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GÖTEBORGS UNIVERSITET Kl: 14.15-18.15
Fysiska institutionen Sal: VV21

Tentamen i LÅGTEMPERATURFYSIK för F4, GU, Master, Ph.D. Students

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Hjälpmedel: Tefyma, Physics Handbook, Stand Math Tables och liknande handböcker, valfri räknedosa.

Answer 5 of the following problems. Motivate your answer in a logical way. You are welcome to illustrate with readable diagrams. Answer in Swedish or English.

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1. Sound of second order in superfluid helium.

(a) Superfluid helium may be described by a two fluid model. What is meant by this model? Describe the temperature dependences of the parameters of the two fluids. (1p)

(b) What is meant by sound of second order in superfluid helium? What kind of wave motion is it and why is it obtained? Suggest an experiment to measure the velocity of 2nd sound (describe the principle and a possible set-up). What result do you expect as the temperature approaches T_λ? Well below T_λ? What happens with the wave propagation at very low temperature? How is heat transfer in superfluid helium affected by sound of second order? (2.5p)

(c) What is meant by 3rd and 4th order sound? Sound of 0th order? (0.5p)

2. Critical velocities in superfluids.

(a) What critical velocity of superfluid flow in liquid ^{4}He would you expect if the only excitations from the condensate were phonons with dispersion law $E = pc$, where $c = 239$ m/s, and rotons, with corresponding relation $E = \Delta + (p-p_o)^2/2\mu$, where $\Delta/k_B \approx 8.65$ K, $p_o/h \approx 1.92$ Å^{-1} and $\mu \approx 0.16$ x the mass of ^{4}He, which, in turn, $\approx 6.69 \cdot 10^{-27}$ kg? (1p)

(b) Similarly for a superconductor like Pb, estimate the critical velocity for Cooper pairs to be broken and corresponding critical current density. Consider Pb as a free electron metal with an electron density, $N/V=1.3 \cdot 10^{29}$ electrons/m^3. Pb becomes superconducting ($T_c=7.2$ K) and the superconducting gap parameter for excitations $\Delta \approx 1.3$ meV. What is the critical velocity and the critical current density that you expect for Pb in this, so called, depairing limit? (For free electrons $k_F=(3\pi^2 N/V)^{1/3}$; $D(\varepsilon_F)=N(0)=3N/2\varepsilon_F$. For Pb, you can, for simplicity, assume that $\varepsilon_F \approx 9.3$ eV.) The free electron mass is $9.1 \cdot 10^{-31}$ kg. Planck's constant/2π = $1.05 \cdot 10^{-34}$ Js. $e=1.6 \cdot 10^{-19}$ As. For Pb, Z=82 and e/a=4 free electrons per atom. (2p)

(c) In practice, one observes values that are much lower than the estimates following from (a) and (b). Why? Are their ways to increase these lower critical values? (1p)

3. Superfluids. Both ^{3}He and ^{4}He display superfluid phases. It is your task, in this problem, to discuss the similarities and the differences, with emphasis on differences, between superfluid ^{3}He and ^{4}He. The discussion should include both theoretical concepts like statistics, symmetry, interactions, different phases, phase diagrams, etc, and experimental properties like heat capacity, viscosity, anisotropy, vortices, etc. (4p)

4. Type II superconductors. Abrikosov employed the Ginzburg-Landau equations to show that superconductivity can be nucleated at a critical magnetic field, H_{c2}, which is

much higher than the thermodynamic critical field, H_c. The Ginzburg-Landau equations can be written (with $e^*=2e$, $m^*=2m$):

$$\alpha\Psi + \beta|\Psi|^2\Psi + (1/2m^*)(-i(h/2\pi)\nabla + e^*A)^2\Psi = 0$$

$$J_s(r) = i(e^*(h/2\pi)/2m^*)(\Psi\nabla\Psi^* - \Psi\nabla\Psi^*) - (e^{*2}/m^*)(\Psi^*\Psi)A$$

(a) Indicate the derivation of the upper critical field, H_{c2}, from the G-L equations. (1p)

(b) Draw a phase diagram of critical field visavi temperature for a type II superconductor and indicate the different phases. Would the corresponding phase diagram be very different for a so called high temperature superconductor? (1p)

(c) Above a lower critical field, H_{c1}, magnetic flux penetrates a type II superconductor in the form of quantized fluxons (flux lines). These form a lattice. Show that the flux is quantized in an area enclosed by a superconductor. What is the value of the flux quantum? (1p)

(d) One tries to prevent flux lines from moving (as they then give dissipation) in a practical superconducting wire. Suggest possible ways of preventing the flux lines from moving. (1p)

5. Josephson effects in superconducting weak links.

(a) Derive and sketch the magnetic field dependence of the Josephson current (maximal current without a voltage drop) in a junction of the following geometry:
Current in the z direction, perpendicular to an insulating barrier between two superconductors. The thickness of the insulator is d. The thicknesses of the superconducting films are both t. The junction width (along the y-axis) is w and its length (along x) is l. Magnetic flux density B along the x-axis. The critical current density is such that the Josephson penetration depth $\lambda_J > w, l,$ and t. (2p)

(b) Calculate the London penetration depth λ_L from the following observation:
For a junction with w=20 μm, l=30 μm, t=2000 Å, and d= 20 Å one found the first zero of the maximal Josephson current at $B_o = 1.32\cdot10^{-3}$ T at 4.2 K. (2p)

6. Single charge tunneling and applications of the phenomenon.

There are similarities between the current transport in Josephson junctions and in ultrasmall tunnel junctions dominated by charging phenomena (Coulomb blockade). The charge Q and the phase Φ in a small superconducting tunnel junction (or number and phase) are conjugate variables.

(a) Consider a small tunnel junction. Coulomb blockade, or charging phenomena can occur in such a junction. Discuss the phenomenon and the conditions that have to be fulfilled in order to observe the Coulomb blockade. What is the threshold voltage? What is the junction capacitance if the threshold voltage is 1 mV? (2p)

(b) In a laboratory exercise, you have studied a recently proposed primary thermometer based upon the temperature dependence of the zero bias conductance of a small tunnel junction. Discuss in principle how the thermometer functions. What is the difference between a primary and a secondary thermometer? (2p)

(c) As an alternative to part (b) above, you can treat the following problem: The thermometer is one application of the Coulomb blockade (or single electron tunneling) effect. If you prefer, you can choose another device (component) based on this effect and describe its principle and performance, the circuit within which the component is used or characterized, and the prospective uses of the device. (2p)

7. High temperature superconductors. Compare ten properties of high temperature (cuprate) superconductors and "conventional" (low temperature) superconductors. Give appropriate (approximate) values of or define parameters for the two cases. Comment if needed. Your comparison can, for example, include transition temperatures, critical fields and currents densities, coherence and penetration depths, directional dependence, energy gap, pseudo-gap, interaction, symmetry, pairing, quantized flux, electron density, crystal structure, phase diagram, heat capacity, susceptibility, kappa-values, vortices, or other properties. (4p)

8. Heat leaks. Low temperature scanning probes are becoming increasingly popular in order to study, e.g., surface topology, tunneling spectroscopy or local magnetic properties of a sample. Assume that you want to install a scanning tunnel microscope (STM) connected to the mixing chamber of a dilution refrigerator in order to study tunnel effects when the capacitance between tip and sample is small.

(a) The refrigerator has a cooling capacity of about 3 microwatt at 50 mK, see the attached curve of cooling capacity vs. temperature. Will the coldest part (the mixing chamber) be affected to a large extent by the microscope and what will the temperature be, considering the heat inputs described in next sentences? The STM is connected to the environment via a coaxial cable that has an inner conductor of copper that is 0.5 mm in diameter and is thermally anchored at the still (0.8 K) which is 20 cm away (the thermal conductivity of the outer conductor can be neglected). The area of the microscope head can be estimated to 10 cm^2. The instrument is surrounded by a heat shield at 4.2 K. The background pressure in the chamber can be neglected. The tunnel current is about 1 nA and the tunnel resistance is typically 1 Mohm. (The average thermal conductivity of Cu in the interval of 0.05-1K can be set to 20W/m·K; the Stefan-Boltzmann constant is $5.67.10^{-8}$ W/m^{2}K^4. You may consider a worst case of black radiation from the 4.2 K enclosure upon the microscope head, which is polished and has an emissivity of ε=0.2. Can you suggest a way to decrease the thermal leak? (3.5 p)

(b) The current density under the tip can be appreciable (1 nA, < 1 nm diameter). Can you suggest an experiment to check that the sample is not heated appreciably just under the tip? (0.5 p)

9. Mechanical coolers become more and more common in research laboratories. Can you describe a refrigerator based on either the McMahon or the pulse tube configuration? Your description should treat principle, realization, performance and advantages of the mechanical refrigerator that you choose to treat. (4p)