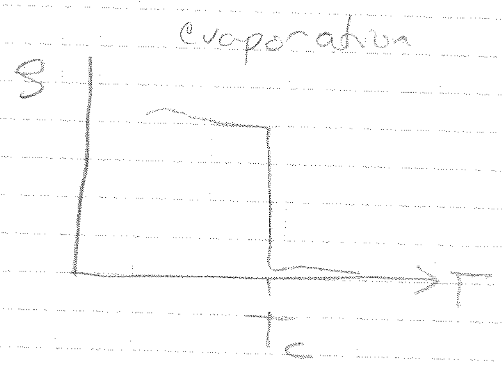
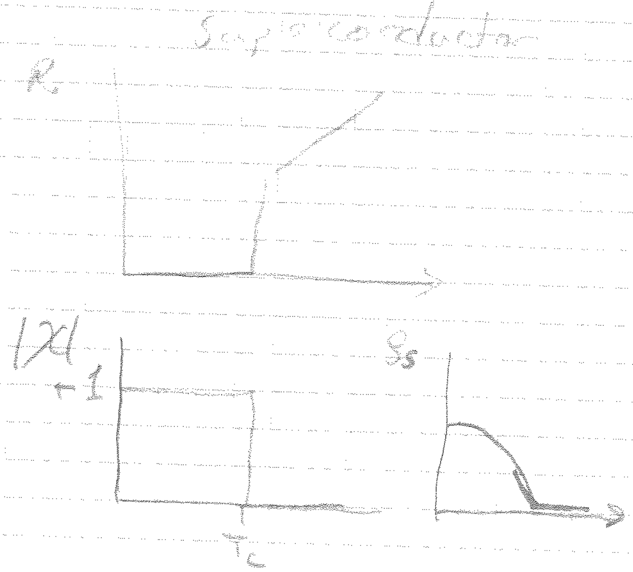
Phase transitions

An abrupt sudden change in one or more physical properties as a function of a thermodynamic variable, e.g. temperature





Classification (Ehrenfest)

1st order phase transition

exhibits a discontinuity in the first derivative of the free energy. Exhibits Latent heat

Ex. All solid/liquid/gas transitions show a discontinuity in the density which is

(per unit volume) $F = U - TS - \rho\mu$ $\quad \rho = \frac{N}{V}$

$$\frac{dF}{d\mu} = \rho$$

2nd order

discontinuity in 2nd derivative of F

e.g. ferromagnetic transition. No latent heat

$F = U - TS - MH$

$\frac{dF}{dH} = M \qquad \frac{d^2F}{dH^2} = \frac{dM}{dH} = \chi$

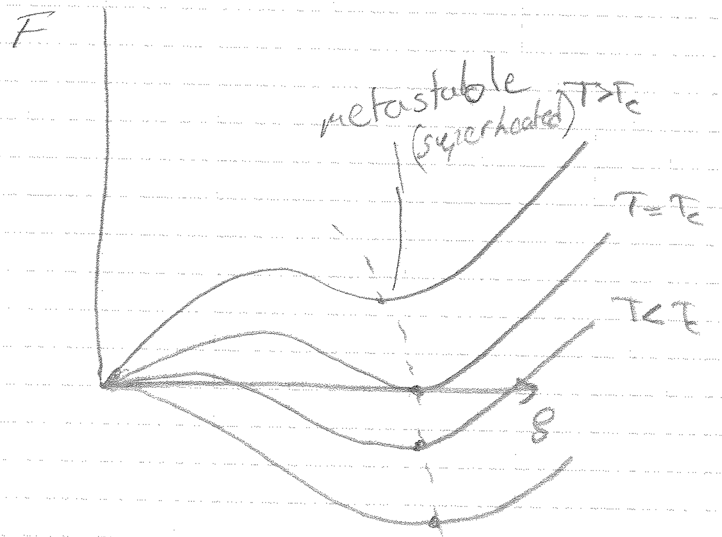
Supercond.

χ goes from 0 → -1

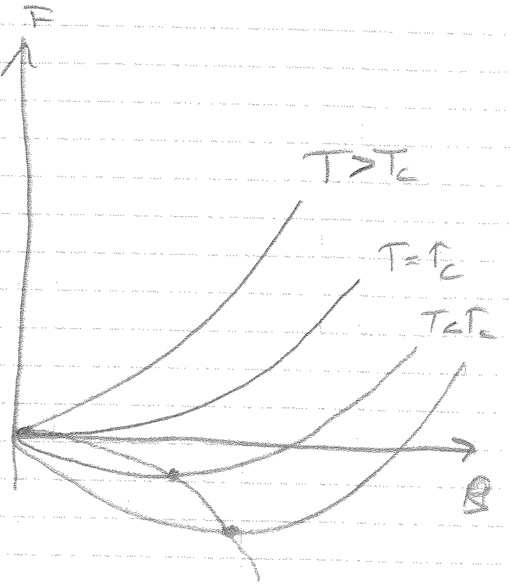
Landau theory for phase transitions

The free energy of a system is a function of its density

First versus second order



Melting boiling



Ferromagnets
SC

$F = (\rho_p, T)$

expand in ρ_p

$$F(\rho_p, T) = \underbrace{F(0,T)}_{F_n(T)} + \alpha(T)\rho_p + \beta(T)\frac{\rho_p^2}{2} + \dots$$

$$\alpha(T) = \frac{\partial F}{\partial \rho_p} \qquad \beta(T) = \frac{\partial^2 F}{\partial \rho_p^2}$$

Ignor 3rd and higher derivatives

Now expand β, α close to T_c

$$\beta = \beta_0 + \beta_1 (T-T_c) + \beta_2 \frac{(T-T_c)^2}{2} \approx \beta_0 \equiv \beta > 0$$

$$\alpha = \alpha_0 + \alpha_1 (T-T_c) + \dots = \alpha(T)$$

$\left.\frac{dF}{d\rho}\right|_{T_c} = 0$ see fig

Temperature dependence of ρ

Thus:

$$F_s(\rho_p, T) = F_n(T) + \alpha(T)\,\rho_p + \beta \cdot \frac{\rho_p^2}{2}$$

$T \not> T_c \;\rightarrow\; \rho_p = 0$ normal

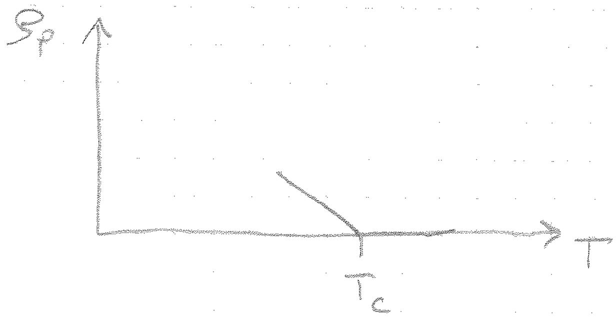
$T > T_c \;\rightarrow\; \rho_p > 0$ SC

In the SC state $\frac{dF}{d\rho} = 0$ (in equilib.)

$$\frac{dF}{d\rho} = \underbrace{\frac{dF_n}{d\rho}}_{0} + \alpha(T) + \beta \cdot \rho_p = 0$$

F_n indep of ρ

$$\rho_p = -\frac{\alpha(T)}{\beta} = \frac{\alpha_1}{\beta}(T_c - T)$$



Free energy difference

$$\Delta F = F_n(T) - F_s(\rho_B T)$$

$$= -\alpha(T)\,\rho_p - \beta\,\frac{\rho_p^2}{2}$$

insert $\rho_p = -\frac{\alpha(T)}{\beta}$

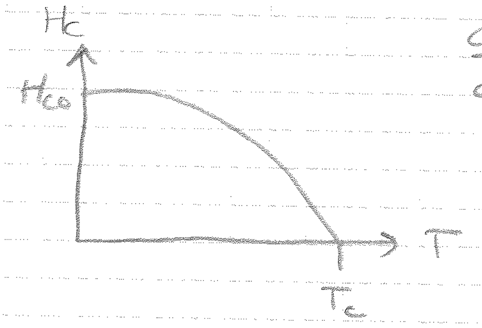
$$\Delta F = \frac{\alpha^2}{\beta} - \beta\,\frac{\alpha^2}{2\beta^2} = \frac{\alpha^2}{2\beta} \quad \propto (T_c - T)^2$$

Critical Field relating β to H_c and γ

$$\Delta F = \frac{\alpha(T)^2}{2\beta} = \frac{\mu_0}{2} H_c^2 = \frac{1}{2\mu_0} B_c^2$$

Last Lecture

$$H_c(T) = \sqrt{\frac{1}{\mu_0 \beta}}\, \alpha(T) \quad \sim T_c - T$$



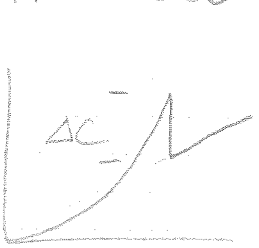
$$\left.\frac{dH_c}{dT}\right| = \frac{1}{\sqrt{\mu_0 \beta}}$$

$$H_c = H_{c0}\left(1 - \left(\frac{T}{T_c}\right)^2\right)$$

$$\frac{dH_c}{dT} = -H_{c0}\frac{2}{T_c^2} T \longrightarrow -\frac{2H_{c0}}{T_c} \quad @T_c$$

$$H_{c0} = \frac{T_c}{2}\frac{1}{\sqrt{\mu_0 \beta}}$$

From BCS



$$\Delta C = T\frac{d^2}{dT^2}\Delta F = 1.43 \cdot \gamma \cdot T_c$$

Thermodynamics BCS

$$T \cdot \frac{d^2}{dT^2}\left(\frac{\alpha^2(T)}{2\beta}\right) = \frac{T}{2\beta}\frac{d^2}{dT^2}\alpha_1 (T_c - T)^2$$

$$\frac{\alpha_1^2 \cdot T}{2\beta}\frac{d}{dT} 2(T_c - T) = +\frac{\alpha_1^2 \cdot T_c}{\beta} = 1.43 \cdot \gamma \cdot T_c$$

$$\frac{\alpha_1^2}{\beta} = 1.43 \cdot \gamma$$

$$\frac{\alpha^2}{\beta} = 1.43 \cdot \gamma (T_c - T)^2$$

Ginzburg - Landau Theory

Add: 1 Magnetic field

2 Variations of ρ_p in space

Inspired by Londons success assume

- $\rho_p = \Psi^* \Psi \qquad \Psi = \sqrt{\rho_p}\, e^{i\theta}$

- F may depend on derivatives of Ψ

The GL Free energy

$$F = F_N + \int_V dV \left[\alpha \Psi^* \Psi + \frac{1}{2} \beta (\Psi^* \Psi)^2 + \frac{1}{4m} \left| i\hbar \vec{\nabla} \Psi - 2e\vec{A}\Psi \right|^2 \right]$$

Kinetic E

$$+ \int \frac{\mu_0}{2} |\vec{M}|^2 \, dV$$

Magnetization E

Minimize F with eespect to variations in ψ (and ψ^*)

$$dF = \frac{\partial F}{\partial \psi} d\psi + \frac{\partial F}{\partial \psi^*} d\psi^* = 0$$

$$\frac{\partial F}{\partial \psi} = 0 \qquad \frac{\partial F}{\partial \psi^*} = 0$$

$$\frac{\partial F}{\partial \psi^*} = \int_V dV \left[\alpha \psi \, d\psi^* + \frac{1}{2}\beta 2 \psi^* \psi \cdot \psi \, d\psi^* + \frac{1}{4m}\left(i\hbar\bar{\nabla} + 2e\bar{A}\right)^2 \psi \, d\psi^* \right]$$

$$+ \frac{i\hbar}{2m} \int_S \left(-i\hbar\bar{\nabla} + 2e\bar{A}\right)\psi \, d\psi \cdot d\bar{S} = 0$$

Valid for arbitrary $d\psi^*$ $\Rightarrow$

1st GLE: $\alpha \psi^* + \beta \psi^* \psi^2 + \frac{1}{4m}\left(-i\hbar\bar{\nabla} + 2e\bar{A}\right)^2 \psi = 0$

GLBC $\left(-i\hbar\bar{\nabla}_n + 2e\bar{A}_n\right)\psi = 0$

Boundary condition $\bar{p}_n \propto \bar{v}_{sn} = 0$ no current flows out from the SC surface S

Variation with respect to magnetic field

$$F = F_N + \int dV \left[\alpha|\Psi|^2 + \frac{1}{2}\beta|\Psi|^4 + \frac{\hbar^2}{4m}\left|\nabla\Psi - \frac{2e}{i\hbar}\bar{A}\Psi\right|^2\right]$$

$$+ \frac{\mu_0}{2}\int_V dV\, |\bar{M}|^2$$

$$F_{kin} = +\int dV \frac{\hbar^2}{4m}\left(\nabla\Psi - \frac{2e}{i\hbar}\bar{A}\Psi\right)\left(\nabla\Psi^* + \frac{2e}{i\hbar}\bar{A}\Psi^*\right)$$

$$= \int dV \frac{\hbar^2}{4m}\left[\nabla\Psi\nabla\Psi^* - \frac{2e}{i\hbar}\bar{A}\Psi\nabla\Psi^* + \frac{2e}{i\hbar}\bar{A}\Psi^*\nabla\Psi + \frac{4e^2}{\hbar^2}\bar{A}^2\Psi^*\Psi\right]$$

$$= \frac{\hbar^2}{4m}\int dV \left[|\nabla\Psi|^2 + \frac{2e}{i\hbar}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*)\bar{A} + \frac{4e^2}{\hbar^2}\bar{A}^2|\Psi|^2\right]$$

$$dF_{kin} = \frac{\partial F}{\partial \bar{A}}\cdot d\bar{A} = \frac{\hbar^2}{4m}\int dV\left[\frac{2e}{i\hbar}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) + \frac{4e^2}{\hbar^2}2\bar{A}|\Psi|^2\right]d\bar{A}$$

$$dF_B = \frac{\partial F}{\partial B}\cdot dB = \frac{\mu_0}{2}\int dV\, 2|M|\frac{dM}{dB}\cdot d\bar{B} = \int dV\, |M|\cdot\nabla\times d\bar{A}$$

$$M = \frac{B}{\mu_0} - H \qquad \frac{\partial M}{\partial B} = \frac{1}{\mu_0} \qquad \nabla\times B = \mu_0 J$$

Amperes Law

$$= \int dV\, \bar{\nabla}\times M\cdot d\bar{A} = \int dV\, \bar{J}_s\cdot d\bar{A}$$

$$dF = dF_{kin} + dF_B = \int dV\left[\frac{-i\hbar e}{2m}(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*) + \frac{2e^2}{m}A|\Psi|^2 + \bar{J}_s\right] = 0$$

leads to the 2nd GL eqn

$$\vec{J}_s = \frac{i\hbar e}{2m}\left(\psi^* \nabla \psi - \psi \nabla \psi^*\right) - \frac{2e^2}{m} A |\psi|^2$$

i.e starting eqn. for LE

this means that we can now compar the LE with the GLE

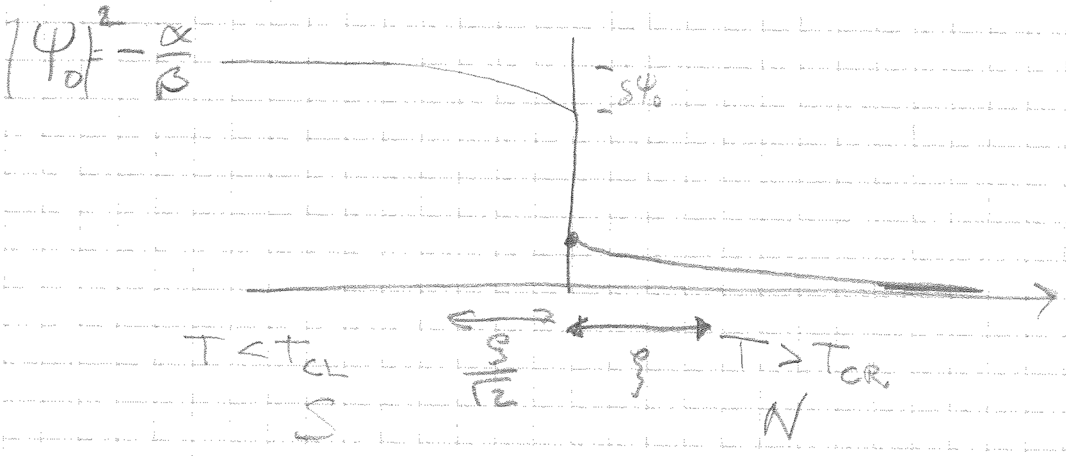
Penetration depth

$$\lambda_L = \sqrt{\frac{m}{\mu_0 2 S_p e^2}} = \sqrt{\frac{m\beta}{2\mu_0 e^2 |\alpha|}}$$

The coherence length

Lets consider a SN interface at B=0

i.e $\bar{A}=0$



If the SC is simply connected no current flows out of the SC

Then we can adopt the Lorenz guage which allows us to choose any θ

Choosing $\theta = 0$ which makes Ψ real

$$\alpha\Psi + \beta\Psi^3 + \frac{1}{4m}(-\hbar^2)\nabla^2\Psi = 0$$

In the normal metal

A slight increase of Ψ at the interface will then give

$$\alpha\Psi + 0 - \frac{\hbar^2}{2m}\nabla^2\Psi = 0 \quad , \quad \Psi(\infty) = 0$$

$$\nabla^2\Psi = \frac{2m\alpha}{\hbar^2}\Psi = \frac{\Psi}{\xi^2} \qquad \xi = \sqrt{\frac{\hbar}{2m|\alpha|}}$$

in x direction

$$\Psi = \Psi(0)\, e^{-x/\xi} \qquad x > 0$$

In the SC

$$\alpha(\Psi_0-\delta\Psi)+\beta(\Psi_0-\delta\Psi)^3-\frac{\hbar^2}{4m}\nabla^2(\Psi_0-\delta\Psi)=0$$

B.C. $\quad \Psi(-\infty)=\Psi_0=\sqrt{\frac{|\alpha|}{\beta}}$

$$(\Psi_0-\delta\Psi)^3=\Psi_0^3-3\Psi_0^2\delta\Psi+\dots$$

neglect

$$\underbrace{\alpha\Psi_0+\beta\Psi_0^3-\frac{\hbar^2}{4m}\nabla^2\Psi_0}_{=0}-\alpha\delta\Psi-3\beta\Psi_0^2\delta\Psi+\frac{\hbar^2}{4m}\nabla^2\delta\Psi=0$$

$$\nabla^2\delta\Psi=\frac{4m}{\hbar^2}\left(\alpha+3\beta\left(-\frac{\alpha}{\beta}\right)\right)\cdot\delta\Psi=\frac{4m}{\hbar^2}$$

$$\nabla^2\delta\Psi=-\frac{8m\alpha}{\hbar^2}\delta\Psi=\frac{\delta\Psi}{\xi^2/2}$$

$$\delta\Psi=\delta\Psi_0\, e^{+x/\frac{\xi}{\sqrt{2}}} \qquad x<0$$

The $\varkappa$-parameter

$$\varkappa=\frac{\lambda}{\xi}=\frac{\sqrt{\frac{m\beta}{2\mu_0 e^2|\alpha|}}}{\sqrt{\frac{\hbar^2}{4m|\alpha|}}}=\frac{m}{\hbar e}\sqrt{\frac{2\beta}{\mu_0}}$$

$\varkappa<\frac{1}{\sqrt{2}}\approx 0.707$ Type I SC

$\varkappa>\frac{1}{\sqrt{2}}\approx 0.707$ Type II SC

Expressing H_c interms of λ and ξ

$$H_c = \sqrt{\frac{\alpha^2}{\mu_0 \beta}}$$

$$\lambda = \sqrt{\frac{m\beta}{2\mu_0 e^2 |\alpha|}} \qquad \frac{\beta}{|\alpha|} = \frac{2\mu_0 e^2}{m}\lambda^2$$

$$\xi = \sqrt{\frac{\hbar^2}{4m|\alpha|}} \qquad |\alpha| = \frac{\hbar^2}{4m\xi^2}$$

$$H_c^2 = \frac{|\alpha|}{\mu_0\beta/|\alpha|} = \frac{\hbar^2}{\mu_0 4m\xi^2}\,\frac{m}{2\mu_0^2 e^2\lambda^2} = \frac{\hbar^2}{4\,2\mu_0 e^2\lambda^2\xi^2}$$

$$H_c = \frac{\hbar}{2e}\,\frac{1}{\sqrt{2\mu_0}\,\lambda\cdot\xi} = \frac{1}{\mu_0}\,\frac{\Phi_0}{2\pi\sqrt{2}\,\lambda\,\xi}$$