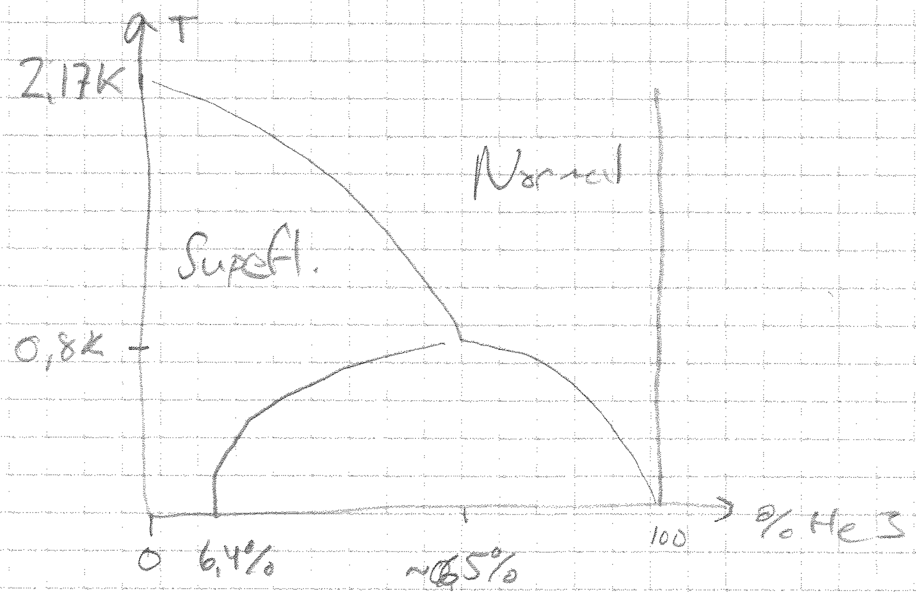
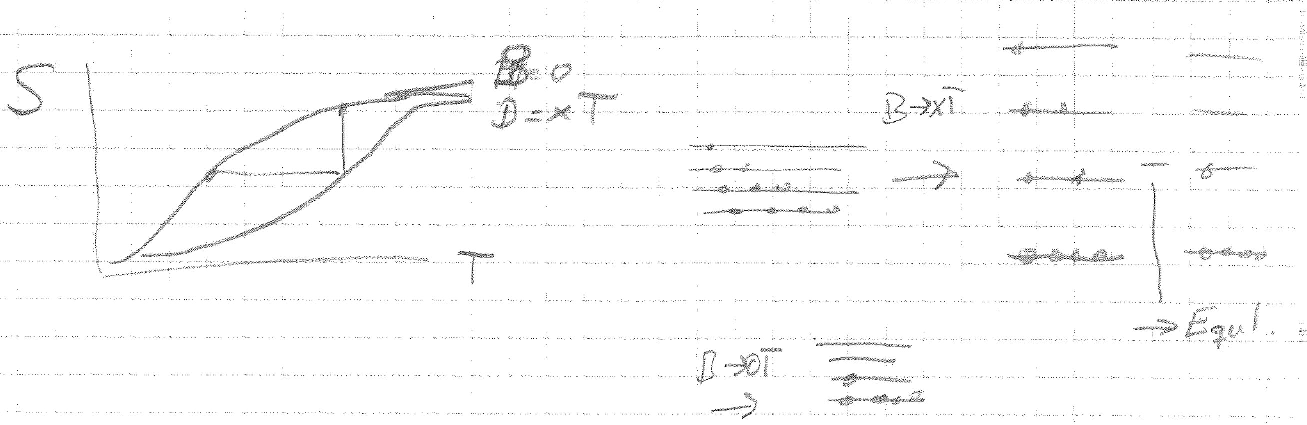
Solutions Exam 2004

1a)



b)

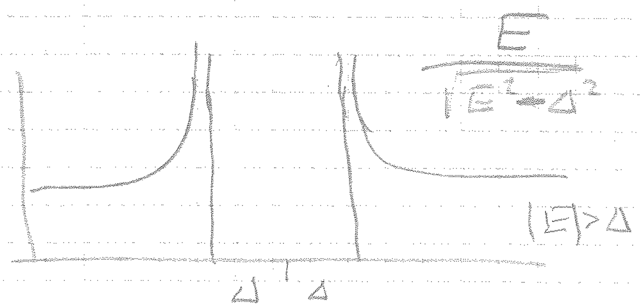


Adiabatic Salt 2mK

Cu ~1μK

c)

$\Delta = 1.76 \cdot k_B T_c$

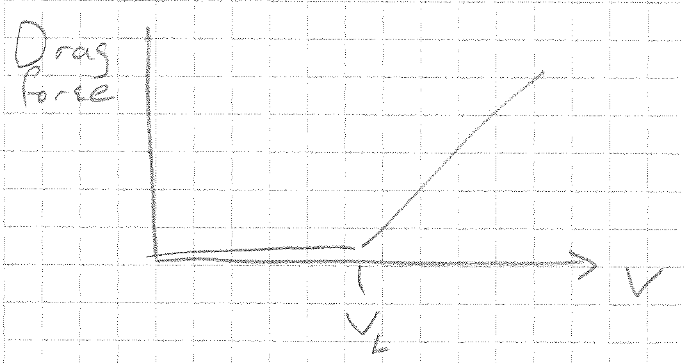


d)

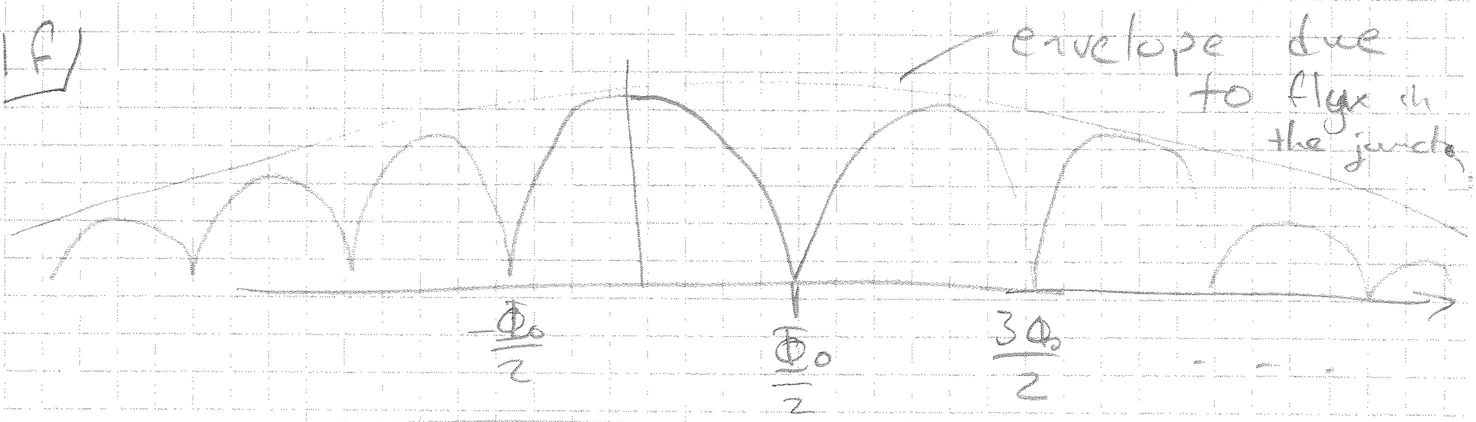
$$m\bar{v}_s = \bar{p} = -i\hbar\nabla\Psi = -i\hbar\nabla\left(\sqrt{\tfrac{\rho_s}{m}}\, e^{i\theta}\right) = \hbar\nabla\theta\,\Psi$$

$$\kappa = \oint_L v_s\, dl = \frac{\hbar}{m}\oint \nabla\theta\, dl = \frac{\hbar}{m}\, n 2\pi = n\frac{h}{m} = n\kappa_0$$

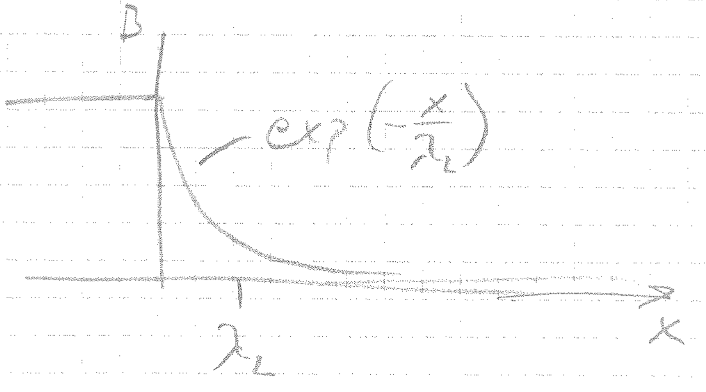
1e) Above a certain velocity excitations are formed. Below this velocity no excitations are formed becaus of the enegy gap



1f)



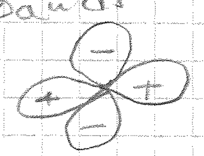
g) The distanance which a magnetic field penetrates into a super conductor



$\lambda_L \simeq 50\,nm$

for elements

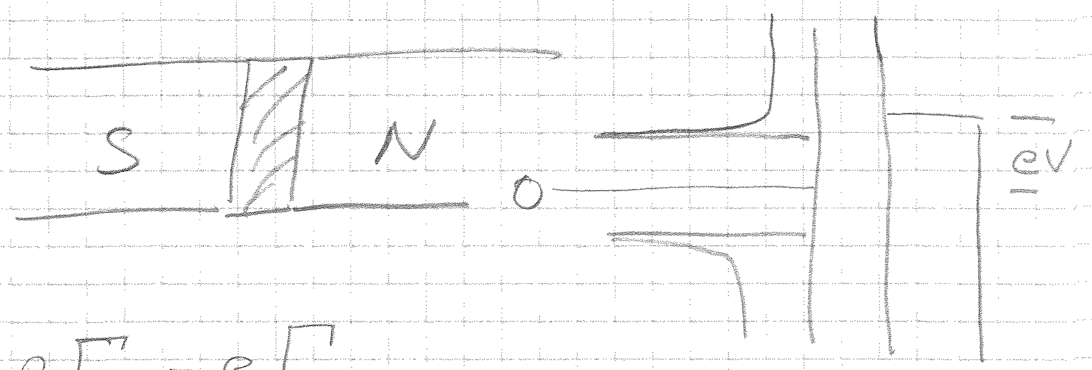
h/ High Tc, typically Cu compounds

Anisotropic d wave 

nodes in the gap

i/ $C_v = \underbrace{\gamma T}_{\text{electronic}} + \underbrace{\alpha T^3}_{\text{phononic}}$

2a)



$$I = e\Gamma_{\leftarrow} - e\Gamma_{\rightarrow}$$

$$\cancel{e}\Gamma_{\leftarrow} = T \int_{-\infty}^{\infty} N_N(E+eV)\, f(E+eV) \cdot N_S(E)\,(1-f(E))\, dE$$

$$\Gamma_{\rightarrow} = T \int_{-\infty}^{\infty} N_N(E+eV)\,(1-f(E+eV))\, N_S(E)\, f(E)\, dE$$

$$I = eT \int_{-\infty}^{\infty} N_N(E+eV) \cdot N_S(E) \left[f(E+eV) - f(E+eV)f(E) + f(E+eV)f(E) - f(E) \right] dE$$

$$= eT \int_{-\infty}^{\infty} N_N(E+eV)\, N_S(E) \left[f(E+eV) - f(E) \right] dE$$

$$= eT N_0^2\, 2 \int_{\Delta}^{\infty} \frac{E}{\sqrt{E^2+\Delta^2}} \left(\frac{1}{1+e^{-\frac{E+eV}{kT}}} - \frac{1}{1+e^{-E/kT}} \right) dE$$

b) $T = 0$ $\quad \Theta(E+eV) \quad \Theta(E$

$$I = eTN_0^2 \cdot 2 \int_{\Delta}^{eV} \frac{E}{\sqrt{E^2-\Delta}}\, dE$$

2b/ cont

$$I = eTN_0^2 \int_{-\infty}^{\infty} N_s(E) \left(\theta(eV) - \theta(0)\right) dE$$

$$I = eTN_0 \int_{0}^{eV} N_s(E)\, dE$$

$$\frac{dI}{dV} = e^2 TN_0 \, N_s(eV)$$

3/ a) ξ = describes over which length scale that the order parameter change

λ = describes how far into a SC ~~the~~ magnetic field penetrats

b) if $\varkappa = \frac{\lambda}{\xi} > \frac{1}{\sqrt{2}}$ negative (sur)face energy (inter)
$\Rightarrow$ Type II

$\varkappa = \frac{\lambda}{\xi} < \frac{1}{\sqrt{2}}$ $\Rightarrow$ positive interface energy

3b)

$$\Psi = \sqrt{n_s}\, e^{i\theta} \rightarrow \text{into GL 2}$$

$$J_s = \frac{ie\hbar}{m}\left(\Psi^* \nabla \Psi - \Psi \nabla \Psi^*\right) - \frac{4e^2}{m} A \Psi^* \Psi$$

$$J_s = \frac{ie\hbar}{m}\left(-i\hbar \nabla \theta \cdot 2 \cdot \Psi^* \Psi\right) - \frac{4e^2}{m} \bar{A}\, \Psi^* \Psi$$

$$J_s = \left(\frac{2e\hbar}{m} n_s \nabla \theta - \frac{4e^2}{m} A n_s\right)$$

$$\frac{m}{4e^2 n_s} J_s = \frac{\hbar}{2e} \nabla \theta - \bar{A}$$

$$\Lambda J_s = \frac{\hbar}{2e} \nabla \theta - \bar{A}$$

$$\nabla \times (\Lambda J_s) = \frac{\hbar}{2e} \underbrace{(\nabla \times \nabla \theta)}_{= 0} - \underbrace{\nabla \times \bar{A}}_{\bar{B}}$$

$$\nabla \times \Lambda J_s = -\bar{B} \qquad \text{LE}$$

$$\nabla \times B = \mu_0 J_s \qquad \text{ME}$$

$$\boxed{\lambda = \sqrt{\frac{\Lambda}{\mu_0}} = \sqrt{\frac{m}{4e^2 \mu_0 n_s}}}$$

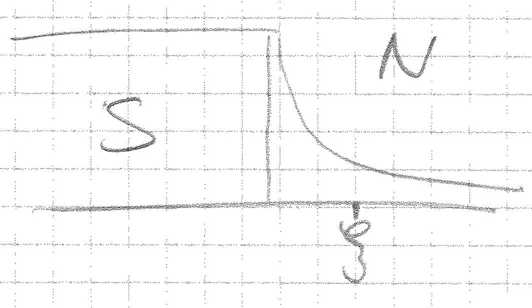
$$\nabla \times (\nabla \times B) = \mu_0 \nabla \times J = \frac{\mu_0}{\Lambda} B$$

$$\nabla \underbrace{(\bar{\nabla} \cdot B)}_{0} - \nabla^2 B = -\frac{\mu_0}{\Lambda} B \Rightarrow \nabla^2 B = +\frac{\mu_0}{\Lambda} B$$

$$B = B_0 e^{-x/\lambda}$$

3d

$$\alpha\Psi + \beta|\Psi|^2\Psi + \frac{1}{2m}\left(-i\hbar\nabla + 2eA\right)^2\Psi = 0$$



B=0, - inside normal Ψ small $\Rightarrow$

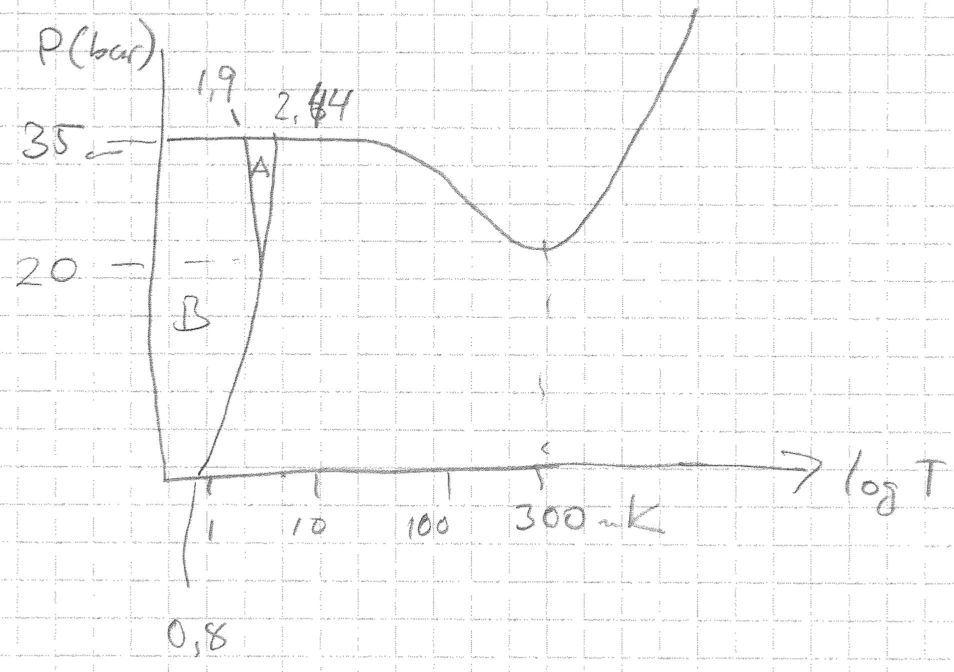
$$\alpha\Psi + \frac{1}{2m}-\hbar^2\nabla^2\Psi = 0$$

$$\nabla\Psi = \frac{2m\alpha}{\hbar^2}\Psi = \frac{\Psi}{\xi^2}$$

$$\Psi = \Psi_0 \cdot e^{-\frac{x}{\xi}}$$

$$\xi = \sqrt{\frac{\hbar^2}{2m|\alpha|}}$$

4a)



b) A anisotropic $|\uparrow\uparrow\rangle$ or $|\downarrow\downarrow\rangle$

B isotropic $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$

c) below 300 mK where $\frac{dp_m}{dT} < 0$

Pressurize to force the liquid to solidify

d) Pomeranchuck and adiab. one shot
Dil continuous

Dil 100–400 μW

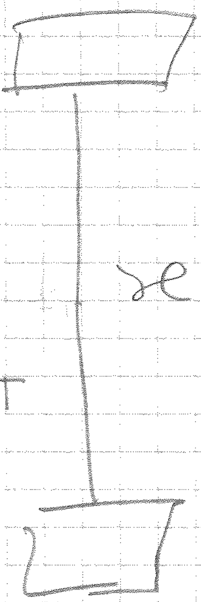
Pomeranchuck more cooling power for the same "flow"

AD	SS	dep size of pill	1mK, 1μK
DIL	cont	100–400	~~[illegible]~~ mK
Pom	SS	bigger	~1mK

6.) a)

$\varkappa_{SS} = 0{,}15\,T \ \frac{W}{K^2 m}$

$\varkappa_T = 0{,}003\,T^2 \ \frac{W}{K^3 m}$



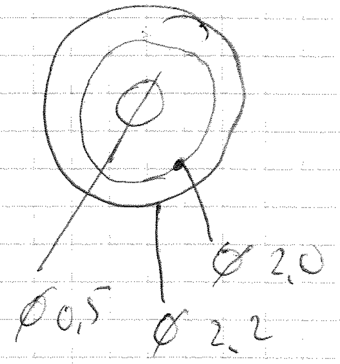
$T_h = 0{,}7$

$\varkappa(T)$ $L = 20\,cm$

T_l

$T,\ dx \; — \; dT$

$P = \varkappa(T) \cdot \frac{A}{dx} \cdot dT$



$$A_{SS} = \frac{\pi\, 2{,}2^2}{4} - \frac{\pi\, 2{,}0^2}{4} + \frac{\pi\, 0{,}5^2}{4} = \frac{\pi}{4}(1{,}09) = 0{,}856\ mm^2$$

$$A_T = \frac{\pi\, 2{,}0^2}{4} - \frac{\pi\, 0{,}5^2}{4} = \frac{\pi}{4}(4 - 0{,}25) = 2{,}95\ mm^2$$

$$P \cdot \int_{x_l}^{x_h} dx = A \int_{T_l}^{T_h} \varkappa(T) \cdot dT$$

$$P = \frac{A}{L} \int_{T_l}^{T_h} \varkappa(T)\, dT$$

$$P_{SS} = \frac{A_{SS}}{L} \left[0{,}15 \frac{T^2}{2} \right]_{0{,}05}^{0{,}7} = \frac{0{,}856 \cdot 10^{-6}}{0{,}2} \cdot \frac{0{,}15}{2} \left[0{,}7^2 - 0{,}05^2 \right]$$

$$= 0{,}15\ \mu W$$

$$P_T = \frac{A_T}{L} \left[0{,}003 \frac{T^3}{3} \right]_{0{,}05}^{0{,}7} = \frac{2{,}95 \cdot 10^{-6}}{0{,}2} \cdot \frac{3 \cdot 10^{-3}}{3} \cdot 0{,}343 \approx 2\, nW$$

b/

$$\frac{P_{ss1}}{P_{ss2}} = \frac{\frac{A_{ss}}{L_1} \; 0{,}15 \; \frac{T_1}{2}^2}{\frac{A_{ss}}{2L_2} \; 0{,}15 \; \frac{T_2}{2}} = \frac{2 \cdot T_1^2}{T_2^2} = \frac{2 \cdot 0{,}7^2}{4{,}2} = 0{,}055$$

$$\frac{P_{ss1}}{P_{ss2}} = 18$$

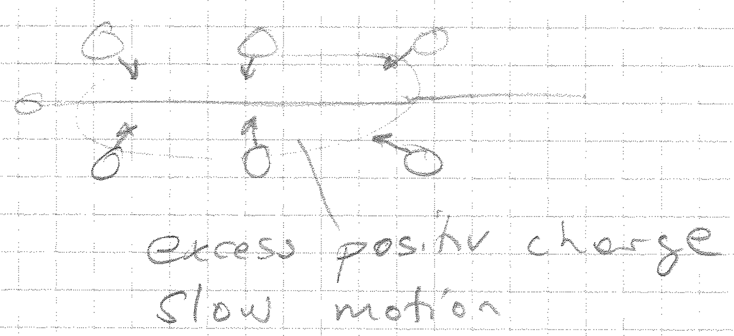
c/

$$P = \underset{0{,}9}{\varepsilon} \cdot \underset{20 \cdot 10^{-4}}{A} \cdot \underset{5{,}67 \cdot 10^{-8}}{\sigma} \cdot \underset{4{,}2^4}{T^4} = 32\,nW$$

7/

a)

$$|BCS\rangle = \prod_k \left(u_k + e^{i\theta} v_k c^{\dagger}_{k+s\uparrow} c^{\dagger}_{-k+s\downarrow}\right)|0\rangle$$



b)

$|0\rangle$ vacuum

θ the supercond phase

$c^{\dagger}_{k+s\uparrow}$ creates a particle in te state $k+s, \uparrow$

u_k pobability amplitude of k beeing unoccupied

v_k prob ampl. of k beeing occupied

c)

$$T_c \propto \frac{1}{\sqrt{M}} \qquad \omega_{ph} \propto \sqrt{\frac{k}{M}} \sim \Theta_D$$

$$T_c \propto 1{,}14 k\Theta_D e^{-1/NV}$$