Physical properties of materials at low temperatures

1 Resistivity / Conductivity

Free electron modell with scattering

$$\sigma = \frac{\bar{J}}{\bar{E}} = \frac{n \cdot e \, \bar{V}_D}{\bar{E}} \qquad n = \frac{N}{V}$$

V_D Drift velocity

$$m \cdot \dot{v} = e \cdot \bar{E}$$

$$\int_0^{V_D} d\bar{v} = \int_0^{\tau} \frac{e\bar{E}}{m} dt$$

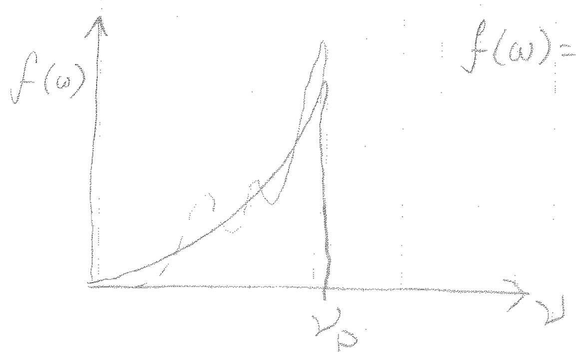
$$\bar{V}_D = \frac{e\bar{E}\tau}{m} \qquad \frac{V_D}{\bar{E}} = \frac{e\tau}{m}$$

$$\sigma = \frac{1}{\rho} = \frac{n e^2 \tau}{m} \qquad \rho = \frac{m}{n e^2 \tau}$$

τ is determined by scattering
the dominating process is

$$\rho = \rho(T) + \rho_0$$

Debey spectrum



$$f(\omega) = \begin{cases} kV\nu^2 & \nu < \nu_D \\ 0 & \nu > \nu_D \end{cases}$$

$$f(\omega) = k \cdot V \cdot \omega^2 = 4\pi\left(\frac{1}{v_l^3} + \frac{2}{v_t^3}\right) \cdot V \cdot \omega^2$$

N atoms in volum V $\Rightarrow$ 3N modes

$$3N = \int_0^{\nu_D} f(\nu)\,d\nu = k \cdot V \frac{\nu_D^3}{3}$$

$$\nu_D^3 = \frac{9n}{k} = \frac{9n}{4\pi\left(\frac{1}{v_l^3} + \frac{2}{v_t^3}\right)}$$

$$\Theta_D = \frac{h\nu_D}{k_B} = \frac{h}{k_B}\sqrt[3]{\frac{9n}{4\pi\underbrace{\left(\frac{1}{v_l^3} + \frac{2}{v_t^3}\right)}_{\frac{3}{v_s^3}}}} \approx \frac{h}{k_B}\sqrt[3]{\frac{3n}{4\pi}}\, v_s$$

sound velocity

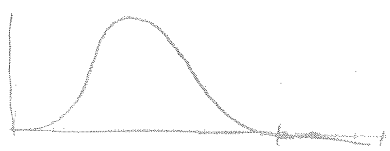
<u>Grüneise-Block equation</u>

$$\rho \propto \frac{T^5}{\Theta_D^6}\int_0^{\Theta/T} \frac{x^5\,dx}{(e^x - 1)(1 - e^{-x})}$$

low T

$x \to \infty$

$$\rho = c\,\frac{T^5}{\Theta_D^6} \cdot \underbrace{\int_0^{\Theta/T} \frac{x^5}{e^x}\,dx}_{\sim \Gamma(6) = 5!} = c\,5!\,T^5$$



high T

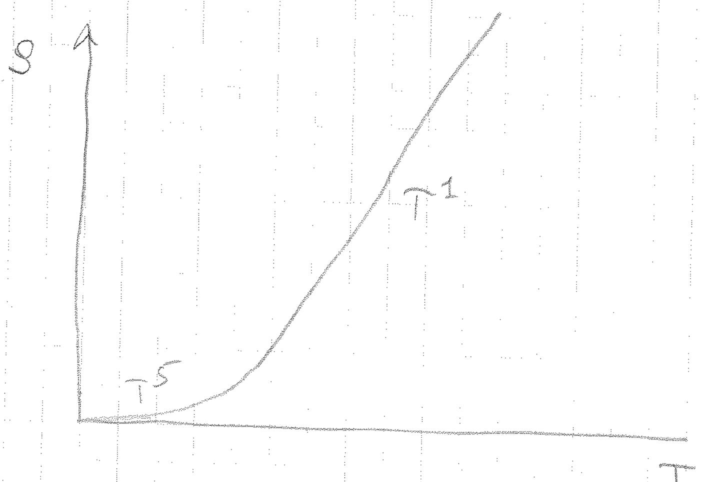
$x \ll 1$

high $T \rightarrow x \ll 1$

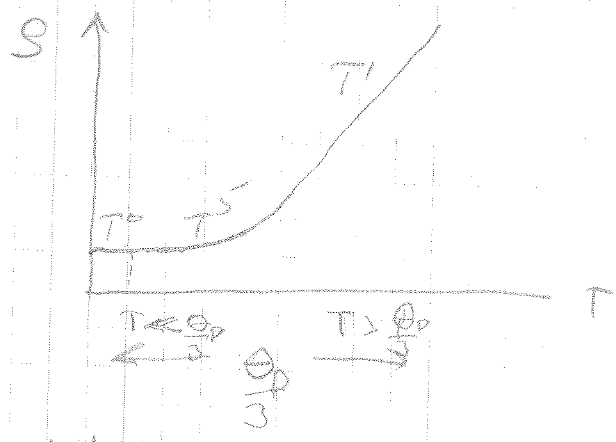
$$e^{x} \approx 1 + x$$
$$e^{-x} \approx 1 - x$$

$$\rho = C \frac{T^5}{\Theta_D^6} \int_0^{\Theta/T} \frac{x^5}{x \cdot x}\, dx = C \frac{T^5}{\Theta_D^6} \left[\frac{x^4}{4} \right]_0^{\Theta_D/T}$$

$$= \frac{C}{4} \cdot \frac{T^5}{\Theta_D^6} \cdot \frac{\Theta_D^4}{T^4} = \frac{C}{4} \cdot \frac{T}{\Theta_D^2} \propto T$$



Add impurities Temperature indep. scattering



$\Theta_D \sim 100 - 400\,K$

Valid also for SC above T_c

Heat capacity

$$C_V = \left.\frac{\partial U}{\partial T}\right|_V = T \left.\frac{\partial S}{\partial T}\right|_V$$ calculated

$$C_P = \left.\frac{\partial U}{\partial T}\right|_P + P \underbrace{\left.\frac{\partial V}{\partial T}\right|_P}_{\text{Thermal expansion}} = \left.\frac{\partial H}{\partial T}\right|_P$$ measured $\boxed{H = U + PV}$

$$C_P = \gamma C_V \qquad \gamma \sim 1$$

$$C_P - C_V = \frac{T \cdot V \beta^2}{\varkappa_T}$$ β — volume exp, $\varkappa_T$ — kompresibility

No big differense if T low

Lattice heeat capacity

High Temp $T > \Theta_D$

$$U = 3 \cdot N \cdot k_B T$$

atoms per unit cell

$$\frac{\partial U}{\partial T} = 3Nk_B = 3 \cdot l \cdot R = 25 \text{ J/mole K}$$

mono atomic

$$C_{Vl} = 9Nk\left(\frac{T}{\Theta_D}\right)^3 \underbrace{\int \frac{x^4}{x \cdot x}}_{\frac{x^3}{3}} = 9Nk_B\left(\frac{T}{\Theta_D}\right)\left(\frac{\Theta_D}{T}\right)^3 \frac{1}{3} = 3Nk_B = 3lR$$

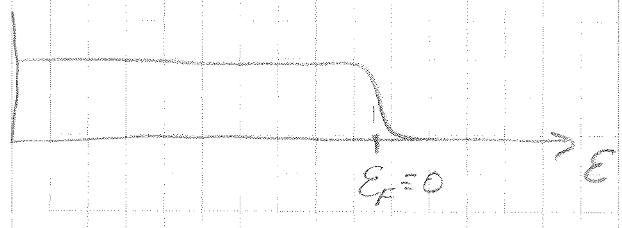
Low temp $T \ll \Theta_D$ $\Rightarrow x \to \infty$

$$C_{Vl} = 9Nk_B\left(\frac{T}{\Theta_D}\right)^3 \int_0^{\Theta/T} \frac{x^4 dx}{(e^x - 1)(1 - e^{-x})}$$

$$C_{Vl} \approx 9Nk\left(\frac{T}{\Theta_D}\right)^3 \int_0^{\Theta/T} \frac{x^3}{e^x} dx$$

Electronic heat capacity

$$U_e = \sum_k \varepsilon_k$$



$$U_e = \int_{-\infty}^{\infty} \varepsilon \cdot N(\varepsilon) \cdot f(\varepsilon, T) \cdot d\varepsilon$$

$$N(\varepsilon) = N(0) = N_0$$

$$f(\varepsilon, T) = \frac{1}{e^{\varepsilon/kT} + 1}$$

$$x = \frac{\varepsilon}{kT} \qquad \varepsilon = kT \cdot x \qquad d\varepsilon = kT\, dx$$

$$U_e = 2 N_0 k_B^2 T^2 \int_0^{\infty} \frac{x}{e^x + 1}\, dx = 2 N_0 k_B^2 T^2 \cdot \frac{\pi^2}{12}$$

$$= \frac{\pi^2}{6} N_0 \cdot k_B^2 T^2 = \frac{\pi^2}{4} N \cdot \frac{k_B^2 T^2}{\varepsilon_F}$$

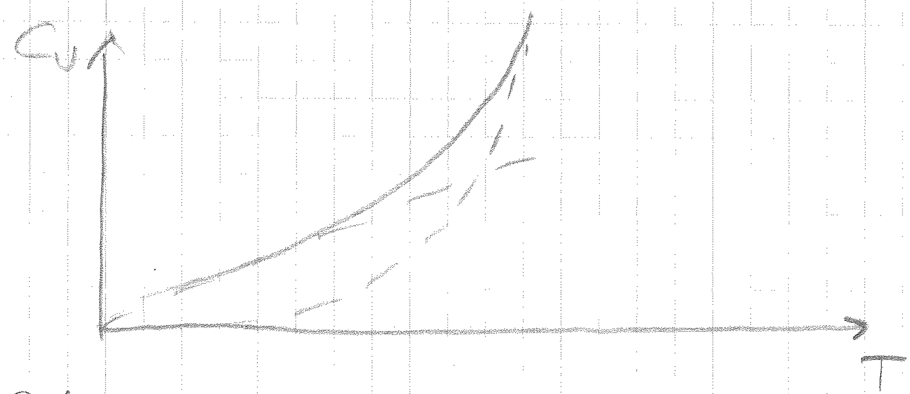
$$N_0 = \frac{3}{2} \frac{N}{\varepsilon_F}$$

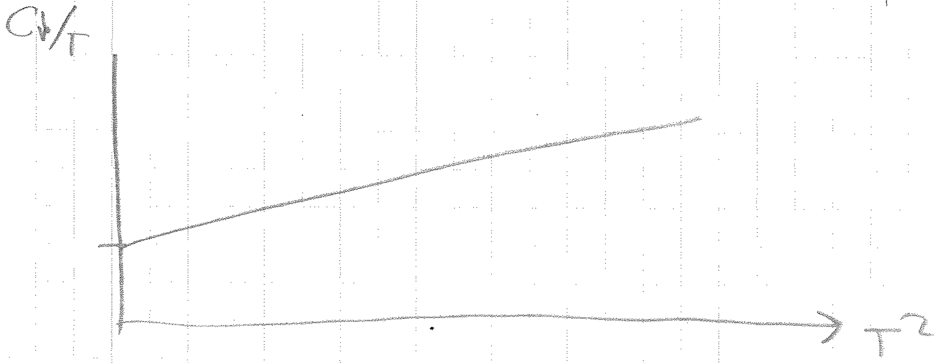
$$C_{ve} = \frac{\pi^2}{2} N k_B \frac{T}{\varepsilon_F} = \frac{\pi^2}{2} N k_B \frac{T}{T_F}$$

Summary C_V for low T

$$C_V = C_{Ve} + C_{Ve} =$$

$$C_V = N \cdot k_B \left(\frac{\pi^2}{2} \frac{T}{T_F} + \frac{3\pi^4}{5} \frac{T^3}{\Theta_D^3} \right) = \gamma T + \alpha T^3$$



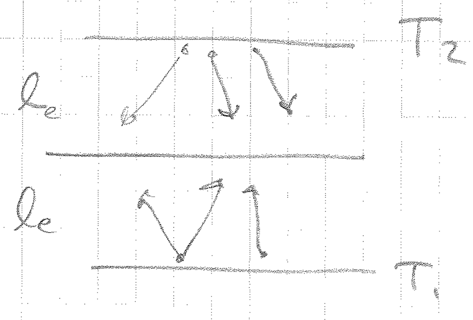


Thermal conductivity of electrons

Dominates at low T for metals

$$\varkappa = \frac{\bar{J}}{\nabla T}$$

consider 3 planes



$$c_v = \frac{C_V}{N}$$

$$\bar{J} = \frac{1}{6} n \cdot v_F \cdot c_v \cdot \Delta T$$

$$\nabla T = \frac{T_2 - T_1}{2 l_e} = \frac{\Delta T}{2 l_e}$$

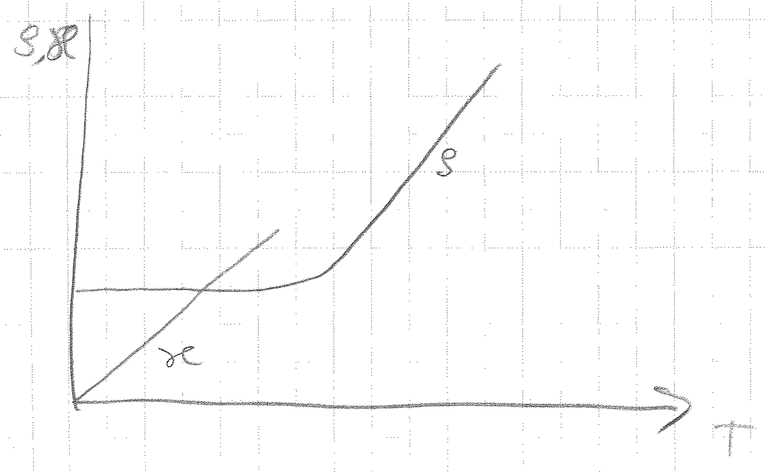
$$\varkappa = \frac{N}{6V} v_F \frac{C_V}{N} \cdot \frac{\Delta T}{\Delta T} \cdot 2 l_e = \frac{C_V}{V} \cdot \frac{v_F}{3} \cdot l_e = \underline{\underline{\frac{C_V}{V} \cdot \frac{v^2 \tau}{3}}}$$

$$= \frac{\pi^2}{\cancel{6}} n k_B^2 v_F^2 \cdot \tau \frac{T}{\varepsilon_F}$$

$$\sigma = \frac{n e^2 \tau}{m}$$

$$\frac{\varkappa}{\sigma} = \frac{\pi^2 \, n k_B^2 v_F^2 \tau \, T m}{\cancel{6} \frac{1}{2} m v_F^2 \cdot n e^2 \tau} = \frac{\pi^2}{3} \frac{k_B^2}{e^2} T = L_0 \cdot T$$

$\boxed{\text{Wideman Franz Law}}$



Lattice Heat conductivity

$$\varkappa = \frac{C_V}{V} \cdot \frac{V_s^2 \tau}{3} = \frac{C_V}{V} \frac{V_s l}{3} \propto T^3 \text{ at low}$$

$T \ll \Theta_D$

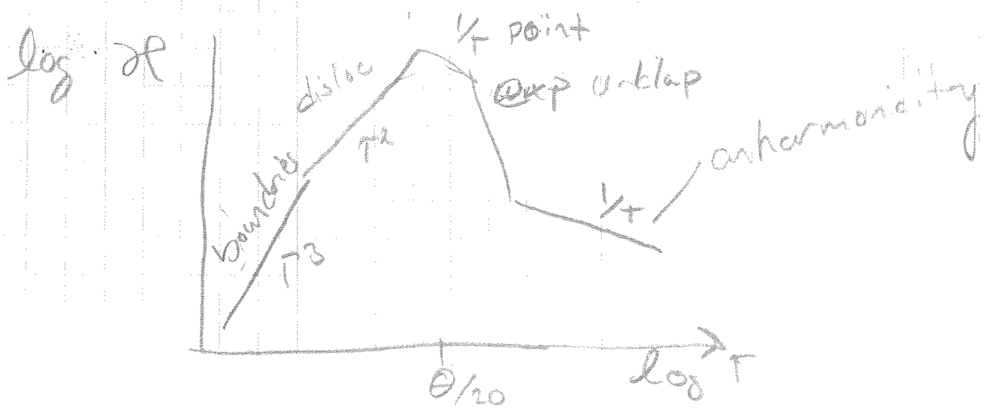
Scattering of phonons is much complicated

τ_{ph-ph}

τ_{ph-e}

τ_{p-def} — many differen types
grain boundaries, vacancies
dislocations ...

These all give different T dep.



Non metals

$$\kappa_T = \frac{1}{R_T} = \frac{1}{(R_B + R_p + R_p + R_U)}$$

$$T^{-3} \quad T^{-2} \quad T \quad T^{-1} e^{-\frac{\Theta}{kT}}$$

Summary how T

Metals

$$\rho = \rho_0 + kT^5$$

$$C_V = \underset{el}{\gamma T} + \underset{Latt}{\alpha T^3}$$

$$\kappa = \underset{el}{a \cdot T} + \underset{Latt}{bT^3}$$

Nonmetals

$$\rho = \infty$$

$$C_V = \alpha T^3$$

$$\kappa = bT^3 \qquad bT^2 \text{ for some}$$

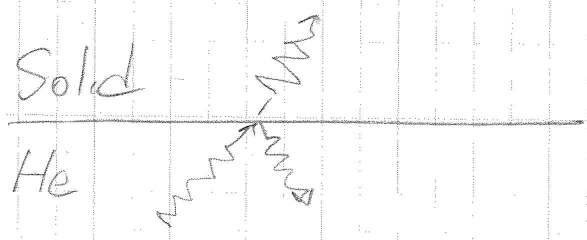
Supercond

$$\rho = 0 \qquad T < T_c$$

$$C_{ve} = C_{ve}(T_c)\, e^{-\frac{\Delta}{kT}} + \alpha T^3$$

$$\kappa = aTe^{-\frac{\Delta}{kT}} + bT^3$$

Kapitza resistance



$$R_K = \frac{15\hbar^3 \, \rho_{solid} \, v_s^3}{2\pi^2 k_B^4 \, \rho_{liq} \, v_l} \, \frac{1}{T^3}$$

Thermal integrals

$$\frac{dP}{dT} = K(T)$$

$$P = \int_{T_1}^{T_2} K(T)\, dT = \overline{K}_{T_1 T_2} \cdot \Delta T$$

$$\overline{K}_{T_1 T_2} = \frac{1}{\Delta T} \int_{T_1}^{T_2} K(T)\, dT$$