CHALMERS TEKNISKA HÖGSKOLA 2004-12-15
Mikroteknologi och nanovetenskap Kl: 08.30-12.30
Sal: Mxx

**Exam in Low Temperature Physics for F4, Master, Ph.D. Students**

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**Hjälpmedel:** Tefyma, Physics Handbook, Stand Math Tables och liknande handböcker, valfri räknedosa, ett A4 ark med handskrivna formler och anteckningar.

Answer problem 1 and four of the following 8 problems. Motivate your answer in a logical way. You are welcome to illustrate with readable diagrams. Answer in Swedish or English.

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**1. Short questions to test the understanding of concepts.**

Give short descriptions or definitions (use diagrams if appropriate) of the following:

(a) Sketch the low temperature phase diagram (T vs concentration) of a $^4$He-$^3$He liquid mixture. Give most important temperatures and concentrations of the phase diagram for the functioning of a dilution refrigerator.
(b) How can adiabatic demagnetization be used for refrigeration, what temperatures can be reached?
(c) What is the BCS-relation between the critical temperature and the energy gap? Sketch the density of states for excitations in an ordinary superconductor such as tin.
(d) Sketch the derivation of quantization of circulation in a superfluid.
(e) What is the meaning of the critical velocity in a superfluid.
(f) Sketch how the critical current of a SQUID changes with external magnetic field.
(g) What is the London penetration depth?
(h) What is characteristic for a d-wave superconductor?
(i) What is the temperature dependence of the heat capacity of cupper at low temperatures ? What are the two main contributions due to ? (0.5p per sub-problem, max 4p)

**2. Tunneling in a Superconductor-Insulator-Normal metal (SIN) tunnel junction.**

(a) Set up the integral expression for the current in the SIN junction at finite temperature, include both the left and the right going currents and simplify the expression. Use a figure to explain the used variables. (2p)
(b) Show how the density of states for the superconductor can be obtained from the current voltage characteristics, and derive the form of the current versus voltage. Assume T=0. (2p)

**3. Characteristic lengths as derived from the Ginzburg-Landau theory.**

The penetration depth $\lambda$ and the coherence length $\xi$ are two important lengths in superconductors that follow from the Ginzburg-Landau equations, which can be written:

$$\alpha\Psi + \beta|\Psi|^2\Psi + \frac{1}{2m^*}\left(-i\hbar\vec{\nabla} + e^*\vec{A}\right)^2\Psi = 0$$

$$\vec{J}_S = \frac{ie^*\hbar}{2m^*}\left(\Psi\vec{\nabla}\Psi^* - \Psi^*\vec{\nabla}\Psi\right) - \frac{2e^*}{m^*}\vec{A}\Psi^*\Psi$$

(where $e^*$ and $m^*$ are charge and mass of an electron pair)

(a) Define and describe the two lengths (1p)
(b) Derive $\lambda$ from the Ginzburg-Landau Equations (1p)
(c) Derive $\xi$ from the Ginzburg-Landau Equations (1p)
(d) How do you know if a superconductor is Type I or Type II, from these length scales? (1p)

**4. Superfluid Helium-3 and Pomeranchuk cooling**

(a) Sketch the (p-T) phase diagram for helium-3 in the range from 1-400 mK, using a logarithmic T scale. Indicate important pressures and temperatures, where the superfluid phases are. (1p)

(b) What are the differences between the different superfluid phases (1p)

(c) In what region can is Pomeranchuk cooling effective ? How do you get cooling in a Pomeranchuk cell ? (1p)

(d) Compare qualitatively the cooling power and minimum achievable temperature of Pomeranchuk cooling with those of dilution refrigeration and adiabatic demagnetization. (1p)

**5. Superconducting devices**

There are many practical applications of superconducting tunneling and Josephson effects. Your task is to pick one possible application and discuss:

- principle (physics behind the application)
- conditions to be fulfilled, design criteria, ultimate performance
- practical set-up
- possible circuit designs
- examples of results and prospects for the future

Pick one of the applications, below:

(a) a dc-SQUID

(b) a voltage standard

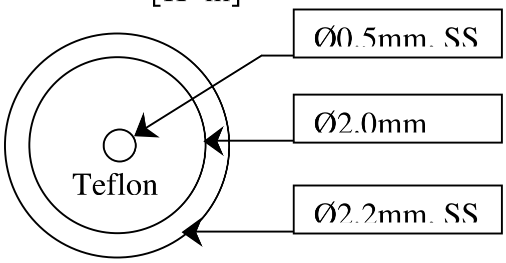
(c) an RSFQ flip-flop

(d) a radio-frequency single electron transistor (4p)

**6. Low temperature techniques.**

A stainless steel (SS) coaxial cable with Teflon isolation is mounted in a dilution refrigerator, it is heat sunk at the Still (0.7K) and at the mixing chamber, which is at 50mK. The coax is 20 cm long and has a cross-section as shown in the figure below.
The heat conductivity for stainless steel is

$\kappa = 0.15 \cdot T \left[\frac{\mathrm{W}}{\mathrm{K \cdot m}}\right]$, and for teflon $\kappa = 0.003 \cdot T^2 \left[\frac{\mathrm{W}}{\mathrm{K \cdot m}}\right]$




(a) Calculate the power with which the coax heats the mixing chamber. (2p)

(b) How much worse would it be if the coax was not heat sunk at the Still, but was mounted directly between the IVC (4.2K) and the mixing chamber but twice as long. (1p)

(c) Calculate the power radiated from a heat shield, which is attached to the Still, assuming an area of the mixing chamber of 20cm$^2$, and an emission/absorbtion coefficient of 0.9. (1p)

**7. BCS theory**

The BCS ground state is given by $|BCS\rangle = \prod_k \left(u_k + e^{i\theta} v_k c^\dagger_{k+s,\uparrow} c^\dagger_{-k+s,\downarrow}\right)|0\rangle$

(a) Explain in simple terms how the electron phonon interaction can give rise to an attractive interaction between electrons that leads to (Cooper-)pairing. (1p)

(b) Explain the different parts of the ground state. What is |0> and θ. Wh . do the two operators $c^\dagger$ do and what would to conjugate operator $c$ do? What is the meaning of $u_k$ and $v_k$? (2p)

(c) What is the isotope effect ? How can it be explained ? (1p)

**8. High Temperature superconductors**

(a) Describe the typical structure of cuprate high temperature superconductors (HTS), *e.g.*, crystal structure, function of different layers, charge carriers, etc. (1p)

(b) Make a drawing, which outlines a typical HTS doping-phase diagram and indicate how the material behaves when it is underdoped (or undoped), optimally doped, and overdoped (1p)

(c) Give at least 2 examples on ways of making Josephson junctions in HTS (1p)

(d) Discuss problems and advantages with the suggested techniques, and what makes things different from working with junctions based on low temperature superconductors. (1p)

**9. <u>Bose Einstein Condensation (BEC)</u>**

Einstein proposed in 1925 that an ideal Bose gas would undergo a condensation to a coherent ground state at low temperature. 70 years later BEC was shown experimentally in dilute, very cold gases of alkali atoms. But well before that, Bose-Einstein type condensations had been proposed for superfluid He-4 and He-3 as well as for superconductors.

(a) What are the conditions for BEC to occur? What is the temperature dependence of the condensate density for temperatures below the condensation temperature? (Sketch the dependence in a diagram.) (1p)

(b) What are the differences between liquid He-4 and an ideal gas? Why would there be a BEC? What are the excitations from the condensate in superfluid helium? What are their temperature dependences? (2p)

(c) The electrons in a superconductor or the atoms in He-3 are fermions. Why should there be a type of BEC for these particles? What are the excitations from the condensed state? (1p)