CHALMERS TEKNISKA HÖGSKOLA
GÖTEBORGS UNIVERSITET
Fysiska institutionen

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Sal: MG

Tentamen i LÅGTEMPERATURFYSIK för Doktorander, F4, GU och Master Students

Lärare: Tord Claeson, tel 772 3304

Hjälpmedel: Tefyma, Physics Handbook, Stand Math Tables och liknande handböcker, valfri räknedosa.

Answer 5 of the following problems. Motivate your answer in a logical way. You are welcome to illustrate with readable diagrams. Answer in Swedish or English.

The Nobel Prize of Physics for 1998 was given to researchers that have developed the concept of fractionally charged quasiparticles in a two-dimensional electron gas. You had the possibility to attend lectures by two of the prize winners where they discussed the features of ”flatland”. The subject was also covered by M Jonson in his guest lectures and by others.

1. A fractional quantum Hall effect (FQHE) was discovered experimentally by transport measurements at high magnetic fields and low temperature on 2DEG samples with high mobility.

(a) What is the Hall effect? What is it used to measure? How is a two-dimensional electron gas (2DEG) realized? (1p)

(b) K von Klitzing (Nobel prize 85) discovered the quantum Hall effect. What is that? Why does it occur? (1p)

(c) What is the fractional quantum Hall effect? What are the conditions of its observance? (Note the difference to the quantum Hall effect.) Why does it occur? (2p)

2. A low temperature is needed to observe the fractional quantum Hall effect. A dilution refrigerator is commonly used to achieve such a low temperature.

(a) Describe the principles of a dilution refrigerator. Sketch how such a refrigerator is built up. What are the most important parts? (2p)

(b) Compare the heat influx to the mixing chamber, which is assumed to be at 10 mK, from the closest heat exchanger at 60 mK, via
(i) 20 copper wires, each of diameter 0.05 mm, length 10 cm, and
(ii) the circulating (incoming) ^{3}He if the the flow is 10^{-4} mole/s.
The average heat conductivities are 2 W/m·K and 0.02 W/m·K for Cu and ^{3}He, resp., within the temperature interval in question. The specific heat of ^{3}He, C_3, is linear in temperature with a propotionality constant of $\approx$ 24 J/mole·K^2 at low temperature. (We simplify and assume, for this problem, that the enthalpy at T=0 K for the incoming, concentrated phase can be neglected as it is balanced by the one of the outgoing dilute phase such that the incoming heat flux $\approx \int_0^{T_{\text{heat exchanger}}} C_3 dT$.) The heat conductivity by liquid ^{3}He through the 0.5 mm ID capillary to the mixing chamber may be neglected. (2p)

3. Critical velocities are central concepts in superfluids and superconductors.

(a) Show that a minimal velocity has to be exceeded in order for superfluidity to be lost in superfluid helium, i.e. to create excitations in the superfluid.What critical velocity of superfluid flow in liquid ^{4}He would you expect if the only excitations from the condensate were rotons, with dispersion law $E = \Delta + (p-p_o)^2/2\mu$, where $\Delta/k_B \approx 8.65$ K, $p_o/\hbar \approx 1.92$ Å^{-1} and the effective mass $\mu \approx 0.16$ x the mass of ^{4}He ≈ 0.16 x $6.69\cdot10^{-27}$ kg? Experimentally, much lower critical velocities are measured. Why? (2p)

(b) Similarly for a superconductor, sketch the dispersion relations for electron and hole like excitations from the ground state. Estimate the critical velocity for Cooper pairs to be broken if half the energy gap $\Delta = 1$ meV, the Fermi velocity $v_F = 10^6$ m/s and the Fermi energy $E_F = 2$ eV. The free electron mass = $9.1\cdot10^{-31}$ kg. (2p)

4. Josephson effects in superconducting weak links.

(a) Consider a Josephson junction consisting of two metals (Pb) separated by a tunnel barrier, which is about 2.0 nm thick (in the x-direction) and has an extension of about 0.5 and 20 micrometers in the y- and z-directions. A magnetic field is applied in the y-direction. The maximal Josephson current of the junction is 0.1 mA without a magnetic field and at 4.2 K. The first zero for the Josephson current at 4.2 K appears at B=1.2 mT. Estimate the London penetration depth, λ_L, at 4.2 K for Pb. Assume that the extension of the junction is much less than the Josephson penetration depth, λ_J. (2p)

(b) One finds a strong absorption of microwaves at a resonance frequency of 10.00 GHz at 4.2 K in the unbiased junction. This is supposedly due to plasma oscillations. Estimate the value of the junction capacitance. The maximal Josephson current at the presently applied magnetic field and microwave radiation has been decreased to 0.010 mA. (Clue: The Josephson junction can be considered to have a Josephson inductance, the value of which can be obtained from the basic Josephson equations.) $h=6.63\cdot10^{-34}$ Js, $e=1.6\cdot10^{-19}$ As. (2p)

5. Both ^{3}He and high T_c superconductors can be fundamentally different from the "low T_c superconductors" described by the BCS theory. This may be valid for both the symetry of the wave function, describing the superconducting order, and the interaction leading to pairing.
Describe (shortly) the main properties of the two phenomena with an emphasis on those properties that indicate that a simple singlet BCS pairing is not sufficient to describe the states. Describe how the attractive interactions may differ from those of "ordinary" superconductors. (4p)

6. Choose either of alternatives (A) or (B) concerning mesoscopic phenomena.

Alt.A You have studied a newly proposed primary thermometer during a laboratory exercise.

(a) What is characteristic for a primary thermometer? How does it differ from a secondary thermometer (list characteristic properties for the latter as well)? (1p)

(b) Discuss "single electron tunneling". What are the conditions that have to be met in order to observe the phenomenon? What are the main features? What is meant by Coulomb blockade? How can the phenomenon be used to measure small shifts in the charge distribution? What is the principle behind the thermometer application? (3p)

Alt.B Andreev reflections in mesoscopic contacts between superconducting and normal metals.

(a) What happens in current transport when electrons in a normal metal encounter a boundary to a superconductor and their energies are less than the gap energy of the superconductor? Describe the so called Andreev reflections. How can you detect them? Compare with the superconducting proximity effect that may occur in such an N/S contact. (2p)

(b) The effect can be used in bolometers used for the detection of high frequency radiation (e.g. in the infrared range). Describe how such a bolometer works, how the temperature of the bolometer can be determined, how the sensitivity is improved by Andreev reflections. (2p)

7. (a) Superfluid ^{4}He. Different values of the viscosity of superfluid ^{4}He are measured, depending upon the type of experiment. Discuss what properties (parameters) that can be determined by the different types of viscosity measurements in ^{4}He. (2p)

(b) Quantized vortices appear in superfluid ^{4}He in rotation for a frequency $f>f_{crit}$. Similarly, quantized fluxons appear in a typeII superconductor for $H>H_{c1}$. Show that the circulation and the magnetic flux are quantized and discuss under which conditions the phenomena occur. Suggest different ways of measuring vortices and fluxes. What results are obtained? (2p)

8. Order of magnitude values of parameters. It is always of value to know rough estimates of different parameter values. Give approximate values of the following parameters:

a. Boiling temperatures at 1 atm. of ^{4}He, H_2 and N_2
b. Lambda temperature of ^{4}He.
c. Poly critical point of ^{3}He (equilibrium between superfluids A and B and Fermi liquid)
d. Fermi temperature of liquid ^{3}He
e. Coherence length of typical "low temperature superconductor"
f. Coherence length of typical "high temperature superconductor", specify supercond.
g. Ratio $2\Delta(0)/k_BT_c$ according to the BCS theory
h. κ-value which separates Type I and Type II superconductors
i. Low temperature typically reached with the aid of a dilution refrigerator
j. Superconducting transition temperature of $YBa_2Cu_3O_{7-\delta}$
k. Energy (in eV) corresponding to a temperature of 1 K
l. Quantized magnetic flux, Φ_o
m. Ratio between ac frequency and applied dc voltage in a Josephson junction
n. Typical critical current density at low temperature for a superconductor
o. Expression for the quantized circulation (vorticity) in superfluid ^{4}He or ^{3}He
p. Lowest temperature achieved with adiabatic demagnetization of nuclear spins (Sum 4p)