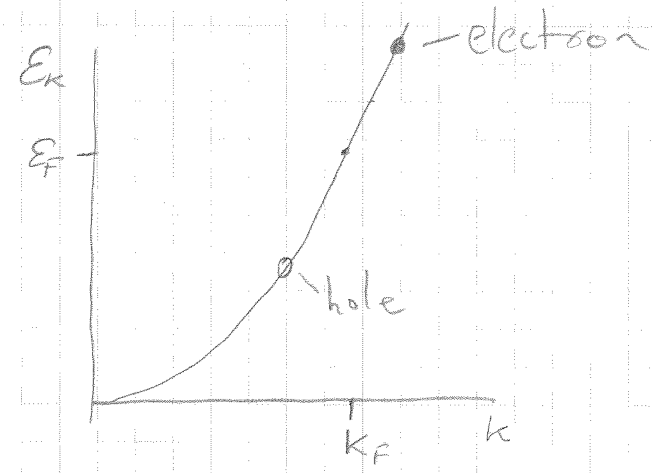
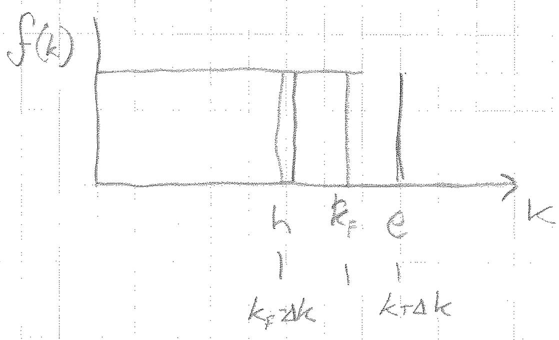
Exitations in SC

First consider a normal metal

$$\mathcal{E} = \frac{\hbar k^2}{2m}$$



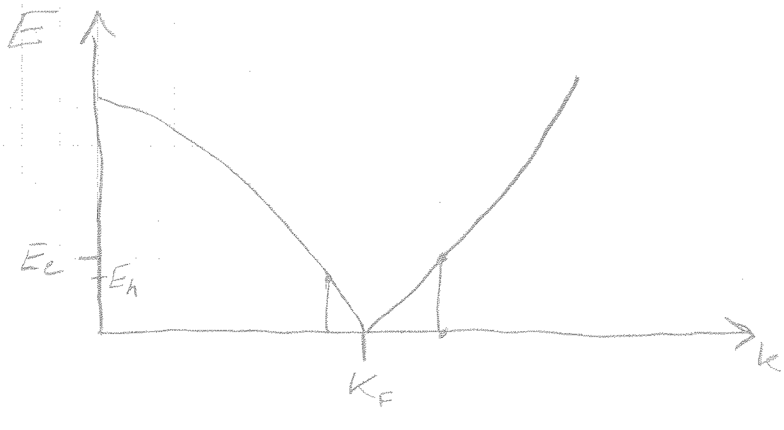


Exitation energy

$$E_e = \mathcal{E}(k_F + \Delta k_e) - \mathcal{E}(k_F) = \frac{\hbar^2}{2m}\left[(k_F + \Delta k)^2 - k_F^2\right] = \frac{\hbar^2}{2m}\left(2k_F \cdot \Delta k_e + \Delta k_e^2\right)$$

$$E_h = \mathcal{E}(k_F) - \mathcal{E}(k_F - \Delta k_h) = \frac{\hbar^2}{2m}\left[k_F^2 - (k_F - \Delta k)^2\right] = \frac{\hbar^2}{2m}\left(2k_F \Delta k_h - \Delta k_e^2\right)$$

$$E_{e,h} \simeq \mathcal{E}_F \left(\frac{2\Delta k_{e,h}}{k_F} \pm \underbrace{\frac{\Delta k^2}{k_F^2}}_{\approx 0}\right) \approx \mathcal{E}_F \frac{2\Delta k}{k_F}$$

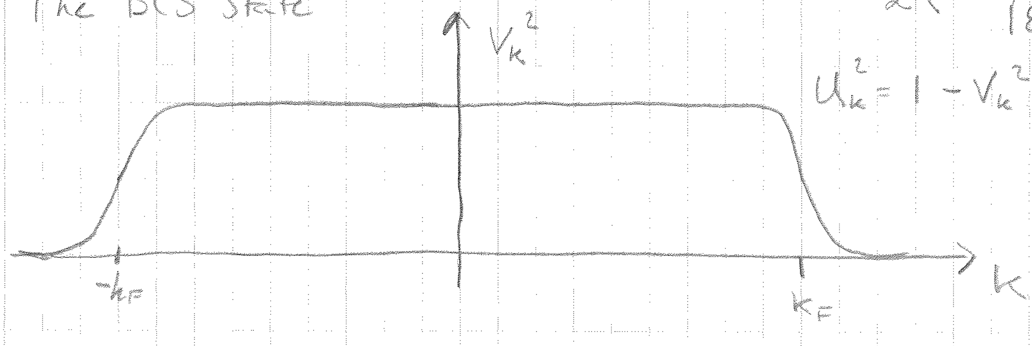


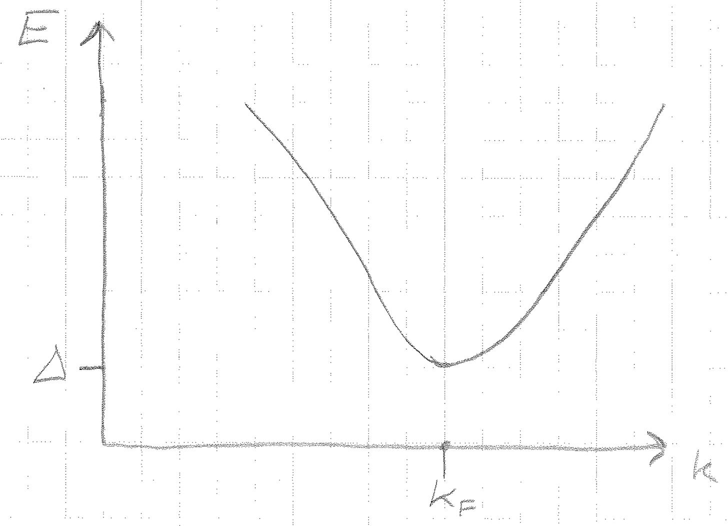
For the SC

The BCS State

$$V_k^2 = \frac{1}{2}\left(1 - \frac{\varepsilon_k}{\sqrt{\varepsilon_k^2 + \Delta^2}}\right)$$

$$U_k^2 = 1 - V_k^2$$

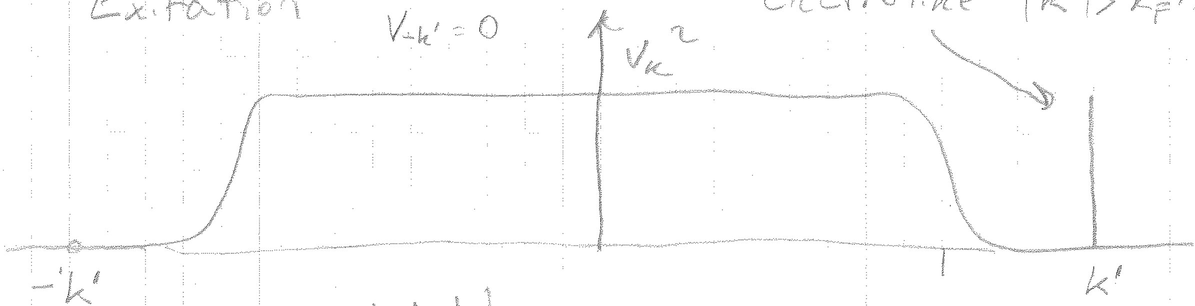


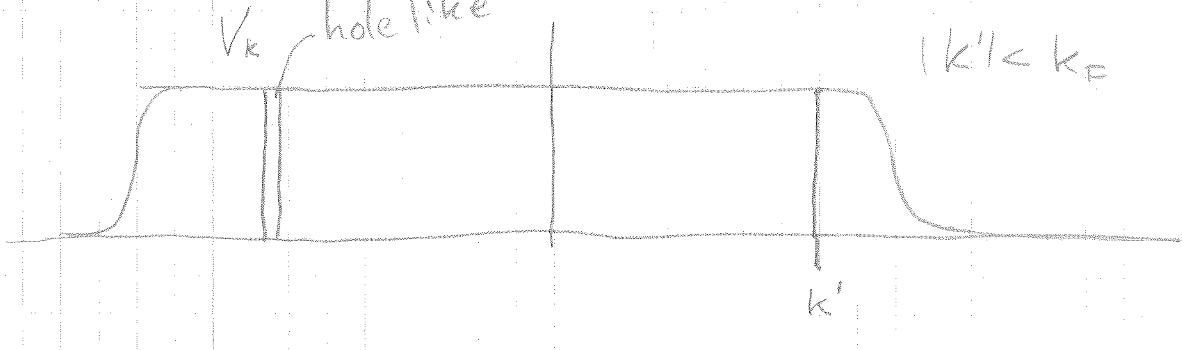


$$E_k = \sqrt{\varepsilon_k^2 + \Delta^2} \qquad \varepsilon_k = \varepsilon(k) - \varepsilon(k_F) = \varepsilon(k) - \varepsilon_F$$

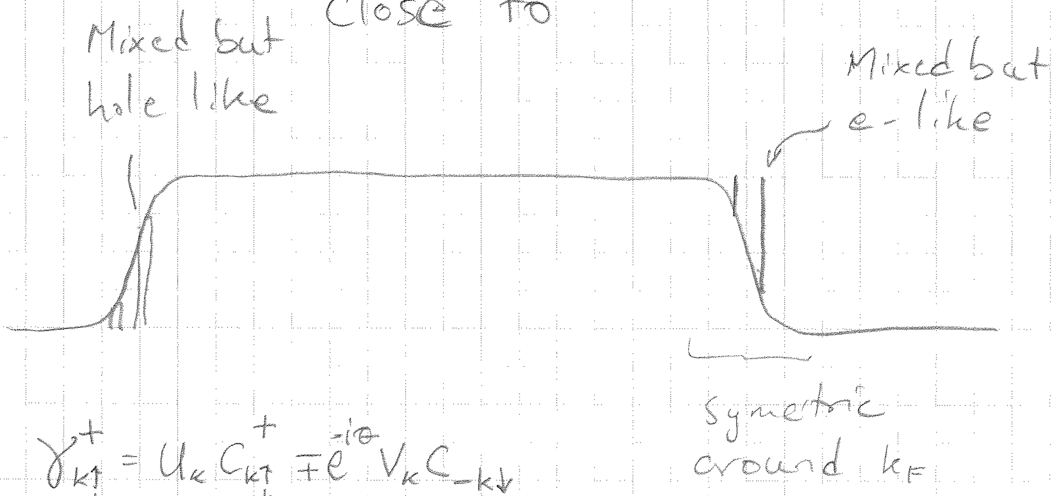
Exitation $V_{k'} = 1$, $V_{-k'} = 0$

electrolike $|k'| > k_F$





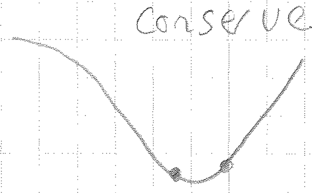
Close to

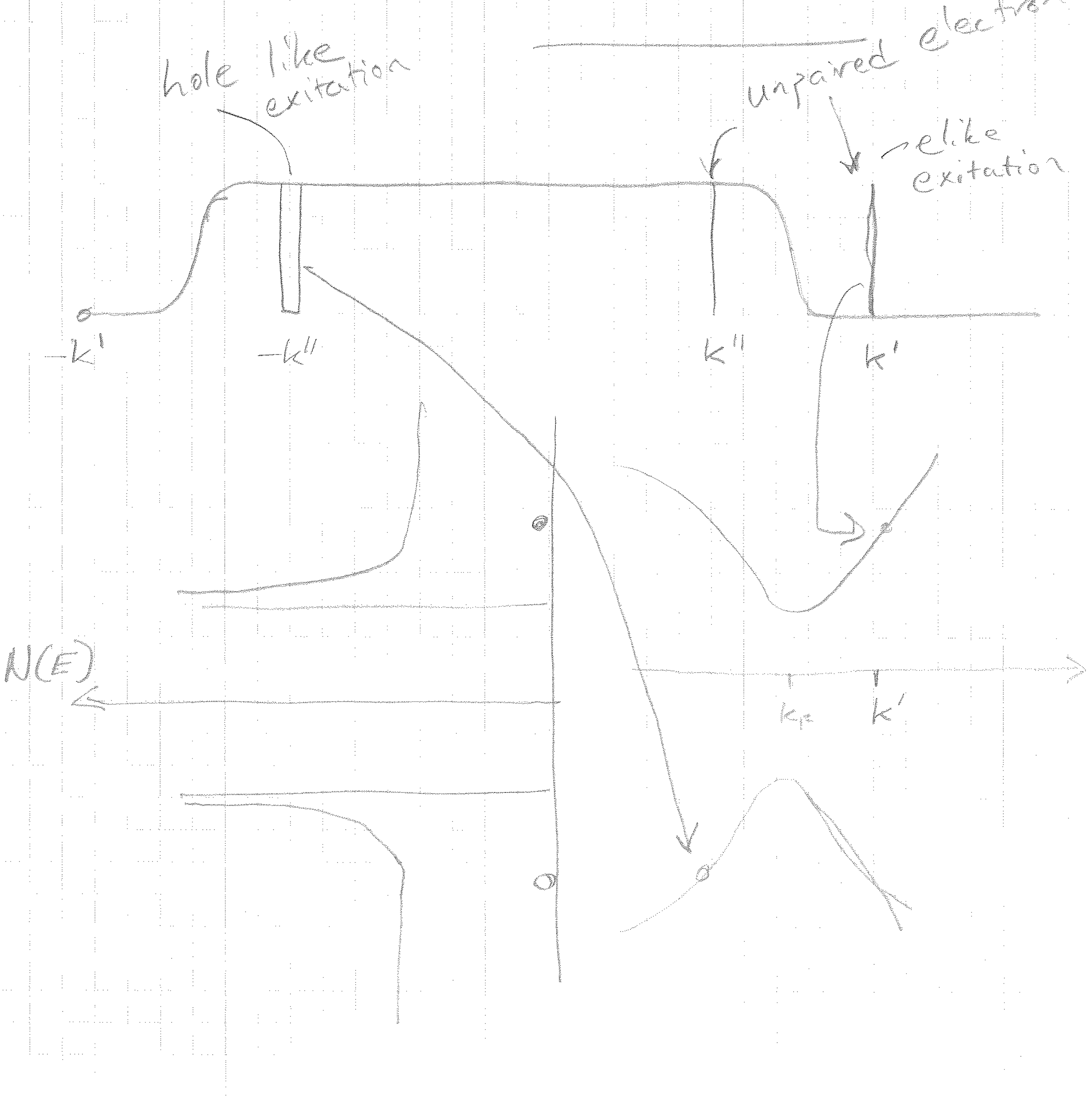


conserves

$$\gamma^{+}_{k\uparrow\downarrow} = U_k C^{+}_{k\uparrow\downarrow} \mp e^{-i\theta} V_k C_{-k\downarrow}$$

$$\gamma_{k\uparrow\downarrow} = U_k C_{k\uparrow\downarrow} \mp e^{i\theta}$$





Bogoliubov operators

Create an exitation

$$\gamma^{+}_{k\uparrow\downarrow} = u_k c^{+}_{k\uparrow\downarrow} \mp e^{-i\theta} v_k c_{-k\downarrow\uparrow}$$

to the extent that the state $k\uparrow$ is empty u_k
Create a fully ocupied state

Anihilate an exitation

$$\gamma_{k\uparrow\downarrow} = u_k c_{k\uparrow\downarrow} \mp e^{i\theta} v_k c^{+}_{-k\downarrow\uparrow}$$

- Exitations always come in pairs
- minimum exitation energy $E_{min} = 2\Delta$
- Symetric exititations around k_F conserves Number of electrons (Needed on average)

$$\{\gamma_i, \gamma_j^{+}\} = \delta_{ij}$$