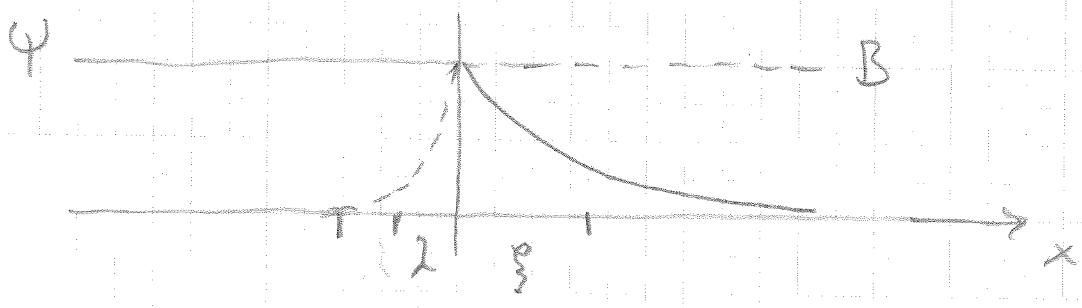
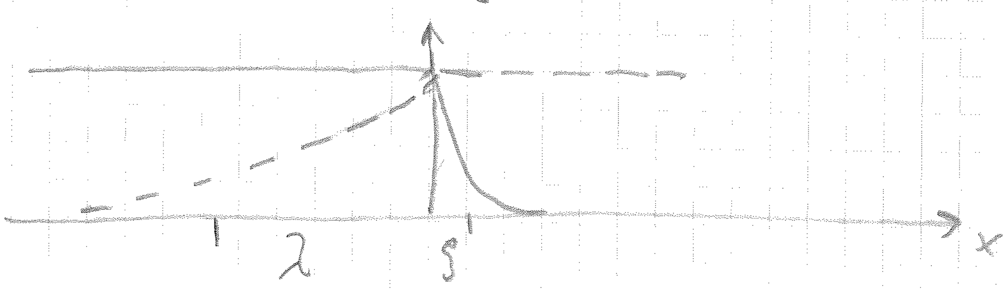
The Energy of th SN interface

$\lambda \ll \xi \quad \Rightarrow \quad \varkappa = \frac{\lambda}{\xi} \ll 1$ Type I



$\lambda \gg \xi$



Energy per unit volume of SC

$$\Delta F_0 = \frac{\mu_0}{2} H_c^2$$

Interface energy

Due to tail of Ψ into N part

$$F_\psi = \Delta F_0 \cdot \xi$$

Due to tail of B into SC

$$F_B = -\Delta F_0 \cdot \lambda$$

$$F_I = F_\psi + F_B = \Delta F_0 (\xi - \lambda)$$

$$F_I \;\begin{cases} > 0 & \xi \gg \lambda \quad \varkappa \ll 1 \\ < 0 & \xi \ll \lambda \quad \varkappa \gg 1 \end{cases}$$

real cross over $\varkappa = \frac{1}{\sqrt{2}}$

Dimension less form of the GL-equations

$$f = \frac{\psi}{\psi_0}$$

$$\bar{R} = \frac{\bar{r}}{\lambda}$$

$$\bar{b} = \frac{\bar{B}}{\sqrt{2}\,B_c} \qquad B_c = \mu_0 H_c$$

$$\bar{a} = \frac{\bar{A}}{\sqrt{2}\,B_c \lambda}$$

$$\phi = \frac{2\pi}{\varkappa} \frac{\Phi}{\Phi_0}$$

$$\bar{j} = \frac{\lambda}{\sqrt{2} \cdot H_c} \cdot \bar{J}$$

GL free energy difference

$$\Delta \mathcal{E} = - f^* f + \frac{1}{2} (f^* f)^2 + \left| \frac{1}{\varkappa} \bar{\nabla} f + i \bar{a} f \right|^2 + (b - \mu_0 h)^2$$

GL 1

$$- f + f^* f^2 + \left(-\frac{i}{\varkappa} \nabla + a \right) f = 0$$

GL 2

$$\bar{j} = \frac{i}{2\varkappa} \left(f^* \bar{\nabla} f - f \nabla f^* \right) - a f^* f$$

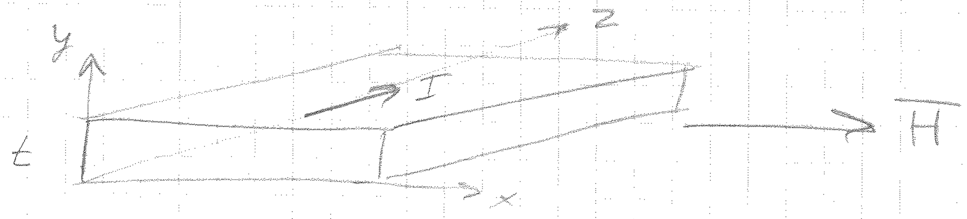
for a simply connected SC with no flux lines

you can choose a guage such tha f is real and we have

$$\begin{cases} \Delta \mathcal{E} = -f^2 + \frac{1}{2} f^4 + \frac{1}{\varkappa} (\nabla f)^2 + |\bar{a}^2| f^2 + (b - \mu_0 h)^2 \\ -f + f^3 - \frac{1}{\varkappa^2} \nabla^2 f + |\bar{a}|^2 f = 0 \\ \bar{j} = -\bar{a} f^2 \end{cases}$$

note: the only parameter is $\varkappa$

J_c of a thin fillm



if $t \ll \lambda$ magnetic flux penetrates the film almost homogeneously therefore J is also constant over the film

$$\bar{j} = -\bar{a} f^2 = \text{const (indep of } x, y)$$

$$\Rightarrow \quad f = \text{const} \quad \Rightarrow \quad \vec{\nabla} f = 0$$

$$\Rightarrow \quad |\bar{a}|^2 + f^2 - 1 = 0$$

$$|\bar{a}|^2 = 1 - f^2 \qquad \bar{a} = |\bar{a}| \cdot \hat{z}$$

$$j^2 = |\bar{j}|^2 = |\bar{a}|^2 \cdot f^4 = f^4 - f^6$$

$$\frac{d j^2}{d f} = 4f^3 - 6f^5 = 0$$

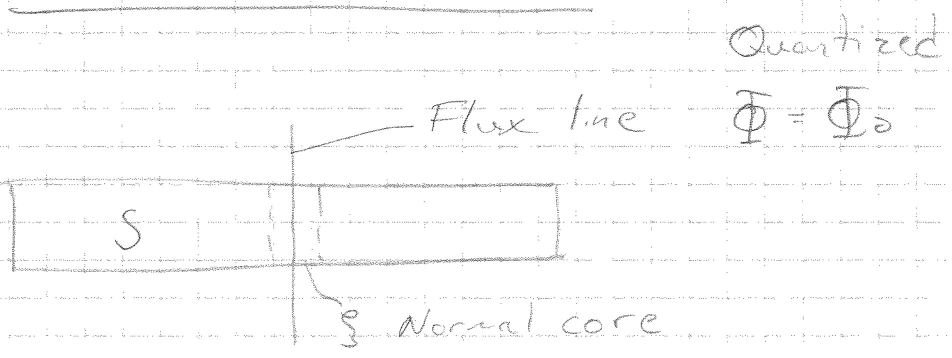
$$4f^3 \left(1 - \frac{3}{2} f^2\right) = 0$$

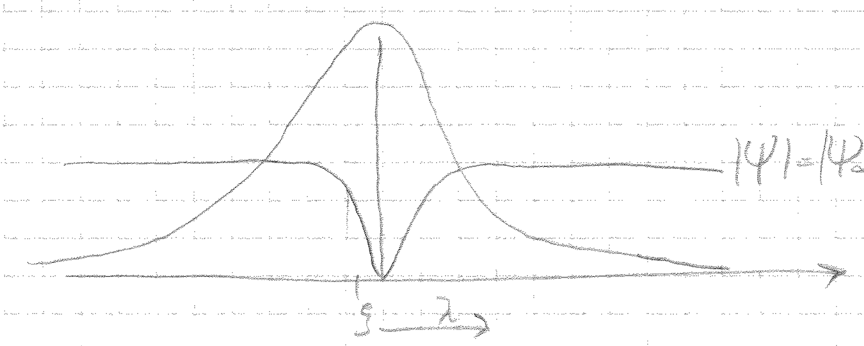
$$f^2 = \frac{2}{3} \quad \Rightarrow \quad j_{max}^2 = \left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3 = \frac{4}{9} \frac{1}{3}$$

$$j_{max} = \frac{2}{3\sqrt{3}} \qquad J_{max} = \frac{\sqrt{2}}{\lambda} H_c \, j_{max} = \frac{2\sqrt{2}}{3\sqrt{3}} \frac{H_c}{\lambda} = 0.54 \frac{H_c}{\lambda}$$

The mixed state of Type II SC

Abrikosov vortecies





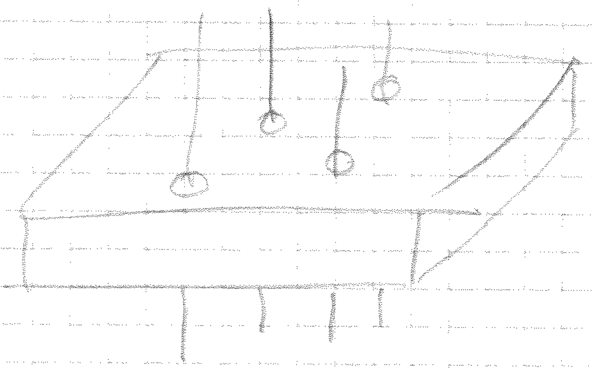
Cylindrical coordinates

$$\nabla^2 B = \frac{1}{r}\frac{d}{dr}\left(r\frac{dB}{dr}\right) = \frac{B}{\lambda^2}$$

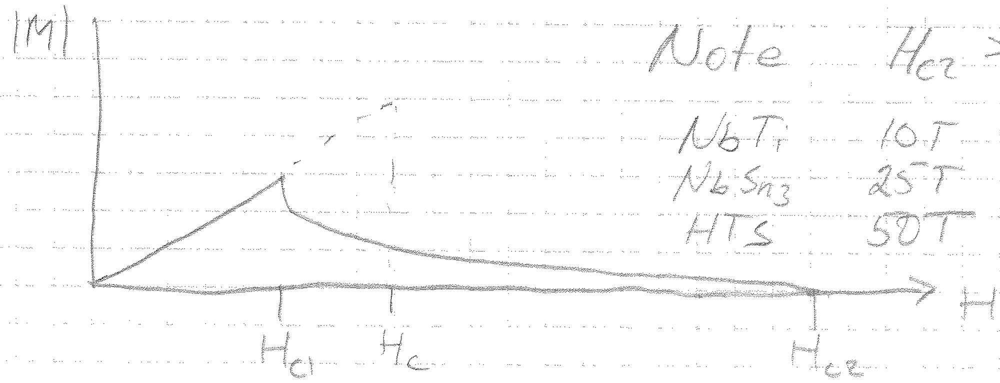
Solution is a modified Bessel function

$$|B| = \frac{\sqrt{2}}{\kappa} B_c \cdot K_0\left(\frac{r}{\lambda}\right)$$

At high field



Forms a triangular flux lattice



Note $H_{c2} \gg H_c$

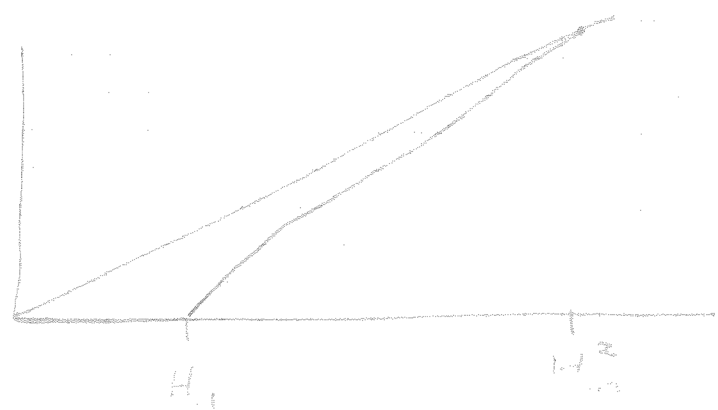
NbTi	10T
Nb_3Sn	25T
HTS	50T

$$B_{tot} = \frac{n \Phi_0}{A}$$

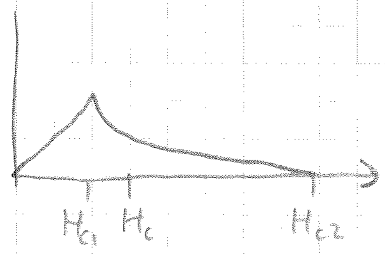
density of votecies $\frac{n}{A} = \frac{B}{\Phi_0}$

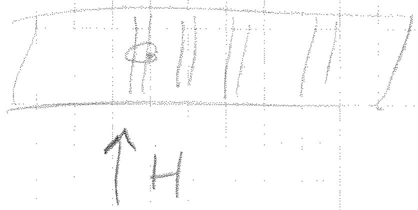
B inte core

$$B_{core} = \frac{\Phi_0}{\pi \lambda^2} \simeq \frac{2 \cdot 10^{-15}}{\pi \cdot (4 \cdot 10^{-8})^2} \simeq \frac{20}{\pi \cdot 16} \sim 0.4\,T$$



Calculating H_{c2}





Strong magnetic field $\parallel \hat{z}$

$\Rightarrow$ Ψ small i.e. linearize GLE

$$\frac{1}{4m}\left(-i\hbar\nabla + 2e\vec{A}\right)\Psi = -\alpha\Psi = E\Psi$$

! Schrödinger Equation for a particle with mass $2m$ and charge $2e$ in a strong field

Landau orbitals cyclotron frequency $\omega_c = \frac{2e}{2m}\mu_0 H$

$$E = -\alpha = \left(n+\tfrac{1}{2}\right)\hbar\omega_c + \hbar\frac{k_z^2}{4m}$$

$$\left(n+\tfrac{1}{2}\right)\hbar\frac{2e}{2m}\mu_0 H + \hbar^2\frac{k_z^2}{4m} = -\alpha$$

Find largest possible $H = H_{c2}$

Happens if $n = 0$, $k_z = 0$

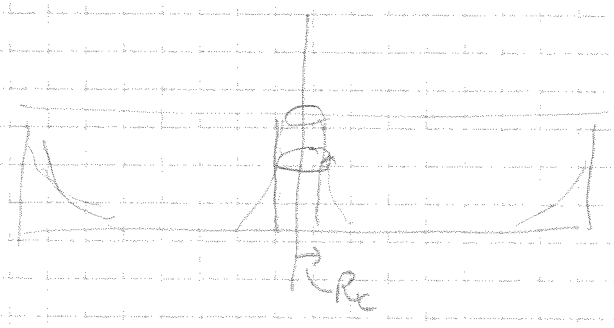
$$\frac{1}{2}\frac{e\hbar}{m}\mu_0 H_{c2} = -\alpha$$

$$H_{c2} = \frac{2m|\alpha|}{e\hbar\mu_0}$$

$$\frac{H_{c2}}{H_c} = \frac{2m|\alpha|}{e\hbar\mu_0}\frac{\sqrt{\mu_0\beta}}{|\alpha|} = \frac{2m}{e\hbar}\sqrt{\frac{\beta}{\mu_0}}$$

$$\boxed{H_{c2} = H_c\sqrt{2}\,\kappa}$$

Calculating the flux line energy



Assume $\varkappa \gg 1$

Then energy due to f is small (small core)

almost all energy due to kinnetic and magnetic energy

Cylindrical coordinates $\quad \nabla \times$

$$\epsilon = F_V - F_S = \left(F_{SFV} + F_{KinV} + F_{MagV}\right) - \left(F_{SF} + F_{Kin} + F_{Mag}\right)$$

$$= \left(F_{KinV} - F_{Kin}\right) + \left(F_{MagnV} - F_{Mag}\right)$$

$$= F_{jV} + F_{MV} =$$

Reduced coordinates $\quad$ energy per unit length

$$\epsilon = \int_{R_c}^{\infty} (j^2 + b^2)\, 2\pi R \cdot dR$$

$$\frac{d}{dR}\left[R \cdot \bar{j}\, \bar{b}\right] = \bar{j} \cdot b + R \frac{d}{dR}\bar{j}\, \bar{b} + R \cdot \bar{j} \cdot \underbrace{\frac{d}{dR} b}$$

$\perp_{110}$

Ampere Law

$\nabla \times \bar{B} = \mu_0 \bar{J}_s$

LE2

$\frac{d}{dR} \bar{j} = b$

$\frac{d}{dR} \bar{b} = \bar{j}$

$\nabla \times \bar{J} = \frac{B}{\lambda}$

$$\frac{d}{dR}\left[R \cdot \bar{j}\, b\right] = R\left(b^2 + j^2\right.$$

$$\epsilon = 2\pi \int_{R_c}^{\infty} \frac{d}{dR}\left[R\, j\, b\right] dR = 2\pi \left[R\, j \cdot b\right]_{R_c}^{\infty}$$

$j = 0 \quad R = \infty \qquad R_c = \frac{r}{\lambda} = \frac{\xi}{\lambda} = \frac{1}{\varkappa}$

$b = 0 \quad R = \infty$

$$= -2\pi\, R_c\, j(R_c) \cdot b(R_c)$$

$b = \frac{1}{\varkappa} K_0(R) \simeq + \frac{1}{\varkappa} \ln\left(\frac{1}{R}\right) \qquad b(R_c = \frac{1}{\varkappa}) = \frac{1}{\varkappa} \ln \varkappa$

$j = \frac{d}{dR} b = \frac{1}{\varkappa} \cdot \frac{1}{1/R} \cdot -\frac{1}{R^2} = -\frac{1}{\varkappa R} \qquad j(R_c = \frac{1}{\varkappa}) = -1$

$$\epsilon = -2\pi \frac{1}{\varkappa} \frac{1}{\varkappa} \ln \varkappa \cdot (-1) = \frac{2\pi}{\varkappa^2} \ln \varkappa$$

including the core gives

$$\epsilon = \frac{2\pi}{\varkappa}(\ln \varkappa + 0.08)$$

$$\mathcal{E}_{FL} = 2\Delta F_0 \cdot \frac{2\pi}{\varkappa^2} \cdot \ln \varkappa \cdot \lambda^2$$

$$= 2 \cdot \frac{\mu_0}{2} H_c^2 \frac{2\pi}{\lambda^2} \xi^2 \lambda^2 = 2\pi\mu_0 H_c^2 \xi^2 \ln(\varkappa + 0.08)$$

Energy per unit Length of a flux line

$$\mathcal{E}_{FL} = 2\pi\mu_0 H_c^2 \xi^2 \cdot \ln(\varkappa + 0.08)$$

First flux line enters
when the energy difference is negativ
i.e whe

$$\varepsilon - B \cdot H \cdot A < 0 \quad \text{equal at } H_{c1}$$

$$2\pi\mu_0 H_c^2 \xi^2 \ln\varkappa = \Phi_0 \cdot H_{c1}$$

$$H_{c1} = 2\pi\mu_0 \frac{H_c^2}{\Phi_0/\xi^2} \ln\varkappa = \underbrace{\frac{\sqrt{2}\, 2\pi\mu_0 \xi \lambda}{\sqrt{2}\, \Phi_0\, \lambda}}_{\frac{1}{H_c}} H_c^2 \cdot \xi \ln\varkappa$$

$$H_{c1} = \frac{1}{\sqrt{2}\varkappa} \ln\varkappa \cdot H_c$$