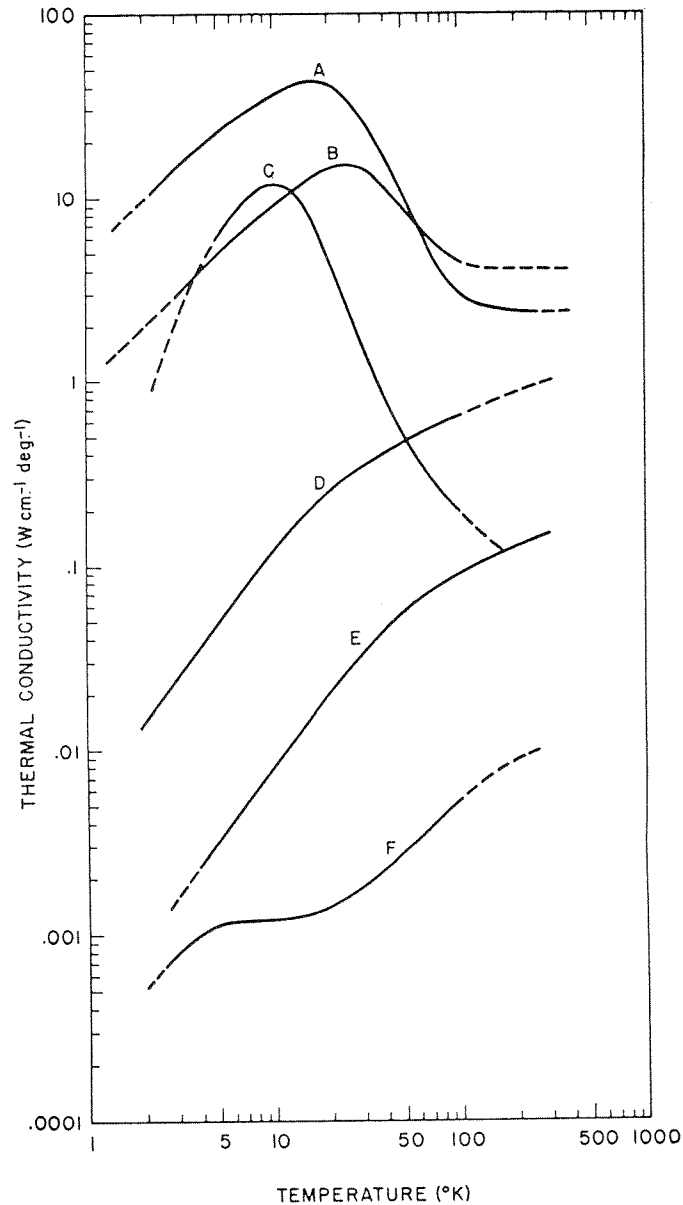
FIG. 96. Thermal conductivity of some solids, from data in Powell and Blanpied (1954). *A*, a metallic element, Al (high purity); *B*, a metallic element, Cu (electrolytic tough pitch); *C*, a dielectric crystal, quartz; *D*, an alloy, annealed brass (70 Cu, 30 Zn); *E*, an alloy, stainless steel; *F*, a glass, quartz glass.

residual electrical resistance (normally at about 4·2° K) enables a good estimate of the heat conductivity to be made.

The knowledge most often needed for practical cryogenic calculations is the effective heat conductance of a solid bar of a few common materials, with certain end temperatures. Commonly encountered pairs of end temperatures may be 300° K (room temperature) and 77° K, 300° and 4·2° K, 77° and 20° K, 77° and 4·2° K, 4·2° and 2° K. In Table XV are given values for the mean heat conductivity,

$$\bar{\lambda} = (T_2 - T_1)^{-1} \int_{T_1}^{T_2} \lambda(T) \, dT,$$

under these temperature conditions for a number of common materials, e.g. Pyrex glass, stainless steel, inconel (hard-drawn), monel (annealed), German silver, constantan, brass, phosphorus deoxidized copper (representing the material from which many copper items are frequently made: pipe, tube, and some rod and bar), electrolytic tough pitch copper (representing the type of copper frequently used in commercially available spools of wire). The values in the table, with the exception of those on coppers and brass, were calculated from the data compiled by Powell and Blanpied (1954). The values for the copper specimens were based on measurements by Powell, Rogers, and Roder (1957) and those for brass were taken from Kemp, Klemens, Tainsh, and White (1957).

It is well to note that values for $\bar{\lambda}$ for most commercial glasses appear to be close (within 30 per cent) to those given for Pyrex glass in the table. Similarly, values for manganin approximate to the constantan values given. Values for annealed inconel and annealed K-monel will approximate to those for annealed monel, and values for hard-drawn monel and K-monel will be not more than about 20 per cent greater than those given for hard-drawn inconel.

For comparatively pure metallic elements such as commercial copper, small variations in chemical or physical purity may affect the conductivity considerably, particularly at low temperatures. This is discussed more fully in Chapter XI. Data

TABLE XV

Mean values of Thermal Conductivity expressed in watts/cm °K

	λ, $T_2 = 300°K$, $T_1 = 77°K$	λ, $T_2 = 300°K$, $T_1 = 20°K$	λ, $T_2 = 300°K$, $T_1 = 4°K$	λ, $T_2 = 77°K$, $T_1 = 20°K$	λ, $T_2 = 77°K$, $T_1 = 4°K$	λ, $T_2 = 20°K$, $T_1 = 4°K$	λ, $T_2 = 4°K$, $T_1 = 2°K$
Pyrex glass	0·0082	0·0071	0·0068	0·0028	0·0025	0·0012	0·0007
Stainless steel†	0·123	0·109	0·103	0·055	0·045	0·009₇	0·0022
Inconel (c. 72 Ni, 14–17 Cr, 6–10 Fe, 0·1 C) hard-drawn	0·125	0·111	0·106	0·061	0·051	0·012	0·003
Monel (c. 66 Ni, 2 Fe, 2 Mn, 30 Cu) annealed	0·207	0·192	0·183	0·133	0·113	0·040	0·007
German silver (47 Cu, 41 Zn, 9 Ni, 2 Pb) as received	0·20	0·19	0·18	0·14	0·12	0·039	0·005
Constantan (60 Cu, 40 Ni) wire as received	0·22	0·21	0·20	0·16	0·14	0·04₆	0·006
Brass (30 Zn, 70 Cu) as received	0·81	0·70	0·67	0·31	0·26	0·078	0·015
Copper (phosphorus deoxidized) as received	1·91	1·71	1·63	0·95	0·80	0·25	0·07
Copper (electrolytic tough pitch) as received	4·1	5·4	5·7	9·7	9·8	10	4

† These figures for stainless steel are calculated from thermal conductivity data which are representative of the behaviour of types 303, 304, 347, and therefore an appropriate composition could be 18% Cr, 9% Ni, traces of Mn, Nb, Si, Ti totalling 2–3% with remainder Fe.

for $\lambda(T)$ of materials such as nylon, Teflon, Perspex, soft solder, Wood's metal, and a silicon bronze† are also given in tabular form in this later chapter.

The values in this table make it evident why alloys of the cupro-nickel family (copper-nickel, constantan, German silver) or of the monel and stainless steel group are so frequently chosen for the tubes in a cryostat where low thermal conductivity is a requirement. Stainless steel and inconel have somewhat lower heat conductivities than the cupro-nickel alloys although the latter are often preferred due to the greater ease with which they may be soft-soldered.

4. Heat transfer by radiation

A perfect black body may be defined as one which absorbs all radiation falling upon it. For such a body the absorptivity a and emissivity ϵ are unity, and so its reflectivity

$$R = 1 - \epsilon = 1 - a$$

is zero.

It may be shown (see, for example, Roberts, 1940) that for a black body at a temperature T the total radiant energy emitted per second per unit area is given by

$$E = \sigma T^4,$$

where the constant σ, Stefan's constant, has an experimental value of $5·67 \times 10^{-12}$ W cm^{-2} deg^{-4}. Such radiant energy is distributed over a range of wavelengths, the energy $E(\lambda)\,d\lambda$ emitted over a narrow interval λ, $\lambda + d\lambda$ being a function of λ and T. The function $E(\lambda)$ at any temperature T has a maximum value for $\lambda = \lambda_m$, and it may be shown that

$$\lambda_m T = \text{constant (Wien's constant)} \tag{45}$$

for which the experimental value is $0·290$ cm deg.

Thus for
$$T = 300° \text{ K}, \quad \lambda_m = 9·67 \ \mu,$$
$$T = 200° \text{ K}, \quad \lambda_m = 14·5 \ \mu,$$
$$T = 77° \text{ K}, \quad \lambda_m = 37·7 \ \mu,$$
$$T = 4·2° \text{ K}, \quad \lambda_m = 690 \ \mu.$$

† A useful low-conductivity alloy which is non-magnetic at low temperatures; contains 96% Cu, 3% Si, 1% Mn.

Most calculations dealing with the radiant heat transfer in low-temperature equipment are not restricted to black bodies but deal with metallic surfaces whose emissivity may be anywhere between 0·01 and 1·0.

Most non-metallic surfaces, of which glass and perhaps baked varnishes are the most important from our viewpoint, do approximate to black bodies in that their emissivities are in the neighbourhood of 0·9. However, for a particular metallic conductor the emissivity or the reflectivity depends on the wavelength of incident radiation and the physical state of the surface.

The classical theory of Drude yields a relation between the reflectivity R, the wavelength λ (microns), and the d.c. electrical resistivity of the metal ρ (ohm cm):

$$1-R = \epsilon = 36\cdot5(\rho/\lambda)^{\frac{1}{2}}. \qquad (46)$$

Evidence for the experimental validity of this formula has been discussed by Worthing (1941), Blackman, Egerton, and Truter (1948), Reuter and Sondheimer (1948), and Ramanathan (1952), etc. At relatively high temperatures the experimental agreement is reasonably good in many instances.

At low temperatures, however, the theoretical relation might be expected to fail even with smooth uncontaminated metallic surfaces, as the relaxation time of the electrons (period of the electron mean free path) becomes comparable with the period of vibration of the incident radiation.

The validity of the Hagen–Rubens equation (46) depends on the assumption that $\nu\tau < 1$ where ν is the frequency of the radiation and τ is the electron relaxation time. In a typical monovalent metal at room temperature the electron velocity (Fermi velocity) and mean free path are respectively about 1×10^8 cm/sec and 1 to 5×10^{-6} cm, so that $\tau \sim 3 \times 10^{-14}$ sec (assuming $l \sim 3 \times 10^{-6}$ cm). For room-temperature incident radiation, $\lambda_m \simeq 10^{-3}$ cm, whence $\nu \sim 3 \times 10^{13}$ (assuming velocity $c = 3 \times 10^{10}$ cm/sec). Therefore $\nu\tau \sim 1$. Therefore for radiation in the visible region of the spectrum we should expect a calculation of the reflection coefficient to require us to take electron relaxation into account. This yields a relation

for R or ϵ which is independent of the wavelength λ (see, for example, Mott and Jones, 1936, Chapter III). Agreement with available experimental evidence appears tolerably good for optical wavelengths when relaxation is considered. The discrepancy is still very marked for infra-red radiation at low temperatures, and it is difficult to believe that surface roughness or a surface electrical resistance higher than the bulk resistance is sufficient to explain discrepancies of a factor of 100 or more which occur with annealed electro-polished specimens at helium temperatures. Reuter and Sondheimer (1948; see also review on electron mean free paths in metals by Sondheimer, 1952) treated theoretically the problem of the anomalous skin effect, i.e. the effect on surface resistance when electron mean free paths become so long that at high frequencies the applied electric field may change appreciably within the extent of an electron mean free path. They assumed specular reflection of electrons at the metal surface and deduced a formula for the reflectivity or absorptivity in considerably better agreement with low-temperature experiments (e.g. Ramanathan, 1952). Dingle, in a series of papers in *Physica* in 1952–3 (see particularly Dingle, 1953 b, but also Dingle, 1952, 1953 a, c, d) deduced a slightly simpler expression and by assuming diffuse reflection showed that reasonable agreement with experiment could be obtained. In Table XVI (Dingle, 1953 b) are compared experimental values of the *percentage* absorptivity for $14\,\mu$ radiation on electro-polished copper, with theoretical values.

TABLE XVI

Temperature	Experimental	Classical theory (incl. relaxation)	Anomalous skin effect (diffuse reflection)
Room temp.	1·2	0·5	0·8
Liquid-oxygen temp.	0·8	0·09	0·5
Liquid-helium temp.	0·6	0·003	0·4

Returning to our practical problem of calculating the approximate radiant heat transfer between surfaces, it is apparent that despite the partial success of more sophisticated theoretical

analyses, it is generally necessary to select experimental data which may be considered most appropriate for the particular surfaces in question. In Table XVII below are collected experimental values for the emissivity of a number of commonly used metals; as indicated in the table these values have been obtained on surfaces in various physical conditions, e.g. electro-polished, normally smooth and clean, highly oxidized, etc. By some intelligent guessing, the most suitable value of ϵ, or a probable

TABLE XVII

Experimental Values of Emissivity

Material	Fulk, Reynolds, & Park (1955) 300° K radn. on 78° K surface	McAdams (1954) room temp.	Ramanathan (1952) 14μ radn. on 2° K surface	Blackman, Egerton, & Truter (1948) 293° K radn. on 90° K surface	Ziegler & Cheung (1957) 273° K radn. on 77°K surface
Al-clean polished foil .	0·02	0·04	0·011†	0·055	0·043‡
Al-plate .	0·03	. .	. .	. .	. .
Al-highly oxidized .	. .	0·31	. .	. .	. .
Brass-clean polished .	0·029	0·03	0·018†	0·046	0·10‡
Brass-highly oxidized .	. .	0·6	. .	. .	. .
Cu-clean polished .	0·015–0·019	0·02	0·0062–0·015†	0·019–0·035	. .
Cu-highly oxidized .	. .	0·6	. .	. .	. .
Cr-plate .	0·08	0·08	. .	0·065	0·084‡
Au-foil .	0·010–0·023	0·02–0·03	. .	0·026	. .
Au-plate .	0·026	. .	. .	. .	. .
Monel .	. .	0·2	. .	. .	0·11‡
Ni-polished .	. .	0·045	. .	. .	. .
Rh-plate .	0·078	. .	. .	. .	. .
Ag-plate .	0·008	0·02–0·03	. .	0·023–0·036	. .
Stainless steel .	0·048	0·074	. .	. .	. .
Sn-clean foil	0·013	0·06	0·013†	0·038	. .
Soft solder .	0·03	. .	. .	. .	0·047‡
Glass . .	. .	0·9	. .	0·87	. .
Wood's metal	. .	. .	. .	. .	0·16

† These surfaces were electro-polished (Ramanathan).
‡ These surfaces were neither highly polished nor heavily oxidized, but as encountered in normal practice (Ziegler). Ziegler observed that a thin layer of oil or Apiezon grease on a low-emissivity surface raised the emissivity to 0·2 or 0·3. He also found that varnishes such as GEC adhesive no. 7031 and bakelite lacquer gave an emissivity $e \simeq 0.87$; similarly, Scotch tape (Sellotape) had an emissivity of about 0.88.

upper limit for ϵ, must then be selected for calculation of heat transfer.

For two plane parallel surfaces each of area A, and emissivities ϵ_1 and ϵ_2, and at respective temperatures T_1 and T_2, the heat transfer by radiation per unit time is

$$\dot{Q} = \sigma A(T_1^4 - T_2^4)\frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2}, \tag{47}$$

whence
$$\dot{Q} \simeq \sigma A(T_1^4 - T_2^4) \quad \text{for } \epsilon_1 = \epsilon_2 \simeq 1. \tag{48}$$

In the case where $\epsilon_2 \ll \epsilon_1$, (47) reduces to

$$\dot{Q} = \sigma A(T_1^4 - T_2^4)\epsilon_2. \tag{49}$$

Similarly, if $\epsilon_1 = \epsilon_2 = \epsilon$, and $\epsilon \ll 1$, (47) becomes

$$\dot{Q} = \sigma A(T_1^4 - T_2^4)\epsilon/2. \tag{50}$$

5. Other causes of heat transfer

While the processes of heat transfer discussed in the preceding sections, viz. low-pressure gas conduction, heat conduction by solids, and radiation transfer, are those chiefly encountered, other less common but quite troublesome sources of energy often arise.

Joule heating

Joule heating in connecting leads and in resistance thermometers, giving rise to a heat input $\dot{Q} = I^2 R = V^2/R$ is not difficult to estimate but may be overlooked as a source of temperature drift or temperature inhomogeneity in a cryostat. The opposing demands of low thermal conduction along electrical leads, and insignificant heat dissipation in these same leads often presents a problem. At temperatures below 7° K this has frequently been solved by using wires of a poor thermal conductor such as constantan, and tinning the surface with a thin lead coating; this becomes superconducting below 7° K and the wire remains a poor heat conductor. As is discussed in a later chapter (XI), most alloys (excluding superconducting alloys) exhibit an electrical resistance which does not decrease appreciably as the temperature falls from room temperature to liquid-helium

temperatures, but their heat conductivity usually decreases by a factor of 10–100. On the other hand, pure metallic elements have an electrical resistance which may fall by a factor of a hundred or more as the temperature is changed from 300°–4·2° K; over the same range the thermal conductivity of the pure metallic element normally increases as T falls below 100° K, passes through a maximum at a temperature in the vicinity of $\theta_D/20$ (say 10°–20° K), and falls linearly towards zero at the absolute zero. As a result a piece of copper wire may have approximately the same heat conductivity at 4° and 300° K while its electrical resistance is different by a factor of 100; a wire of constantan may have approximately the same electrical resistance at 4° and 300° K but a heat conductivity which has changed by a factor of 100. It may be noted that at both low temperatures (~ 4° K) and high temperatures (~ 300° K) the ratio $\rho\lambda/T$ for most metals has a value which at least approximates to the theoretical Lorenz value of $2\cdot45\times10^{-8}$ W ohm/deg². In selecting electrical lead wires to place in a cryostat, these factors should be borne in mind.

Gas adsorption

Another cause of temperature drift of concern to the low-temperature calorimetrist is the adsorption or desorption of residual gas. The thermal energy required to desorb a layer of adsorbed gas from a surface is of the same order of magnitude as the latent heat of vaporization. As the temperature of the solid surface is raised, e.g. crystals in a calorimeter vessel, gas desorption begins and part of any electrical energy fed to the calorimeter is employed in this desorption process. Keesom (1942, p. 127) has given some adsorption data for helium gas on surfaces of glass and charcoal at low temperatures. Between 2° and 3° K glass adsorbs about 30×10^{-10} mole/cm² for which the heat of adsorption is ~ 70 cal/mole. A simple calculation indicates that the desorption of such a layer from 100 cm² of glass requires energy of $\sim 1,000$ ergs. It appears likely that with surfaces of baked lacquers and powdered materials, the amount of adsorption is very much greater than with a clean glass or metal

surface. Thus it has been found by some experimenters that in high-precision calorimetry at low temperatures, it is undesirable to expose the sample or calorimeter to exchange gas at temperatures below about 10° K, because of the adsorption which occurs and subsequent slow temperature drifts which may be attributed to gradual desorption of the helium layer when the space is pumped to a high vacuum. This process has also been named as a contributing factor to 'heat leaks' in adiabatic demagnetization cryostats; when a salt pill is cooled to a temperature of $\frac{1}{100}$° K and is surrounded by a 1° K wall, helium gas (remaining from exchange gas used in cooling) slowly desorbs and becomes adsorbed on the colder pill, transferring thermal energy in an amount which may cause a serious temperature rise, owing to the small heat capacities of the system.

Mechanical vibration

In most cryogenic experiments gas desorption is not a very serious factor, nor is mechanical vibration. Quantitative information on the amount of heat released in an otherwise thermally isolated system by its being in mechanical vibration—a result of mechanical linkage to its surroundings, i.e. external vibrations of pumps, building vibration, etc.—is rather lacking. However, it has been frequently noted by workers in the field of temperature below 1° K, where heat inputs of 100 erg/min may be considered excessive, that the rate of temperature rise of their salt pills after demagnetization was affected by the operation of pumps. In a recent interesting series of experiments on thermal contact and insulation below 1° K, Wheatley, Griffing, and Estle (1956) mechanically coupled their cryostat to a motor whose speed could be adjusted to vary the vibration frequency from 0 to 13 per second; they found at the maximum frequency a heat leakage to a pill (mounted on a rather rigid support) of 235 erg/min, compared with about 10 erg/min when the motor was switched off. It is apparent that a fairly rigid suspension, e.g. glass rods as opposed to nylon or cotton threads, is less susceptible to external vibrations as its resonant frequency is much higher than the frequency of most normal building or

pump vibrations. A salt pill suspended by fine threads is much more likely to be set in resonant vibration by the action of external sources, and heat leaks of the order of 1,000 erg/min have been encountered in some such instances.

Darby *et al.* (1951) reported that in their two-stage de-magnetization equipment, in which the two pills were suspended by nylon threads, vacuum pumps produced a serious vibrational heat source; even the action of mercury 'bumping' in a diffusion pump attached to the cryostat caused 'heat leaks' in excess of 300 erg/min to the upper salt pill. On the other hand, Malaker (1951) used a nylon thread suspension and calculated the heat leak to be only ~ 4 erg/min at very low temperatures. The exact process of such energy release in the pill, although presumably frictional, appears somewhat obscure.

An excellent discussion of the problems of thermal insulation at very low temperatures was given some years ago by Cooke and Hull (1942).

6. Example of heat-transfer calculation

As an illustration of the calculations that may be necessary in cryostat design, consider the magnitudes of the heat inflows from various sources in the following example (see Figure 97): A polished copper inner chamber of surface area 500 cm² is supported by a German silver tube (2·0 cm diameter, 0·3 mm wall thickness) inside a tarnished brass vacuum chamber in which the measured pressure (of helium gas) is 10^{-5} mm; the length of the tube separating the two chambers is 6 cm. Twelve electrical leads (eight leads of 38 B. & S. gauge copper and four of 32 B. & S. constantan) enter the outer chamber through the pumping tube and are effectively thermally anchored on a

FIG. 97. Diagram of a cryostat.

copper bush at 77° K; the leads are then taken to the inner chamber, each lead having a length of about 12 cm between its points of attachment on the respective chambers.
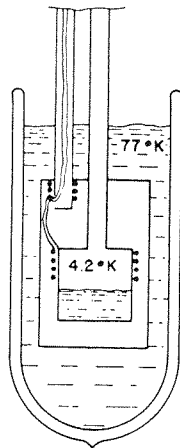
We require to calculate the following:

(i) Radiant heat inflow from the brass to the copper chamber.
(ii) Radiant heat inflow down the German silver tube to the inner chamber when no radiation baffle is present.
(iii) Heat conducted down the German silver tube from the outer to the inner chamber.
(iv) Heat conducted down the electrical leads from one chamber to the other.
(v) Heat conducted through the low-pressure helium gas in the inter-chamber space.
(vi) And to examine whether Joule heating due to a current of 5 mA in the electrical leads is serious in view of the other heat inflows.

Firstly,

(i) Assuming that for tarnished brass $\epsilon_1 \to 1$, and for polished copper $\epsilon_2 \simeq 0.03$,

from (49) $\dot{Q}_r = \sigma A (T_1^4 - T_2^4) 0.03$

$$\simeq 5.67 \times 10^{-12} \times 500 \times 0.03 \times (77)^4$$

$$= 3 \times 10^{-3} \text{ W.}$$

(ii) Any calculation of the radiant heat reaching the inner space by 'funnelling' of room temperature radiation down the German silver tube must yield a very crude approximation to the true experimental result, at best. An upper limit $\dot{Q}_f(\text{max})$ may be calculated easily, assuming 'complete funnelling' i.e. perfect specular reflection from the inner wall of the tube, so that all the radiation $\dot{Q} (= \sigma \pi 1^2 (295)^4)$ reaches the inner space.

$$\dot{Q}_f(\text{max}) = 5.67 \times 10^{-12} \times \pi \times (295)^4$$

$$= 0.135 \text{ W.}$$

Alternatively, if we assume almost complete absorption of the radiation which reaches the inner wall of the tube, $\dot{Q}_f(\text{min})$ is merely the radiation from a small solid angle subtended by the lower exit of the tube at its upper (room temperature) end. If

the length of the tube be about 30 cm between $4 \cdot 2°$ K end and that part at room temperature

$$\dot{Q}_f(\text{min}) \simeq \frac{\pi \cdot 1^2}{2\pi \cdot (30)^2} \times \sigma A (295)^4$$

$$\sim 10^{-4} \text{ W}.$$

In practice this minimum value for radiant heat inflow via the pumping tube may be approached by painting the inner wall of the German silver tube with an optical black paint, 'Aquadag', etc. More complete isolation from room-temperature radiation is usually obtained by means of a bend in the tube or insertion of a small radiation baffle. By this means the room-temperature radiation is adsorbed by the tube wall near the bend or at the baffle and transmitted into the liquid-nitrogen bath; the inner experimental space then only sees radiation emitted from a relatively small area of surface at $77°$ K which is transmitted by a direct path or successive internal reflections down the tube.

(iii) For the heat conducted by the German silver tube

$$\dot{Q}_c = \lambda \frac{A}{l} \Delta T$$

$$= 0 \cdot 12 \times \frac{2\pi \times 0 \cdot 03 \times 73}{6} \quad (\lambda = 0 \cdot 12 \text{ from Table XV})$$

$$= 0 \cdot 275 \text{ W}.$$

(iv) The heat conducted down the electrical leads is given by $\dot{Q}_c^* = \dot{Q}$ (eight copper leads of $0 \cdot 010$ cm diameter) $+ \dot{Q}$ (four constantan leads of $0 \cdot 020$ cm diameter); using data from Table XV, λ (copper) $= 9 \cdot 8$, λ (constantan) $= 0 \cdot 14$, therefore

$$\dot{Q}_c^* = 8 \times 9 \cdot 8 \times \frac{0 \cdot 786}{12} \times 10^{-4} \times 73 + 4 \times 0 \cdot 14 \times \frac{0 \cdot 314}{12} \times 10^{-3} \times 73$$

$$= 6 \cdot 08 (0 \cdot 00616 + 0 \cdot 000176)$$

$$= 0 \cdot 0385 \text{ W}.$$

(v) For gas conduction $\dot{Q}_g$ is given by (42), which in the case of helium gas reduces to

$$\dot{Q}_g = 0 \cdot 028 p_{\text{mm}} a_0 \Delta T \text{ W cm}^{-2}.$$

Assuming that the respective surface areas are approximately equal and parallel, and that

$$a_{\text{max}} \simeq 0 \cdot 5 = a_1 = a_2,$$

then

$$a_0 = 0 \cdot 5 / (2 - 0 \cdot 5)$$

$$= 0 \cdot 333.$$

Since $\Delta T = 73°$ K, $p = 10^{-5}$ mm,

therefore $\quad \dot{Q}_g = 0 \cdot 028 \times 0 \cdot 333 \times 73 \times 10^{-5}$ W cm^{-2},

and since $\quad A \simeq 500$ cm^2,

$$\dot{Q}_g = 0 \cdot 0034 \text{ W}.$$

(vi) (a) Eight copper leads of 38 B. & S. wire each have an approximate room-temperature resistance of $0 \cdot 024$ ohm/cm. Hence, assuming for normal commercial wire that $\rho_r / \rho_{0 \, °C} \simeq 10^{-2}$, the resistance will be about $0 \cdot 0046$ ohm/cm at $77°$ K and $0 \cdot 0003$ ohm/cm at $4 \cdot 2°$ K.

If the mean resistance is $0 \cdot 0025$ ohm/cm, the total heat produced by 5 mA is

$$(5 \times 10^{-3})^2 \times 0 \cdot 0025 \times 12 = 0 \cdot 75 \times 10^{-6} \text{ W per wire.}$$

A rather crude calculation based on a mean thermal conductivity of 10 W/cm °K shows that the temperature rise in the wire will be $< 0 \cdot 01°$ K at any point. If all the Joule heat were transferred to the inner chamber this would only amount to about 6 μW.

(b) In the case of constantan (four wires of 32 B. & S.) the electrical resistance of each wire is about $0 \cdot 14$ ohm/cm and is not particularly sensitive to change in temperature.

Total Joule heat produced is

$$4 \times (5 \times 10^{-3})^2 \times 12 \times 0 \cdot 14 = 1 \cdot 68 \times 10^{-4} \text{ W}.$$

Again, assuming $\lambda \simeq 0 \cdot 14$ W/cm deg, it may be calculated that the temperature rise will not be greater than about $5°$ K.

Summarizing, we note that the major source of heat leakage is via the supporting tube, and provided that precautions are taken to prevent room-temperature radiation entering the inner chamber via the tube this leak amounts to about $0 \cdot 275$ W. Radiant heat of 10^{-3} W reaches the inner chamber from the

surrounding 77° K chamber and heat leakage through the residual 10^{-5} mm pressure of helium gas is less than 0·004 W; by comparison a 5 mA current through each electrical lead could contribute a maximum heat inflow of about 100 μW.

The total heat leak of about 0·28 W is equivalent to 240 cal/hour, which would evaporate nearly 400 cm³ of liquid helium per hour; this assumes that the cold evaporating gas does not play any useful role in cooling the wall of the German silver tube and thereby reduce the heat leak.

7. Heat transfer through pressed contacts

Elsewhere we make passing reference to the problem of heat transfer across the boundary between two solid surfaces in contact. This has a direct application to at least two practical cryogenic problems, namely to the design of mechanical heat switches (for example, as applied to calorimetry by Westrum, Hatcher, and Osborne, 1953; Webb and Wilks, 1955; Ramanathan and Srinivasan, 1955; and Rayne, 1956) and in the design of insulating supports for low-temperature equipment (as applied to the internal supporting members in large dewar vessels by Birmingham *et al.*, 1955).

The first comprehensive investigation of thermal contact has been that of Berman[†] (1956) who studied the thermal conductance of various solid contacts at liquid-helium and liquid-nitrogen temperatures, using loads of 50–250 lb. His results suggest that:

(i) The measured thermal conductance is always greater than that calculated by the Wiedemann–Franz–Lorenz law from the measured electrical conductance. This discrepancy, often a factor of 100 or more, is as high as 10^5 when contact and measurement are made at 4·2° K. The only possible conclusion seems to be that the majority of the heat is carried across the interface by thermal waves rather than by electrons.

(ii) The thermal conductance varies nearly linearly with the pressure, and hence is not sensitive to change in area for a given total load.

† In this paper Berman includes a brief account of some earlier work in this field by Jacobs and Starr, Fulton, Zavaritski, etc.

(iii) At liquid-helium temperatures (1°–4° K), the conductance varies as T^2 but becomes less temperature-sensitive at higher temperatures, the change in conductance between 64° and 77° K being only about 10 per cent.

(iv) Some typical figures given by Berman for the thermal conductance of the contact between (*a*) two copper rods, (*b*) two 0·001 in. steel disks are:

(*a*) $1·02 \times 10^{-2}$ W deg^{-1} at 4·2°K; $32·5 \times 10^{-2}$ W deg^{-1} at 77°K;

(*b*) $0·54 \times 10^{-2}$ W deg^{-1} at 4·2° K; 26×10^{-2} W deg^{-1} at 77° K;

in each case the contact was made at room temperature and the load applied was 100 lb.

The recent experimental work on heat conduction through insulating supports by Mikesell and Scott (1956) is of less fundamental interest than Berman's experiments but of great practical significance in cryogenic design. Their work was concerned with the internal support of large dewar vessels by using stacks of thin disks. Although the mechanical strength of the stack is great, the thermal contact resistance between disks is such that the total thermal resistance of a stack may be a hundred times greater than that of a solid rod of the same material and dimensions. For example, measurements of the thermal conductance of a stack of about 315 stainless steel plates of thickness 0·0008 in. under a pressure of 1,000 p.s.i. showed that: with end temperatures of 296° and 76° K, the conductance is 0·93 W cm^{-2} and with end temperatures of 76° and 20° K it is 0·062 W cm^{-2}; assuming the length of the stack is $315 \times 0·0008$ in. $= 0·25$ in., these figures correspond to effective mean conductivities of 2·7 mW/cm °K and 0·71 mW/cm °K respectively. These may be compared with values of $\lambda = 123$ mW/cm °K (300°–77° K) and $\lambda = 55$ mW/cm °K (77°–20° K) from Table XV (§ 6.3) for stainless steel in solid form. Mikesell and Scott found that the conductance of the stack of disks could be decreased substantially by using a thin layer of manganese dioxide dust between each pair of plates.

Such stacked disks provide an excellent means of transmitting

high pressures into a cryostat, as a stack can have very high compressional strength, and yet retain a low heat conductivity (see, for example, their use by Berman, 1956).

The use of the mechanical contact as a heat switch for cooling specimens prior to a specific heat determination has already been mentioned (§ 5.2). Such a 'switch' obviates the necessity for using exchange gas at low temperature with the entailed problems of slow gas desorption. Cone and socket contacts have been used for this purpose but they appear to generate considerable frictional heat on breaking the contact and the double-jaw type (Webb and Wilks, 1955) or flat plates (Ramanathan and Srinivasan, 1955; Rayne, 1956) seem more successful. Webb and Wilks report that the heat flow across their closed contact was of the order of 10^{-2} J/deg min, this being sufficient to cool their specimens from 4° to 1° K in a few minutes. It is probable that the conductance of the flat-plate type of switch is somewhat greater.

Experiments on contact resistance also confirm the reasons for the frequent difficulties that are encountered in thermally anchoring electrical leads effectively at low temperatures. When wires are brought down through a pumping tube into a cryostat, they are often wrapped around a copper post or tube which is in close thermal contact with the refrigerant; despite this, they often seem to be the source of heat inflow into a specimen (calorimeter, etc.) to which they are connected and which is otherwise isolated. The only method of avoiding this trouble appears to be to wrap as great a length of the wire as possible in close contact with the anchoring pillar and then to cement it firmly in place; glyptal, bakelite varnish, Formel varnish, or nail polish are all useful cements for this purpose.

Although the heat transfer between liquid helium and a solid is not properly a problem of pressed contacts, it seems appropriate to mention here some experimental values obtained for the flow of heat $\dot{Q}$ across a liquid-solid interface. As part of an investigation of the heat conductivity of liquid helium below 1° K, Fairbank and Wilks (1955) observed that between copper and the liquid helium, $\dot{Q}$ was proportional to the temperature

gradient and varied nearly as the square of the temperature; in fact

$$\dot{Q} = 2 \cdot 20 \times 10^{-2} T^2 \text{ W cm}^{-2} \text{ deg}^{-1}.$$

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