CHALMERS UNIVERSITY OF TECHNOLOGY 2005-12-10
Mikrotechnology och nanoscience Kl: 08.30-12.30
Building V

Exam in FMI036, Superconductivity and low temperature physics
Teacher: Per Delsing, phone: 772 3317, 070-3088317
Allowed aids: Tefyma, Physics Handbook, Stand Math Tables and similar handbooks, calculator, and one A4 sheet with handwritten notes.

Answer problem 1 and 4 of the following 6 problems (i.e. problem 2-7). Motivate your answer in a logical way. You are welcome to illustrate with readable diagrams. Answer in Swedish or English.

---

1. Short questions to test the understanding of concepts.
Give short descriptions or definitions (use diagrams if appropriate) of the following:

(a) Sketch the low temperature phase diagram (P vs T) for ^{3}He. Which are the different phases. Give the most important temperatures and pressures.
(b) How can a dilution refrigerator be used for cooling, what temperatures and cooling powers can be reached?
(c) Sketch the dispersion relation (E versus p) for excitations in superfluid ^{4}He, point out the two most important types of excitations?
(d) What are the three most important figures of merit for an RSFQ circuit?
(e) What does u_k and v_k describe in the BCS theory?
(f) What is the meaning of second sound in a superfluid? What is different from ordinary sound?
(g) Sketch how the critical current of a Josephson junction changes with external magnetic field.
(h) Sketch the thermal conductivity for a superconductor above and below the transition temperature. What happens with the electronic and phononic contribution below Tc
(i) The high-T_c cuprates, e.g. the $Bi_2Sr_2CaCu_2O_8$ compound, are highly anisotropic. What is meant by this? Discuss how charge transport and superconductivity influences the anisotropy.

(0.5p per sub-problem, max 4p)

2. Characteristic lengths as derived from the Ginzburg-Landau theory.
The penetration depth λ and the coherence length ξ are two important lengths in superconductors that follow from the Ginzburg-Landau equations, which can be written:

$$\alpha\Psi + \beta|\Psi|^2\Psi + \frac{1}{2m}\left(-i\hbar\vec{\nabla} + 2e\vec{A}\right)^2\Psi = 0$$

$$\vec{J}_S = \frac{ie\hbar}{m^*}\left(\Psi\vec{\nabla}\Psi^* - \Psi^*\vec{\nabla}\Psi\right) - \frac{4e}{m}\vec{A}\Psi^*\Psi$$

(a) Define and describe the two length scales? How do you know from these two length scales if a superconductor is Type I or Type II? (1p)
(b) Derive λ from the Ginzburg-Landau Equations (2p)
(c) Show that the critical current density, $\vec{J}_S$ falls off exponentially with the characteristic length λ. (1p)

3. Cryogenics
The cooling power of a dilution refrigerator is given by $P = a\cdot(T^2 - T_{\min}^2)$, where T is the mixing chamber temperature, $T_{\min}$=12mK is the mixing chamber temperature without any heat load, and $a = 13.6$ [mW/K^2]. A mechanical manipulator made of stainless steel in the shape of a rod and with the dimensions L=0.4m and diameter 2mm, is mounted between the IVC-flange (T=4.2K) and the mixing chamber. The heat conductivity of stainless steel is given by $\kappa = 0.12\cdot T\left[\frac{\mathrm{W}}{\mathrm{m\cdot K^2}}\right]$.

(a) Calculate the power with which the manipulator heats the mixing chamber. (1.5p)
(b) What is the new minimum temperature with the manipulator installed. (1p)
(c) What minimum temperature do you get if you can heat sink the rod at 1.4K with an effective length of 0.3 m. (1.5p)

4. **Josephson tunneling.**

A Josephson junction can be described by the following set of coupled time-dependent Schrödinger equations:

$$-i\hbar\frac{\partial}{\partial t}\Psi_1 = \frac{qV}{2}\Psi_1 + K\Psi_2$$

$$-i\hbar\frac{\partial}{\partial t}\Psi_2 = -\frac{qV}{2}\Psi_2 + K\Psi_1$$

where K describes the coupling between left and right electrode, q is the charge of a Cooper-pair and V is the voltage applied to the junction. Derive the two Josephson equations (4p)

5. **Superconducting devices**

The RF-SET is an instrument that has an extremely high charge sensitivity (less than $5\times10^{-6}e/\sqrt{Hz}$), is fast and has a large bandwidth. It is complementary to the so called RF-SQUID that has a similar performance measuring changes in a magnetic flux. This is only one of several complementary effects between low and high resistance, superconducting junctions. There are a number of similarities between the electrical transport in Josephson junctions and ultrasmall tunnel junctions dominated by charging phenomena. The charge Q and the phase ϕ of a small superconducting tunnel junction are conjugated variables.

(a) What is the Josephson effect? What are the conditions for it to occur? What are the main properties (e.g., current/voltage/phase dependence, critical current, magnetic field response, high frequency properties, etc)?

(b) What is the Coulomb blockade of tunneling? What are the conditions for it to occur? What are the main characteristics (e.g., current-voltage, influence of charging, high frequency properties, etc.) you can choose any Coulomb blockade device.

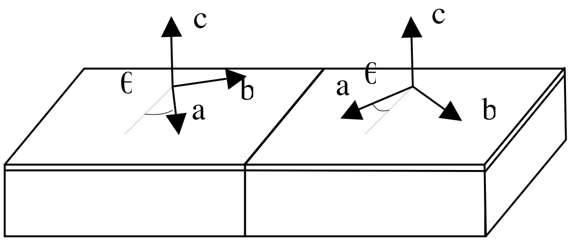
(c) Discuss some complementary entities, effects, or applications of the two phenomena. (4p)

6. **High Temperature superconductors**

Nature gives at least two types of excellent high-T_c Josephson junctions "for free", namely the "grain boundary" Josephson junction and the "intrinsic" Josephson junction. The first type has been developed into artificial grain boundary junctions by using bi-crystal substrates, on which high-T_c thin films have been grown epitaxially. Various types of grain boundaries have been used, resulting in quite different properties.

(a) The figure below shows an epitaxial c-axis oriented $YBa_2Cu_3O_{7-\delta}$ thin film grown on a $SrTiO_3$ bi-crystal substrate (a symmetric c-axis tilt boundary). Discuss for which angles 2θ the critical current across the boundary is highest and lowest, respectively. Argue in detail how the gap parameter plays a role in this (draw a figure) (2p)

(b) How does the "intrinsic" Josephson junction work and why is the same phenomena a problem for electrotechnical applications in *e.g.* high magnetic fields? (2p)



7. **Superfluid Helium**

(a) Describe how a Bose Einstein condensation comes about ? What is the condensate ? (1p)

(b) Derive the value of the quantum of circulation in a superfluid vortex. (1p)

(c) Describe what different symmetry breaking mechanisms that can be found in superfluid Helium 3 and explain how the superfluid A and the B phases relate to these mechanisms. (2p)