CHALMERS TEKNISKA HÖGSKOLA 2005-02-10
Mikroteknologi och nanovetenskap Kl: 09.00-13.00
Sal: xxx

Exam in Low Temperature Physics for F4, Master, Ph.D. Students

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Hjälpmedel: Tefyma, Physics Handbook, Stand Math Tables och liknande handböcker, valfri räknedosa, ett A4 ark med handskrivna formler och anteckningar.

Answer problem 1 and four of the following 8 problems. Motivate your answer in a logical way. You are welcome to illustrate with readable diagrams. Answer in Swedish or English.

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1. Short questions to test the understanding of concepts.

Give short descriptions or definitions (use diagrams if appropriate) of the following:

(a) Sketch the low temperature phase diagram (T vs concentration) of a ^{4}He-^{3}He liquid mixture. Give most important temperatures and concentrations of the phase diagram for the functioning of a dilution refrigerator.
(b) How can a dilution refrigerator be used for cooling, what temperatures and cooling powers can be reached?
(c) Sketch the density of states for excitations in an ordinary superconductor such as tin. What is the BCS-relation between the critical temperature and the energy gap?
(d) Sketch the derivation of quantization of circulation in a superfluid.
(e) What is the meaning of the critical velocity in a superfluid.
(f) Sketch how the critical current of a SQUID changes with external magnetic field.
(g) What is the Josephson penetration depth?
(h) What is characteristic for a d-wave superconductor?
(i) What is the temperature dependence of the heat capacity of cupper at low temperatures ? What are the two main contributions due to ? (0.5p per sub-problem, max 4p)

2. Tunneling in a Superconductor-Insulator-Normal metal (SIN) tunnel junction.

(a) Set up the integral expression for the current in the SIN junction at finite temperature, include both the left and the right going currents and simplify the expression. Use a figure to explain the used variables. (2p)
(b) Show how the density of states for the superconductor can be obtained from the current voltage characteristics, and derive the form of the current versus voltage. Assume T=0. (2p)

3. Characteristic lengths as derived from the Ginzburg-Landau theory.

The penetration depth λ and the coherence length ξ are two important lengths in superconductors that follow from the Ginzburg-Landau equations, which can be written:

$$\alpha\Psi + \beta|\Psi|^2\Psi + \frac{1}{2m^*}\left(-i\hbar\vec{\nabla} + e^*\vec{A}\right)^2\Psi = 0$$

$$\vec{J}_S = \frac{ie^*\hbar}{2m^*}\left(\Psi\vec{\nabla}\Psi^* - \Psi^*\vec{\nabla}\Psi\right) - \frac{2e^*}{m^*}\vec{A}\Psi^*\Psi$$

(where e^* and m^* are charge and mass of an electron pair)

(a) Define and describe the two lengths (1p)
(b) Derive λ from the Ginzburg-Landau Equations (1p)
(c) Derive ξ from the Ginzburg-Landau Equations (1p)
(d) How do you know if a superconductor is Type I or Type II, from these length scales? (1p)

4. **Superfluid Helium-3 and Pomeranchuk cooling**

(a) Sketch the (p-T) phase diagram for helium-3 in the range from 1-400 mK, using a logarithmic T scale. Indicate important pressures and temperatures, where the superfluid phases are. (1p)

(b) What are the differences between the different superfluid phases (1p)

(c) In what region can is Pomeranchuk cooling effective ? How do you get cooling in a Pomeranchuk cell ? (1p)

(d) Compare qualitatively the cooling power and minimum achievable temperature of Pomeranchuk cooling with those of dilution refrigeration and adiabatic demagnetization. (1p)

5. **Superconducting devices**

The RF-SET is an instrument that has an extremely high charge density (less than $5\text{x}10^{-6}e/\sqrt{\text{Hz}}$), is fast and has a large bandwidth. It is complementary to the so called RF-SQUID that has a similar performance measuring changes in a magnetic flux. This is only one of several complementary effects between low and high resistance, superconducting junctions. There are a number of similarities between the electrical transport in Josephson junctions and ultrasmall tunnel junctions dominated by charging phenomena. The charge Q and the phase ϕ of a small superconducting tunnel junction are conjugated variables.

(a) What is the Josephson effect? What are the conditions for it to occur? What are the main properties (e.g., current/voltage/phase dependence, critical current, magnetic field response, high frequency properties, etc)?

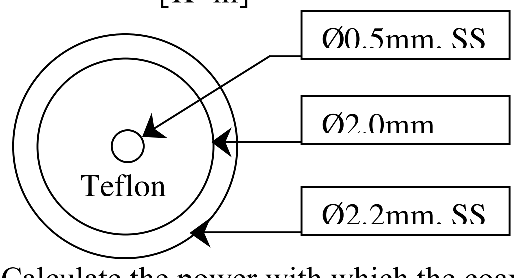
(b) What is the Coulomb blockade in tunneling? What are the conditions for it to occur? What are the main characteristics (e.g., current-voltage, influence of charging, influence of environment, high frequency properties, etc

(c) Discuss some complementary entities, effects, or applications of the two phenomena. (4p)

6. **Low temperature techniques.**

A stainless steel (SS) coaxial cable with Teflon isolation is mounted in a dilution refrigerator, it is heat sunk at the Pot (1.2K) and at the mixing chamber, which is at 50mK. The coax is 20 cm long and has a cross-section as shown in the figure below.
The heat conductivity for stainless steel is

$\kappa = 0.25 \cdot T \left[\frac{\text{W}}{\text{K} \cdot \text{m}}\right]$, and for teflon $\kappa = 0.005 \cdot T^2 \left[\frac{\text{W}}{\text{K} \cdot \text{m}}\right]$



(a) Calculate the power with which the coax heats the mixing chamber. (2p)

(b) How much worse would it be if the coax was not heat sunk at the Still, but was mounted directly between the IVC (4.2K) and the mixing chamber but twice as long. (1p)

(c) Calculate the power radiated from a heat shield, which is attached to the Still, assuming an area of the mixing chamber of 15 cm^2, and an emission/absorbtion coefficient of 0.95. (1p)

7. **Theory**

Two theories have been particularly successful in describing (at least conventional, low temperature) superconductors – the Bardeen-Cooper-Schrieffer (BCS) theory and the Ginzburg-Landau theory (with the extension by Abrikosov to type II superconductors). Describe the advantages, assumptions, approximations, and main results of the two theories. (4p)

8. **<u>High Temperature superconductors</u>**

Compare ten properties of high temperature (cuprate) superconductors and "conventional" (low temperature) superconductors. Give appropriate (approximate) values of or define parameters for the two cases. Comment if needed. Your comparison can, for example, include transition temperatures, critical fields and currents densities, coherence and penetration depths, directional dependence, energy gap, pseudo-gap, interaction, symmetry, pairing, quantized flux, electron density, crystal structure, phase diagram, heat capacity, susceptibility, kappa-values, vortices, or other properties. (4p)

9. **<u>Second sound.</u>**

You have measured the velocity of second sound in superfluid helium during a laboratory session.

(a) What is second sound? What are the conditions that must be fulfilled for its occurrence? (1p)

(b) How do you measure it? Discuss at least one experimental set-up to detect second sound waves and the speed of their propagation. What is the speed at, say, 1.5K? (1p)

(c) Imagine that you rotate superfluid helium. Will there be any difference between the propagation (and damping) of second sound waves propagating parallel and perpendicular to the axis of rotation? If so, why? If not, why? (1p)

(d) There are also other "orders of sound". What is zero sound? third sound? fourth sound? What are the conditions for these to occur? (1p)